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The Origin and Causes for the Observed Accelerated Expansion of the Universe*

A. Teleki and B. Lacsný

Jednota slovenskych matematikov a fyzikov, Farska 3, SK 949 01 Nitra, Slovakia
JSMFteleki@gmail.com; *borislacsny@hotmail.com*

M. Noga

Comenius University in Bratislava, Faculty of Mathematics, Physics and Computer Sciences,
Department of Theoretical Physics, Mlynská dolina F2, SK-842 48 Bratislava, Slovakia
milan.noga1@gmail.com
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Abstract: Proposed theories for the observed accelerated expansion of the Universe are analyzed from viewpoints of Feynman's criteria on physical theories. On their basis we have derived the most general set of equations between fields which can be generated by point like particles with masses and with given various charges. This general set of equations is formally identical with laws of motion of the general theory of relativity completed by the dark energy and by the cosmological constant. The dark energy density is produced by a tensor field due to the existence of the cosmological constant. The derived laws of motion are exactly solved for a spherically symmetric gravitational field generated by a heavy body with the mass M. Its metric corresponds to a black hole which must have three horizons. An oversimplified model of the Universe in which galaxies are regarded as test particles moving in the gravitational field of this black hole provides the simple explanation for understanding of the origin and causes for the observed accelerated expansion of the Universe. The theory provides a generalized Hubble law and all physical observables are determined uniquely by three dimensionful fundamental constants the presence of which is dictated by the internal structure of the theory.

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1. Introduction

The proper understanding of the origin for the observed accelerated expansion of the Universe represents one of the most fundamental open problem in physics even after 20 years of these discoveries [1, 2]. Since then in cosmology several physical models and theories have been proposed for possible explanations of this phenomenon. The existence of the dark energy [3] seems to be the most accepted hypothesis for the explanation of this acceleration. The proposed two forms for the dark energy are represented by introducing the cosmological constant into Einstein equations of general theory of relativity (GTR) [4], or by a scalar field such as a kind of a quintessence whose energy density can depend on coordinates in a metric space [5, 6]. The hypothesis for the dark energy is, in fact, very much dependent on a way how one modifies GTR. Therefore, there are possibilities that

*Dedicated to Professor Peter Prešnajder on the occasion of his 70th birthday modifications to GTR completely eliminate the need for dark energy [7, 8]. There exist, of course, skeptical alternatives to dark energy in such strong statements that the dark energy does not actually exist and is merely an improper measurement artifact [9–12].

All of these models and theories for the explanation of the observed accelerated expansion of the Universe are, in fact, empirically equivalent theories accounting for the explanation of the same observations on a basis of different assumptions. All of them they have the common feature by which they tie directly effects with causes on the basis of perfect logical arguments so in order their assumptions were consistent with observations. Thus these assumptions bring no additional empirical content to be tested and by this fact these theories are unfalsifiable in the sense defined by Popper[13]. In our program we want to identify at least one of them which could be the best candidate corresponding to the existing physical reality by applying Feynman's criteria on physical theories. According to Feynman [14]: "*The modern physics had banished any possibility of discovering a system of laws unambiguously tying effects to causes; or a system of laws deduced and conjoined with perfect logical consistency; or a system of laws rooted in the objects that people can see and feel.*"

Led by these Feynman criteria, our aims are to find the most general set of equations between various fields which can be generated by point like particles with masses and with various charges moving along worldlines of time-like characters in a given metric space. We will show explicitly that the set of these most general equations is formally identical with all laws of motion of the standard general theory of relativity completed by the cosmological constant and by an antisymmetric tensor field $G \quad (x)$. The tensor field $G \quad (x)$ does not interact with an electromagnetic field and therefore its effects are invisible by instruments of optical and radar astronomers. The field $G \quad (x)$ does not interact with any charges except for masses and its energy density contributes to the stress energy tensor of fields in the Einstein equations of GTR. It represents the dark energy density in a contradistinction to the theories [5, 6].

In the IV. Section of this paper we demonstrate the exact solution to the derived equations corresponding to the gravitational field generated by a heavy mass at the center of the Universe. We will explicitly show how test bodies far from the center of the Universe are accelerated away from its center. On this simplified physical system one can understand the origin of the observed accelerated expansion of the Universe [1, 2].

2. General properties of worldlines

According to Okun [15] "in the modern language of relativity theory there is only one mass, the Newtonian mass m, which does not depend on a particle state or on a reference frame." The mass m of a particle is an internal and invariant characteristic of the particle like its electric charge e, baryon number, hypercharge and so on. In order to elucidate the role of mass we consider a curve of time like character, i.e., a worldline corresponding to a massive particle in a metric space with a given metric tensor $g \quad , \quad , \quad 0,1,2,3.$ Each such worldline

$$z \quad (z^0(s), z(s)) \tag{II.1}$$

can be parametrized by the invariant parameter s, i.e., by its "length" defined by the relation

$$\mathrm{d}s^2 \quad g \quad \mathrm{d}z \quad \mathrm{d}z \tag{II.2a}$$

and the expression

$$w \quad \frac{\mathrm{d}z}{\mathrm{d}s} \tag{II.2b}$$

can be regarded as the tangential vector of this worldline. From the last two definitions one gets the relation

$$g \quad \frac{\mathrm{d}z}{\mathrm{d}s}\frac{\mathrm{d}z}{\mathrm{d}s} \quad g \quad w \quad w \quad 1 \tag{II.3}$$

which is the universal property of the worldlines in the metric space, namely, that tangential vectors w are always unit vectors in the sense (II.3).

In the metric space with the given metric tensor g we define the vector

$$p \quad mc\frac{\mathrm{d}z}{\mathrm{d}s} \quad mcw \;, \tag{II.4a}$$

where m is the mass of particle and c is an universal constant. For the sake of simplicity and for a moment we consider the Minkowski space-time with the metric $g \quad , \quad , \quad 0,1,2,3,$ $g_{00} \quad 1$ and $g_{ik} \quad {}_{ik}$, $i, k \quad 1,2,3$ and the worldline is parametrized by the time t, i.e., $z \quad (ct,\ z(t))$. In this simple case

$$\mathrm{d}s^2 \quad c^2 \;\; 1 \quad \frac{\dot{z}^2}{c^2} \;\; \mathrm{d}t^2 \quad c^2 \mathrm{d}\tau^2$$

and the four vector $p \quad (p^0, \boldsymbol{p})$ has the components

$$p^0 \quad \frac{mc}{\sqrt{1 \quad \frac{\dot{z}^2}{c^2}}}\,, \qquad \boldsymbol{p} \quad \frac{m\dot{z}}{\sqrt{1 \quad \frac{\dot{z}^2}{c^2}}}\,,$$

which are identical with the mechanical four-momentum of the particle with the mass m in the special theory of relativity.

Next we introduce the dimensionful entity known as the energy of the particle $E \quad p^0 c$ in general metric space and by this fact the fundamental constant c has the dimension of velocity. For this reasons the vector p defined by (II.4a) is the vector of energy-momentum of the particle in each metric space. The vector p is therefore proportional to the tangential vector w and satisfies the generally valid relation

$$g \quad p \quad p \quad m^2 c^2 \tag{II.4b}$$

under any circumstances. On the last equation (II.4b) we apply the covariant derivative $\frac{D}{Ds}$ along the worldline $z(s)$ to get the relation

$$g \quad \frac{\mathrm{d}p}{\mathrm{d}s} \quad p \quad \frac{\mathrm{d}z}{\mathrm{d}s}\frac{\mathrm{d}z}{\mathrm{d}s} \quad 0\,, \tag{II.5}$$

where is affine connection [16] defined by

$$\frac{1}{2} g \quad g \quad g \quad g \tag{II.6}$$

and evaluted on the worldline $z(s)$. The generally valid relation (II.5) dictates that the vector

$$\frac{1}{c}F \quad (z) \quad \frac{\mathrm{d}p}{\mathrm{d}s} \qquad p \ \frac{\mathrm{d}z}{\mathrm{d}s} \tag{II.7}$$

must be under any circumstances an orthogonal vector to the tangential vector w of the given worldline.

For any given worldline $z(s)$ in the given metric space with the metric g the right hand site of the expression (II.7) can be explicitly calculated with some result

$$\frac{\mathrm{d}p}{\mathrm{d}s} \qquad p \ \frac{\mathrm{d}z}{\mathrm{d}s} \quad \frac{1}{c}F \ (z), \tag{II.8}$$

by which one derives the vector $F \ (z)$ defined on the worldline $z(s)$. If we denote the proper time τ by the relation $\mathrm{d}s \quad c\mathrm{d}\tau$, then we can rewrite (II.8) in the form

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} \qquad p \ \frac{\mathrm{d}z}{\mathrm{d}\tau} \quad F \ (z) \tag{II.9}$$

and interpret it as the relativistic law of motion of the particle moving under the influence of two forces

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} \quad F_g \ (z) \quad F \ (z), \tag{II.10}$$

where

$$F_g \ (z) \qquad p \ \frac{\mathrm{d}z}{\mathrm{d}\tau}$$

is interpreted as the "gravitational force" due to the existence of metric space with the metric tensor g dependent on coordinates and $F \ (z)$ can be regarded as an external force. It means that one can always assign an external force $F \ (z)$ to the given worldline in the metric space according to Eq. (II.9) and the influence of metric space on the motion of the particle can be regarded as the gravitational force. However, the forces $F \ (z)$ cannot be arbitrary, but must be orthogonal to the tangential vectors w, i.e. they must satisfy the equations

$$g \ F \ (z)\frac{\mathrm{d}z}{\mathrm{d}\tau} \quad 0 \tag{II.11}$$

under any circumstances. We develop the general method for the explicit determination of the forces $F \ (z)$ satisfying the last relation in general for any existing interactions between particles.

If one selects from all worldlines such one which gives $F \ (z) \quad 0$, then for this worldline one gets the relation

$$\frac{\mathrm{d}^2 z}{\mathrm{d}s^2} \qquad \frac{\mathrm{d}z}{\mathrm{d}s}\frac{\mathrm{d}z}{\mathrm{d}s} \quad 0, \tag{II.12}$$

which represents the equation for geodetics in the metric space. In this case the motion of the particle is without the presence of external forces and therefore it is a free motion in the metric space.

Next we consider a system of N worldlines in a given metric space and each worldline z_n, $n = 1, 2, \ldots, N$, is parametrized by one out of coordinates of the metric space, let us choose the coordinate $x^0 \quad ct$, i.e., $z_n(t) \quad (ct, z_n(t))$.

The general relations (II.1)–(II.11) must be satisfied for each curve corresponding to the worldline of a particle with the mass m_n absolutely and for any existing interactions between these particles. In this paper we develop the general method by which the external forces $F \ (z)$ defined by Eq. (II.8) are explicitly determined.

3. Field theory, the mechanical energy-momentum tensor and field strength tensors

The laws of motion of relativistic mechanics of massive particles (II.8), generally valid for every worldline $z_n(t) \quad (ct,\ z_n(t)),\ n \quad 1,2,3,\ldots,N,$ in the form

$$\frac{\mathrm{d}p_n}{\mathrm{d}\tau_n} \qquad (z_n)p_n \frac{\mathrm{d}z_n}{\mathrm{d}\tau_n} \quad F_n \ (z_n) \tag{III.1}$$

will be formulated in field theory for a given set of worldlines with the aim to determine the forces $F_n \ (z_n)$ explicitly. In what follows, for the sake of simplicity, we restrict ourselves to the 4-dimensional metric space – spacetime, however, our considerations are valid in metric spaces of any dimensions. By the definition we denote the left hand sides of (III.1) as "forces" $F_n \ (z_n)$ and for these forces we derive their density $f \ (x)$.

In 4-dimensional metric space we have the invariant 4-volume element

$$\mathrm{d} \ ^4 \quad \sqrt{g}\,\mathrm{d}^4 x \quad \sqrt{g}\,c\,\mathrm{d}t\,\mathrm{d}^3\mathbf{x}\,; \quad g \quad \left|\det g \ \right| \tag{III.2a}$$

and in each its point we define the invariant 3-volume element

$$\mathrm{d} \ ^3 \quad \sqrt{g}\,\frac{\mathrm{d}x^0}{\mathrm{d}s}\,\mathrm{d}^3\mathbf{x} \quad c\sqrt{g}\,\frac{\mathrm{d}t}{\mathrm{d}s}\,\mathrm{d}^3\mathbf{x}\,; \quad \mathrm{d}s^2 \quad g \ \ \mathrm{d}x \ \mathrm{d}x \ . \tag{III.2b}$$

Our aim is to find the force density $f \ (x)$ in the form of covariant divergence of the symmetric mechanical energymomentum tensor $T_m \ (x) \quad T_m \ (x)$, i.e.

$$f \ (x) \quad T_m \ (x)_{;} \ , \tag{III.3a}$$

where

$$T_m \ (x)_{;} \qquad \frac{1}{\sqrt{g}} \quad [\sqrt{g}T_m \ (x)] \qquad T_m \ (x)$$

so in order to get the relation

$$\mathrm{d} \ ^3 f \ (x) \quad \sum_{n \ 1}^{N} \frac{\mathrm{d}p_n}{\mathrm{d}\tau_n} \qquad p_n \frac{\mathrm{d}z_n}{\mathrm{d}\tau_n} \quad . \tag{III.3b}$$

Such tensor indeed exists and is determined by curve integrals along the world lines $z_n(t) \quad (ct,\ z_n(t))$ in the form

$$T_m \ (x) \qquad \sum_{n \ 1}^{N} m_n c^2 \quad \frac{\mathrm{d}x}{\sqrt{g}}\frac{\mathrm{d}x}{\mathrm{d}s} \quad ^4(x \quad z_n(t)) \quad T_m \ (x)\,. \tag{III.4a}$$

By the simple integration over $x^0 \quad ct$ we get the result

$$T_m \ (x) \qquad \sum_{n \ 1}^{N} \frac{c}{\sqrt{g}}\frac{\mathrm{d}x}{\mathrm{d}s} m_n \frac{\mathrm{d}x}{\mathrm{d}t} \quad ^3(\mathbf{x} \quad \mathbf{z}_n(t))\,, \tag{III.4b}$$

which indeed satisfies Eq. (III.3b). Next we note that the quantity

$$\sum_{n\ 1}^{N} \frac{m_n}{\sqrt{g}} \frac{\mathrm{d}x}{\mathrm{d}t}\ {}^{3}(\mathbf{x}\quad \mathbf{z}_n(t)) \quad J_m(x) \tag{III.4c}$$

is a 4-vector which satisfies the invariant conservation law

$$J_m(x)_{;} \quad 0 \tag{III.4d}$$

by its definition (III.4c), because the invariant conservation law is satisfied by every individual term in the sum (III.4c). Thus the masses m_n are indeed internal and invariant characteristics of particles and the mechanical energymomentum tensor $T_m\ (x)$ can be expressed in the elegant covariant form

$$T_m\ (x) \quad c\frac{\mathrm{d}x}{\mathrm{d}s} J_m(x) \tag{III.4e}$$

as the direct product of two four vectors giving the scalar function

$$D(x) \quad g \quad T_m\ (x)\,. \tag{III.4f}$$

The studied system of worldlines exists in the metric space with the given g , where one has the Riemann curvature tensor defined by

$$R \qquad \frac{1}{2} \qquad g \qquad g \qquad g \qquad g \quad g \qquad , \tag{III.5a}$$

from which one can derive the only symmetric tensors of the second rank

$$R \quad g \ R \quad , \tag{III.5b}$$

i.e. Ricci tensor and $g\ \ L$, where L is, in general, a function of fourteen curvature scalars which can be constructed out of R and g .

From the definitions (III.5) it is clear that the Ricci tensor has the dimension $1/\ell^2$, where ℓ is a fundamental dimensionful constant with the dimension of a length.

Next we want to define the most general form for the density $f\ \ (x)$ of the forces $F_n\ \ (z_n)$ present on the right hand side of Eq. (III.1). Its most general form is given as the sum of curve integrals along the worldlines $z_n(t)$, i.e.,

$$f\ \ (x) \quad \sum_{n\ 1}^{N} \frac{\mathrm{d}x}{\sqrt{g}} g \quad A\ \ (x) \quad S\ \ (x) \quad {}^{4}(x \quad z_n(t))\,, \tag{III.6}$$

where $A\ \ (x)$ is an antisymmetric and $S\ \ (x)$ is a symmetric tensor. The only symmetric tensors which are associated with the metric space are the Ricci tensor R and $g\ \ L$. With the help of them we construct the most general symmetric tensor

$$S \quad \tilde{}\,R \quad g\ L \quad \mathcal{H} \quad , \tag{III.7}$$

where $\mathcal{H}\ \ (x)$ denotes yet an unspecified symmetric tensor which will be determined later on.

There exist many antisymmetric tensors $A\ \ (x)$, but in our further considerations, for the sake of simplicity, we consider only one antisymmetric tensor $F\ \ (x)$ and with it we write the most general force density (III.6) in the form

$$f\ \ (x) \quad \sum_{n\ 1}^{N} \frac{\mathrm{d}x}{\sqrt{g}} g \quad e_n F \qquad k(R \quad g\ L \quad \mathcal{H}\ \ ) m_n^2 \quad {}^{4}(x \quad z_n)\,, \tag{III.8a}$$

where e_n and m_n are internal and invariant characteristics of particles as their some charges e_n and masses m_n and k is a fundamental dimensionful constant. By the direct integration of the last integral over $x^0 \quad ct$ we get the result

$$f\ \ (x)\quad \frac{1}{c}\sum_{n\ \ 1}^{N}\frac{1}{\sqrt{g}}g\quad e_n F\quad\ k(R\quad g\ \ L\ \ \mathcal{H}\ \ )m_n^2\ \frac{\mathrm{d}x}{\mathrm{d}t}\ \ ^3(\mathbf{x}\ \ \mathbf{z}_n(t))\,. \tag{III.8b}$$

The force density $f\ \ (x)$ must have the dimension $[E][\ell]\ \ ^4$, what implies that dimensions of its terms in the sum (III.8b) are as given by

$$[e_n F\ \ ]\quad [E][\ell]\ ^1,\quad [k]\quad [\ell]^3\ \mathrm{kg}^{-1}\mathrm{s}^{-2}. \tag{III.9}$$

Thus, the dimensionful fundamental konstant k has exactly the same dimension as the Newton gravitational constant G. For the sake of brevity and from now on we use the notation

$$S\quad\ R\quad\ g\ \ L\ \ \mathcal{H}\ \ . \tag{III.10}$$

We note that quantities entering the expression (III.8b)

$$\sum_{n\ \ 1}^{N}\frac{1}{\sqrt{g}}e_n\frac{\mathrm{d}x}{\mathrm{d}t}\ \ ^3(\mathbf{x}\ \ \mathbf{z}_n(t))\quad j\ \ (x) \tag{III.11a}$$

and

$$\frac{k}{\ell}\sum_{n\ \ 1}^{N}\frac{1}{\sqrt{g}}\ \frac{m_n}{c}\ ^2\frac{\mathrm{d}x}{\mathrm{d}t}\ \ ^3(\mathbf{x}\ \ \mathbf{z}_n(t))\quad J\ \ (x) \tag{III.11b}$$

are four-vectors which satisfy the invariant conservation laws

$$j\ \ (x)_{;}\quad J\ \ (x)_{;}\quad\ 0 \tag{III.11c}$$

and the 4-vector (III.11b) has the same dimension as the 4-vector (III.4c) associated with the mechanical energymomentum tensor $T_m\ \ (x)$.The 4-vectors $j\ \ (x)$ and $J\ \ (x)$ are used to express $f\ \ (x)$ in the elegant explicitly covariant form

$$f\ \ (x)\quad \frac{1}{c}g\ \ \{F\ \ j\quad\ \ell c^2 S\ \ J\ \ \}\,. \tag{III.12}$$

In the next step we find such symmetric and traceless tensor $\ \ (x)$, the stress energy tensor of the fields F and S, the covariant divergence of which gives the force density $f\ \ (x)$, i.e.,

$$f\ \ (x)\qquad (x)_{;}\ \ . \tag{III.13}$$

Such tensor indeed exists in the form

$$\begin{aligned} &\frac{1}{4}g\ \ F\ \ F\quad\ g\ \ F\ \ F \\ &\frac{1}{k}\ \frac{1}{4}g\ \ G\ \ G\quad\ g\ \ G\ \ G\quad\ \ell c^2 g\ \ [S\ \ G\quad\ S\ \ G\ \ ]\ , \end{aligned} \tag{III.14}$$

where G is an antisymmetric tensor and the konstant k guarantees that the dimensions of all terms in (III.14) are the same and given by

$$[\quad]\quad [E][\ell]\ ^3. \tag{III.15}$$

The relations (III.12) and (III.13) are satisfied if and only if the tensor fields F , G and S satisfy the following equations

$$F\ \ (x)_{;}\quad \frac{1}{c}j\ (x)\,,\qquad F\ \ _{;}\quad F\ \ _{;}\quad F\ \ _{;}\quad\ 0\,, \tag{III.16}$$

$$G\ \ (x)_{;}\quad\ \ \frac{k}{c}J\ (x)\,, \tag{III.17}$$

and

$$\frac{k}{c} g \quad G \quad J \qquad \frac{1}{2} g \quad G \quad [G \qquad G \qquad G \quad]$$

$$\ell c^2 g \qquad S_{;} \; G \qquad S_{;} \; G \qquad S \; G_{;} \qquad 0 . \tag{III.18}$$

The equations (III.16) are identical with Maxwell's equations of electrodynamics and e_n are electric charges of particles. The tensor $F \quad (x)$ is the electromagnetic field strength tensor which is gauge invariant under the U(1) gauge group transformations. The expressions (III.17) are similar to the first of Eq. (III.16), however the fields G and S must satisfy Eq. (III.18).

From Eqs. (III.9)–(III.18) it follows that electric charges e_n and masses m_n must have dimensions

$$[e^2] \quad [k][m^2] \quad [E][\ell] . \tag{III.19}$$

It means that general properties of worldlines corresponding to massive particles dictate the existence of three dimensionful fundamental constants c, ℓ and G, as a complete set of units by which one can measure masses m, electric charges e, and energy E in units

$$[m] \quad \frac{[\ell][c]^2}{[G]}, \quad [e] \quad \frac{[\ell][c]^2}{\sqrt{[G]}}, \quad [E] \quad \frac{[\ell][c]^4}{[G]} . \tag{III.20}$$

It is therefore necessary and sufficient to have three dimensionful units in order to reproduce in an empirically meaningful way the dimensions of all quantities entering our theory. The presented origin of the fundamental constants of physics is perfectly consistent with the analysis made by Weinberg [17] and Okun [18].

Next we assume that we have found the fields F , G and S as solutions to Eqs. III.16)–(III.18). In this case the total energy-momentum tensor

$$T \quad (x) \quad T_m \, (x) \qquad (x) \tag{III.21}$$

satisfies the important equation

$$T \quad (x)_{;} \qquad 0 \tag{III.22}$$

which is equivalent to the relation $f \quad (x) \quad f \quad (x)$ representing the generally valid relations (III.1) formulated in the presented field theory. Now if we carry out the integration of the integral

$$\mathrm{d} \quad {}^3 T \quad (x)_{;} \qquad 0 ,$$

then we get the explicit result

$$\sum_{n \; 1}^{N} \frac{\mathrm{d} p_n}{\mathrm{d}\tau_n} \qquad p_n \frac{\mathrm{d} z_n}{\mathrm{d}\tau_n} \qquad \frac{1}{c} \sum_{n \; 1}^{N} g \qquad e_n F \quad (z_n) \quad kS \quad (z_n) m_n^2 \; \frac{\mathrm{d} z_n}{\mathrm{d}\tau_n} . \tag{III.23}$$

Thus we have obtained laws of motion (III.1) generally valid for every worldline in which the forces $F_n \; (z_n)$ are explicitly determined by (III.23), i.e.

$$F_n \; (z_n) \quad \frac{1}{c} g \qquad e_n F \quad (z_n) \quad kS \quad (z_n) m_n^2 \; \frac{\mathrm{d} z_n}{\mathrm{d}\tau_n} . \tag{III.24}$$

However, the force $F_n \; (z_n)$ must be under any conditions always orthogonal to the vector $\mathrm{d} z_n / \mathrm{d}\tau_n$, i.e., it must satisfy the relation

$$g \quad F_n \;(z_n)\frac{\mathrm{d}z_n}{\mathrm{d}\tau_n} \quad 0\,. \tag{III.25}$$

The last relation generally valid for every worldline can be satisfied if and only if the symmetric tensor field $S \quad (x) \quad 0$, i.e.

$$R \qquad g \;\; L \quad \mathcal{H} \quad . \tag{III.26}$$

In the presented theory we have at our disposal only one symmetric tensor and it is $T \quad (x)$ defined by (III.21). By this fact $\mathcal{H} \qquad KT$, where K is a constant expressed by the universal constants c, G and ℓ. The dimensional analysis determines the constant $K \quad G/c^4$, where is a dimensionless numerical constant, and the relation (III.26) can be written in the form

$$R \qquad g \;\; L \quad \frac{G}{c^4}T \quad . \tag{III.27}$$

The equation (III.22) with $S \quad (x) \quad 0$ and $S \quad (x)_{;} \quad 0$ imply that Eq. (III.27) must be of the form

$$R \qquad \frac{1}{2}g \;\; R \;\; g \qquad \frac{G}{c^4}T \quad , \tag{III.28}$$

where $R \quad g \quad R$ is the scalar curvature and is the so called cosmological constant with the dimension l . The last equations are formally identical with Einstein equations in general theory of relativity known from textbooks providing that 8 .

By Eq. (III.28), i.e., $S \quad (x) \quad 0$ and $S \quad (x)_{;} \quad 0$, the relation (III.18) is simplified to the form

$$\frac{k}{c}g \;\; G \;\; J \qquad \frac{1}{2}g \;\; G \;\; [G \qquad G \qquad G \quad] \quad 0\,. \tag{III.29}$$

The expression in the bracket is an antisymmetric tensor of the third rank which has only four independent components and can be written in the form

$$G \qquad G \qquad G \qquad \frac{k}{c}\sqrt{g} \qquad \tilde{J} \;\; , \tag{III.30}$$

where $\tilde{J} \quad (x)$ is a 4-vector and stands for the totally antisymmetric Levi-Civitta symbol with the convention $^{0123} \qquad _{0123} \quad 1$. The last relation and the law of motion (III.17) for the field $G \quad (x)$ can remind one laws of motion of an electrodynamics with magnetic monopoles with the current density $\tilde{J} \quad (x)$ [19]. However, in our case the current density $\tilde{J} \quad (x)$ is not a given 4-vector, but it is the 4-vector generated by the tensor field $G \quad (x)$ which satisfies the law of motion (III.17). By inserting the definition (III.30) into (III.29) and by defining the dual tensor field

$$\tilde{G} \qquad \frac{1}{2}\sqrt{g}G \tag{III.31}$$

we can rewrite Eqs.(III.17) and (III.29) in the simple covariant forms

$$G \;\; (x)_{;} \qquad \frac{k}{c}J \;\; (x); \qquad G \;\; (x)J \;\; (x) \quad \tilde{G} \;\; (x)\tilde{J} \;\; (x)\,. \tag{III.32}$$

Similarly, one can define the dual tensor $\tilde{F}$ $\frac{1}{2g^{1/2}}$ F (x) to the electromagnetic field strength tensor F and express the Maxwell equations (III.16) and the Lorentz force density (III.12) in the covariant forms

$$F \ (x)_{;} \quad \frac{1}{c} j \ (x); \quad \tilde{F} \ (x)_{;} \quad 0; \quad f \ (x) \quad \frac{1}{c} g \ F \ (x) j \ (x) . \qquad \text{(III.33)}$$

The last equations represent laws of motion of classical electrodynamics which were derived as rigorous consequences of differential geometry in the four dimensional metric space – spacetime.

By comparing Eqs. (III.32) and (III.33) one sees that only first equations in them are similar, however the second equation in (III.32) represents a nonlinear constraint on the field G (x) and a "Lorentz" force density corresponding to the field G (x) does not exist.

From now on we accept the numerical value for 8 to have Eq. (III.28) in the form

$$R \quad \frac{1}{2} g \ R \ g \quad \frac{8 \ G}{c^4} T \ , \qquad \text{(III.34)}$$

with the cosmological constant . The last equations are only formally identical with the Einstein equations of the standard general theory of relativity, because in our case general properties of worldlines imply the existence of the field G which always gives a contribution to the tensor T standing on the right hand side of Eq. (III.34). However, as one knows, no such tensor field G exists in the standard Einstein general theory of relativity. If the presented theoretical analysis of the necessary existence of the tensor field G corresponded to physical reality, then its contribution into T could represent the dark energy in the spacetime.

Let us suppose that we have found exact solutions to Eqs. (III.32)–(III.34) determining F (x) and g (x) as explicit functions of coordinates x in the spacetime, then we get laws of motion of a test particle with electric charge e and mass m in the form

$$\frac{\mathrm{d}p}{\mathrm{d}\tau} \quad (z) p \ \frac{\mathrm{d}z}{\mathrm{d}\tau} \quad \frac{e}{c} g \ F \ (z) \frac{\mathrm{d}z}{\mathrm{d}\tau} . \qquad \text{(III.35)}$$

The last equations represent laws of motion of the relativistic mechanics in the general theory of relativity. The Eqs. (III.32)–(III.35) are classical laws of motion of physics derived as rigorous consequences of differential geometry in the spacetime and they represent the closed set of most general equations between fields generated by particles with masses and electric charges. The presented theory has induced the existence of three fundamental dimensionful constants c, G and the cosmological constant as a natural set of units in cosmology. By this fact the unit of length ℓ and the constant k entering Eqs. (III.11b)–(III.32) are replaced by the relations

$$\ell \quad \frac{4}{\sqrt{\ }} , \qquad k \quad 4 \ G. \qquad \text{(III.36)}$$

The classical tests of Einstein general theory of relativity: the gravitational red-shift of spectral lines, the precession of perihelia of orbits of planets, the deflection of light by the Sun and the time delay of radar echoes passing the Sun are in the perfect agreement with empirical observations [16, 20]. All four tests were carried out in the empty space, i.e., without the dark energy and with the cosmological constant 0 in the spherically symmetric gravitational field with the metric

$$\mathrm{d}s^2 = B(r)c^2\mathrm{d}t^2 - A(r)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{III.37}$$

where where r, θ and φ are the spherical polar coordinates.

According to Birkhoff theorem [16] every spherically symmetric gravitational field in empty space must be static and with the Schwarzschild metric

$$B(r) = 1 - \frac{r_S}{r}; \quad A(r) = \frac{1}{B(r)}; \quad r_S = \frac{2GM}{c^2}, \tag{III.38}$$

where M is the mass of Sun.

However, general properties of worldlines corresponding to massive particles require the necessary existence of the field $G_{\mu\nu}(x)$ which can never be zero. Thus the spacetime must contain the dark energy corresponding to this field and by this fact a gravitational field in empty space cannot exist. If the presented analysis with the necessary existence of the dark energy contradicted to the classical tests of the standard general theory of relativity, then we would regard content of our paper as an academic and meaningless exercise in differential geometry or as a sophisticated way for a derivation of equations which are accidentally identical with laws of motion of classical physics. For these reasons in the next section we find exact solutions to Eqs. (III.32)–(III.35) for a point like particle with mass M and electric charge Q having the worldline $z(t) = (ct, 0)$ with the aim to elucidate the origin and causes for the observed accelerated expansion of the Universe [1, 2].

4. Static spherically symmetric gravitational field with the dark energy in the space time

We derive the properties of the metric space with the gravitational field generated by a point like particle with the mass M and electric charge Q having the worldline $z(t) = (ct, 0)$. We use the spherical polar coordinates r, θ and φ with the diagonal metric in the standard form [16]

$$\begin{aligned}\mathrm{d}s^2 &= g_{00}c^2\mathrm{d}t^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2 \\ &= B(r)c^2\mathrm{d}t^2 - A(r)\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2.\end{aligned} \tag{IV.1a}$$

The indices μ, ν of tensors are identified so that $0 \to t, 1 \to r, 2 \to \theta$ and $3 \to \varphi$. It is very convenient to use the function $A(r)$ expressed in the form

$$A(r) = \frac{f^2(r)}{B(r)} \tag{IV.1b}$$

with the function $f(r)$ to be determined and which gives $g = -f^2(r)\,r^4\sin^2\theta$. In this notation the affine connection $\Gamma^{\lambda}_{\mu\nu}$ is computed from the formula (II.6). Its only nonvanishing components are

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{B'}{2B},$$

$$\Gamma^1_{00} = \frac{BB'}{2f^2}; \quad \Gamma^1_{11} = \frac{B}{2f^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{f^2}{B}\right); \quad \Gamma^1_{22} = -\frac{rB}{f^2}; \quad \Gamma^1_{33} = -\frac{rB}{f^2}\sin^2\theta$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r}; \quad \Gamma^2_{33} = -\sin\theta\cos\theta; \quad \Gamma^3_{23} = \Gamma^3_{32} = \frac{\cos\theta}{\sin\theta}, \tag{IV.2a}$$

where $B(r)$ means differentiation with respect to r. Similarly the Riemann tensor R is calculated from the formula (III.5a) and its only nonvanishing components are

$$R_{0101} \quad R_{0110} \quad R_{1001} \quad R_{1010} \quad \frac{1}{2} \quad B \frac{f}{f} \quad B \quad ,$$

$$R_{0202} \quad R_{0220} \quad R_{2002} \quad R_{2020} \quad \frac{rBB}{2f^2} \, ,$$

$$R_{0303} \quad R_{0330} \quad R_{3003} \quad R_{3030} \quad \frac{BB}{2f^2} r \sin^2 \quad ,$$

$$R_{1212} \quad R_{1221} \quad R_{2112} \quad R_{2121} \quad \frac{B}{2B} \quad \frac{f}{f} \quad r \, ,$$

$$R_{1313} \quad R_{1331} \quad R_{3113} \quad R_{3131} \quad \frac{B}{2B} \quad \frac{f}{f} \quad r \sin^2 \quad ,$$

$$R_{2323} \quad R_{2332} \quad R_{3223} \quad R_{3232} \quad \frac{B}{f^2} \quad 1 \; r^2 \sin^2 \quad . \tag{IV.2b}$$

The nonvanishing Riemann tensor is the general and plausible representation of the existence of a gravitational field. For the metric (IV.1) the Riemann tensor (IV.2b) has, in fact, only four independent components. The Ricci tensor R is computed from the last formulae and its only nonvanishing components are

$$R_0^0 \quad \frac{1}{2fr^2} \frac{\mathrm{d}}{\mathrm{d}r} \frac{r^2 B}{f} \quad ; \quad R_1^1 \quad R_0^0 \quad \frac{2B}{r} \frac{f}{f^3} \, ; \quad R_2^2 \quad R_3^3 \quad \frac{1}{fr^2} \frac{\mathrm{d}}{\mathrm{d}r} \frac{rB}{f} \quad \frac{1}{r^2} \tag{IV.3a}$$

which give the scalar curvature R in the form

$$R \quad \frac{1}{fr^2} \quad \frac{\mathrm{d}}{\mathrm{d}r} \frac{1}{f} \frac{\mathrm{d}}{\mathrm{d}r}(r^2 B) \quad 2rB \frac{f}{f^2} \quad \frac{2}{r^2} \, . \tag{IV.3b}$$

The point-like particle with the mass M and electric charge Q with the worldline $z(t) \quad (ct, \mathbf{0})$ has the mechanical 4-momentum $p \quad (p^0, \mathbf{0})$ which must satisfy the condition (II.4b), i.e.,

$$g \;\; p \;\; p \quad B(p^0)^2 \quad M^2 c^2 ,$$

which implies the solution

$$p^0 \quad \frac{Mc}{\sqrt{B}}$$

under the necessary condition

$$\lim_{r \;\; 0} B(r) \quad 0 \, . \tag{IV.4}$$

This particle generates the mechanical energy-momentum tensor T_m as given by the formula (III.4e) with the only one nonvanishing component T_m^{00} and the scalar D in the forms

$$T_m^{00} \quad \frac{Mc^2}{4 \; f\sqrt{B}} \frac{(r)}{r^2} \, , \quad D \quad \frac{Mc^2 \sqrt{B}}{4 \; fr^2} \quad (r) \, . \tag{IV.5}$$

Thus, the condition (IV.4) is the absolutely necessary one for the meaningful definitions of p^0, T_m^{00} and D. As one can notice the Schwazschild metric (III.38) contradicts to the condition (IV.4). This fact does not disturb us at all, because the Schwarzschild metric (III.38) was derived in the theory permitting the existence of gravitational field even in an empty space, i.e., even if $p^0 \quad T_m^{00} \quad 0$, while in our theoretical analysis a gravitational field in an empty space cannot exist. Thus, we have two distinct theories which are autonomous and must be internally self-consistent accounting for the same empirical observations as two empirically equivalent theories. For Feynman [14], especially, "the tensions and contradictions between alternative theories served as a creative force, an engine, for generating new knowledge". These contradictions will simply disappear under the presence of dark energy.

The corresponding current densities $j(x)$ and $J(x)$ as defined by (III.11) are given by

$$j \quad (j^0,\mathbf{0}), \quad j^0 \quad \frac{Qc}{4 \; fr^2} \; (r); \qquad J \quad (J^0,\mathbf{0}), \quad J^0 \quad \frac{\sqrt{\;\;G}\; M^2}{4 \; c \;\; fr^2} \; (r). \tag{IV.6}$$

They generate the field strength tensors F and G satisfying Eqs. (III.32) and (III.33) with their only nonvanishing components as given by

$$F^{10} \quad F^{01} \quad \frac{Q}{4 \; fr^2} \quad (r) \quad a \quad (r) \quad 1 \;\; ^2 \;\; ;$$

$$G^{01} \quad G^{10} \quad \frac{\sqrt{\;\;}}{fr^2} \; \frac{GM}{c} \;^2 \quad (r) \quad a \quad (r) \quad 1 \;\; ^2 \;\; ; \tag{IV.7a}$$

$$G^{23} \quad G^{32} \quad \frac{\sqrt{\;\;\;b}}{fr^4 \sin} \; \frac{GM}{c} \;^2 ,$$

where a and $b > 0$ are dimensionless parameters to be determined later on. Here (r) denotes the step function defined so that $(r) \quad 1$ for $r \quad 0$, $(r) \quad 0$ for $r \quad 0$ and $(r) \quad (r)$.

We regard the step function in (IV.7a) as the distribution rigorously defined on a set of real analytic functions $(r; \;)$ of the variable r for every value of the parameter $\quad 0$ by the relation

$$(r) \quad \lim \; (r; \;), \tag{IV.7b}$$

where $(r; \;)$ can be represented in the form

$$(r; \;) \quad 1 \quad \frac{1}{\sqrt{\;\;}} e^{\;\; r} \;\; ^1 . \tag{IV.7c}$$

We determine the current density $\tilde{J} \quad (\tilde{J}^0,\mathbf{0})$,

$$\tilde{J}^0 \quad \frac{G\sqrt{\;\;/b}}{4 \; c} \frac{M^2}{fr^2} \frac{f}{f^2},$$

and Eqs. (III.32) imply the differential equation for the function $f(r)$ in the form

$$f(0) \;\; (r) \quad \frac{1}{b} \frac{f}{f^2}. \tag{IV.8a}$$

The last equation has the general solution in the form of the distribution

$$f(r) \quad 1 \;\; b \quad (r) \;\; 1 \quad ^1 \tag{IV.8b}$$

under the condition $f(0)\quad 1$. Thus, $f(r)$ is the distribution rigorously defined on the set of real analytic functions $f(r;\quad)$ of the variable r for every value of the parameter $\quad 0$ by the relation

$$f(r)\quad \lim f(r;\quad),$$

where $f(r;\quad)$ is given by (IV.8b) in which $\quad(r)$ is replaced by $\quad(r;\quad)$. In this case the function $f(r)$ has the properties $f(r)\quad 1$ for every $r\quad 0$ and $f(r)\quad (1\quad b)^{\quad 1}$ for every r < 0.

The field strength tensors (IV.7a) imply the only nonvanishing diagonal components of the stress energy tensor as given by

$$\begin{matrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{matrix}$$

$$\frac{\quad}{8\ G}\frac{GM}{cr}^{4}\frac{\quad}{bf}\quad r\quad a^2[\ r\quad]^2\quad \frac{1}{2}\ \frac{Q}{4\ r^2}^{2}\quad r\quad a^2[\ r\quad]^2\ . \tag{IV.9a}$$

By the last formula the total energy-momentum tensor T as defined by (III.21) is explicitely determined by the relations

$$T_0^0\quad \frac{Mc^2\sqrt{B}}{4\ fr^2}\ (r)\quad {}^0_0;\qquad T_1^1\quad T_2^2\quad T_3^3\quad {}^0_0. \tag{IV.9b}$$

The Einstein equations (III.34) are written in the form

$$R\quad \frac{1}{2}\quad R\qquad \frac{8\ G}{c^4}T$$

and they imply the following set of equations:

$$R_0^0\quad R_1^1\quad 2\frac{GM}{c^2}\frac{\sqrt{B}}{fr^2}\ (r);\quad \frac{f}{f^2}\quad \frac{GM}{c^2}\frac{(r)}{r\sqrt{B}}, \tag{IV.10a}$$

$$R\quad 4\quad 2\frac{GM}{c^2}\frac{\sqrt{B}}{fr^2}\ (r)\quad \frac{8\ G}{c^4}D, \tag{IV.10b}$$

$$R_0^0\qquad \frac{GM}{c^2}\frac{\sqrt{B}}{fr^2}\ (r)\quad \frac{8\ G}{c^4}\ {}^0_0. \tag{IV.10c}$$

The Eqs. (IV.10a) and (IV.8) require the condition

$$\lim_{r\quad 0} r\sqrt{B}\quad \frac{GM}{c^2 b} \tag{IV.11}$$

to be satisfied. Eq. (IV.10b) determines the scalar curvature R of the spacetime by the relation

$$R\quad 4\quad 2\ \frac{GM}{c^2}^{2}\ \frac{(r)}{br^3}$$

which with the expression for R (IV.3b) leads to the second order differential equation

$$2f\quad 4\ fr^2\quad \frac{\mathrm{d}}{\mathrm{d}r}\ \frac{1}{f}\frac{\mathrm{d}}{\mathrm{d}r}(r^2B)\quad 0 \tag{IV.12}$$

for the function $B(r)$. This differential equation can be easily integrated with the result

$$B(r)\quad f^2(r)\quad \frac{C_1 f(r)}{r}\quad \frac{C_2}{r^2}\quad \frac{1}{3}\quad f^2(r)r^2, \tag{IV.13}$$

where C_1 and C_2 are integration constants. By taking into account the relation (IV.11), then the integration constant C_2 is given by the formula

$$C_2 \quad \frac{GM}{c^2}^{\,2} \frac{1}{b^2}.$$

The explicitly known form (IV.13) for $B(r)$ used in Eq. (IV.10c) implies the algebraic equation

$$\frac{1}{f^2b^2} \quad \frac{GM}{c^2}^{\,2} \quad \frac{\quad}{bf} \quad (r) \quad a^2[\ (r)\ 1]^2 \quad \frac{1}{4}\frac{Q^2}{GM^2} \quad (r) \quad a^2[\ (r)\ 1]^2$$

which must be satisfied for every r. Its solutions determine the parameters

$$\frac{1}{b} \quad \frac{\quad}{2} \frac{GM}{c^2}^{\,2} \quad \frac{^2}{4} \frac{GM}{c^2}^{\,4} \quad \frac{GM}{c^2}^{\,2} \quad \frac{1}{4}\frac{Q^2}{GM^2}^{\,1/2} \tag{IV.14a}$$

and

$$a^2 \quad 1 \quad \frac{1}{b}^{\,2} \quad 1 \quad b \frac{GM}{c^2}^{\,2} \quad \frac{GM}{c^2}^{\,2} \quad \frac{1}{4}\frac{Q^2}{GM^2}^{\,1} \tag{IV.14b}$$

uniquely.

At this point we want to emphasise that the Einstein equations (IV.10) do not specify the integration constant C_1 and by this fact they do not determine the metric uniquely. The value of C_1 must be determined from the motion of a test particle with the mass m and electric charge e in the static gravitational and electric fields with the metric (IV.13) for $r \quad 0$. The laws of this motion are given by (III.35), i.e.,

$$\frac{dp}{d\tau} \quad p \frac{dz}{d\tau} \quad \frac{e}{c} g \quad F \quad \frac{dz}{d\tau}, \tag{IV.15}$$

where $z \quad (ct, r, \ , \ )$, $p \quad m\frac{dz}{d\tau}$ and the affine connection is explicitly determined by the known functions $f(r)$ and $B(r)$ as given by the relations (IV.8) and (IV.13) respectively. Since the gravitational and electric fields are isotropic, the motion of the test particle must be confined to planes passing through the origin $r \quad 0$, i.e., either to meridian planes const., or to the equatorial plane $\frac{\quad}{2}$. One can convince himself from (IV.15) that orbits of the test particle are exactly the same in both meridian planes and in equatorial plane.

We may consider the orbit of our particle confined to the equatorial plane, that is

$$\frac{\quad}{2}, \quad p^2 \quad m\frac{d}{d\tau} \quad 0.$$

In this case Eqs. (IV.15) for $\quad 0$ and $\quad 3$ have the forms

$$\frac{dp^0}{d\tau} \quad \frac{B}{B} p^0 \frac{dr}{d\tau} \quad \frac{eQ}{4\ cr^2} \frac{1}{B} \frac{dr}{d\tau}; \quad \frac{dp^3}{d\tau} \quad \frac{2}{r} p^3 \frac{dr}{d\tau} \quad 0 \tag{IV.16a}$$

which permit their simple integrations with the results

$$p^0 B \quad \frac{eQ}{4\ cr} \quad mc\ ; \quad p^3 \quad \frac{J}{r^2}, \tag{IV.16b}$$

where 0 and J are integration constants which represent integrals of motion of the test particle. The constant J is its angular momentum. The motion of the test particle must be along its world line and it must satisfy the relation

$$g \quad p \quad p \qquad B(r)(p^0)^2 \quad \frac{1}{B(r)}(p^1)^2 \quad r^2(p^3)^2 \quad m^2c^2. \tag{IV.16c}$$

Eq. (IV.15) for 1 has the explicit form

$$\frac{\mathrm{d}p^1}{\mathrm{d}\tau} \quad \frac{B}{2m} \quad B(p^0)^2 \quad \frac{1}{B}(p^1)^2 \qquad \frac{B}{m}\frac{J^2}{r^3} \qquad \frac{eQ}{4 \quad cr^2}\frac{Bp^0}{m}. \tag{IV.17}$$

By exploiting the results (IV.16) in the last equation we rewrite it in the form

$$\frac{\mathrm{d}p^1}{\mathrm{d}\tau} \quad \frac{\mathrm{d}}{\mathrm{d}r}U(r) \quad 0, \tag{IV.18a}$$

where

$$U(r) \quad m \quad _g(r) \quad \frac{J^2}{2mr^2}B(r) \quad e \quad _C(r)$$

with

$$_g(r) \quad \frac{1}{2}c^2B(r); \qquad _C \quad \frac{Q}{4 \quad r} \qquad \frac{1}{2mc^2}\frac{eQ}{4 \quad r} \tag{IV.18b}$$

can be regarded as the total potential energy of the test particle. Here in the first term $_g(r)$ denotes the gravitational potential, the second term represents the potential energy due to the centrifugal force and in the third term $_C(r)$ is identified with the effective Coulomb potential.

By multiplying Eq. (IV.18a) with $\frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \frac{\mathrm{d}r}{\mathrm{d}t}$ we may write it as

$$\frac{\mathrm{d}}{\mathrm{d}t} \quad \frac{1}{2m}(p^1)^2 \quad U(r) \qquad 0$$

and our last constant of the motion is therefore $\mathcal{E} \quad \frac{1}{2}mc^2 \quad ^2$ as given by

$$\frac{1}{2m}(p^1)^2 \quad U(r) \quad \frac{1}{2}mc^2 \quad ^2. \tag{IV.18c}$$

The last formula is identical with the equation relating the kinetic energy $(p^1)^2/(2m)$ and the effective potential energy $U(r)$ in the nonrelativistic mechanics [21] for a particle with the mass m moving in a spherically symmetric potential. This means that the effective gravitational potential $_g(r)$ as defined by (IV.18b) must reduce on its nonrelativistic form in the nonrelativistic limits 0 and c , i.e.,

$$\lim_{c} \lim_{0} \frac{1}{2}c^2B(r) \qquad \frac{GM}{r} \quad \text{const.} \tag{IV.18d}$$

The last condition determines the value of the constant C_1 uniquely in the form

$$C_1 \qquad 2\frac{GM}{c^2} \qquad r_S,$$

where r_S is the Schwarzschild radius corresponding to the mass M. The condition (IV.18d) represents the empirical requirement without of which one could not determine the metric of the system under consideration uniquely. By the last relation and with the expression (IV.14) we have determined the metric (IV.1) explicitly in the form

$$B(r) \quad 1 \quad \frac{r_S}{r} \quad \frac{1}{4b^2}\frac{r_S^2}{r^2} \quad \frac{1}{3} \quad r^2, \quad r \quad 0 \tag{IV.19}$$

which represents the generalization and modification of the Reissner-Nordstr m metric [20, 22] under the presence of dark energy and cosmological constant .

Intuitively it may seem as an unacceptable fact that the effective Coulomb potential $_C(r)$ as defined by (IV.18b) could be dependent on a state ε of test particle. We briefly analyse this unexpected situation on the Coulomb system similar to the hydrogen-like atom with $Q \quad eZ, \; Z \quad 0$, and e is the elementary charge of electron. By ignoring gravitational effects with respect to Coulomb forces we may write Eq. (IV.18c) in the form

$$\frac{1}{2m}(p^1)^2 \quad \frac{J^2}{2mr^2} \quad \frac{Ze^2}{4 \; r} \qquad \frac{1}{2mc^2}\frac{Ze^2}{4 \; r} \quad \frac{1}{2}mc^2(\; ^2 \quad 1) \quad E$$

which, with the relations

$$m\frac{\mathrm{d}r}{\mathrm{d}\tau} \quad p^1 \; ; \qquad m\frac{\mathrm{d}}{\mathrm{d}\tau} \quad \frac{J}{r^2},$$

represents the exactly solvable Keppler problem [21] for any permitted values of constants 0 and J.

Only for the sake of short notations we allow to use the Planck constant $\hbar$ for the definitions of the following dimensionless and real parameters

$$\frac{Ze^2}{4 \; \hbar c}; \qquad 1 \quad {}^2\frac{\hbar^2}{J^2}\;^{1/2}; \qquad 1 \quad 1 \quad \frac{1}{^2}\;\frac{J^2 \; ^2}{^2\hbar^2}\;^{1/2}.$$

The states of the electron are enumerated by two numbers 0 and J. For $0 \qquad 1$, i.e., $E \quad 0$, the electron has bound state orbits, as given by the exact solution of the above equations, in the form

$$\frac{1}{r(\;)} \quad \frac{Ze^2}{4}\frac{m}{^2J^2} \; 1 \qquad \cos \qquad .$$

At perinuclei and apnuclei, r reaches its minimum and maximum values r and r as given by

$$r \qquad \frac{4}{Ze^2}\frac{^2J^2}{m}\frac{1}{1 \mp}$$

and the electron orbit precesses in each revolution by the angle

$$2 \quad \frac{1}{} \quad 1 \; .$$

Thus, the electron orbit is, in general, not a closed ellipse. By the assumption, which can be empirically tested by its consequences, we may assume that the electron orbits $r(\;)$ are closed curves. This assumption is satisfied if and only if the parameter is a rational number

$$\frac{1}{} \qquad \frac{}{},$$

where and are integers. The last condition implies immediately that parameters , and $J/\hbar$ must have discrete spectra of values consisting of rational numbers. There exist infinitely many solutions satisfying these criteria and from them we select one solution in the following form

$$\frac{J}{\hbar} \quad n \quad \frac{\;^2}{4n}; \qquad n \quad \frac{\;^2}{4n} \quad n \quad \frac{\;^2}{4n} \quad ^1, \quad n \quad 0,$$

where n are positive and negative integers.

The bound state energies $E \quad \frac{1}{2} mc^2 (\quad ^2 \quad 1)$ become minimal for states $(\;, J)$ if and only if $(p^1)^2 \quad 0$. In these states the electron orbits have eccentricities $\quad 0$ implying the quantization of the parameter $\;$,

$$_n \quad n \quad \frac{\;^2}{4n} \quad n \quad \frac{\;^2}{4n} \quad ^1,$$

and the orbits become circles with radii

$$r_n \quad \frac{4 \quad \hbar^2}{Ze^2 m} \quad n^2 \quad \frac{\;^2}{4n} \quad ^2 \qquad 0.$$

The corresponding bound state energies are

$$E_n \quad \frac{m}{2\hbar^2} \quad \frac{Ze^2}{4} \quad ^2 \quad n \quad \frac{\;^2}{4n} \quad ^2 \qquad \frac{m}{2\hbar^2} \quad \frac{Ze^2}{4} \quad ^2 \qquad \frac{1}{2} mc^2$$

which become identical with spectra of hydrogen-like atoms in nonrelativistic quantum mechanics in the nonrelativistic limit c . Note that the parameters and can differ from one only by relativistic terms of the second order.

The last inequalities are satisfied if and only if the following constraint is satisfied

$$Z^2 \quad \frac{e^2}{4 \quad \hbar c} \quad ^2 \qquad 1$$

which is exactly the same one as that following from the relativistic quantum mechanics governed by the Dirac equation.

Thus, the general theory of relativity predicts that point like particle charges $|Q|$ are limited from above and ground state energies of Coulomb systems are limited from bellow. The explicit dependence of the effective Coulomb potential $\;_C(r)$ on a state of test particle does not bring any contradictions to empirical observations.

In what follows the metric (IV.1) with $B(r)$ as given by (IV.19) will be applied for understanding of the origin and causes for the observed accelerated expansion of the Universe [1, 2]. For this reason we may consider an oversimplified model of a Universe having a very heavy body in its center with the mass M and with $Q \quad 0$ and its galaxies are regarded as test particles. The empirical value [23] for the cosmological constant

$$1.11 \quad 10^{\;52} \text{ m}^{\;2}$$

implies $\quad r_S^2 \quad 1$ for any meaningful Schwarzschild radius r_S. Even if r_S were about one light year, $r_S \quad 10^{16}$ m, then $\quad r_S^2$ would be $\quad r_S^2 \quad 10^{\;20}$. Thus, we are permitted to retain only the linear terms in $\quad r_S^2$ in the formula (IV.19) for $B(r)$, i.e.,

$$B(r) \quad 1 \quad \frac{r_S}{r} \quad \frac{1}{16} \quad r_S^2 \frac{r_S^2}{r^2} \quad \frac{1}{3} \quad r^2. \tag{IV.20}$$

The plot of $B(r)$ is schematically depicted on Fig. 1.

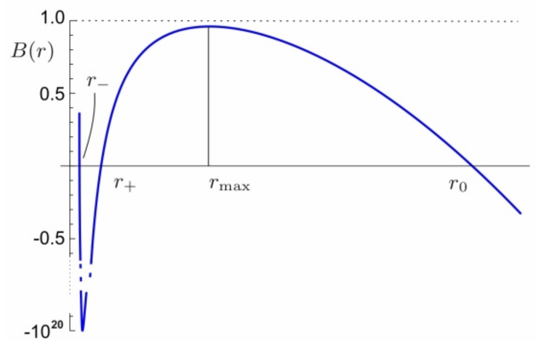

Fig. 1. The schematical plot of $B(r)$.

The metric with $B(r)$ as given by (IV.20) corresponds to a black hole with two inner horizons

$$r \quad \frac{r_S}{2}\; 1 \quad 1 \quad \frac{1}{4}\; r_S^2 \quad {}^{1/2} \quad , \tag{IV.21a}$$

and one outer horizon

$$r_0 \quad \frac{3}{r_S^2}^{\;1/2} \quad r_S \quad \frac{3}{\;}^{\;1/2} , \tag{IV.21b}$$

where $B(r)$ 0 for r $(0, r$) and r $(r$, $r_0)$, and $B(r)$ 0 for r $(r$, r) and r r_0. The metric (IV.1) and the Riemann tensor (IV.2b) are singular on the horizons. However, these singularities do not appear in the Ricci tensor (IV.3) and in invariants constructed out of Riemann tensor (IV.2b).

The function $B(r)$ has a very narrow and very deep minimum at r $r_{\min}$ $\frac{1}{8}$ r_S^3 with $B(r_{\min})$ 1 $4/(\;r_S^2)$ 10^{20} and a very broad maximum at r $r_{\max}$,

$$r_{\max} \quad \frac{3}{2\; r_S^2}^{\;1/3} \quad r_S \quad \frac{r_S^2}{12}^{\;1/6} \quad r_0\, ; \quad r \quad r_{\max} \quad r_0\, , \tag{IV.21c}$$

with $B(r_{\max})$ 1 $(\frac{9}{4\; r_S^2})^{1/3}$ 1. The approximative values r , r_0, $r_{\min}$ and $r_{\max}$ as given by (IV.21) do not differ essentially from those following from exact numerical calculations.

Next we examine properties of galaxy orbits. Their radial components p^1 satisfy Eqs. (IV.18c) written in the form

$$(p^1)^2 \quad m^2c^2\;{}^2 \quad m^2c^2 \quad \frac{J^2}{r^2}\; B(r) \quad m^2c^2\;{}^2 \quad 2mU(r)\, . \tag{IV.22a}$$

The function $2mU(r)$ has a similar behaviour as the function $B(r)$ depicted on Fig. 1. Thus, galaxies with ^{2} 1 and with r $(r_S, r_{\max})$ have

$$p^1(r) \qquad m^2c^2\;{}^2 \quad 2mU(r) \quad {}^{1/2} \tag{IV.22b}$$

with two turning points at which $p^1(r)$ 0 and they have bound orbits. Note that for r $(r_S, r_{\max})$, the function $B(r)$ reduces to the Schwarzschild metric $B(r)$ 1 r_S/r with an extraordinary high accuracy. Thus the results of all classical tests of general theory of relativity within the solar system [16] must be perfectly consistent with the Schwarzschild metric even under the existence of dark energy and the presence of cosmological constant.

We next analyse galaxy orbits with $r \quad r_{\max}$, i.e., in the space region, where one has the effective metric

$$B(r) \quad 1 \quad \frac{r}{r_0}^{2} . \tag{IV.23a}$$

In this case the radial component $p^1 \quad m\frac{\mathrm{d}r}{\mathrm{d}\tau}$ satisfies the relation (IV.22) written in the form

$$\frac{\mathrm{d}r}{\mathrm{d}\tau}^{2} \quad c^2 \quad \frac{r_0}{r}^{2} \quad \quad \frac{r}{r_0}^{2\;2} \quad (\;^{2} \quad) , \tag{IV.23b}$$

where

$$\frac{1}{2}(\;^{2} \quad 1 \quad); \qquad \frac{J^2}{m^2 c^2 r_0^2} .$$

In this space region a galaxy orbit can have only one turning point at $r \quad \tilde{r}$, satisfying the following relations

$$\frac{\tilde{r}}{r_0}^{2} \quad \quad (\;^{2} \quad)^{1/2} \quad \frac{r_{\max}}{r_0}^{2} \quad 1 , \tag{IV.23c}$$

at which $p^1 \quad 0$ and represents an unbound orbit. The last relations require the parameters and to have the following approximative values

$$\frac{J^2}{2m^2c^2r_{\max}^2} , \qquad ^{2} \quad 1 \quad \frac{J^2}{m^2c^2r_{\max}^2} . \tag{IV.23d}$$

The differential equation (IV.23b) has the exact solution in the form

$$\frac{r}{r_0}^{2} \quad \sqrt{\;^{2} \quad } \;\mathrm{ch}\; \frac{2c}{r_0}(\tau \quad \tau_0) \quad , \tag{IV.24a}$$

where τ_0 is the integration constant chosen so that $r \quad \tilde{r}$ for $\tau \quad \tau_0$. Thus for $\tau \quad \tau_0$, these unbound orbits have radial velocities v_r and radial accelerations a_r expressed by the formulae

$$v_r \quad \frac{\mathrm{d}r}{\mathrm{d}t} \quad \frac{\mathrm{d}r}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \frac{c}{r_0} r \;1 \quad \frac{r}{r_0}^{2} \tag{IV.24b}$$

$$a_r \quad \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} \quad \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}r} \quad \frac{\mathrm{d}r}{\mathrm{d}t}^{2} . \tag{IV.24c}$$

We identify the Hubble constant $H_0 \quad c / r_0$ and we define the effective Hubble constant $H \quad H_0 /$ with $^{2} \quad 1 \quad J^2 / (mcr_{\max})^2$ in order to express $_r$ and a_r in the forms

$$v_r \quad Hr \;1 \quad \frac{r}{r_0}^{2} , \tag{IV.25a}$$

$$a_r \quad H^2 r \;1 \quad \frac{r}{r_0}^{2} \quad 1 \quad 3\frac{r}{r_0}^{2} . \tag{IV.25b}$$

The last formulae can be regarded as a generalized Hubble law in which the effective Hubble constant H is slightly dependent on individual properties of galaxies represented by their angular momenta J and their masses m. Thus, the empirical values of H corresponding to various galaxies are expected to exhibit a small dispersion of them bellow the value H_0. The empirical data as presented in [24, 25] show that it is indeed so. The Hubble time $t_H \quad 1/H_0 \quad 5.48 \quad 10^{17}$ s is very close to the value presented in [24, 25]. The formula (IV.25b) predicts the accelerated expansion of the Universe by galaxies with $r \quad (r_{\max}, r_0/\sqrt{3})$ and $r \quad r_0$, and the deaccelerated compression of the Universe by galaxies with $r \quad (r_0/\sqrt{3}, r_0)$.

Similarly we determine circular velocities of galaxies with unbound orbits by the formula

$$v \quad r\frac{\mathrm{d}}{\mathrm{d}t} \quad r\frac{\mathrm{d}}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t} \quad \frac{J}{m\;r} \quad 1 \quad \frac{r}{r_0}^{2} \qquad \text{(IV.26)}$$

which predicts the anticlockwise direction of galaxy rotations by galaxies with $r \quad (r_{\max}, r_0)$ and clockwise rotations by galaxies with $r \quad r_0$ provided that $J \quad 0$. Thus, the presented theory brings empirical elements which can be tested by empirical observations. The last result is qualitatively consistent with analysis of rotation curves of galaxies [26–30]. According to Feynman [14] "*. . . the only true test of a theory is its ability to produce good numbers, numbers agreeing with experiment*". It seems that the presented theory satisfies this criterion.

5. Discussion

We have explicitly demonstrated that general properties of curves corresponding to worldlines of massive particles in the 4-dimensional metric space – spacetime imply the existence of tensor fields and ecessary relations among them which are identical with laws of motion of classical physics. By the same way they require the existence of three dimensionful fundamental constants by which one can reproduce in an empirically meaningful way all quantities entering the presented theory in order to be compared with empirical observables. Theoretical predictions of the theory are perfectly consistent with the classical tests of general theory of relativity and provide the simple explanation for understanding of the origin and causes for the observed accelerated expansion of the Universe.

Even if we have not expected that the presented theoretical analysis could be accepted as a possible way for completing the standard Einstein general theory of relativity by the dark energy with the cosmological constant, the developed methods can be used for solving the following problems as simple exercises in differential geometry.

One can solve exactly corresponding equations in order to find the modification of the Kerr-Newmann metric [20, 22] under the presence of the dark energy in the space. In the relatively simple problem one can consider instead of one tensor field $F \quad (x)$ four tensor fields $F_a \quad (x)$, $a \quad 1, 2, 3, 4,$ with the non-abelian gauge group SU(2)×U(1) symmetry in order to derive laws of motion of Weinberg's unified theory of electro-weak interactions [31] and their implications on the gravitational field and its metric similarly as we have derived Maxwell's equations.

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