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Remarks upon Neutrino Mixing Hypothesis

V. Pažma[1], J. Vanko[2]

[1]Department of Theoretical Physics and Didactics of Physics
[2]Department of Nuclear Physics and Biophysics, Comenius University, Mlynská dolina F1, 842 48 Bratislava, Slovakia

Abstract: It is shown that various versions of the neutrino mixing hypothesis and its theoretical descriptions are in contradiction with generally accepted facts and principles. The possible alternative formulation of the neutrino oscillation theory there is also presented and it is shown under what conditions this theory reproduces the known oscillation probability formula. In our approach (flavor) neutrinos are Dirac particles. In the case of Majorana neutrinos, or the nonrelativistic neutrinos (i.e. relic neutrinos), the problem could be more complicated.

1. Introduction

The neutrino mixing hypothesis is the cornerstone of the contemporary formulation of the neutrino oscillation theory (see e.g. [1 5). In accordance with this hypothesis the states $| \;\; \rangle$, $\;\; e, \;,\;$ of the free flavor neutrinos $_{e}, \;\;,\;$ are defined by means of the states $| \;_{i} \rangle, (i \;\; 1,2,3)$ of the so-called massive free neutrinos $_{i}$ by the relation

$$| \;\; \rangle \;\; U_{\;i} | \;\; \rangle, \tag{1}$$

where $U_{\;i}$ are the elements of unitary matrix U (U is the constant matrix) and the summation over repeated indices is assumed.

The massive free neutrino states $| \;_{i} \rangle$ have the masses M_i, the helicity (1/2) and they are eigenstates of the generator H of the time development. The definition (1) is too formal and one can encounter its various versions in the literature:

$$| \;\; ; \vec{p} \rangle \;\; U_{\;i} | \;\; ; \vec{p}, M_i \rangle . \tag{2}$$

The equation (2) represents the so-called equal momentum case. It is well-known that (2) is not compatible with the Loretz invariance (see e.g. [1]). Namely, if some observer O registers $U_{\;i} | \;\; ; \vec{p}, M_i \rangle$ as the flavor state of than another observer O', who moves with the velocity $\vec{\;} \;\; 0$ with respect to O, will register the state

$$U_{\;j} | \;\; ; \vec{p}_i, M_i \rangle \tag{3}$$

where $\vec{p}_i \;\; \vec{p}_j$, if $M_i \;\; M_j$. Thus, in accordance with (2), the observer O' will not consider (3) as the state of flavor neutrino .

$$| \;\; \rangle \;\; U \;\; | \;\; ; \vec{p}_i, M_i \rangle \tag{4}$$

where

$$E \;\; \sqrt{\vec{p}_i^{\,2} \;\; M_i^2} \quad \text{for all } i \;\; 1,2,3,$$

Devoted to Prof. Pavel Povinec 65-th anniversary (the equal energy case (see e.g. [1])). The definition (4) is not compatible also with the Lorentz invariance. Moreover, it is evident that thde right-hand side of (4) is the eigen-state of *H*, so transitions to other states do not exist. In [1] there is the following problem solved: "How to take into account the different momenta of massive neutrinos in the derivations of the oscillation probability?". Hence, the author considers the following version of (1)

$$| \ \rangle \ U \ | \ ; \vec{p}_i, M_i \rangle \quad (5)$$

This definition of $| \ \rangle$ can be hardly accepted. Let us consider two trinities $(\vec{p}_1, \vec{p}_2, \vec{p}_3)$, $(\vec{p}_1, \vec{p}_2, \vec{p}_3 \quad \vec{p}_3)$. In the first case we have, e.g.,

$$| \ _e \rangle \ U_{ei} | \ , \vec{p}_i, M_i \rangle$$

whereas, for the second it is

$$| \ \rangle \ \sum_{i \ 1}^{2} U_{\ i} | \ , \vec{p}_i, M_i \rangle \ U_{\ 3} | \ , \vec{p}_3, M_3 \rangle.$$

Then $\langle \ _e | \ \rangle \ \sum_{i \ 1}^{2} U^*_{ei} U_{\ i} \ \ 0$.

Moreover, as the kernel of the considerations presented in [1] is the oscillation probability formula of the form

$$P \qquad (L,T) \ \left| U^*_{\ k} e^{ip_k L \ \ iE_{kT}} U_{\ k} \right|^2$$

(6)

where *L* is the source-detectordistance and *T* is the time which passed from the production of till the detection of . This result evokes a certain suspicion. Namely, for $T \quad 0$, we get

$$P \qquad (L,0) \ \left| U^*_{\ k} e^{ip_k L} U_{\ k} \right|^2 \qquad ,$$

whereas it is natural to expect that no transition is possible for $T \quad 0$. This simply means that considerations presented in [1] are not consistent. Hence, the neutrino mixing hypotheses (2) or (4) are reference frame dependent.

In [6] there was the reference frame independent definitions of the flavor states presented. It reads

$$| \ \rangle \ U \ | \ , \vec{p}_i, M_i \rangle \quad (7)$$

where

$$\frac{\vec{\mathrm{p}}_\mathrm{i}}{E_\mathrm{i}} \quad \frac{\vec{\mathrm{p}}_\mathrm{i}}{\sqrt{\vec{\mathrm{p}}_\mathrm{i} \quad M_i^2}} \quad \vec{\mathrm{v}} = \mathrm{const.}$$

for all $i \quad 1,2,3$ (equal velocity case). Let us now consider the interaction of with charged lepton l . Considering the case when l gained the momentum $\vec{k}$. The initial state $|l \ , \vec{p}, m \ \rangle U_{\ i} | \ _i, \vec{p}_i, M_i \rangle_{\vec{v}}$ will convert to the final state

$$|l \ , \vec{p} \quad \vec{k}, m \ \rangle A(t, \vec{p}, m \ , \vec{p}_i, M_i, \vec{k}) U_{\ i} | \ _i, \vec{p}_i \quad \vec{k}, M_i \rangle. \quad (8)$$

All $\vec{p}_i$ are parallel, and if $\vec{k}$ is not parallel with $\vec{p}_i$ then $\vec{p}_i \quad \vec{k}$ $(i \quad 1,2,3)$ are not parallel and then (8) cannot be written in the form

$$\left| l \ , \vec{p} \quad \vec{k}, m \ \right\rangle \widetilde{A} U \ _i \left| \ _i, \vec{p}_i \quad \vec{k}, M_i \right\rangle_{\vec{v}} .$$

Hence, if interaction with l is defined by means of interactions with l , then, in the final state, we shall not obtain the flavor neutrino defined by (7). The previous remarks evoke the question: What version of the neutrino mixing hypothesis can be acceptable and in what sense? In the next section we shall show that (2) (equal momentum case) can be accepted as a good approximation in the case of ultra-relativistic neutrinos. Naturally instead of (2) we have to write

$$| \quad \rangle \ U \ | \quad , \vec{p}, M_i \rangle .$$

2. Alternative approach to neutrino oscillations

In this section we shall regard $_e$, , as Dirac particles and formulate (phenomenologically) the theory of their oscillations. We shall also show that this theory reproduces the results following from the standard one (based on (2)) in the region $\vec{p}^{\,2}$ >> squared mass of any neutrino.

Let us now define the (flavor) neutrino wave functions $\langle p | \ _i \rangle$ (we work in the p-representation and, instead of indices e, , , we shall use the indices i $1,2,3$) as the eigenfunctions of the hamiltonian

$$H_0 \quad \vec{} \ \vec{p} \quad M_d \tag{9}$$

where M_d $diag(m_1, m_1, m_3)$, m_i's are the masses of $_i$ and the standard meaning of other symbols is assumed. In the standard representation of Dirac matrices and for $\vec{p}$ $(0, 0, p \quad 0)$ we can choose $\langle p | \ _i \rangle$ as

$$\langle p | \ _i \rangle \quad \frac{U_i}{\sqrt{2} \ _i}, \qquad (\ _i \quad \sqrt{p^2 \quad m_i^2}\,),$$

where for i $1,2,3$ we have

$$U_i \quad \begin{matrix} _{1i} \ u_i \\ _{2i} \ u_i \\ _{3i} \ u_i \end{matrix} , \qquad u_i \quad \begin{matrix} \sqrt{\ _i \quad m_i} \quad w \\ \sqrt{\ _i \quad m_i} \quad w \end{matrix} , \qquad _3 w \quad w$$

These eigenfunctions correspond to positive eigenvalues of H_0 and the negative helicity and for i $4,5,6$

$$U_i \quad \begin{matrix} _{4i} \ u_i \\ _{5i} \ u_i \\ _{6i} \ u_i \end{matrix} , \qquad u_i \quad \begin{matrix} \sqrt{\ _i \quad m_i} \quad w \\ \sqrt{\ _i \quad m_i} \quad w \end{matrix} , \qquad m_i \quad m_{i\ 3} , \qquad _i \quad _{i\ 3}$$

These eigenfunctions correspond to negative eigenvalues of H_0 and the negative helicity.

Now if

$$H \quad H_0 \quad M \quad \vec{} \ \vec{p} \quad M$$

where $(M \ )$ M and M_{ii} 0 for all i, is the generator of the time-development, then the transitions $_i$ $_j$ will occur in the theory. (As far as we know this approach was considered in [5, 6]). Now the question arises. Is there any point of contact between this theory and the standard one? In the next we shall show that the reply is yes.

Let C be matrix satisfying $CMC \quad diag(M_1, M_2, M_3)$ where M_i are the eigenvalues of M. We can choose the eigenfunctions of H corresponding to negative helicity in the form

$$\langle p | \;{}_i \rangle \quad \frac{V_i}{\sqrt{2E_i}} \quad \frac{C\ V_i}{\sqrt{2E_i}}, \qquad (E_i \quad \sqrt{p^2 \quad M_i^2}),$$

where for $i \quad 1,2,3$

$$V_i \quad \begin{matrix} {}_{1i} & u_i \\ {}_{2i} & u_i \\ {}_{3i} & u_i \end{matrix}, \qquad u_i \quad \begin{matrix} \sqrt{E_i \quad M_i} & w \\ \sqrt{E_i \quad M_i} & w \end{matrix}$$

(they correspond to positive eigenvalues of H) and for $i \quad 4,5,6$ $(E_4 \quad E_1,\ E_5 \quad E_2,\ E_6 \quad E_3)$

$$V_i \quad \begin{matrix} {}_{4i} & u_i \\ {}_{5i} & u_i \\ {}_{6i} & u_i \end{matrix}, \qquad u_i \quad \begin{matrix} \sqrt{E_i \quad M_i} & w \\ \sqrt{E_i \quad M_i} & w \end{matrix}$$

(they correspond to negative eigenvalues of H). Because it holds that

$$\frac{U_i\, U_j}{2\sqrt{\;{}_i\;{}_j}} \quad \frac{V_i\, V_j}{2\sqrt{E_i E_j}} \quad {}_{ij}\,,$$

$$\sum_{i\ 1}^{6} \frac{U_i U_i}{2\ {}_i} \quad \sum_{i\ 1}^{6} \frac{V_i V_i}{2E_i} \quad diag(010101010101)\,.$$

Then we can write

$$\frac{U_i}{\sqrt{2\ {}_i}} \quad T_{ij} \frac{V_j}{\sqrt{2E_j}} \tag{10}$$

or

$$T_{ij} \quad \frac{V_j\, U_i}{2\sqrt{E_j\ {}_i}}\,. \tag{11}$$

Putting

$$T \quad \begin{matrix} T^{(1)} & T^{(2)} \\ T^{(3)} & T^{(4)} \end{matrix}$$

where $T^{(a)}$ are $3 \quad 3$ matrices, then from (11), we get

$$T_{ij}^{(1)} \quad C_{ji} \frac{1}{2\sqrt{E_j\ {}_i}} \left(\sqrt{(\ {}_i \quad m_i)(E_j \quad M_j)} \quad \sqrt{(\ {}_i \quad m_i)(E_j \quad M_j)}\right),$$

$$T_{ij}^{(2)} \quad C_{ji} \frac{1}{2\sqrt{E_j \quad _i}} (\sqrt{(\ _i \quad m_i)(E_j \quad M_j)} \quad \sqrt{(\ _i \quad m_i)(E_j \quad M_j)}),$$

(similar for $T^{(3)}$ and $T^{(4)}$). Thus, in the region $\vec{p}^{\,2} \quad m_i^2, M_i^2$ (for all $i \quad 1,2,3$) we can write

$$T^{(1)} \quad C^T \quad T^{(4)}, \qquad T^{(2)} \quad T^{(3)} \quad 0.$$

Hence, in the ultrarelativistic case the equation (10) can be approximated by

$$\frac{U_i}{\sqrt{2 \quad _i}} \quad (C^T)_{ij} \frac{V_i}{\sqrt{2E_i}}, \qquad (i, j \quad 1,2,3), \tag{12}$$

or in a more familiar form the last equation can be written as

$$|\ _i\rangle \quad (C^T)_{ij} |\ _j\rangle. \tag{13}$$

where $|\ _i\rangle$ are the states of flavor neutrinos and $|\ _i\rangle$ can be interpreted as so-called massive neutrinos states (eigenstates of the generator H of the time-development). We remark that for $\vec{p} \quad 0$ we get $|\ _i\rangle \quad (C^T)_{ij} |\ _j\rangle$

3. Concluding remarks

Within the framework of the presented considerations the neutrino oscillations probability formula can be derived by the standard way. Having the state $|\ _i\rangle_0$ (corresponding to the momentum $\vec{p}$) at time $t \quad 0$ then the time-development of this state is given by

$$|\ _t\rangle \quad e^{\ iHt} |\ _i\rangle_0 \quad \sum_{j\ 1}^{6} \sum_{k\ 1}^{6} T_{ij} e^{\ iE_j t} T_{jk} |\ _k\rangle_0 .$$

Hence, the amplitude A of the transition $\ _i \quad \ _k$ at time t is equal to

$$A(\ _i \quad \ _k; t) \quad \sum_{j\ 1}^{6} T_{ij} e^{\ iE_j t} T_{jk} .$$

In the region $\vec{p}^{\,2} \quad m_i^2$ (for all $i \quad 1,2,3$) the last formula can be approximated by

$$A(\ _i \quad \ _i; t) \quad \sum_{j\ 1}^{3} U_{ij} e^{\ \frac{iM_j}{2\ p} t} U_{jk} \quad e^{\ ipt}$$

where $U \quad C^T$ and C are given by the equation

$$CMC \qquad diag(M_1, M_2, M_3)$$

Our previous considerations generate several questions and we want to mention some of them at least. First of all, what does H describe actually? Does it describe the oscillations of $\ _e, \ , \ $ or a triplet of free Dirac particles with masses M_1, M_2, M_3? . If $M_i \quad 0$ then H evidently describes a triplet of free particles with masses M_i and oscillations in our approach are something artificial (The case when M_i is negative requires a more detailed study.).

Let us consider for a while that the transitions $\ _i \quad \ _j$ are caused by the interaction of $\ _i$ with some fields $\ _{ij}$ ($\ _{ij} \quad 0$) and the interaction term is $\sim (\bar{\ }_i \ _{ij} \ _j \quad h.c.)$. If we shall replace $\ _{ij}$ by some constants, say M_{ij}, then we obtain (semi)phenomenological theory outlined above. This possibility that neutrino oscillations are caused by the interaction of neutrinos with some field seems to be very tempting.

We are not sure if all these questions are meaningful but we feel that a detailed analysis of basic postulates of the theory in question is entitled and desirable.

References

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