Acta Physica Universitatis Comenianae
Vol. XLIV–XLV, (2003–2004) 23–35

Simplification of a Pipeline Network for a Steady-State Real Gas Transport

R. Hajossy

Faculty of Mathematics, Physics and Informatics, Comenius University,
Mlynská dolina, F2 842 15 Bratislava, Slovak Republic

Abstract: It is shown how to simplify a complex pipeline network at numerical calculations of a steady gas flow. Even a network which cannot be analyzed in terms of series and parallel combinations can be replaced by a single pipe. The heat exchange with the surrounding soil, the Joule-Thomson effect and the influence of the pipeline altitude profile have been taken into account.

1. Introduction

A complexity of a gas transport calculation is proportional to a number of pipes in a pipeline network. A computational time could be decreased by a reduction of the pipe number. The time reduction is very important at optimizing an international gas transport since in this case a multiple repeating of the network calculation for different transport conditions is necessary. For example, the Slovak pipeline network for a natural gas transport from the Ukraine to Bohemia and Austria consists of four parallel pipelines (approximately, 400 km length, 1200 and 1400 mm diameter) mutually interconnected (after about each 20 km) with shorter pipes. (There is also a looping of an unfinished fifth parallel pipeline of 1400 mm). It is reasonable to simplify the network by a substitution of several pipes connected in parallel or serial for one substitutional tube, similarly as it is done in electrical circuites.

Unfortunately, the similarity to the electrical analogue is imperfect as the equations of a turbulent gas flow are nonlinear. As a consequence, the usually applied substitution of pipes used to be too simplified [1]:

A gas flow is supposed to be isothermal. (To justify the simplification, the pipes are supposed to be deep in a soil, 90 cm at least. The temperature in the depth changes insignificantly during a year.)

Parallel pipes are supposed to be horizontal.

The substitution is only limited to parallel and serial connections of pipes. (An analytical formula for a substitution of a general pipeline dipole, such as bridged parallel pipes, could not be derived due to the nonlinearity of a pipeline model.)

The aim of this article is:

to derive a pipeline model taking into account: the heat exchange between the pipe and a surrounding soil; the temperature changes of the transported gas due to the Joule-Thomson effect; the influence of the pipeline altitude profile.

to substitute only one pipe for a general pipeline dipole at numerical calculations of a steady gas flow in a complex pipeline network.

2. The Model of a Turbulent Real Gas Flow in an Inclined Pipe

A turbulent real gas flow in an element of an inclined pipe is described by the conservation laws of mass, momentum and energy [2–6]. The laws have the following one-dimensional form.

Conservation of mass (continuity equation)

$$(\ )_t + (G/S)_x = 0. \tag{1}$$

is the density; G = v S is the mass flow across the pipe cross-section area S at the gas velocity v; subscripts $_t$ and $_x$ denote the partial derivative with respect to time t or to the coordinate x which determines position (along the pipe axis) of the observed part of gas.

Conservation of momentum

$$(G/S)_t + [(G/S)^2/\ ]_x = -(P)_x - \ (G/S)|G/S| / (2D\ ) - \ g\ (z)_x \tag{2}$$

P is the gas pressure, z is the altitude of the observed pipeline part; is the friction coefficient (hydraulic resistance of th pipe) depending on the Reynolds number of the turbulent gas flow and on the roughness and diameter D of the pipe; g is the gravitational accelleration.

Conservation of energy

$$\{[(G/S)^2/(2\ ^2)] + h + gz\}_t + (G/S)\{[(G/S)^2/(2\ ^2)] + h + (gz)]\}_x - (P)_t =$$

$$= -\ _{ef}\ T - T_w)\ \ D/S \tag{3}$$

where h is the specific entalpy (enthalpy of 1 kg of gas); $_{ef}$ J $m^{-2}s^{-1}K^{-1}$ is the effective heat transport coefficient; T and T_w are the temperatures of the gas and the pipe wall.

The system of three conservation laws for five unknown variables P, , T, h and G has to be completed by two termodynamical equations for a real gas [5–9] (To overcome friction, during a long distance transport spanning hundreds of kilometers, it is necessary to apply high pressure gradients – several MPa across the pipe length. The transport regime: 3–6 MPa, 10–30 C, 40–50 kg m^{-3} are far from the ideal gas conditions.)

State equation of a real gas

$$P = Z\ \ RT$$

or

$$[(1/P) - (Z)_P/Z)]\ dP = d\ \ /\ \ + [(1/T) + (Z)_{T/}Z]\ dT$$

where parameter Z(P,T) is the gas compressibility; the gas constant for the methane $R = 519{,}6\ J\ K^{-1}kg^{-1}$.

Change of a real gas enthalpy

$$dh = (h)_T\ dT + (h)_P\ dP = c_p\ dT - c_p\ \ \ dP \tag{5}$$

where $c_p(P,T) = h_T$ is the specific heat capacity at a constant pressure. The temperature change (in an isoentalpic flow due to a unit pressure drop) is determined by the Joule-Thomson coefficient

$$(P,T) = RT^2\ (Z)_T\ /(c_pP) \tag{6}$$

In steady-state conditions according to equations (1) (5), the mass flow, pressure and temperature changes are determined by relations

$$(G)_x = 0 \tag{7}$$

$$(P^2)_x + [2g\ (z)_x/(ZRT)]\ P^2 = -\ \ G|G|\ ZRT/\ (DS^2) - (2PG^2/S^2)\ (\ ^{-1})_x \tag{8}$$

$$(T)_x + T/\ \ = T_w\ /\ \ +\ \ \ (P)_x - (g/cp)\ (z)_x - [G^2/(2c_pS^2)]\ (\ ^{-2})_x \tag{9}$$

where the length parameter

$$= (c_pG)/(\ _{ef}\ \ D)\ \ \ 100\ km \tag{10}$$

characterizes cooling of the flowing gas due to the heat losses to a surrounding soil.

As the influence of the kinetic energy (the last terms proportional to a constant G^2) in the equations (8), (9) is small, the inhomogenious system of the linear equations (for P^2 and T) can be solved iteratively.

Temperature and pressure profile

For a pipe of a length L and a constant inclination; with initial pressure, temperature, density, altitude (P_b, T_b, $_b$, z_b) at the beginning of the pipe (x = 0) and the similar end parameters (P_e, T_e, $_e$, z_e) at the end of the pipe (x = L), the following *profiles of temperature and pressure* along the pipe are obtained from the conservation laws (7) (9)

$$G = G_b = G_e = \text{constant}$$

$$T(x) = T_b - [T_b - T_w + \underline{(v^2/2 + gz)}_x\ (\ \ /\underline{c}_p) + \_\ \ (P_b{}^2 - P_e{}^2)/(2L\ \underline{P})]\ (1 - e^{-x/}\ ) \tag{11}$$

$$P(x)^2 = [P_b{}^2 + B(x)]\ \exp\{-\ \ (z - z_b)\ \} \tag{12}$$

where function B(x) is determined as

$$B(x) = -F\ L\ [\exp\{\ (z_e - z_b)x/L\} - 1]/[\ (z_e - z_b)] \qquad (13)$$

with parameter F, connected to the right side of the equation (8)

$$F = \ G|G|\ \underline{Z}R\underline{T}/(DS^2) + (2\underline{P}G|G|/S^2)\ |\ {}_e^{-1} - \ {}_b^{-1}|\ /L \qquad (14)$$

and parameter is given by the relation

$$= 2\ g\ /(\underline{Z}R\underline{T}) \qquad (15)$$

The underlined parameters $\underline{P}$, $\underline{T}$ correspond to the mean values of these parameters obtained by averaging along the whole length L of the pipe. Values of the pressure and temperature dependent parameters $\underline{c}_p$, $\underline{\ }$, $\underline{Z}$ can be estimated as the values at the mean pressure and temperature: $\underline{c}_p = c_p(\underline{P}, \underline{T})$; $\underline{\ } = \ (\underline{P}, \underline{T})$; $\underline{Z} = Z(\underline{P}, \underline{T})$.

The mean values of temperature and pressure

The *mean temperature* in an inclined pipe of length L, calculated from the temperature profile (11), is determined by the formula

$$T = T_b - \qquad (16)$$

$$- [T_b - T_w + (\underline{v^2/2 + gz})_x\ (\ /\underline{c}_p) + \underline{\ }\ \ (P_b^2 - P_e^2)/(2L\ \underline{P})]\ [1 - (\ /L)(1 - e^{-L/\ })]$$

The mean value is influenced by the soil cooling and by the work of the intermolecular forces described by the Joule-Thomson effect. Moreover, the mean temperature is also affected by changes of the kinetic and gravitational energy. The mean gradients of both energies can be assessed as

$$(\underline{v^2/2 + gz})_x = G^2(\ {}_k^{-2} - \ {}_p^{-2})/(2LS^2) + g\ (z_k - z_p)/L \qquad (17)$$

According to equation (8), a value of the parameter $= 2\ g\ /(\underline{Z}R\underline{T})$ in the pressure profile is determined by the mean value of the variable $(\underline{T}^{-1})$. (A change of compressibility Z along the pipe is much smaller than that of the temperature therefore it does not influence the value of the parameter significantly.)

As it follows from the temperature profile (11), *the mean value of the variable* 1/T(x) is given by the formula [12]

$$L\ /\ \underline{T(x)} = \{(L/\mathbf{b}) - (\mathbf{ab})^{-1}\ln[\mathbf{b} + \mathbf{c}\ \exp(\mathbf{a}L)] + (\mathbf{ab})^{-1}\ln[\mathbf{b} + \mathbf{c}]\} \qquad (18)$$

where

$\mathbf{a} = -1/$

$\mathbf{b} = T_w - (\underline{v^2/2 + gz})_x\ (\ /\underline{c}_p) - \underline{\ }\ \ (P_b^2 - P_e^2)/(2L\ \underline{P})]$

$\mathbf{c} = T_b - T_w + (\underline{v^2/2 + gz})_x\ (\ /\underline{c}_p) + \_\ \ (P_b^2 - P_e^2)/(2L\ \underline{P})$

The *mean value of the pressure* for an *ascending* gas flow ($z_e - z_b > 0$) can be calculated from the pressure profile (12)–(15) as [12]

$$\underline{P} = [2\mathbf{C}^{1/2}\ /(\mathbf{LB})]\ \mathrm{arctg}\ \{(\mathbf{A}\ e^{-\mathbf{B}L} - \mathbf{C})/\mathbf{C}\ \}^{1/2} - [2/(L\mathbf{B})]\ (\mathbf{A}\ e^{-\mathbf{B}L} - \mathbf{C})^{1/2} +$$

$$+ [2\mathbf{C}^{1/2}\ /(L\mathbf{B})]\ \mathrm{arctg}\ \{(\mathbf{A} - \mathbf{C})/\mathbf{C}\ \}^{1/2} + [2/(L\mathbf{B})]\ (\mathbf{A} - \mathbf{C})^{1/2} \qquad (19)$$

where constants **A**, **B**, **C** are given by the following relations

$$\mathbf{A} = P_b^2 + F\ L\ /[\ \ (z_e - z_b)]$$

$$\mathbf{B} = \ \ (z_e - z_b)/L \qquad (20)$$

$$\mathbf{C} = F\ L\ /[\ \ (z_e - z_b)] = F/\mathbf{B} = \mathbf{A} - P_b^2$$

while F and   are determined by (14), (15).

The mean pressure for a *descending* gas flow ($z_e - z_b < 0$) can be also calculated from the pressure profile (12)–(15) but in this case the constants (20) have negative values and therefore the mean pressure is determined by the formula [12]

$$\underline{P} = (\mathbf{C}^{1/2}/(L\mathbf{B}))\ \ln\{[\mathbf{C}^{1/2} + (\mathbf{C} - \mathbf{A}\ e^{-\mathbf{B}L})^{1/2}]\ /\ [\mathbf{C}^{1/2} - (\mathbf{C} - \mathbf{A}\ e^{-\mathbf{B}L})^{1/2}]\} -$$

$$- (\mathbf{C}^{1/2}/(L\mathbf{B}))\ \ln\{[\mathbf{C}^{1/2} + (\mathbf{C} - \mathbf{A})^{1/2}\ ]\ /\ [\mathbf{C}^{1/2} - (\mathbf{C} - \mathbf{A})^{1/2}]\} -$$

$$- (2/(L\mathbf{B}))[(\mathbf{C} - \mathbf{A}\ e^{-\mathbf{B}L})^{1/2} - (\mathbf{C} - \mathbf{A}\ )^{1/2}] \qquad (21)$$

In the case of a *horizontal pipe* ($z_e - z_b = 0$; $\mathbf{B}\ \ \ 0$) both relations (21) and (19) change to the well known *mean value of the pressure* [10,11]

$$\underline{P}\ \ \ (2/3)\ (P_p^3 - P_k^3)\ /(FL) = (2/3)\ (P_p^3 - P_k^3)\ /(\ P_p^2 - P_k^2) \qquad (22)$$

The limit formula (22) can be obtained from the relation (19) by the approximation of [12, 6]

$$\mathrm{arctg}\ x\ \ \ \ x - x^3/3$$

and from the relation (21) by

$$2\ \mathrm{Arth}\ x = \ln\{(1 + x\ )\ /\ (1 - x)\}\ \ \ \ 2x + (2/3)x^3$$

Obtaining the formula (22) for a mean pressure of a horizontal pipe as a limit of the relations for the ascending and descending pipes is a partial proof of the correctness of the relations (19) and (22).

The end pressures and mass flow in a pipe with a constant slope

According to (12), (13) *the end pressures* P_e and P_b *in a pipe with a constant slope* are related as

$$P_e^2 = [P_b^2 + B(L)] \exp\{- \ (z_e - z_b)\}$$
where (23)
$$B(L) = -F\,L\,[\exp\{ \ (z_e - z_b)\} - 1]/[\ (z_e - z_b)]$$

while parameters F and   are determined by (14) and (15).

The relation (23) between the end pressures can be arranged to a simple form, used to *solve the problems of a steady-state gas flow*

$$P_b^2 = P_e^2\, a + G|G|\, b \qquad (24)$$

with the constants

$$a = \exp\{ \ (z_e - z_b) \} \qquad (25)$$

$$b = \{[\ \underline{Z}R\underline{T}\, L/(DS^2)] + [(2\underline{P}/S^2)\, | \ _e^{-1} - \ _b^{-1}|]\}\, [\exp\{ \ (z_e - z_b)\} -$$
$$- 1]/[\ (z_e - z_b)] \qquad (26)$$

where the parameter   is given by the formula (15); the mean values of temperature $\underline{T}$ and inverse temperature ($\underline{1/T}$) are determined by the equations (16), (18); depending on the pipe inclination, the mean pressure $\underline{P}$ can be calculated from the relations (19)–(22).

From the end pressure relation (24) at a sufficiently high pressure difference ($P_b^2 > P_e^2\, a$), a *mass flow* G can be calculated as

$$G = (P_b^2 - P_e^2\, a)^{1/2}\, b^{-1/2} \qquad (27a)$$

In the opposite case, ($P_b^2 - P_e^2\, a$), the mass flow changes its direction and sign. Then it is determined as

$$G = (P_b^2 a - P_e^2)^{1/2}\, b^{-1/2} \qquad (27b)$$

According to relations (25) and (15), for an ascendent pipe with a difference of 100 m in the pipe end altitudes, the value of the parameter *a* can be estimated as

$$a = \exp\{ 2\, g(z_e - z_b) /(\underline{Z}R\underline{T})\} \quad \exp (2.10.100/(1.520.300)) = 1 + 0{,}012$$

The value *a* differs little from the number 1. In consequence, the parameter $a = 1 + x$ can be expressed by the small number *x*. Then a *mass flow can be estimated* from (27a) as

$$G = (P_b^2 - P_e^2\, a)^{1/2}\, b^{-1/2} = (P_b^2 - P_e^2\, (1 + x))^{1/2}\, b^{-1/2} =$$

$= \{(P_b^{\,2} - P_e^{\,2})[1 - xP_e^{\,2}/(P_b^{\,2} - P_e^{\,2})]\}^{1/2}\, b^{-1/2}$

$$(P_b^{\,2} - P_e^{\,2})^{1/2}\, b^{-1/2} - 0{,}5\, x\, b^{-1/2}\, P_e^{\,2}(P_b^{\,2} - P_e^{\,2})^{-1/2} \tag{28}$$

3. Simplification of a pipeline network

Set of different pipes in serial

Two pipes in a serial, characterized by constants a_1, b_1; a_2, b_2, have *the same gas flow* G but the different end pressures determined by the equation (24)

$$P_{b1}^{\,2} = P_{e1}^{\,2}\, a_1 + G|G|\, b_1$$

$$P_{b2}^{\,2} = P_{e2}^{\,2}\, a_2 + G|G|\, b_2$$

Due to a joint node, the pressures $P_{e1} = P_{b2}$. Then

$$P_{b1}^{\,2} = P_{e1}^{\,2}\, a_1 + G|G|\, b_1 = (P_{e2}^{\,2}\, a_2 + G|G|\, b_2)\, a_1 + G|G|\, b_1$$

$$P_{b1}^{\,2} = P_{e2}^{\,2}\, (a_2 a_1) + G|G|\, b_2(b_2\, a_1 + b_1) \tag{29}$$

Comparing the equations (24) and (29), it is evident that the last equation represents a substitutional pipe with the same gas flow G and the same end pressure difference

$$P_{b1} - P_{e2} = (P_{b1} - P_{e1}) + (P_{b2} - P_{e2})$$

as there is across the ends of these two serial pipes. The substitutional pipe is characterized by the parameters

$$a = a_2\, a_1 \qquad b = b_2\, a_1 + b_1 \tag{30}$$

The shown procedure can be *generalized* to a set of *n* different pipes in serial. In this case the serial set of pipes with parameters a_1, b_1; ...; a_n, b_n is equivalent to a single pipe given by parameters

$$a = a_1\, a_2 \ldots a_{n-1}\, a_n \qquad b = b_1\, a_2\, a_3 \ldots a_n + b_2\, a_3\, a_4 \ldots a_n + \ldots + b_{n-1}\, a_n + b_n \tag{31}$$

Set of different pipes in parallel

Two *closely laid on parallel pipes* (a_1 , b_1 ; a_2 , b_2) have the same parameter $a_1 = a_2 = a$, the same end pressures P_b, P_e but the different gas flow G_1, G_2. Then, in accordance with (27a) the resulting flow has the following value

$$G = G_1 + G_2 = (P_b^{\,2} - P_e^{\,2}\,a)^{1/2}\, b_1^{-1/2} + (P_b^{\,2} - P_e^{\,2}\,a)^{1/2}\, b_2^{-1/2}$$

$$G = (P_b^{\,2} - P_e^{\,2}\,a)^{1/2}\,(b_1^{-1/2} + b_2^{-1/2})$$

To obtain this equation, a sufficiently high pressure difference ($P_b^{\,2} > P_e^{\,2}$ a) is supposed.

The substitutional pipe for a close pair of parallel pipes has the following characteristics: original $a = a_1 = a_2$ and

$$b^{-1/2} = (b_1^{-1/2} + b_2^{-1/2}) \qquad (32)$$

The result can be *generalized* for a set of *n different closely laid on pipes in parallel*. In this case the parallel set of pipes with parameters a_1, b_1; ...; a_n, b_n can be replaced by a single pipe with parameters

$$a = a_1 = a_2 = \ldots = a_n$$

$$b^{-1/2} = (b_1^{-1/2} + b_2^{-1/2} + \ldots + b_n^{-1/2}) \qquad (33)$$

When *a geometry* (length and diameter) *of parallel pipes differs sufficiently*, then (due to a different heat losses) their mean temperatures and consequently, the parameters a of the pipes, differ as well.

Then a *set of two parallel pipes of different geometry* is characterized by parameters $a_1 = 1 + x_1$, b_1; $a_2 = 1 + x_2$, b_2. According to the approximative relation (28), the resulting flow through the parallel pipes

$$G = G_1 + G_2 \quad (P_b^{\,2} - P_e^{\,2})^{1/2}\,(b_1^{-1/2} + b_2^{-1/2}) -$$

$$- 0{,}5\,(x_1\, b_1^{-1/2} + x_2\, b_2^{-1/2})\,P_e^{\,2}(P_b^{\,2} - P_e^{\,2})^{-1/2} \qquad (34)$$

Comparing the equations (34) and (28), it is evident that *the equivalent pipe for a pair of parallel pipes with a different geometry* is characterized by parameters

$$b^{-1/2} = b_1^{-1/2} + b_2^{-1/2}$$

$$x\, b^{-1/2} = x_1\, b_1^{-1/2} + x_2\, b_2^{-1/2} \qquad (35)$$

or

$$a = 1 + (a_1 - 1)(b/b_1)^{1/2} + (a_2 - 1)(b/b_2)^{1/2}$$

The result can be easily *generalized* for a set of *n* different pipes in parallel. In this case the parallel set of pipes with parameters a_1, b_1; ...; a_n, b_n can be replaced by a single pipe with parameters

$$a = 1 + (a_1 - 1)(b/b_1)^{1/2} + (a_2 - 1)(b/b_2)^{1/2} + ... + (a_n - 1)(b/b_n)^{1/2}$$
$$b^{-1/2} = (b_1^{-1/2} + b_2^{-1/2} + ... + b_n^{-1/2}) \qquad (36)$$

The mass flow through parallel pipes at small pressure gradients

It is necessary to take into account that the relations (34) –(36) are correct only at the condition

$$1 \gg (x\, P_e^2)/(P_b^2 - P_e^2) \qquad (37)$$

which is not fulfilled at very small pressure differences $P_b - P_e$ or in the case of the zero difference. In these cases it is inevitable to use the original, not simplified equation

$$G = G_1 + G_2 + \dots + G_n = [(P_b^2 - a_1 P_e^2)^{1/2}\, b_1^{-1/2}] + ...$$
$$\dots + [(P_b^2 - a_n P_e^2)^{1/2}\, b_n^{-1/2}] \qquad (38)$$

For more clarity, the individual flows in (38) are ordered in agreement with the descending sequence of the constant values $a_1 \quad a_2 \quad ... \quad a_m \quad ... \quad a_n$. Then at the unchanged initial pressure P_b and the increasing end pressure $P_e < P_b$, the mass flow G_1 through the first pipe decreases finally to zero. In such a case

$$G_1 = [(P_b^2 - a_1 P_e^2)^{1/2}\, b_1^{-1/2}] = 0$$

so the end pressure P_e and the total flow $G = G_t$ reach the values

$$P_{e1} = P_b / a_1^{1/2}$$
$$G_{t1} = [(1 - a_2 a_1^{-1})^{1/2}\, b_2^{-1/2} + \dots + (1 - a_n a_1^{-1})^{1/2}\, b_n^{-1/2}]P_b \qquad (39)$$

Naturally, in the formula (39) the term $(1 - a_1 a_1^{-1})^{1/2}\, b_1^{-1/2}$ is missing. Of course, if there are several constants equal to the value a_1, then also the flows through the corresponding pipes are zero.

At lower end pressures ($P_e < P_{e1}$), the flow G_1 through the first pipe (and also through the additional pipes with the same contants a) change the direction ($G_1 < 0$). In agreement with (27b), the changed flow G_1 must be determined from the altered relation

$$G_1 = -(a_1 P_e^2 - P_b^2)^{1/2}\, b_1^{-1/2} \qquad (40)$$

Gradually limiting the total flow, the end pressure rise up to the value when the flow G_m through the *m*-th pipe becomes to be zero. In such a case the flows from G_1 to G_{m-1} are already opposite. Such end pressure and total mass flow are calculable from

$$P_{em} = P_b \,/\, a_m^{1/2}$$

$$G_{tm} = -[(a_1 a_m^{-1} - 1\,)^{1/2}\, b_1^{-1/2} - \ldots + (1 - a_{m+1}\, a_{m-1})^{1/2}\, b_{m+1}^{-1/2} + \ldots +$$

$$+ (1 - a_n\, a_m^{-1})^{1/2}\, b_n^{-1/2}]P_b \qquad (41)$$

Naturally, in the formula (41) the term $(1 - a_m a_m^{-1})^{1/2}\, b_m^{-1/2}$ is missing. Of course, if there are several constants equal to the value a_m, then also the flows through the corresponding pipes are zero.

At the end the zero mass flow reaches the last (*n*-th) pipe. It happens at the end pressure and total mass flow

$$P_{en} = P_b \,/\, a_n^{1/2}$$

$$G_{tn} = -[(a_1 a_n^{-1} - 1\,)^{1/2}\, b_1^{-1/2} - \ldots - (a_{m+1}\, a_n^{-1} - 1)^{1/2}\, b_{m+1}^{-1/2} - \ldots -$$

$$- (a_{n-1}\, a_n^{-1} - 1\,)^{1/2}\, b_{n-1}^{-1/2}]P_b \qquad (42)$$

The sequences of the pressures $P_{e1}, \ldots, P_{en}$ and of the corresponding total flows $G_{t1}, \ldots, G_{tn}$, determined by relations (39), (41) and (42), enable us (by means of interpolation or extrapolation) to calculate another pressures and flows (in the region of small pressure differences and correspondingly small mass flows).

General passive dipole of horizontal pipelines

A part of a pipeline network connected to the rest of the network only in two nodes (input and output) can be denoted as a pipeline dipole. The treated sets of serial and paralell network are examples of a pipeline dipole. Unfortunately, not all the pipeline dipoles can be separated to a combination of serial and parallel networks and finally be replaced by a single pipe. (For example, a network of pipes, connected in form of a ladder, is inseparable.) There is a practical question: “Can any pipeline dipole be replaced by a single tube which is described by the equation (24)?” The answer is positive, at least in the case of *not strongly inclined pipes* with the parameter a 1. In this case, according to (24), transport through a pipe is described by the only constant

$$b \quad (P_b^2 - P_e^2)/G^2 = [(kP_b)^2 - (kP_e)^2]/(kG)^2 = k^2(P_b^2 - P_e^2)/(k^2G^2) \qquad (43)$$

It means, the higher are the end pressures (P_b kP_b; P_e kP_e) the proportionaly higher is the mass flow (G kG). (It is an analog of Ohm´s law in a pipeline network.) Since multiplication of the numerator and of the denominator by the same number does not change the value of the fraction (43), it can be shown that a similar proportionality between the end pressures and the mass flow is typical also for any horizontal pipeline dipole. Evidently, such a dipole made of N nodes interconnected by pipes can be described by steady-state mass flows in its nodes and by pressure differences at its pipes: Generaly, the nodes can be numbered as: 1 = input, ..., n, ..., j, ... , N = output . The pressure in a node “n” is denoted as P_n. A mass flow through a pipe between nodes “n” and “j” is signed as G_{nj}. External mass flows into the dipole G_{in} and out of the dipole G_{out} are equal at a steady-state: $G_{in} = -G_{out} = G$. (The negative sign denotes a mass outflow of a node.) Then conservation of mass in the steady-state flow can be described in nodes as

$$k\,(\ {}'G_{1,j} = G_{in} = +G\ )$$

$$k\,(\ {}'G_{N,j} = G_{out} = -G\ ) \qquad (44)$$

...

$$k\,(\ {}'G_{n,j} = 0\ )$$

The prime above the sum (´) excludes a non-existing flow of a type G_{nn} from the sum. Multiplication by “k” does not change equations (44). Similarly, the pressure equations for tubes are also not influenced by a likewise multiplication

$$\ldots\ k^2(\,P_n^2 - P_j^2 \quad b_{nj}\,G_{nj}^2)\ \ldots.. \qquad (a_{nj} \quad 1) \qquad (45)$$

As the shown multiplication of the equations is equivalent to the change of variables

$$G \quad kG;\ \ldots\ G_{nj} \quad kG_{nj}\ \ldots;\quad P_n \quad kP_n$$

then the change in $G \quad kG$ leads inevitably to the changes of the individual mass flows kG_{nj} and pressures kP_n, including the end pressures kP_{in}, kP_{out}. Consequently, the value of the initial fraction

$$b_{dip} = (P_{in}^2 - P_{out}^2)/G^2 = [(kP_{in})^2 - (kP_{out}^2)]\,/\,(k\,G^2) = \text{constant}$$

does not vary with a variation of the mass flow through the pipeline dipole. Evidently, the dipole has the same attributes as a single pipe, described by the relation (43). Thus a dipole of little tilted pipes can be replaced by a single tube given by the following characteristic parameters

$$a \quad 1$$

$$b \quad (P_{in}^2 - P_{out}^2)/G^2 \qquad (46)$$

It is a *disadvantage* that the formula (46) shows no simple way how to calculate the parameter b of the substitutional pipe from the parameters a_{nj}, b_{nj} of the individual pipes of which the dipole is made. According to (46), for a given pair of the three values (P_{in},P_{out} and G), the third value has to be calculated laboriously for the real not simplified pipeline dipole. However, such a *procedure has to be done only once*. That is an indisputable *advantage* of the relation (46). At repeating calculations for different transport conditions, a very complex dipole can be replaced by a single tube with parameters (46), determined by this initial numerical computation.

4. Application of the simplified model of a complex pipeline network

The shown formulas for simplification of complex pipeline networks have been implemented to a computational program MARTI, designated to solve the problems of steady-state transport of natural gas. Simplification of a real transit network is done by the program automatically in three successive steps:

simplification of parallel pipelines, utilizing the relations (36), (39)–(42)
simplification of serial pipelines by the equations (31)
simplification of a complex pipeline dipole by the procedure (46)

Computation of pressures, temperatures and gas flows in nodes and pipes goes in the opposite direction: from the most simplified network complex dipoles serial pipelines parallel pipelines and finally, to the pipes and nodes of the original network. Such a simplifying procedure can spare the computational time importantly. In dependence on the actual complexity of the analyzed network, there can be even hundredfold time saving in comparison to the computation of the original, not simplified network [13].

The proposed pipeline model – the formulas (11)–(26) describing pressure and temperature in tilted pipes – takes into account all the decissive attributes of a turbulent flow in a long distance transport of a real gas. Such a model can solve transport problems at very extreme conditions which are projected for the new gas transmission pipeline *Goluboj Potok* (*The Blue Stream*) [10]. The pipeline provides transmission of a Russian gas to Turkey under the Black Sea. The whole length of the undersea route is 340 km. The deepest place is 2100 m under the sea level. The drop to the seabed is steep. It occurs within 80 km at both Russian and Turkish shore. The pipe has the external diameter 610 mm, wall thickness 38 mm, inner wall roughness 0,03 mm, the heat transport coefficient 7,0 W $m^{-2}K^{-1}$ to the surrounding water of temperature 8 C, an expected pressure drop from initial 20 MPa to the end 5,5 MPa at a projected mass flow 150 kg s^{-1}.

Supposing the projected conditions and utilizing the relations (11)–(26), the temperature drop from the initial 37 C to the end –6 C is calculated. In spite of the surrounding water temperature, +8 C, the pipeline could be encrusted with ice along the last 70 km. It is a consequence of the Joule-Thomson effect (a distinct property of a real gas). The ice could cause technical problems as it could lead to floating of the encrusted pipe.

At the steep slope near the Russian shore, the gravitational force exceeds the friction. As a result, the pressure of the flowing gas should increase and not decrease as it is usual. After the initial 50 km, the original pressure 20 MPa should rise up to 21 MPa.

The calculated termal and pressure effects are in agreement with the published results [10]. However, it is necessary to notify that the actual transmission regime at the Blue Stream is far from the projected extreme conditions. During this summer, the actual pressure dropped from 5,64 to 5,43 MPa; temperature increased from16,4 to 19,2 C due to heating by a hot surface water of 26 C at a slow mass flow 17,5 kg s^{-1} [14].

5. Conclusions

A model of a turbulent real gas flow trough an inclined pipe has been developed. The model enables to solve the steady-state problems of gas transmission even at extreme conditions by a superlong undersea transit along a unique altitude profile.

Simultaneously, a procedure for simplification of complex pipeline networks has been shown. The procedure permits to simplify also a network that cannot be distributed to a set of parallel or serial pipes. Such a simplification of a real network providing international gas transport can significantly spare the time for computation of the pressure, temperature and mass flow at a steady-state transit regime.

Reference

[1] M. Gančo a kolektív: Prúdenie plynu v potrubných systémoch , STU Bratislava 1998, ISBN 80-227-1151-9, pp. 98-112.

[2] L. D. Landau, E. M. Lifšic: Gidrodinamika, vyd. 4, Moskva, Nauka 1988, pp. 71, 304.

[3] L. Prandtl: Gidroaeromechanika, Iževsk - Moskva , NIC 2000, pp. 157, 243, 295.

[4] G. Šlichting: Teorija pograničnogo sloja, Moskva, Nauka, 1969, pp. 553.

[5] R. Hajossy a kolektív: Zjednodušený model nestacionárneho prúdenia plynu určený pre rýchle výpočty MÚ SAV-FMFI UK Bratislava, Výskumná správa, 2002.

[6] R. Hajossy a kolektív: Kompletný model turbulentného prúdenia reálneho plynu MÚ SAV-FMFI UK Bratislava, Výskumná správa, 2002.

[7] R. C. Reid, J. M. Prausnitz, T. K. Sherwood: The Properties of Gases and Liquids, 3rd ed. McGraw-Hill Book Comp., New York (preklad Svojstva gazov i židkostej Leningrad, Chimija 1982 pp. 133, 537; 205, 538; 128, 36.

[8] L. D. Landau, E. M. Lifšic: Statističeskaja fizika (Teoretičeskaja fizika V), Moskva, Nauka 1964, pp.71, 75.

[9] J. P. Novák, A. Maljevský, J. Dědek: Fyzikální a chemické vlastnosti plynů, par a plynných směsí pro aplikace v čs. plynárenství IV. Termodynamické vlastnosti reálných plynů a reálných plynných směsí. Plyn 61 (1981) č. 10, pp. 300–304.

[10] M. G. Sucharev, A. M. Karasevič: Technologičeskij rasčet i obespečenie nadežnosti gazo- i nefteprovodov Neft i gaz, Moskva 2000.

[11] Obščesojuznye normy technologičeskogo proektirovanija. Magistraľnye truboprovody. Č. 1. Gazoprovody. ONTP 51-1-85. Mingazprom Moskva 1985 pp. 220.

[12] I. N. Bronštejn, K. A. Semendjaev: Spravočnik po matematike,Gosizdat matfizlit, Moskva 1954 pp. 379, 351, 328, 329.

[13] R. Hajossy, A. Huček, P. Somora, T. Žáčik: Acceleration of the computations in gas transport optimization; 1st International Conference on Global Research and Education, Conference Proceedings of Inter-Academia 2002, UK Bratislava, JSMF, ISBN:80-968253-6-4, pp. 90–94.

[14] V. A. Plesniaev: Experience in costruction and adopting applications for general dispatching service needs, Abstracts of 2nd Int.Conf. AMADEUS 2003 Complex Pipeline Systems/High precission gas dynamics computations, Smolenice, Slovakia, October 6–9, 2003.