

Neutrino Oscillations and Neutrinoless Double Beta Decay

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Abstract: The developments in neutrino oscillations and neutrinoless double-beta decay ($0\nu\beta\beta$ -decay) are reviewed focusing on what can be learned about the three light neutrinos in future $0\nu\beta\beta$ -decay experiments. Recent values of the mixing angles from atmospheric, solar, accelerator and reactor neutrino oscillation experiments are used to predict values of the effective Majorana mass of the electron neutrino. It is stressed that the study of the $0\nu\beta\beta$ is the most important source of information about the Majorana nature of neutrinos.

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1. Introduction

Neutrinos are one of the fundamental particles which make up the Universe. Tremendously large numbers of neutrinos are produced in various places in the Earth and in the Universe. They are made mostly naturally, by cosmic rays, by natural radioactive decay and by the thermonuclear reactions occurring inside the Sun and other stars as well as during the formation of the neutron stars. Nuclear power plants produce large numbers of neutrinos as by product of nuclear fissions in reactors. It is also possible to produce neutrinos using beams of high energy protons generated by large accelerators.

In number, neutrinos exceed the constituents of ordinary matter (electrons, protons, neutrons) by a factor of ten billion. They account for at least as much energy in the Universe as all the stars combined and, depending on their exact masses, might also account for a substantial fraction of the so-called ‘dark matter’.

Neutrinos are important in stellar processes. Neutrinos govern the dynamics of supernovae, and hence the production of heavy elements in the Universe. Furthermore, if there is CP violation in the neutrino sector, the physics of neutrinos in the early Universe might ultimately be responsible for baryogenesis. If we are to understand ‘why we are here’ and the basic nature of the Universe in which we live, we must understand the basic properties of the neutrino.

Neutrinos are one of the least understood particles. Neutrinos are similar to the more familiar electron, with one crucial difference: neutrinos do not carry electric charge. Since neutrinos are electrically neutral, they are not affected by the electromagnetic forces which act on electrons. Neutrinos are affected only by a ‘weak’ sub-atomic force of much

shorter range than electromagnetism, and are therefore able to pass through great distances in matter without being affected by it.

Three types of neutrinos are known. There is a strong evidence that no additional neutrinos exist, unless their properties are unexpectedly very different from the known types. Each type or ‘flavor’ of neutrino is related to a charged lepton. There are various open questions about neutrinos that need both theoretical and experimental exploration as the Majorana or the Dirac nature of neutrinos, their electromagnetic properties, or the possible existence of CP violation in the leptonic sector.

Neutrinos are very special particles. Studies of these particles have played a crucial role in the understanding of the laws of elementary particles and their interactions. Until recently, there was no evidence that neutrinos have masses, and therefore the Standard model in elementary particle physics assumed that neutrinos are massless. Small but non-zero masses of neutrinos have been first clearly discovered by studying neutrinos produced by cosmic ray interactions in the atmosphere. The small neutrino masses have profound implications for our understanding of particle physics and the Universe.

The recent observation of neutrino oscillations has now established beyond doubt, the non-zero masses of neutrinos, the flavor change and mixing of neutrinos. The existence of neutrino masses is in fact the first solid experimental fact requiring physics beyond the Standard Model. The observed small neutrino masses have profound implications for our understanding of particle physics and the Universe. It has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties.

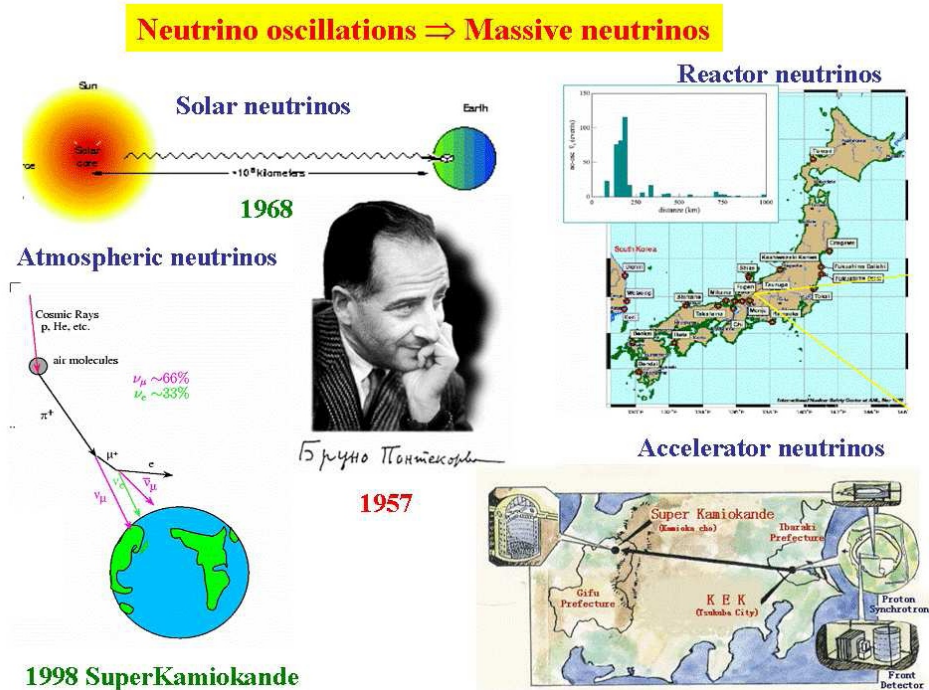


Fig. 1. The various experiments showing neutrino oscillations, as suggested by Bruno Pontecorvo.

As the most intriguing and fascinating fundamental particle, the neutrino is so important that neutrino physics has become one of the most significant branches of modern physics. In this paper the fields of neutrino oscillations and neutrinoless double beta decay are presented.

2. Neutrino oscillations

A. A brief history of neutrino oscillations

The first experimental observation of the neutrino (nuclear reactor neutrino) was made by Frederick Reines and Clyde Cowan in 1956 [1].

In 1957 the first idea of neutrino masses, mixing and oscillations was put forward by Bruno Pontecorvo [2]. He thought that there is an analogy between leptons and hadrons and believed that in the lepton world a phenomenon does exist, which is analogue to the well-known $K^0\bar{K}^0$ oscillations.

In 1957–58 Ray Davis was running an experiment searching for production of ^{37}Ar in the reaction $\bar{\nu}_e + {}^{37}\text{Cl} \rightarrow e + {}^{37}\text{Ar}$ (proposed by Pontecorvo) with antineutrinos from a reactor. A rumor reached Pontecorvo that this process had been observed by Davis. He assumed that production of ^{37}Ar might be due to antineutrino $\bar{\nu}_e$ to neutrino ν_e (only one neutrino species ν_e was known at that time) transitions in vacuum [3].

In 1962 there was an indication from the process of muon disintegration $\mu \rightarrow e + \bar{\nu}_e + \nu_\mu$ that ν_e and ν_μ are different particles. This inspired Shoichi Sakata, Ziro Maki and Masami Nakagawa to introduce two-neutrino mixing in the context of the Nagoya model [4]. This paper was practically unknown for many years. After the second muonic neutrino ν_μ was discovered in the Brookhaven experiment Pontecorvo generalized his idea of neutrino oscillations for the case of two flavor neutrinos [5].

In 1969 the first phenomenological theory of two neutrino mixing was proposed by Gribov and Pontecorvo [6]. They assumed that the left-handed fields ν_{eL} and $\nu_{\mu L}$ enter not only into the weak interaction but also into a mass term.

Electron neutrinos are produced in the thermonuclear reactions occurring inside the Sun. Each reaction is characterized by a specific energy spectrum of emitted neutrinos, which can be predicted by theory. The first experimental observation made with solar neutrinos and compatible with the hypothesis of neutrino oscillations originated from a discrepancy between the measured detection rate in the solar neutrino experiment by Davis [7]. This effect is commonly referred as the solar neutrino problem. Three years before the first results of Davis experiment were announced, Pontecorvo pointed out that due to neutrino oscillations the observed flux of solar neutrinos could be twice smaller than the expected flux. So far, five different experiments confirmed and consolidated the measurement of the solar neutrino deficit.

If neutrino oscillations are invoked to explain the solar neutrino problem, there are two possibilities: oscillations in vacuum, or oscillations in matter. In 1978 Wolfenstein found out [8] that matter can change the pattern of neutrino oscillations drastically. In particular, a resonance enhancement of oscillations and resonance flavor conversion become possible. In 1985 Mikheyev and Smirnov applied this to the solar neutrino problem [9].

In 2001 the Mikheyev-Smirnov-Wolfenstein (MSW) effect was confirmed in the Sudbury Neutrino Observatory (SNO) [10], where the solar neutrino problem was finally solved. It was shown there that only $\sim 34\%$ of the electron neutrinos (measured with one charged current reaction of the electron neutrinos) reach the detector, whereas the sum of rates for all three neutrinos (measured with one neutral current reaction) agrees well with the expectations.

The MSW effect is important for high energy solar neutrinos. For the low energy solar neutrinos, on the other hand, the matter effect is negligible and the vacuum oscillation should be considered. This is consistent with the experimental observations on such neutrinos by the Homestake Experiment of Davis, the first experiment to reveal the solar neutrino problem, followed by those of Gallex/GNO [11] and SAGE [12] (collectively, gallium experiments) that measured the lowest energy neutrinos and provided a strong support to Homestake. These results are supported by the results of the reactor antineutrino experiment KamLAND [13], which alone is able to provide also a measurement of the parameters of oscillation that is consistent with all other measurements.

Atmospheric neutrinos are produced by primary cosmic rays interacting with the atmosphere and creating a large number of secondary pions and K-mesons, mainly with energies of a few GeV. Due to the low density of the atmosphere essentially all the mesons decay before interacting via reaction $\pi^+ \rightarrow K^+ + \mu^+ + \nu_\mu$ ($\pi^- \rightarrow K^- + \mu^- + \bar{\nu}_\mu$). Most of the muons decay before reaching the ground: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ ($\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$). If all muons decayed one would expect $N(\nu_\mu)/N(\nu_e) = 2$ in the absence of physics beyond the Standard Model. This ratio is known with an error of $\sim 5\%$ and is a subject of experimental interest. The distance traveled by neutrinos before reaching the detector is related to the zenith angle. Neutrino oscillation could be therefore identified by detecting the zenith angle dependence. The Super-Kamiokande experiment has proven, with large statistical significance, this dependence making a strong claim for the observation of neutrino oscillation in 1998 [14]. The observed updown asymmetries of the detected muons showed that, with the three known neutrino flavors, the solution to this anomaly is $\nu_\mu \leftrightarrow \nu_\tau$ oscillations. In agreement with the result of the reactor experiment CHOOZ [15] significant ν_e oscillations have been excluded.

The idea of an accelerator neutrino beam was proposed by Pontecorvo a long time ago. In brief, a proton beam strikes a thick nuclear target, producing π secondaries, such as pions and kaons. For conventional neutrino beams, the neutrino spectra may be derived from the π/K meson spectra and the kinematics of meson decay in flight. The long baseline oscillation experiments K2K [16] and MINOS [17] using man-made neutrinos at accelerators has confirmed the atmospheric neutrino deficit as well as a distortion of the energy spectrum in agreement with the oscillation hypothesis.

B. Neutrino mixing and masses

The discovery of neutrino oscillations in the atmospheric Super-Kamiokande experiment [14], in the solar SNO experiment [10], in the reactor KamLAND experiment [13] in the accelerator K2K experiment [16] and other neutrino experiments [11, 12, 18, 19] is one of the most compelling evidence in favor of new physics beyond the Standard Model. All the existing neutrino oscillation data, with the exception of the data of the short base-

line accelerator experiment LSND [20] (the LSND result will be checked by the running MiniBooNE experiment [21] soon) are described by the three-neutrino mixing scheme

$$\nu_{lL}(x) = \sum_{i=1}^3 U_{li} \nu_{iL}(x); \quad l = e, \mu, \tau. \quad (1)$$

Here, $\nu_{iL}(x)$ is the field of the neutrino with mass m_i ($i = 1, 2, 3$) and $\nu_{lL}(x)$ is a flavor neutrino field which enters into the standard charged and neutral currents

$$\begin{aligned} j^{CC}(x) &= 2 \sum_l \bar{\nu}_{lL}(x) \gamma_\mu l_L(x), \\ j^{NC}(x) &= \sum_l \bar{\nu}_{lL}(x) \gamma_\mu \nu_{lL}(x), \end{aligned} \quad (2)$$

U is the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [2, 3, 22] mixing matrix. For massive Dirac neutrinos the PMNS matrix U^D in the standard parameterization has the form

$$U^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ s_{12}c_{23} & c_{12}c_{23} & s_{23}c_{13} \\ s_{12}s_{23} & c_{12}s_{23} & c_{23}c_{13} \end{pmatrix} e^{i\delta} \begin{pmatrix} c_{13} & s_{13} & 0 \\ -s_{13} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

Here $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$, θ_{ij} (i, j) is the neutrino mixing angle and δ is the CP violating phase.

It was obtained from the analysis of the Super-Kamiokande atmospheric neutrino data for the neutrino mass squared difference m_{23}^2 and the parameter $\sin^2 2\theta_{23}$ [14]:

$$\begin{aligned} \text{best fit:} \quad & m_{23}^2 = 2.1 \cdot 10^{-5} \text{ eV}^2, \quad \sin^2 2\theta_{23} = 0.92 \\ 90\% \text{ C.L.:} \quad & 1.5 \cdot 10^{-5} \text{ eV}^2 < m_{23}^2 < 10^{-4} \text{ eV}^2, \quad \sin^2 2\theta_{23} > 0.92 \end{aligned} \quad (4)$$

The global analysis of the data of the solar neutrino experiments and KamLAND experiment yields the following best fit values and 90 % C.L. ranges of the relevant neutrino oscillation parameters [13]:

$$\begin{aligned} \text{best fit:} \quad & m_{12}^2 = 7.9 \cdot 10^{-5} \text{ eV}^2, \quad \tan^2 \theta_{12} = 0.26 \\ 90\% \text{ C.L.:} \quad & 7.4 \cdot 10^{-5} \text{ eV}^2 < m_{12}^2 < 10^{-4} \text{ eV}^2, \quad 0.33 < \tan^2 \theta_{12} < 0.50 \end{aligned} \quad (5)$$

Notice that neutrino mass-squared difference is determined as $m_{ik}^2 = m_k^2 - m_i^2$. For the angle θ_{13} only the upper bound is known. From the exclusion plot obtained from the data of the reactor experiment CHOOZ [15, 23], we have

$$\sin^2 \theta_{13} < 5 \cdot 10^{-2} \text{ (90\% C.L.)}. \quad (6)$$

The CP-violating phase δ remains undetermined. A recent global analysis of the oscillation data leads to the following bound: $\sin^2 \theta_{13} < 0.9 \cdot 10^{-2}$ (95% C.L.) [24].

At present the structure of the neutrino mass spectrum is not known as well. Two types of spectra are possible:

1. Normal spectrum:

$$m_1 < m_2 < m_3; \quad m_{12}^2 < m_{23}^2. \quad (7)$$

2. Inverted spectrum:

$$m_3 > m_1 > m_2; \quad m_{12}^2 > |m_{13}^2| \quad (8)$$

We note that it is common to label neutrino masses differently in the case of the normal and the inverted spectra. For both spectra we have $m_2 > m_1$. But in the case of the normal spectrum m_3 is the mass of the heaviest neutrino and in the case of the inverted hierarchy m_3 is the mass of the lightest neutrino. This convention allows to keep the same notation of the mixing angles for both spectra. Existing oscillation data are compatible both with normal and the inverted spectra.

The lightest neutrino mass $m_0 = m_1 (m_3)$, which determines the absolute values of neutrino masses, is currently also unknown. It was found from an analysis of the data of the Mainz [25] and Troitsk [26] tritium experiments

$$m_0 < 2.3 \text{ eV}. \quad (9)$$

A more stringent bound on the sum of neutrino masses can be found from the measurement of the matter power spectrum $P(k)$. Depending on the data which were taken into account, the cosmological upper bound on the sum of neutrino masses was obtained as (see [27, 28] and references therein)

$$\sum_i m_i < (0.5 - 1.7) \text{ eV}. \quad (10)$$

An important evidence that masses and mixing of neutrinos are of a nature beyond the Standard model (SM) would be that massive neutrinos are Majorana particles. If ν_i are Majorana particles

1. Neutrino fields $\nu_i(x)$ satisfy the Majorana conditions

$$\nu_i^c(x) = \nu_i(x), \quad (11)$$

where $\nu_i^c(x) = C^{-1} \nu_i(x)$ is the conjugated field (C is the charge conjugation matrix).

2. The neutrino mixing matrix has the form [29]

$$U = U^D S(\theta) \quad (12)$$

where $S(\theta)$ is the diagonal phase matrix. In the case of three neutrino mixing the matrix $S(\theta)$ is characterized by two Majorana CP-violating phases. The matrix $S(\theta)$ can be presented in the form

$$S_{ik} = e^{i\theta_{ik}}; \quad \theta_{33} = 0. \quad (13)$$

The unitary matrix U^D , which is characterized by the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ , was already introduced in Eq. (3).

If in the lepton sector CP invariance holds, for the Majorana mixing matrix, we have [30]

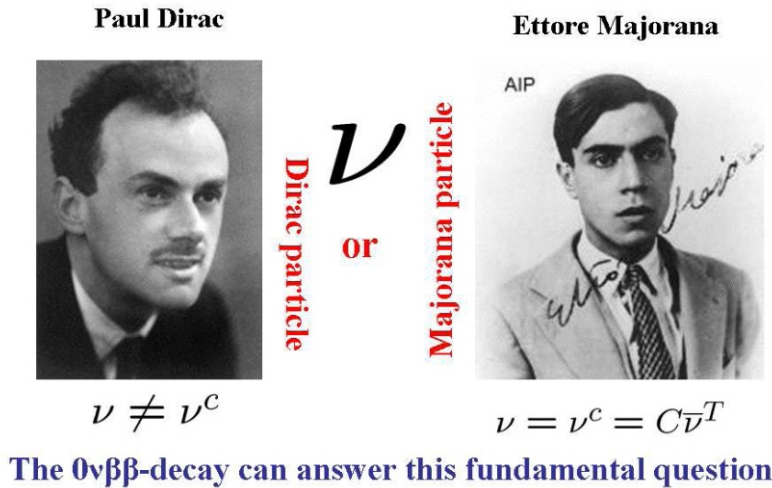
$$U_{li} = U_{li}^* \nu_i, \quad (14)$$

where $\nu_i = \nu_i$ is the CP parity of the Majorana neutrino ν_i . The condition (14) can be presented in the form

$$U_{li}^2 = |U_{li}|^2 e^{i(\pi/2)\nu_i}, \quad (15)$$

where $\nu_i = \pm 1$.

What is the nature of neutrinos?



The $0\nu\beta\beta$ -decay can answer this fundamental question

Fig. 2. Is the neutrino a Dirac or a Majorana particle?

Investigations of neutrino oscillations in vacuum and in matter do not allow to distinguish massive Dirac from massive Majorana neutrinos [29, 31, 32]. In order to reveal the Majorana nature of ν , it is necessary to study processes in which the total lepton number is violated. Because the standard electroweak interaction conserves helicity the probabilities of such processes are proportional to the squares of the neutrino masses, and, consequently, they are strongly suppressed. The best sensitivity on small Majorana neutrino masses can be reached in the investigation of neutrinoless double β -decay ($0\nu\beta\beta$) of some even-even nuclei.

3. Neutrinoless double beta decay

After the discoveries of oscillations of atmospheric, solar and reactor neutrinos the physics community worldwide is embarking on the next challenging problem, finding whether neutrinos are indeed Majorana particles (i.e. identical to its own antiparticle), as many particle models suggest, or Dirac particles (i.e. is different from its antiparticle).

All known fermions (e.g. electron, proton) have a distinct anti-particle (positron, antiproton). Anti-particle is a particle with the same mass and spin but opposite electric charge. An exception might be neutrino, which is a neutral particle. We note that Majorana neutrinos fit very naturally in modern particle physics scenarios. The total lepton number violating neutrinoless double beta decay ($0\nu\beta\beta$ -decay),

$$(A, Z) \rightarrow (A, Z - 2) + 2e^-, \tag{16}$$

is the most powerful tool to clarify if the neutrino is a Dirac or a Majorana particle. Since the $0\nu\beta\beta$ -decay gives practically the only possibility of distinguishing between Majorana

and Dirac neutrinos much effort has been devoted to the problem of $0\nu\beta\beta$ -decay (for reviews see refs. [33]).

There are different possible modes of the double beta decay, which differ from each other by the light particles accompanying the emission of two electrons. We distinguish the $0\nu\beta\beta$ -decay and the two-neutrino double beta decay ($2\nu\beta\beta$ -decay) modes with and without lepton number violation, respectively. The $2\nu\beta\beta$ -decay involves the emission of two electrons and two antineutrinos:

$$(A, Z) \rightarrow (A, Z - 2) 2e^- 2\bar{\nu}_e. \quad (17)$$

This process does not violate the lepton number, it is fully consistent with the standard model (SM) of electroweak interaction. The $2\nu\beta\beta$ -decay has been observed so far in ten nuclides (^{48}Ca , ^{76}Ge , ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{128}Te , ^{130}Te , ^{150}Nd and ^{238}U) and in two excited states [34].

Double beta decay is a very rare process both in the case of the two-neutrino and of the neutrinoless mode.

A. A brief history of double beta decay

The early period

Double beta decay was first considered in publication [35] of Maria Goeppert-Mayer in 1935. It was Eugene Wigner, who suggested this problem to the author about one year after the Fermi weak interaction theory appeared. In the work of Maria Goeppert-Mayer [35] an expression for the $2\nu\beta\beta$ -decay rate was derived and a half-life of 10^{17} years was estimated by assuming a Q-value of about 10 MeV.

Two years later (1937) Ettore Majorana formulated theory of Majorana neutrinos (neutrino and antineutrino $\bar{\nu}$ are indistinguishable) and suggested antineutrino induced $0\nu\beta\beta$ -decay for experimental verification of this hypothesis [36]. Giulio Racah was the first, who proposed testing Majorana's theory with the $0\nu\beta\beta$ -decay for processes with real neutrinos [37]. In 1939, Wolfgang Furry discussed a double beta decay without emission of neutrino ($0\nu\beta\beta$ -decay with virtual neutrino) [38]. In 1952 Henry Primakoff [39] calculated the electron-electron angular correlations and electron energy spectra for both the $2\nu\beta\beta$ -decay and the $0\nu\beta\beta$ -decay, producing a useful tool for distinguishing between the two processes.

At that time nothing was known about the chirality suppression of the $0\nu\beta\beta$ -decay. It was believed that due to a considerable phase-space advantage the $0\nu\beta\beta$ -decay mode dominates the double beta decay rate. Starting from 1950 this phenomenon was exploited in early geochemical, radiochemical and counter experiments. It was found that the measured lower limit on the $0\nu\beta\beta$ -decay half-life exceeds far the values expected for this process, $T_{1/2} \sim 10^{12} - 10^{15}$ years. In 1955 the Raymond Davis experiment [40], which searched for the antineutrinos from reactor via nuclear reaction $\bar{\nu}_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$, produced a zero result. The above experiments were interpreted as proof that the neutrino was not a Majorana particle, but a Dirac particle. This prompted the introduction of the lepton number to distinguish the neutrino from its antiparticle. The assumption of lepton number conservation allows the $2\nu\beta\beta$ -decay but forbids the $0\nu\beta\beta$ -decay, in which lepton number is changed by two units.

In 1949 Fireman reported observing the β -decay of ^{124}Sn in a laboratory experiment [41], but disclaimed it later [42]. The first geochemical observation of the β -decay, with an estimated half-life $T_{1/2}(^{130}\text{Te}) = 1.4 \times 10^{21}$ years, was announced by Ingram and Reynolds in 1950 [43].

The period of scepticism

Shortly after Lee and Yang formulated the parity violation in the weak interaction, it has been established by two epochal experiments. In 1957 Wu et al. discovered the asymmetry in the angular distribution of the β -particles emitted relative to the spin orientation of the parent nucleus ^{60}Co . A year later Goldhaber et al. found the complete polarisation of neutrinos by measuring the photon spin direction determined by the deexcitation of a $^{152}\text{Eu}^*$ nucleus after K-capture. In 1958 seemingly confused situation was simplified in the form of the vector-axial vector (V-A) theory of weak interactions describing maximal parity violation in agreement with available data. In order to account for the chiral symmetry breaking of the weak interaction only left-handed fermions participate and the mediating particles must be vectors of spin 1 and left-handed, as well.

The maximal parity violation is easily realized in the lepton sector by using the two-component theory of a massless neutrino, proposed in 1957 by L. Landau, T. D. Lee, C. N. Yang and A. Salam (This idea was first developed by H. Weyl in 1929, but it was rejected by Pauli in 1933 on the grounds that it violates parity.) In this theory, neutrinos are left-handed and antineutrinos are right-handed, leading automatically to the V-A couplings.

With the discovery of parity violation, it became apparent that the Majorana/Dirac character of the electron neutrino was still in question. The particles that participate in the $0 \rightarrow 0$ -decay reaction at nucleon level are right-handed antineutrino $\bar{\nu}_e$ and left-handed neutrino ν_e :

$$\begin{matrix}
 n & p & e & \bar{\nu}_e^{-RH} \\
 \nu_e^{LH} & n & p & e
 \end{matrix}
 \tag{18}$$

Thus, even if the neutrino is a (massless) Majorana particle, the absence of the $0 \rightarrow 0$ -decay, as the first neutrino has the wrong helicity for absorption on a neutron, implies neither a Dirac electron neutrino nor a conserved lepton number.

The requirement that both lepton number conservation and the G_{51} invariance of the weak current had to be violated in order the $0 \rightarrow 0$ -decay to occur discouraged experimental searches.

The period of GUT

The maximal violation of parity (and of charge conjugation) symmetry is accommodated in the standard model (SM), which describes jointly weak and electromagnetic interactions. This model was developed largely upon the empirical observations of nuclear beta decay during the latter half of the past century. Despite the phenomenological success of the SM, the fundamental origin of parity violation has been found unknown. In spite of the fact that the SM represents the simplest and the most economical theory, it has been not considered as the ultimate theory of nature. It was assumed likely to describe the effective interaction at low energy of an underlying more fundamental theory.

With the development of modern gauge theories at the beginning of the seventies of the previous century, perceptions began to change. In the SM it became apparent that the assumption of lepton number conservation led to the neutrino being strictly massless, thus preserving the $U(1)_L$ -invariance of the weak current. With the development of Grand Unified Theories (GUTs) of the electroweak and strong interactions, the prejudice has grown that lepton number conservation is not the result of an exact global symmetry. The modern GUTs and supersymmetric (SUSY) extensions of the SM suppose that the conservation laws of the SM may be violated to some small degree. The lepton number may only appear to be conserved at low energies because of the large grand unified mass scale M_{GUT} governing its breaking. Within the proposed see-saw mechanism one expects the neutrino to acquire a small Majorana mass of a size $\sim (light\ mass)^2/M_{GUT}$, where “light mass” is typically that of a quark or charged lepton. The considerations of a sensitivity of the $0\nu\beta\beta$ -decay experiments to a neutrino mass $m_\nu \sim 1$ eV became the genesis of a new interest to the double beta decay.

Neutrino masses either require the existence of righthanded neutrinos or require violation of the lepton number (LN) so that Majorana masses are possible. So, one is forced to go beyond the minimal models again, whereby LF and/or LN violation can be allowed in the theory. A good candidate for such a theory is the left-right symmetric model of Grand Unification (GUT) inaugurated by Salam, Pati, Mohapatra and Senjanović [44] (especially models based on $SO(10)$ which have first been proposed by Fritzsch and Minkowski [45]) and its supersymmetric version [46]. The left-right symmetric models, representing generalization of the $SU(2)_L \times U(1)_Y$ SM, predict not only that the neutrino is a Majorana particle, that means it is up to a phase identical with its antiparticle, but automatically predict the neutrino has a mass and a weak right-handed interaction.

In the left-right symmetric models the LN conservation is broken by the presence of the Majorana neutrino mass. The LN violation is also inbuilt in those SUSY theories where R-parity, defined as $R_p = (-1)^{3B-L+2S}$ (S, B, and L are the spin, baryon and lepton number, respectively) is not a conserved quantity anymore.

The $0\nu\beta\beta$ -decay which involves the emission of two electrons and no neutrinos, has been found as a powerful tool to study the LN conservation. Schechter and Valle proved that the $0\nu\beta\beta$ -decay takes place only if the neutrino is a Majorana particle with non-zero mass [47]. It was recognized that the GUT's and R-parity violating SUSY models offer a plethora of the $0\nu\beta\beta$ -decay mechanisms triggered by exchange of neutrinos, neutralinos, gluinos, leptoquarks etc. [48].

Another approach in handling the $0\nu\beta\beta$ -decay problem based on consideration of particles other than the nucleons present in the nuclear soup was proposed as a remark by the genius of Pontecorvo. He introduced the double beta decay of pions in flight between nucleons. This idea was revived by the Bratislava-Dubna-Tuebingen groups in the context of R-parity violating interactions [49], i.e. scalar, pseudoscalar and tensor currents arising out of neutralino and gluino exchange. It was found that the pion-exchange mechanism clearly dominates over the conventional two-nucleon mechanism for this type of interactions.

The experimental effort concentrated on high Q isotopes, in particular on ^{48}Ca , ^{76}Ge , ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe and ^{150}Nd . In 1987 the first actual laboratory observation of the $0\nu\beta\beta$ -decay was done for ^{82}Se by M. Moe and collaborators, who used a time projection chamber. Within the next few years, experiments employing counters were

able to detect 2ν -decay in many nuclei. In addition, the experiments searching for the signal the 2ν -decay pushed by many orders of magnitudes the experimental lower limits for the 2ν -decay half-life of different nuclei.

The period of massive neutrinos - the current period

Starting 1998 we have a convincing evidence about neutrino masses due to SuperKamiokande and SNO experiments. Contrary to the implications of some popular press reports, most physicists have been expecting such results for several years. Non-zero neutrino mass can be accommodated by fairly straightforward extensions of the “standard model” of particle physics. Earlier measurements of neutrinos produced in the Sun, in the atmosphere, and by accelerators suggested that neutrinos might oscillate from one “flavor” (electron-, muon-, and tau-) to another expected consequence of non-zero mass. Neutrino mass gives additional data in constructing the Grand Unified Theory (GUT) of physics. It also provides additional data for cosmologists and established perspectives for observation of the 2ν -decay.

So far the 2ν -decay has been recorded for ten nuclei (^{48}Ca , ^{76}Ge , ^{82}Se , ^{96}Zr , ^{100}Mo , ^{116}Cd , ^{128}Te , ^{130}Te , ^{150}Nd , ^{238}U). In addition, the 2ν -decay of ^{100}Mo and ^{150}Nd to 0^+ excited state of the daughter nucleus has been observed and the ECEC process in ^{130}Ba was recorded. Experiments studying the 2ν -decay are presently approaching a qualitatively new level, where high-precision measurements are performed not only for half-lives but also for all other parameters of the process. As a result, a trend is emerging toward thoroughly investigating all aspects of 2ν -decay, and this will furnish a very important information about the values of nuclear matrix elements, the parameters of various theoretical models, and so on. In this connection, one may expect advances in the calculation of nuclear matrix elements and in the understanding of the nuclear-physics aspects of double beta decay.

Neutrinoless double beta decay has not yet been confirmed. Some authors of the Heidelberg-Moscow (HM) collaboration have claimed the experimental observation of the 2ν -decay of ^{76}Ge with half-lifetime $T_{1/2}^0 = (0.8 - 18.3) \cdot 10^{25}$ years (best-fit value of $1.5 \cdot 10^{25}$ years) [48, 50]. This work has attracted a lot of attention of both experimentalists and theoreticians due to important consequences for particle physics and astrophysics [51]. Several researchers of the 2ν -decay community re-examined and criticized the paper, suggesting a definitely weaker statistical significance of the peak [52–54]. It is also worth mentioning that the Moscow participants of the HM collaboration performed a separate analysis of the data and presented the results at NANP 2003 (Dubna, June 24, 2003) [55]. They found no indication in favor of the evidence of the 2ν -decay. In any case the disproof or the confirmation of the claim will come from future experiments. A good candidate for a cross-check of the claimed evidence of the 2ν -decay of ^{76}Ge is the Cuoricino/CUORE experiment [56] in which 2ν -decay of ^{130}Te is investigated.

There is a hope that the period of Majorana neutrinos is not far. This period should start by a direct and undoubtable observation of the 2ν -decay. It would establish that neutrinos are Majorana particles, and a measurement of the decay rate, when combined with neutrino oscillation data and a reliable calculation of nuclear matrix elements, would yield insight into all three neutrino mass eigenstates.

B. Effective neutrino mass and nuclear matrix elements

The $\beta\beta$ -decay can be observed because the pairing force renders the even-even nuclei with even number of protons and neutrons more stable than the odd-odd nuclei with broken pairs. Thus, the single beta decay transition from the even-even parent nucleus (A, Z) to the neighbouring odd-odd nucleus $(A, Z + 1)$ is forbidden energetically or at least strongly suppressed due to a large change of spin (e.g. ^{48}Ca nucleus) and the $\beta\beta$ -decay to the daughter nucleus $(A, Z + 2)$ is the only possible decay channel. There are few tens of nuclear systems [34], which offer an opportunity to study it.

The inverse half-life of $2\nu\beta\beta$ -decay is free of unknown parameters on the particle physics side and is expressed as a product of a phase-space factor $G_2(Q, Z)$ and squared double Gamow-Teller nuclear matrix element $M_{GT}^2(A, Z)$,

$$(T_{1/2}^0)^{-1} = G_2(Q, Z) |M_{GT}^2|^2.$$

The measured $2\nu\beta\beta$ -decay half-lives give us directly the value of the corresponding nuclear matrix elements, i.e, the $2\nu\beta\beta$ -decay offers a severe test of nuclear structure calculations.

By assuming the dominance of the light neutrino mass mechanism the inverse value of the $2\nu\beta\beta$ -decay half-life for a given isotope (A, Z) is given by

$$(T_{1/2}^0)^{-1} = G_0(Q, Z) |M_0|^2 \left| \sum m_i \right|^2.$$

Here, $G_0(Q, Z)$ and M_0 are, respectively, the known phase-space factor and the nuclear matrix element, which depends on the nuclear structure of the particular isotope under study. The main aim of the experiments on the search for $2\nu\beta\beta$ -decay is the measurement of the effective Majorana neutrino mass $m_{\beta\beta}$.

Under the assumption of the mixing of three massive Majorana neutrinos the effective Majorana neutrino mass $m_{\beta\beta}$ takes the form

$$m_{\beta\beta} = U_{e1}^2 m_1 + U_{e2}^2 m_2 + U_{e3}^2 m_3. \quad (19)$$

Here, U_{ei} and m_i ($i = 1, 2, 3$) are elements of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix and masses of neutrinos, respectively. The predictions for $m_{\beta\beta}$ can be obtained by using the present data on the oscillation parameters. Its value depends strongly on the type of neutrino mass spectrum, minimal neutrino mass and Majorane CP-violating phases [57].

From the measurement of the half-life of the $2\nu\beta\beta$ -decay only the product $|m_{\beta\beta}| |M_0|^2(A, Z)$ can be determined. Thus, it is not possible to reach qualitative conclusions about neutrino masses and the type of neutrino mass spectrum without accurate calculation of nuclear matrix elements [57]. The calculation of the $2\nu\beta\beta$ -decay matrix elements is a difficult problem because the ground and many excited states of open-shell nuclei with complicated nuclear structure have to be considered.

Two basic methods are used in the evaluation of $2\nu\beta\beta$ -decay nuclear matrix elements, the quasiparticle random phase approximation (QRPA) [58] with its various modifications and the nuclear shell model (NSM) [59]. Both methods have the same starting point, namely a Slater determinant of independent particles. However, there are substantial dif-

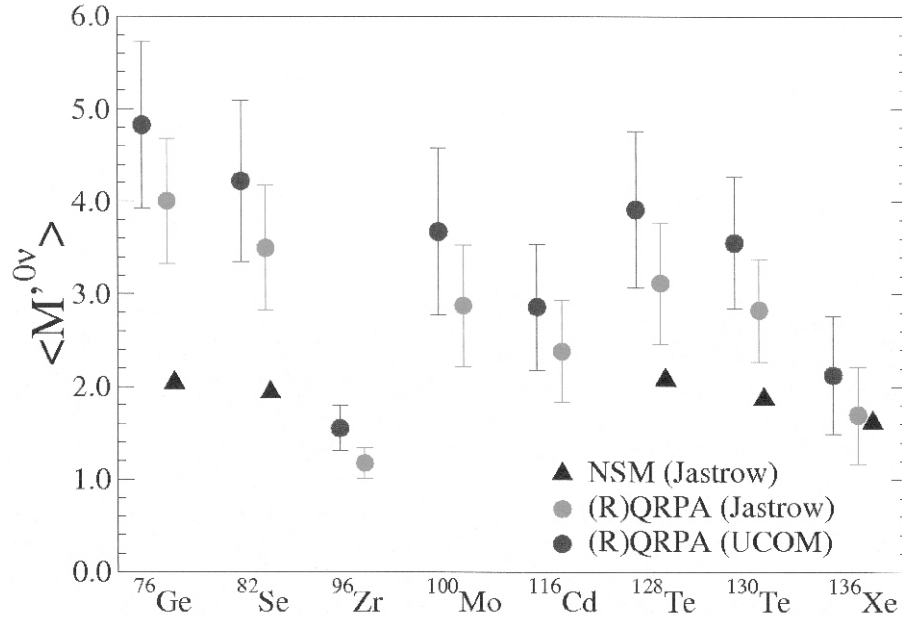


Fig. 3. The $0\nu\beta\beta$ -decay nuclear matrix elements $M^0(A, Z)$ (and their variance) obtained within the (R)QRPA [60] and the Large Scale Nuclear Shell Model [61]. Two alternative treatments of the short range correlations are considered, namely Jastrow-type and UCOM-type.

ferences between both approaches, namely the kind of correlations they include are complementary. The QRPA treats a large single particle model space, but truncates heavily the included configurations. NSM, by a contrast, treats a small fraction of this model space, but allows the nucleons to correlate in arbitrary ways.

The overwhelming majority of the published NME's were obtained within the QRPA. However, various implementations of the QRPA introduced by different authors have produced a spread of results with a factor of three or as much as five. In Ref. [58] a list of main reasons leading to a spread of the previous QRPA NME's was presented. It was shown that in most, albeit not all, cases the differences among them can be understood.

Given the interest in the subject the $0\nu\beta\beta$ -decay NME's of most nuclei of experimental interest are presented in Fig. 3. They were evaluated using the QRPA and the RQRPA within the procedure of fixing g_{pp} to known $M_{GT}^2(A, Z)$ [58, 60]. Their variance includes the error coming from the experimental (statistical and systematic) uncertainty in $M^2(A, Z)$ and uncertainty from theory itself. For ${}^{136}\text{Xe}$ the error bars encompass the whole interval related to the unknown rate of the $2\nu\beta\beta$ decay. These results provide hope that with a consistent treatment 20–30% uncertainties are possible.

After some break of about 10 years the Nuclear Shell Model (NSM) Strasbourg-Madrid group presented new results for the $0\nu\beta\beta$ -decay NME's [59], which are based on good spectroscopy for parent, intermediate and daughter nuclei. The NSM code can deal with problems involving basis of 10^{10} Slater determinants, using relatively modest computational resources. The comparison of recent NME's of [60] based on the QRPA and the

RQRPA with those of the available most recent NSM results [61] is presented in Fig. 3. The NSM values are practically the same for all isotopes unlike the (R)QRPA results. One should keep in mind, however, the discrepancy between the (R)QRPA and NSM approaches as well as systematic effects that might elude either or both calculations.

It is clear that further progress in the calculation of the $0^+ \rightarrow 0^+$ -decay NME's is needed. In particular, the problem of the two-nucleon short-range correlations should be better understood. An open issue is what is the effect of deformation on the $0^+ \rightarrow 0^+$ -decay NME's [62].

The problem of the uncertainty in the $0^+ \rightarrow 0^+$ -decay NME might be better understood by observation of the $0^+ \rightarrow 0^+$ -decay of at least three different nuclei. The ratios of corresponding $0^+ \rightarrow 0^+$ -decay half-lives can be a model independent test of the theoretically calculated NME's.

C. Perspectives of the $0^+ \rightarrow 0^+$ -decay experiments

Experiments on the double beta decay can be divided into two categories:

1. Geochemical determinations of total $0^+ \rightarrow 0^+$ decay half lives by measuring the abundance of a daughter isotope in ore sample containing the parent isotope.
2. Direct detection of electron pairs known to have been emitted at the same time and/or from the same point in the sample of parent isotope. Kinematic data on the electrons provides information on the mechanisms ($0^+, 2^+, \dots$) governing $0^+ \rightarrow 0^+$ decay.

The geochemical method of measuring $0^+ \rightarrow 0^+$ decay depends on the detection of the radiogenic daughter that has accumulated in geologically old minerals of the parent nuclide. However, the geochemical method does not determine the decay mode; it indicates only the total decay probability, $\lambda_{0^+ \rightarrow 0^+} = \lambda_0$. The positive result was obtained by this method with ^{130}Ba for 2^+ EC/EC decay on the level $T_{1/2}^2 = (2.2 \pm 0.5) \times 10^{21}$ y [63].

We can differentiate between two classes of direct $0^+ \rightarrow 0^+$ decay experiments :

- active source experiments (source = detector),
- passive source experiments (measuring the energy of the electrons or tracking + energy detectors).

In the first class of experiments the $0^+ \rightarrow 0^+$ process is usually identified only on the basis of the distribution of the total energy of the electrons. The second class of experiments yields a more complete information on the $0^+ \rightarrow 0^+$ events by measuring time coincidence, tracks and vertices of the electrons and their energy distribution. The extremely low expected counting rates corresponding to the half lives $T_{1/2} > 10^{25}$ years require extreme low-background conditions (measurements in underground laboratories, purification of the source and materials of the detector), the necessity to use enriched samples, the possibility to extend the detector size to ~ 100 kg scale, good energy resolution, long term stability of the detector, usage of samples made of isotopes with large Q (faster $0^+ \rightarrow 0^+$ rate, signal above many potential backgrounds), ability to construct every event (tracks of particles, timing information) to eliminate the backgrounds and reliable calculation of NMEs for selected isotope. The half-life limit $T_{1/2}$ [y] to be extracted from the background fluctuation if no peak is present after measuring time t [y], is:

$$T^{1/2} = (4.18 \cdot 10^{24} \text{ kg}^{-1}) \frac{a}{f} \sqrt{\frac{Mt}{B \cdot E}},$$

where a is the isotopical abundance of ^{76}Ge , M is the active mass of the detector [kg], B is the average background at energy of the peak [counts/keV y kg], E is the energy resolution (FWHM) [keV]. The factor f connects the limit to a confidence level (CL); $f = 3.62$ (1.35) for 90 (68 %) CL, if the minimum detectable count rate is estimated. It could be emphasized that the enrichment factor a is the only parameter not connected through the square root to the sensitivity.

Due to their peculiar interdisciplinarity, experiments on double beta decay involve different fields of science from nuclear, subnuclear and astroparticle physics to radioactivity, material sciences, geochronology etc. Experimental effort on detection of the $0\nu\beta\beta$ -decay has a strong impact, in particular, on development of purification and different special detection techniques as well as on progress in enrichment techniques.

It is worth to mention some of the most important experiments for $0\nu\beta\beta$ decay using different experimental techniques. GERDA collaboration is devoted to the measurement with enriched ^{76}Ge using novel technique of cooling the bare Ge detectors in liquid argon. Advantage of such approach is the best experimental resolution (~ 3 keV at the decay energy of 2039 keV). The detectors are made of germanium enriched to 86 % in ^{76}Ge . Another experiment CUORICINE (and its future extension CUORE) uses individual bolometers made of $5 \times 5 \times 5$ cm³ single crystals of TeO₂. They are made from natural abundance Te (33.8 %) and operating at very low temperatures (8–10 mK). Energy deposition of a few keV results in a measurable temperature increase T which is measured by high resistance germanium thermistors. The CUORE detector will be an array of 988 bolometers (~ 200 kg of ^{130}Te) [72]. Various $0\nu\beta\beta$ decay modes of Cd, Zn and Te isotopes are explored with the help of CdTeZn semiconductor detectors by collaboration COBRA [71]. The most important nucleus for COBRA experiment is ^{116}Cd as it has the highest Q value. The proposal is to operate 64 000 1 cm³ detectors with a total mass of 418 kg.

All these experiments are so called active source experiments. On the contrary, the collaboration NEMO constructed tracking detector NEMO3 [73]. The detector has 20 segments with about of 10 kg of different source materials (e.g. ^{100}Mo 6 914 g, ^{82}Se 932 g). It provides information about the tracks of charged particles (6 180 open cells operating in Geiger mode) and energy of all the particles (1 940 plastic scintillator detectors). Based on the reached results the collaboration SuperNEMO was formed and now it is in R&D phase. The SuperNEMO detector will be again a tracking modular detector with ~ 100 kg of enriched material (^{150}Nd or ^{82}Se). The construction of the first SuperNEMO module will start in 2009 and be completed in 2010.

The $0\nu\beta\beta$ -decay is a process known for almost 70 years, which has been searched for, but not seen yet. The most stringent lower bound on the half-life of the $0\nu\beta\beta$ -decay was measured for ^{76}Ge in the Heidelberg-Moscow experiment [74]:

$$T_{1/2}^0(^{76}\text{Ge}) > 1.9 \cdot 10^{25} \text{ years}. \quad (20)$$

Table 1. Sensitivities of future $\beta\beta$ -decay experiments to the effective Majorana neutrino mass $|m|$ calculated with the RQRPA nuclear matrix elements $M^0(A, Z)$ of Ref. [58]. For the axial coupling constant g_A the value $g_A = 1.25$ was assumed. $T_{1/2}^{0 \text{ exp}}$ is the maximal half-life, which can be reached in the experiment and $|m|$ is the corresponding upper limit of the effective Majorana neutrino mass.

Nucleus	Experiment	Source	$T_{1/2}^{0 \text{ exp}}$ [yr]	Ref.	$M^0(A, Z)$	$ m $ [eV]
^{76}Ge	GERDA(I)	15 kg of $^{\text{enr}}\text{Ge}$	$3 \cdot 10^{25}$	[64]	3.92	0.270
	GERDA(II)	100 kg of $^{\text{enr}}\text{Ge}$	$2 \cdot 10^{26}$	[64]	3.92	0.100
	Majorana	0.5 t of $^{\text{enr}}\text{Ge}$	$4 \cdot 10^{27}$	[65]	3.92	0.023
^{82}Se	SuperNEMO	100 kg of $^{\text{enr}}\text{Se}$	$2 \cdot 10^{26}$	[66]	3.49	0.055
^{100}Mo	MOON	3.4 t of $^{\text{nat}}\text{Mo}$	$1 \cdot 10^{27}$	[67]	2.78	0.024
^{116}Cd	CAMEO	1 t of CdWO_4 crystals	10^{26}	[67]	2.42	0.085
^{116}Cd	COBRA	420 kg of CdZnTe crystals	$3 \cdot 10^{26}$	[68]	2.42	
^{130}Te	CUORE	750 kg of TeO_2	10^{27}	[69]	2.95	0.023
^{136}Xe	XMASS	10 t of liq. Xe	$3 \cdot 10^{26}$	[67]	1.97	0.062
					1.67	0.073
	EXO	1 t $^{\text{enr}}\text{Xe}$	$2 \cdot 10^{27}$	[70]	1.97	0.024
					1.67	0.028

In the near future this limit is expected to be improved in GERDA experiment [64] by 1–2 orders of magnitude. For the $\beta\beta$ -decay of ^{100}Mo and ^{130}Te in the two running experiments, NEMO3 [75] and CUORICINO [56], the following sensitivities have been achieved:

$$\begin{aligned} T_{1/2}^0(^{100}\text{Mo}) & \approx 5.8 \cdot 10^{23} \text{ years} \\ T_{1/2}^0(^{130}\text{Te}) & \approx 3.0 \cdot 10^{24} \text{ years}. \end{aligned} \quad (21)$$

An extraordinary sensitivity of double beta decay-experiments makes them also a unique laboratory tool to probe physics beyond the standard model (SM) underlying the possible lepton number violation (LNV).

The main aim of the experiments on the search for $\beta\beta$ -decay is the measurement of the effective Majorana neutrino mass m . Using recently calculated nuclear matrix elements with significantly reduced theoretical uncertainties [58] from these data the following upper bounds for the effective Majorana mass can be inferred

$$\begin{aligned} |m| & \leq 0.34 \text{ eV} \quad (\text{Heidelberg–Moscow}), \\ |m| & \leq 0.55 \text{ eV} \quad (\text{CUORICINO}). \end{aligned} \quad (22)$$

Several future experiments on the search for $\beta\beta$ -decay will be sensitive to values of the effective Majorana mass in the range, which corresponds to the inverted mass hierarchy (see Table 1). There are many ambitious projects in preparation, in particular, CAMEO, CUORE, COBRA, EXO, GEM, GERDA, MAJORANA, MOON, XMASS. In the $\beta\beta$ -decay detectors a few tons of the the next generation $\beta\beta$ -decay material will be used.

This is a very big improvement as the current experiments use only a few tens of kilograms for the source. The future double beta decay experiments stand to uncover the fundamental nature of neutrinos Dirac or Majorana, probe the mass pattern, and perhaps determine the absolute neutrino mass scale and look for possible CP violation.

The current claim and the sensitivities of the upcoming experiments are indicated in the compilation displayed in Figs. 4 and 5. It shows the estimated effective Majorana neutrino mass characterizing the light neutrino exchange contribution to $\beta\beta$ -decay versus the lightest neutrino mass. The calculation takes into account the current neutrino oscillation parameters and state-of-the-art nuclear matrix elements [60]. The data from neutrino oscillation experiments allow ranges of possible values of the effective Majorana mass for different neutrino mass spectra to be predicted. Figs. 4 and 5 values of a few tens of units or units of meV are expected in the case of the two different ordering of neutrino masses, named “inverted” and “normal” hierarchy, respectively.

We conclude from the Fig. 4 that in the case of inverted hierarchy of neutrino masses a very good potential for discovery of the $\beta\beta$ -decay have the CUORE, EXO, Majorana and MOON experiments. Let us stress that it is very important to achieve a high sensitivity in several experiments using different nuclei as a radioactive source. It will allow to obtain important information about the accuracy of nuclear matrix elements involved and to discuss the effect of the CP-Majorana phases.

The effective Majorana neutrino mass, which determines the half-life of the $\beta\beta$ -decay, crucially depends on the character of the neutrino mass spectrum. All the possible physical neutrino mass spectra (hierarchy, inverted hierarchy, and quasidegenerate) are viable at present. However, in the framework of the seesaw mechanism, which is a plausible explanation of the smallness of neutrino masses, hierarchy is a favorable spectrum. Generically, the neutrino mass hierarchy naturally appears in GUT models like SO(10) which unify quarks, leptons, and neutrinos. We see by glancing Fig. 5 that in the case of the normal hierarchy of neutrino masses the observation of the $\beta\beta$ -decay is beyond the scope of next generation of the $\beta\beta$ -decay experiments.

4. Conclusions

The recent discovery, that neutrinos have masses, opens a wide new field of experimentation. Future neutrino oscillation experiments will lead to precision measurements of neutrino mass splittings and mixings. Therefore the flavor structure of the lepton sector will, at some point, become better known than that of the quark sector. Accelerator-made neutrinos are also essential in this program. Ideas for future facilities include Superbeams, Beta-beams, or Neutrino Factory, each associated with one or several options for detector systems.

After 70 years the brilliant hypothesis of Ettore Majorana is still valid and is strongly supported by the discovery of neutrino oscillations and by the construction of the Grand Unified Theories. Double beta decay is currently the most powerful tool to clarify if the neutrino is a Dirac or a Majorana particle. This issue is intimately related with the origin of neutrino masses having a strong impact also on astrophysics and cosmology.

The most ambitious future $\beta\beta$ -decay experiments aim to probe the inverted hierarchy of neutrino masses which requires a symmetry in the neutrino mass matrix. In a near

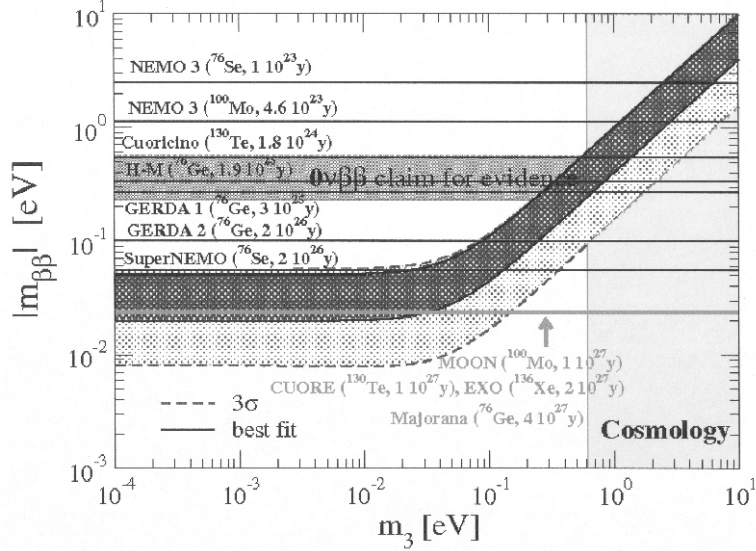


Fig. 4. The $0\nu\beta\beta$ -decay versus current neutrino oscillations data for the case of the inverted hierarchy of neutrino masses. The best fit results (the region with solid line boundary) and the 3σ results (the region with dashed line boundary) for effective Majorana neutrino mass $m_{\beta\beta}$ as a function of lightest neutrino mass m_3 are presented. The sensitivities of the future experiments on the search for the $0\nu\beta\beta$ -decay of different isotopes are indicated with horizontal solid bold lines.

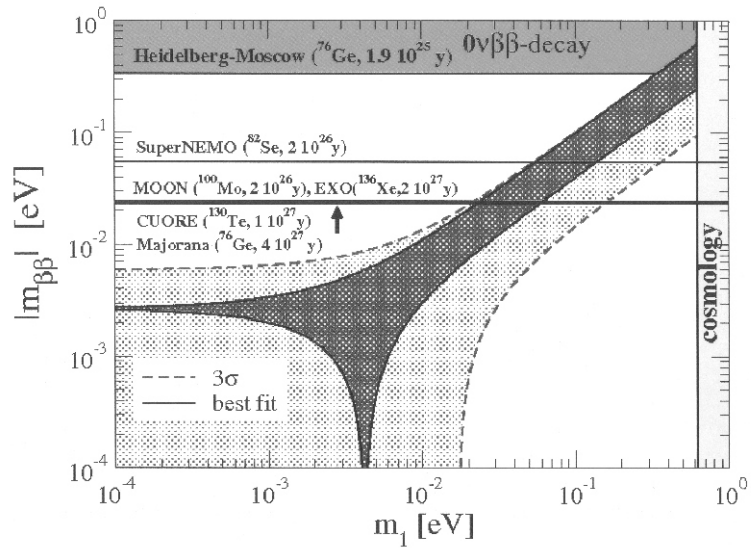


Fig. 5. The $0\nu\beta\beta$ -decay versus current neutrino oscillations data for the case of the normal hierarchy of neutrino masses. The best fit results (the region with solid line boundary) and the 3σ results (the region with dashed line boundary) for effective Majorana neutrino mass $m_{\beta\beta}$ as a function of lightest neutrino mass m_3 are presented. The sensitivities of the future experiments on the search for the $0\nu\beta\beta$ -decay of different isotopes are indicated with horizontal solid bold lines.

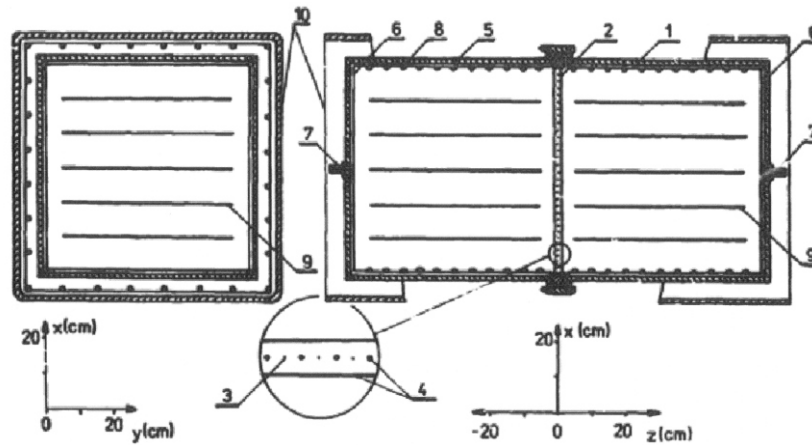


Fig. 6. A time projection chamber for double beta decay experiments proposed by experimental groups at Comenius University Bratislava and JINR Dubna [76, 77].

future (2009–2011) a new generation of detectors will start to operate with about 100 kg of different $\beta\beta$ -radioactive isotopes. Such experiments will probe the effective neutrino mass m down to 10–50 meV. The observation of $\beta\beta$ -decay will allow to reveal the type of the neutrino mass spectrum, to determine the mass of the lightest neutrino and, possibly, Majorana CP phases. For that purpose nuclear matrix elements need to be evaluated with uncertainty of less than 30% to establish the neutrino mass spectrum and CP violating phases. The improvement of the calculation of the nuclear matrix elements is a very important and challenging problem.

Neutrino physics remains one of the most exciting fields of fundamental physics today. The neutrino's position at the intersection of particle physics, astrophysics, and nuclear physics ensures continuing interest in the subject. Major activities at accelerators like Fermilab, KEK and CERN, in addition to underground facilities like Gran Sasso, Kamioka and Sudbury, continue to enhance our understanding of the origins and properties of neutrinos, and their implications for the Standard Model and cosmology.

Appendix A: Double beta decay activities of Slovak scientists

Pavel Povinec was the first in Slovakia, who organized the research related to physics at deep underground laboratories. The main effort of experimental group under his leadership was concentrated on preparation of the double beta decay experiment. He managed to establish fruitful international collaborations with leading scientists in the field (Ettore Fiorini, Alexander Pomansky). In addition, he was also successful in attracting young scientists (J. Lác, P. Kubinec, J. Masarik, F. Šimkovic) to this subject.

An important achievement was the 14th EPS Conf. on Nuclear Physics: Rare Nuclear Decays and Fundamental Processes, which was held in Bratislava in 1990. Unfortunately, few years later Pavel Povinec left Department of Nuclear Physics, Comenius University, Bratislava due to his duties at IAEA. This fact stopped experimental activity in this field in

Slovakia and many of his collaborators changed for high energy physics. After Pavel Povinec came back, there is a revived interest to the fundamental rare nuclear processes and low activities at the Faculty of Mathematics, Physics and Informatics in Bratislava.

Experiment

The group under the P. Povinec leadership participated in several double beta decay experiments. The first experiment involved multielement proportional chambers, which were originally developed in Bratislava. P. Povinec together with R. Janik participated in a double beta decay experiment of ^{136}Xe , which was carried out by A. Pomansky group in the Baksan underground laboratory [78]. A similar multielement proportional chamber for ^{136}Xe was constructed by E. Fiorini group at the University of Milano, and a search for ^{136}Xe double beta decay was carried out in the Gran Sasso laboratory with participation of P. Povinec and J. Szarka, who spent 2 years in Milano and Gran Sasso [79]. Another development was a construction of a drift chamber for double beta decay experiments with solid sources, which was carried out in collaboration with JINR Dubna. Several colleagues from the Comenius University of Bratislava participated on these developments (P. Povinec, B. Sitár, R. Janik, V. Hlinka, P. Kubinec, Ľ. Lúčan, J. Masarik, I. Melo, M. Pikna, J. Staníček, P. Strmeň, I. Sýkora and P. Vojtyla [76,77]). It is worth mentioning that wire chamber technology is currently used successfully by the NEMO collaboration, which succeeded to observe $\beta\beta$ -decay for different isotopes and to establish the strongest constraint on the half-lives of the $\beta\beta$ -decay of ^{100}Mo and ^{82}Se .

Theory

The theoretical work concerning the double beta decay was performed by the author and his students (M. Jandel, P. Domin, F. Knapp, M. Šmotlák, P. Ledňa, R. Dvornický, P. Beneš) mainly in collaboration with physicists from University of Tuebingen, JINR Dubna, University of Lublin, University of Ioannina, California Institute of Technology, University of Valparaiso and ICTP Trieste. The subject of interest has been the particle and nuclear physics aspects of the nuclear double beta decay and related processes. The main results are as follows:

1. Mechanisms of the $\beta\beta$ -decay

Various lepton number violating mechanisms of the neutrinoless double beta decay were studied in details, in particular light and heavy Majorana neutrino exchange [81], right-handed currents [80] and R-parity violating mechanisms [49, 82, 83]. A new contribution of the R-parity violating supersymmetry to neutrinoless double beta decay via the pion-exchange between decaying neutrons was suggested [49, 83]. The importance of gluino and neutralino exchange mechanisms were discussed. A possibility to distinguish between different mechanisms of the neutrinoless double beta decay was proposed [84].

2. Double beta decay nuclear matrix elements

Nuclear matrix elements associated with different mechanisms of the $\beta\beta$ -decay were calculated in QRPA-like approaches. The effects of proton-neutron pairing [85], Pauli exclusion principle violation and deformation [62] on the NME were studied. It was found that when the strength of the particle-particle interaction is adjusted so that the $\beta\beta$ -decay

rate is correctly reproduced, the resulting values of the $\beta\beta$ -decay NME become essentially independent of the size of the basis, and of the form of different realistic nucleon-nucleon potentials [58, 87]. It was also shown that the competition between pairing and the neutron-proton particle-particle interaction causes contributions to the $\beta\beta$ -decay matrix element to nearly vanish at internucleon distances of more than 2 or 3 fermis [60]. Currently, the obtained $\beta\beta$ -decay are considered to be the most reliable [58, 60, 87]. In addition, many-body approximation schemes were studied within the exactly solvable models [88]. It allowed to propose a new many-body approach, namely “QRPA with non-linear phonon operator” [89].

3. Neutrino mass matrix and related processes to the $\beta\beta$ -decay

The effective Majorana neutrino mass $m_{\beta\beta}$ was evaluated by using the latest results of neutrino oscillation experiments. The problems of the neutrino mixing pattern, the absolute mass scale of neutrinos, and the effect of CP phases was addressed. A connection to the next generation of neutrinoless double beta decay was discussed [90]. Arguments were presented that in the framework of the see-saw mechanism the normal hierarchy is favorable for the neutrino mass spectrum. For this spectrum, we presented a detailed calculation of the half-lives of neutrinoless double for several nuclei of experimental interest. The dependence of the half-lives on $\sin^2 \theta_{13}$ and the lightest neutrino mass was studied [91]. Next, the connection between the entries of neutrino mass matrix and the transitional magnetic moment of Majorana neutrinos was studied in supersymmetry without R-parity in light of neutrino oscillations [92]. For that purpose the elements of phenomenological neutrino mass matrix were reconstructed by using the neutrino oscillation data and the lower bound on the $\beta\beta$ -decay half-life. Further, the first realistic treatment of nuclear structure aspects of processes of the muonic analogue of the $\beta\beta$ -decay ($\mu \rightarrow \mu^+$) [93] and the muon to positron conversion ($\mu \rightarrow e^+$) [94] in nuclei were performed. The corresponding rates of these processes utilizing the existing experimental constraints on the parameters of the underlying lepton number violating interactions were calculated. It was found that the $(\mu \rightarrow \mu^+)$ and the $(\mu \rightarrow e^+)$ conversions will hardly be detectable in the near future experiments.

4. Single state dominance hypothesis

The hypothesis of the single state dominance (SSD) in the calculation of the (anti)neutrino accompanied double beta decays of nuclei of interest was tested by an exact consideration of the energy denominators of the perturbation theory. Both transitions to the ground state as well as to the 0^+ and 2^+ excited states of the final nucleus were considered. It was demonstrated that, by experimental investigation of the single electron energy distribution and the angular correlation of the outgoing electrons, the SSD hypothesis can be confirmed or ruled out by a precise measurements of the $\beta\beta$ -decay of ^{100}Mo and ^{116}Cd as well as $^+/\text{EC}$ -electron capture in ^{106}Cd , ^{130}Ba and ^{136}Ce [95]. The suggested effect is currently investigated in the NEMO3 experiment.

5. Bosonic neutrino and two-neutrino double beta decay

We assumed that the Pauli exclusion principle is violated for neutrinos, and thus, neutrinos obey, at least partly, the Bose-Einstein statistics. The parameter $\sin^2 \theta$ that characterizes the bosonic (symmetric) fraction of the neutrino wave function was introduced. Consequences of the violation of the exclusion principle for the two-neutrino double beta decays were considered. It was shown that this violation strongly changes the rates of the decays and modifies the energy and angular distributions of the emitted electrons. The possibility of pure bosonic neutrinos was excluded by the present data. In the case of partly bosonic

(or mixed-statistics) neutrinos the analysis of the existing data allowed to put the conservative upper bound $\sin^2 \theta < 0.6$ [96].

It is worth mentioning that theoretical group from the Department of Nuclear Physics and Biophysics, Comenius University is an integral part of international collaborations searching for the μ -decay, in particular of NEMO Collaboration, COBRA Collaboration and TGV Collaboration. It is also involved in the ILIAS (Integrated Large Infrastructures for Astroparticle Science, 6-th framework EU program) project of the European Union involving 22 leading institutions in the field in Europe. It is expected that both experimental and theoretical groups from Comenius University will play an active role in solving the tasks of the ILIAS-next (7th framework EU program) project in preparation.

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