

# Simple Model of Particle Detector and Arrival Time

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**Abstract:** We present a very simple model of a particle detector and the proposal for calculation of the average value of the time of arrival.

## 1. Introduction

In physics there are questions which have remained unanswered for a long time. Besides others the definitions of the arrival time of particles in a quantum mechanical sense has belonged among them. There exist many approaches to this problem but none of them seems to be generally accepted [1–4].

The close relation between the arrival time and a certain model of the detector is demonstrated in [2]. This fact stimulated us to study the relation between the quantity  $P_D(\langle t_0, t \rangle)$ , which means the probability that a particle entered the detector during the time interval  $\langle t_0, t \rangle$  and the dynamics of a detector.

In [5] the detector is taken to be a two-state system (one state stands for detection and another for no-detection). This system is coupled to the environment consisting of large number of oscillators in their ground states. The coupling between the detector and oscillators is proportional to  $x$ . This means that if a particle enters the region  $x > 0$  the detector becomes coupled to the environment. It is shown [2] that the transition of the detector from no-detection state to detection one is irreversible (i.e. returning to the no-detection state is not possible) and the probability of detection at time  $t$  is proportional to

$$\int_0^t dt \int dx |\psi(x, t)|^2,$$

where  $\psi$  is the wave function of the considered particle.

Similar models of particle detectors can be found, e.g., in [6–8].

This paper is organized as follows. In Sec. 2 we shall present a very simple model of particle detector. The expression for probability that a particle entered the detector during the time interval  $\langle t_0, t \rangle$  will be proposed in Sec. 3. Sec. 4 contains the expression for average value of the arrival time. Sec. 5 contains several notes on problems we studied.

## 2. Simple Particle Detector Model

The model we shall present here is very simple, but in our opinion, it exhibits irreversible behaviour, too. The essence of our considerations consists in the analysis of the following situation.

Let a particle had been emitted at time  $t_0$  by a point source placed at  $\bar{x}_0$ . By this phrase we mean that the wave function of the particle at time  $t_0$  is  $\bar{x} = \bar{x}_0$  and moreover, we shall assume that  $\psi = 0$  in very small region surrounding the point  $\bar{x}_0$  only. The detector is placed at a large distance  $L$  from  $\bar{x}_0$ . Its volume is  $V_D$  and defines the spaceangle  $\Omega_0$  with respect to the point  $\bar{x}_0$ .

The detector is taken to be a two state system as mentioned above. One state,  $\psi_0$ , stands for no-detection and another,  $\psi_1$ , for detection. What quantity has forced the detector to the transition from  $\psi_0$  to  $\psi_1$ ? First of all, an emitted particle can enter the detector during the time interval  $\langle t_0, t \rangle$  with the probability  $P_D(\langle t_0, t \rangle)$  (The expression for  $P_D$  will be proposed in Sec. 3). The detector will register a particle only if that particle will interact with it. However, that interaction happens with the probability  $P_D(\langle t_0, t \rangle)$  only. On the basis of this we stand for opinion that the “power”, which has forced the detector to the transition from  $\psi_0$  to  $\psi_1$ , is the quantity

$$\frac{d}{dt} P_D(\langle t_0, t \rangle)$$

and the probability of the transition  $\psi_0 \rightarrow \psi_1$  cannot be greater than  $P_D(\langle t_0, t \rangle)$ . Moreover, it is natural to assume that  $P_D(\langle t_0, t \rangle)$  is not a decreasing function.

In the next we shall represent states  $\psi_0, \psi_1$  by columns

$$\psi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \psi_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and postulate the equation

$$i \frac{d}{dt} \psi = H \psi \quad (1)$$

with

$$H = A(t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

as the equation describing the dynamics of the detector (Up to now  $A(t)$  is unknown function.). As the initial state we shall always choose  $\psi_0$ .

Now from (1) we get

$$\psi(t) = \psi_0 \cos B(t) + i \psi_1 \sin B(t),$$

where

$$B = \int_{t_0}^t dt A(t).$$

The probability  $P_{reg}$  of finding the detector in the state  $|1\rangle$  is

$$P_{reg}(t) = \sin^2 B(t). \quad (2)$$

If the probability that a particle entered the detector during  $\langle t_0, t \rangle$  is negligible, one can hardly expect that  $P_{reg}$  will be large. Moreover, it seems to be natural to assume that

$$P_{reg} = P_D(\langle t_0, t \rangle).$$

On account of that we postulate the equation

$$P_{reg} = k P_D(\langle t_0, t \rangle), \quad (3)$$

where  $0 < k < 1$ .

As one can see

$$A(t) = \frac{d}{dt} \arcsin \sqrt{k P_D(\langle t_0, t \rangle)}.$$

The requirement (3) ensures the irreversible behaviour of the detector.

Evidently, this model is too simple. Yet it offers another look on the problem. Moreover, it inspired us to look for the expression for the quantity  $P_D(\langle t_0, t \rangle)$ .

### 3. The Expression for $P_D(\langle t_0, t \rangle)$

Let us now consider two events  $E_1$  and  $E_2$ . The event  $E_1$  means that the emitted particle has momentum  $\vec{p} = \vec{p}_D$  (it means that the half-line  $\vec{x} = \vec{x}_0 + \vec{p}s$  ( $s \in (0, \infty)$ ) passes through the detector). The event  $E_2$  means that the particle in question had been situated in an arbitrary  $t \in \langle t_0, t \rangle$  in the volume  $V_D$ . Now the probability  $P_D(\langle t_0, t \rangle)$  can be expressed

$$P_D(\langle t_0, t \rangle) = P(E_2 | E_1) = P(E_2 / E_1)P(E_1), \quad (4)$$

where  $P(E_1)$  is the probability that a particle has momentum  $\vec{p} = \vec{p}_D$  and  $P(E_2 / E_1)$  means that if  $E_1$  set in then  $E_2$  set in with the probability  $P(E_2 / E_1)$ .

If we confine ourselves to a non-relativistic particle and the time development of  $(\vec{x} = \vec{x}_0)$  is given by  $(t = t_0)$

$$(\vec{x}, t) = e^{iH(t-t_0)} (\vec{x} = \vec{x}_0) = \frac{d^3 \vec{p}}{(2\pi)^{3/2}} C(\vec{p}) e^{iE(t-t_0) - i\vec{p}(\vec{x} - \vec{x}_0)}$$

$$d \left( \bar{n} \right) \frac{p^2 dp}{(2)^{3/2}} C(p\bar{n}) e^{iE(t-t_0) - i\bar{p}(\bar{x}-\bar{x}_0)} = d \left( \bar{n} \right)^{(\bar{n})} (\bar{x}, t),$$

where we put  $\bar{p} = |\bar{p}| \bar{n} = p\bar{n}$

$$P(E_1) = \int_0^t d \left( \bar{n} \right) p^2 dp |C(p\bar{n})|^2 \quad (5)$$

and we propose, for  $P(E_2 / E_1)$ , the following expression

$$P(E_2 / E_1) = \frac{\int_{t_0}^t dt \int_{V_D} d^3 \bar{x} |d_D(\bar{x}, t)|^2}{\int_{t_0}^t dt \int_{V_D} d^3 \bar{x} |d_D(\bar{x}, t)|^2}, \quad (6)$$

where

$$d_D(\bar{x}, t) = \int_D d \left( \bar{n} \right)^{(\bar{n})} (\bar{x}, t).$$

As for the expression (6) we stand up for opinion that if a particle has momentum  $\bar{p} = p\bar{n}$  with the density of probability  $|C(\bar{p})|^2$ , then it can be described by the wave function  $d_D(\bar{x}, t)$  and can occur in the detector in time  $t$  with the probability density (with respect to time)

$$\int_{V_D} d^3 \bar{x} |d_D(\bar{x}, t)|^2.$$

It is evident that  $P_D(\langle t_0, t \rangle)$  is not a decreasing function and in the case  $C(p\bar{n}) = C(p)$  we get, for  $P_D(\langle t_0, \rangle)$ , the expression

$$P_D(\langle t_0, \rangle) = \frac{D}{4}$$

as necessary.

#### 4. Arrival time

If we reduce the detector to the point, say  $\bar{x}_D$ , then we can write

$$P(E_2 / E_1) = \frac{\int_{t_0}^t dt |d^{(\bar{n}_D)}(\bar{x}_D, t)|^2}{\int_{t_0}^t dt |d^{(\bar{n}_D)}(\bar{x}_D, t)|^2},$$

where

$$\bar{n}_D = \frac{\bar{x}_D - \bar{x}_0}{|\bar{x}_D - \bar{x}_0|}.$$

The quantity

$$\frac{dP_D(E_2/E_1)}{dt} \sim \left| {}^{(\bar{n}_D)}(\bar{x}_D, t) \right|^2$$

can be interpreted as the density (with respect to  $t$ ) of probability that a particle emitted at time  $t_0$  entered the detector at time  $t$ . Now we can define the average value of the time  $T = t - t_0$  of the arrival to the point  $\bar{x}_D$  by

$$\langle t - t_0 \rangle = \frac{\int_{t_0}^{\infty} dt (t - t_0) \left| {}^{(\bar{n}_D)}(\bar{x}_D, t) \right|^2}{\int_{t_0}^{\infty} dt \left| {}^{(\bar{n}_D)}(\bar{x}_D, t) \right|^2}. \quad (7)$$

## 5. Concluding remarks

The presented particle detector model is too simple to be considered realistic. Although it illustrates some basic features of a particle detector yet many questions have remained open. For example, if a considered source had emitted  $N$  particles at time  $t_0$  what is the probability that the detector will register  $n$  of them? This question is not resolved also in the other more realistic models. At the present time we are not able to state to what extent our approach can be useful for solving problems we study.

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