Contextual Representation and Reasoning with Description Logics

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1 Introduction

Despite most of the information available in the Semantic Web (SW) is context dependent, there is a lack of mechanism to qualify knowledge with the context in which it is supposed to hold. In the current practice, contextual information is often crafted in the ontology identifier or in the annotations, non of which affects reasoning. Extensions of the SW languages with specific mechanisms that allow to qualify knowledge, e.g., w.r.t. its provenance [6] or w.r.t. time and events [17], were proposed. Among other works that offer possible solutions [8,22,13], the most interesting are $\mathcal{ALC}_{\mathcal{ALC}}$ [14] and Metaview [24], however, a widely accepted approach has not yet been reached.

Instead of extending the current SW languages, we propose a shift of approach: to adapt the theories of context proposed by McCarthy [18] and well studied in AI [7,15,3]. We adopt the context-as-a-box metaphor [3] to represent context, in which a context is seen as a "box" containing knowledge in form of logical statements, whose boundaries are determined with contextual attributes (called *dimensions*) qualifying the knowledge inside the box. An example context representing knowledge about football in Italy in year 2010 is depicted in Fig. 1. We will most often rely on three dimensions: time, location and topic; but others were considered as well [15].

time = 2010, location = Italy, topic = football

$\mathsf{Team} \sqsubseteq {=}22\mathsf{has\_player}.\mathsf{Player}$
$\mathsf{Player} \sqsubseteq {\leqslant}1\mathsf{plays\_for}.\mathsf{Team}$
$\mathsf{Team}(\mathsf{Milan})$
$\mathsf{plays\_for}(\mathsf{Cassano}, \mathsf{Milan})$
...

Fig. 1. Italian national football league under the context as a box metaphor

To clarify the requirements for contextual representation in the SW, consider a scenario from the domain of football. Knowledge will be qualified with time, location, and the following topics: football (FB), FIFA world cups (FWC), national football leagues (NFL), world news (WN), and national news (NN). Suppose that all information about FWC and NFL should be included in FB, and for each nation all facts about its NFL should be included in its NN. Also all information about FWC should be included in WN. On the other hand, only a part of information about NFL should be included in WN (only that of worldwide interest). A well designed contextual representation formalism should support the following requirements:

knowledge about context: knowledge about contexts such as contextual dimensions and relations between contexts as for instance that one context is more specific than some other, should be explicitly represented and reasoned about. For example, we should be able to assert that the context of FWC in 2010 is more specific than the context of FB and WN in the same year;

contextually bounded facts: in each context we should be able to state facts with local effect that do not necessarily propagate everywhere, e.g., an axiom like "a player is a member of only one team" should be true in some contexts (e.g., FWC, NFL, for each year) but not in more general contexts like FB;

reuse/lifting of facts: be able to include "automatically" all the information contained in more specific contexts. For example, facts in FWC should be lifted up into the WN, and FB. This lifting should be done without spoiling locality of knowledge;

overlapping and varying domains: objects can be present in multiple contexts, but not necessarily in all contexts, e.g., a player can exist in both the FWC context and in the NFL contexts, but many players present in NFL will not be present in FWC;

inconsistency tolerance: two contexts may possibly contain contradicting facts. For instance NN of Italy could assert that "Cassano is the best player of the world", while at the same time the world news report that "Rooney is the best player of the world", without making the whole system inconsistent;

complexity invariance: the qualification of knowledge by context should not increase the complexity.

Based on these requirements, we propose a framework called *Contextualized Knowledge Repository* (CKR), build on top of the expressive description logic $\mathcal{SROIQ}$[3] [10] that is behind OWL 2 [26]. A CKR knowledge base is composed of DL knowledge bases, called *contexts*, each qualified by a set of contextual attributes that specify the boundaries within which the knowledge base is assumed to be true. Contexts are organized by a hierarchical *coverage* relation that regulates the propagation of knowledge between them. The paper defines the syntax and semantics of CKR; shows that concept satisfiability and subsumption are decidable with the complexity upper bound of 2NEXPTIME (i.e., same as for $\mathcal{SROIQ}$); and finally it provides a sound and complete Natural Deduction calculus that characterizes the propagation of knowledge between contexts. Proofs of our statements are available in [21].

2 Contextualized Knowledge Repository

Logical representation of contextual knowledge is based on two classes of formulae: one class to specify knowledge *within* contexts, and another to predicate *about* contexts. McCarthy [18] proposed to use a unique language for both types of knowledge, namely quantified modal logic. While this is optimal from the representational perspective, it easily leads to undecidability. At the opposite extreme there are approaches such

[3] Although we are able to represent any $\mathcal{SROIQ}$ axioms in CKR, to maintain decidability the framework currently excludes reflexivity and role disjointness axioms. See [21] for discussion.

as multi-context systems [7], distributed [4] or package-based description logics [2], where context structure is fixed and it is not possible to specify knowledge about contexts, which limits their practical applicability. We therefore propose an intermediate approach, by allowing to specify the context structure and properties in a (simple) logical *meta-language*, but avoiding to mix it with the *object-language* used within each context in order to maintain good computational properties.

The meta-language is used to specify context structure. It uses a *meta-vocabulary* Γ, a standard DL vocabulary that contains: (a) a set of individuals called *context identifiers*; (b) a finite set of roles $\mathbf{A} = \{A_1, \ldots, A_n\}$ called *dimensions*; (c) for each dimension $A \in \mathbf{A}$ a set of individuals D_A called *dimensional values* and a role $\prec_A$ called *coverage relation*. The number of dimensions $n = |\mathbf{A}|$ is assumed to be a fixed constant. This will be important in order not to introduce additional complexity blow up. Also, relevant research on contextual dimensions suggests that their number is usually very limited [16]. The meta-assertions of the form $A(\mathcal{C}, d)$ for a context identifier $\mathcal{C}$ and some $d \in D_A$ (e.g., $\mathsf{time}(\mathsf{c0}, 2010)$), state that the value of the dimension A of the context $\mathcal{C}$ is d. The meta-assertions of the form $d \prec_A e$ (e.g., $\mathsf{Italy} \prec_{\mathsf{space}} \mathsf{Europe}$) state that the value d of the dimensions A is covered by the value e. Depending on the dimension, the coverage relation has different intuitive meanings, e.g., if A is space then the coverage relation is topological containment, if A is topic then it is topic specificity.

A (full) *dimensional vector* $\mathbf{d}$ is a set of assignments $\{A_1{:=}d_{A_1}, \ldots, A_n{:=}d_{A_n}\}$, with $d_{A_i} \in D_{A_i}$ for each $1 \leq i \leq n$. Note that d_{A_i} $(e_{A_i}, \ldots)$ denotes the actual value that $\mathbf{d}$ $(\mathbf{e}, \ldots)$ assigns to the dimension A_i. $\mathfrak{D}_\Gamma$ is the set of all dimensional vectors of Γ. For any $\mathbf{B} \subseteq \mathbf{A}$, $\mathbf{d_B} = \{B{:=}d_B \mid B \in \mathbf{B}\}$ and if $\mathbf{B} \subset \mathbf{A}$, then $\mathbf{d_B}$ is called *partial dimensional vector*. Note that $\mathbf{d_A} = \mathbf{d}$. Given two (partial) dimensional vectors $\mathbf{d_B}$ and $\mathbf{e_C}$, the completion of $\mathbf{d_B}$ w.r.t. $\mathbf{e_C}$ is $\mathbf{d_B}{+}\mathbf{e_C} = \mathbf{d_B} \cup \{(A{:=}e_A) \in \mathbf{e_C} \mid A \notin \mathbf{B}\}$.

The object-language is used to specify knowledge inside the contexts. It uses an *object-vocabulary*, obtained from any standard DL vocabulary Σ (containing individuals, concepts, and roles) by closing it w.r.t. what we call *concept/role qualification*. That is, for every concept/role symbol X of Σ and every (partial) dimensional vector $\mathbf{d_B}$, a new concept/role symbol $X_{\mathbf{d_B}}$, called the *qualification of X w.r.t.* $\mathbf{d_B}$, is added to Σ. Qualified symbols are necessary for cross context semantic reference, e.g., the concept of "Italian professor" in the context of France will be formalized by $\mathsf{Professor}_{\mathsf{location}:=\mathsf{Italy}}$. If not ambiguous we will omit the attribute name, using e.g. $\mathsf{Professor}_{\mathsf{Italy}}$ instead of $\mathsf{Professor}_{\mathsf{location}:=\mathsf{Italy}}$.

Definition 1 (Context). *A* context $\mathcal{C}$ *on the meta/object-vocabulary pair* $\langle \Gamma, \Sigma \rangle$ *is a triple* $\langle \mathrm{id}(\mathcal{C}), \dim(\mathcal{C}), \mathrm{K}(\mathcal{C}) \rangle$ *where:*

1. $\mathrm{id}(\mathcal{C})$ *is a* context identifier *of* Γ*;*
2. $\dim(\mathcal{C})$ *is a full dimensional vector of* $\mathfrak{D}_\Gamma$*;*
3. $\mathrm{K}(\mathcal{C})$ *is a DL knowledge base over* Σ*.*

Note that while symbols appearing inside contexts can possibly be qualified with partial dimensional vectors, $\dim(\mathcal{C})$, the dimensional vector of the context $\mathcal{C}$, is never partial. We use the notation $\mathcal{C}_\mathbf{d}$ to denote a context with $\dim(\mathcal{C}) = \mathbf{d}$.

Definition 2 (Contextualized Knowledge Repository). *A* contextualized knowledge repository (CKR) *on a meta/object-vocabulary pair* $\langle \Gamma, \Sigma \rangle$ *is a pair* $\mathfrak{K} = \langle \mathfrak{M}, \mathfrak{C} \rangle$ *where:*

1. *$\mathfrak{C}$ is a set of contexts on $\langle \Gamma, \Sigma \rangle$, one for each context identifier of Γ;*
2. *$\mathfrak{M}$, called meta-knowledge, is a DL knowledge base on Γ where*
   (a) every $A \in \mathbf{A}$ is a functional role;
   (b) for every $\mathcal{C}_{\mathbf{d}} \in \mathfrak{C}$, and every $A \in \mathbf{A}$, $\mathfrak{M} \models A(\mathrm{id}(\mathcal{C}_{\mathbf{d}}), d_A)$;
   (c) for every $A \in \mathbf{A}$, the relation $\{d \prec_A d' \mid \mathfrak{M} \models \prec_A(d, d')\}$ is a strict partial order on D_A.

For a CKR $\mathfrak{K}$, $\mathbf{B} \subseteq \mathbf{A}$, dimensional vectors $\mathbf{d}$, $\mathbf{e}$, and contexts $\mathcal{C}$, $\mathcal{C}'$ we say: (a) $\mathbf{e}$ *covers* $\mathbf{d}$ w.r.t. $\mathbf{B}$ (denoted $\mathbf{d} \prec_{\mathbf{B}} \mathbf{e}$) if $\mathfrak{M} \models \prec_B(d_B, e_B)$ for every $B \in \mathbf{B}$; (b) $\mathbf{e}$ *covers* $\mathbf{d}$ (denoted $\mathbf{d} \prec \mathbf{e}$) if $\mathbf{d} \prec_{\mathbf{A}} \mathbf{e}$; (c) $\mathcal{C}'$ covers $\mathcal{C}$ (denoted $\mathcal{C} \prec \mathcal{C}'$) if $\dim(\mathcal{C}) \prec \dim(\mathcal{C}')$.

If one context covers another it means that its perspective is broader. We will see that this is reflected in the semantics and the domain of the broader context always contains the domain of the narrower context. The coverage induces a hierarchical organization of contexts in each CKR. For instance Fig. 2 depicts the context coverage induced from the following coverage relations between dimensional values:

$$\begin{array}{lll} \mathsf{FWC} \prec_{\mathsf{topic}} \mathsf{WN} & \mathsf{NFL} \prec_{\mathsf{topic}} \mathsf{FB} & \mathsf{africa} \prec_{\mathsf{space}} \mathsf{world} \\ \mathsf{FWC} \prec_{\mathsf{topic}} \mathsf{FB} & \mathsf{NFL} \prec_{\mathsf{topic}} \mathsf{NN} & \mathsf{italy} \prec_{\mathsf{space}} \mathsf{world} \end{array}$$

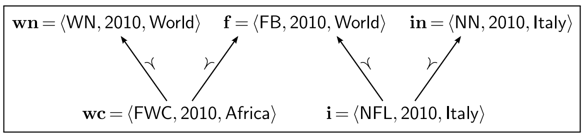

Fig. 2. Coverage relation between contexts

Besides for the coverage relation, which is explicitly expressed in CKR, there are other relations between contexts [3]. We chose to represent the coverage relation because many other relations between contexts can be axiomatized on top of it. For instance the temporal relation between contexts can be axiomatized via GCI axioms in a broader context, e.g., to assert that everyone who is a professor in 2011 typically is a professor also in 2012 (i.e., none of the years covers the other, but instead they are consecutive), we can add the axiom $\mathsf{Professor}_{2011} \sqsubseteq \mathsf{Professor}_{2012}$ into some context that covers both 2011 and 2012 (e.g., one associated with the decade 2011–2020).

A model of a CKR is composed of local models for each context that must satisfy some additional restrictions. Given a CKR $\mathfrak{K}$, a model for a context $\mathcal{C}_{\mathbf{d}}$ is a pair $\mathcal{I}_{\mathbf{d}} = \left\langle \Delta_{\mathbf{d}}, \cdot^{\mathcal{I}_{\mathbf{d}}} \right\rangle$ such that $\mathcal{I}_{\mathbf{d}} \models \mathrm{K}(\mathcal{C}_{\mathbf{d}})$ in the usual DL sense [10] with two exceptions: (a) $\Delta_{\mathbf{d}}$ may also be empty; (b) $\cdot^{\mathcal{I}_{\mathbf{d}}}$ is not required to interpret individuals of Σ that do not occur in $\mathrm{K}(\mathcal{C})$. In the rest of the paper, whenever we write $\phi^{\mathcal{I}_{\mathbf{d}}}$ for any expression ϕ, we will also mean that $\mathcal{I}_{\mathbf{d}}$ is defined on all constants occurring in ϕ.

Definition 3 (CKR Model). *A model of a CKR $\mathfrak{K}$ is a family $\mathfrak{I} = \{\mathcal{I}_{\mathbf{d}}\}_{\mathbf{d} \in \mathfrak{D}_\Gamma}$ of local models such that for all $\mathbf{d}$, $\mathbf{e}$, and $\mathbf{f}$, for every atomic concept A, atomic role R, atomic concept/role X and individual a:*

1. $(\top_{\mathbf{d}})^{\mathcal{I}_{\mathbf{f}}} \subseteq (\top_{\mathbf{e}})^{\mathcal{I}_{\mathbf{f}}}$ *if* $\mathbf{d} \prec \mathbf{e}$
2. $(A_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}} \subseteq (\top_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}}$
3. $(R_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}} \subseteq (\top_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}} \times (\top_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}}$

4. $a^{\mathcal{I}_{\mathbf{d}}} = a^{\mathcal{I}_{\mathbf{e}}}$, *given* $\mathbf{d} \prec \mathbf{e}$, *either if* $a^{\mathcal{I}_{\mathbf{d}}}$ *is defined,*
   or if $a^{\mathcal{I}_{\mathbf{e}}}$ *is defined and* $a^{\mathcal{I}_{\mathbf{e}}} \in \Delta_{\mathbf{d}}$
5. $(X_{\mathbf{d_B}})^{\mathcal{I}_{\mathbf{e}}} = (X_{\mathbf{d_B}+\mathbf{e}})^{\mathcal{I}_{\mathbf{e}}}$
6. $(X_{\mathbf{d}})^{\mathcal{I}_{\mathbf{e}}} = (X_{\mathbf{d}})^{\mathcal{I}_{\mathbf{d}}}$ *if* $\mathbf{d} \prec \mathbf{e}$
7. $(A_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}} = (A_{\mathbf{f}})^{\mathcal{I}_{\mathbf{e}}} \cap \Delta_{\mathbf{d}}$ *if* $\mathbf{d} \prec \mathbf{e}$
8. $(R_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}} = (R_{\mathbf{f}})^{\mathcal{I}_{\mathbf{e}}} \cap (\Delta_{\mathbf{d}} \times \Delta_{\mathbf{d}})$ *if* $\mathbf{d} \prec \mathbf{e}$
9. $\mathcal{I}_{\mathbf{d}} \models \mathrm{K}(\mathcal{C}_{\mathbf{d}})$

The semantics takes care that local domains respect the coverage hierarchy (condition 1). Given contexts $\mathcal{C}_{\mathbf{d}} \prec \mathcal{C}_{\mathbf{e}}$, if an individual a occurs in the narrower context then it must be defined also in the broader context with the same meaning; if a only occurs in the broader context however, it does not have to be defined in the narrower one (condition 4). The interpretation of any concept or role qualified with some $\mathbf{f} \in \mathfrak{D}_{\Gamma}$ is always roofed under $(\top_{\mathbf{f}})^{\mathcal{I}_{\mathbf{d}}}$ in any context $\mathcal{C}_{\mathbf{d}}$, i.e., in a sense $\top_{\mathbf{f}}$ represents the $\top$ of $\mathcal{C}_{\mathbf{f}}$ inside $\mathcal{C}_{\mathbf{d}}$ (conditions 2 and 3). This is always true regardless of the relation between $\mathcal{C}_{\mathbf{f}}$ and $\mathcal{C}_{\mathbf{d}}$. If $\mathcal{C}_{\mathbf{d}} \prec \mathcal{C}_{\mathbf{e}}$ then the interpretation of any concept and role $X_{\mathbf{f}}$ in these contexts must be equal modulo the domain of the narrower context (conditions 7 and 8). Treatment of partially qualified symbols is done in condition 5: missing values are always taken from the current context in which the symbol appears. Therefore in the end all symbols (even those with empty qualifying vector) are treated as fully qualified by the semantics. Finally, for each CKR model we require that each local interpretation $\mathcal{I}_{\mathbf{d}}$ is a model of $\mathcal{C}_{\mathbf{d}}$ in the usual sense for DL (condition 9). Finally notice that $\perp_{\mathbf{d}}$ is always interpreted in the empty set. Therefore we can simplify the notation by using just $\perp$.

3 Reasoning in CKR

In the following we devise a proof theoretical characterization of CKR entailment in the natural deduction (ND) style [20], with special focus on the rules for transferring knowledge across contexts. We decide to characterize CKR entailment with ND, since ND provides a clear intuition on how knowledge propagates across contexts. This allows to show interesting properties of CKR reasoning like the fact that (a) in consistent CKR, unconnected contexts do not interact (b) propagation always follows the coverage relation, and other similar properties. ND formalisms also provide a first base for the development of a forward reasoning algorithm, which constitutes a natural extension of the forward local reasoning supported by OWLIM, the platform on top of which a first version of CKR with limited expressive capacity of RDFS has been implemented [9].

We now briefly introduce ND, for more details see [20]. A ND calculus is a set of inference rules of the form:

$$\frac{\alpha_1 \ \cdots \ \alpha_n \quad \begin{matrix}[B_{n+1}] \\ \alpha_{n+1}\end{matrix} \quad \cdots \quad \begin{matrix}[B_{n+m}] \\ \alpha_{n+m}\end{matrix}}{\alpha}\,\rho \tag{1}$$

with $n, m \geq 0$, where α_i and α are formulae and B_i are sets of formulae. The α_i's are the *premises* of ρ, α is the conclusion and the B_i's are the *assumptions discharged* by ρ. A *deduction* of α depending on a set of formulae Φ is a tree rooted in α inductively constructed starting from a set of assumptions in Φ by applying the inference rules. More formally: a formula α is a deduction of α depending on $\{\alpha\}$; if for each $1 \leq i \leq n+m$, Π_i is a deduction of α_i depending on Φ_i and the calculus contains a rule of the form (1), then $\frac{\Pi_1 \cdots \Pi_{n+m}}{\alpha}$ is a deduction of α depending on $(\bigcup_{i=1}^{n} \Phi_i) \cup \left(\bigcup_{i=n+1}^{n+m} (\Phi_i \setminus B_i)\right)$. A formula α is derivable from Φ if there is a deduction of α depending on a subset of Φ.

CKR reasoning tasks are, as in any DL, concept satisfiability and entailment; however in CKR these tasks are relativized w.r.t. a context. A CKR $\mathfrak{K}$ is $\mathbf{d}$-satisfiable (a concept C is $\mathbf{d}$-satisfiable w.r.t. $\mathfrak{K}$) if there exists a model $\mathfrak{I}$ of $\mathfrak{K}$ with $\Delta_{\mathbf{d}} \neq \emptyset$ ($C^{\mathcal{I}_{\mathbf{d}}} \neq \emptyset$). A formula ϕ is $\mathbf{d}$-entailed by $\mathfrak{K}$ (denoted $\mathfrak{K} \models \mathbf{d} : \phi$) if $\mathcal{I}_{\mathbf{d}} \models \phi$ in every model $\mathfrak{I}$ of $\mathfrak{K}$.

Reasoning rules in the ND calculus for CKR allow to deduce conclusions in one of the contexts based on evidence from other contexts, they are therefore a kind of *bridge rules* [7]. As an example consider the following simple bridge rule:

$$\frac{\mathbf{d} : A \sqsubseteq B \quad \mathbf{d} \prec \mathbf{e}}{\mathbf{e} : A_{\mathbf{d}} \sqsubseteq B_{\mathbf{d}}} \tag{2}$$

It implies that whenever $A \sqsubseteq B$ is true in a context $C_{\mathbf{d}}$ such that $\mathbf{d} \prec \mathbf{e}$, then $A_{\mathbf{d}} \sqsubseteq B_{\mathbf{d}}$ should be true in $C_{\mathbf{e}}$. The rule is indeed sound thanks to conditions 5 and 6 of Definition 3 that impose that in any CKR model $\mathfrak{I}$ the interpretation of A and B in $\mathcal{I}_{\mathbf{d}}$ coincide respectively with the interpretations of $A_{\mathbf{d}}$ and $B_{\mathbf{d}}$ in $\mathcal{I}_{\mathbf{e}}$. The rationale of rule (2) is that a statement in a narrower context, can be embedded into a larger context, by applying a transformation that preserves its semantics.

We generalize this idea by introducing the notion of embedding between DL knowledge bases. An embedding is a function that translates expressions from one vocabulary to another in a suitable manner. The input vocabulary Σ will be split into Σ_c (symbols fully specified w.r.t. the current context) and Σ_e (symbols external to the current context) and each of the sets of symbols will be translated differently. More formally: let Σ and Σ' be two DL alphabets, $\Sigma = \Sigma_c \uplus \Sigma_e$, $\top \in \Sigma_c$. A *DL embedding* is a total function $f : \Sigma \to \Sigma'$ that maps individuals, atomic concepts, and atomic roles of Σ to individuals, atomic concepts, and atomic roles of Σ' respectively. For every embedding f the extension f^* that maps complex expressions and axioms over Σ into complex expressions and axioms over Σ' is recursively defined on top of f as given in Table 1.

Two DL-interpretations $\mathcal{I}$ and $\mathcal{I}'$ of Σ and Σ' respectively are said to *comply with* the DL embedding f if: (a) $a^{\mathcal{I}} = f(a)^{\mathcal{I}'}$ for each individual a of Σ such that $a^{\mathcal{I}}$ is defined; (b) $X^{\mathcal{I}} = f(X)^{\mathcal{I}'}$ for each concept/role $X \in \Sigma_c$; (c) $A^{\mathcal{I}} = f(A)^{\mathcal{I}'} \cap f(\top)^{\mathcal{I}'}$ for each concept $A \in \Sigma_e$; (d) $R^{\mathcal{I}} = f(R)^{\mathcal{I}'} \cap f(\top)^{\mathcal{I}'} \times f(\top)^{\mathcal{I}'}$ for each role $R \in \Sigma_e$.

Lemma 1. *If $\mathcal{I}$ and $\mathcal{I}'$ comply with the DL-embedding $f : \Sigma \to \Sigma'$ then: (a) for every concept/role X, $X^{\mathcal{I}} = (f^*(X))^{\mathcal{I}'}$; (b) for every axiom ϕ, $\mathcal{I} \models \phi$ iff $\mathcal{I}' \models f^*(\phi)$.*

The specific embedding that will be instrumental in order to characterize the logical consequence in CKR is now introduced as the $@\mathbf{d}$ operator.

Definition 4 **($@\mathbf{d}$ operator).** *Given a CKR $\mathfrak{K}$ over $\langle \Gamma, \Sigma \rangle$, for every $\mathbf{d} \in \mathfrak{D}_\Gamma$, the operator $(\cdot)@\mathbf{d}$ is defined as $f^*_{\mathbf{d}}(\cdot)$, using the embedding $f_{\mathbf{d}}$ of Σ into itself such that: (a) $f_{\mathbf{d}}(a) = a$ for every individual a; (b) $f_{\mathbf{d}}(X_{\mathbf{d}'_{\mathbf{B}}}) = X_{\mathbf{d}'_{\mathbf{B}}+\mathbf{d}}$ for every concept/role $X_{\mathbf{d}'_{\mathbf{B}}} \in \Sigma$; (c) $\Sigma_c = \{X_{\mathbf{d}'_{\mathbf{B}}} \in \Sigma \mid \mathbf{d}'_{\mathbf{B}} \preceq \mathbf{d}_{\mathbf{B}}\}$; (d) $\Sigma_e = \Sigma \setminus \Sigma_c$.*

$f^*(A) = \begin{cases} f(A) & \text{if } A \in \Sigma_c \\ f(\top) \sqcap f(A) & \text{if } A \in \Sigma_e \end{cases}$	$f^*(\bot) = \bot$
$f^*(R) = \begin{cases} f(R) & \text{if } R \in \Sigma_c \\ f(I) \circ f(R) \circ f(I) & \text{if } R \in \Sigma_e \end{cases}$	$f^*(C \sqcap D) = f^*(C) \sqcap f^*(D)$
$f^*(\neg C) = f(\top) \sqcap \neg f^*(C)$	$f^*(C \sqcup D) = f^*(C) \sqcup f^*(D)$
$f^*(\exists R.C) = \begin{cases} \exists f(R).f^*(C) & \text{if } R \in \Sigma_c \\ f(\top) \sqcap \exists f(R).f^*(C) & \text{if } R \in \Sigma_e \end{cases}$	$f^*(\{a\}) = \{f(a)\}$
$f^*(\forall R.C) = \begin{cases} f(\top) \sqcap \forall f(R).f^*(C) & \text{if } R \in \Sigma_c \\ f(\top) \sqcap \forall f(R).(\neg f(\top) \sqcup f^*(C)) & \text{if } R \in \Sigma_e \end{cases}$	$f^*(\leqslant nR.C) = f(\top) \sqcap \leqslant nf(R).f^*(C)$
$f^*(\geqslant nR.C) = \begin{cases} \geqslant nf(R).f^*(C) & \text{if } R \in \Sigma_c \\ f(\top) \sqcap \geqslant nf(R).f^*(C) & \text{if } R \in \Sigma_e \end{cases}$	$f^*(R^-) = (f(R))^-$
$f^*(\exists R.\text{Self}) = \begin{cases} \exists f(R).\text{Self} & \text{if } R \in \Sigma_c \\ f(\top) \sqcap \exists f(R).\text{Self} & \text{if } R \in \Sigma_e \end{cases}$	$f^*(R \circ S) = f^*(R) \circ f^*(S)$
	$f^*(C(a)) = f^*(C)(f(a))$
	$f^*(R(a,b)) = f(R)(f(a), f(b))$
	$f^*(C \sqsubseteq D) = f^*(C) \sqsubseteq f^*(D)$
	$f^*(R \sqsubseteq S) = f^*(R) \sqsubseteq f(S)$
	$f^*(a = b) = f(a) = f(b)$
	$f^*(a \neq b) = f(a) \neq f(b)$

Table 1. DL-embedding on complex expressions and axioms; note: I is the identity role, which can be easily added to $\mathcal{SROIQ}$ using the axioms $\top \sqsubseteq \exists I.\text{Self}$ and $\top \sqsubseteq \leqslant 1 I.\top$

For instance if the concept Team occurs in $\mathcal{C}_{\mathbf{d}}$ with $\mathbf{d} = \langle \mathsf{FWC}, 2010, \mathsf{Africa} \rangle$, it belongs to Σ_c as $\mathbf{d}'_{\mathbf{B}} \preceq \mathbf{d}_{\mathbf{B}}$ for $\mathbf{B} = \emptyset$. Hence $\mathsf{Team}@\mathbf{d} = \mathsf{Team}_{\mathsf{FWC,2010,Africa}}$. This is natural, as in a context wider than $\mathcal{C}_{\mathbf{d}}$ the concept $\mathsf{Team}_{\mathsf{FWC,2010,Africa}}$ is fully defined by Team in $\mathcal{C}_{\langle \mathsf{FWC,2010,Africa} \rangle}$. But $\mathsf{NationalTeam}_{\mathsf{FB}} \notin \Sigma_c$ as $\mathsf{FB} \not\preceq \mathsf{FWC}$. Hence $\mathsf{NationalTeam}_{\mathsf{FB}}@\langle \mathsf{FWC}, 2010, \mathsf{Africa} \rangle = \mathsf{NationalTeam}_{\mathsf{FB,2010,Africa}} \sqcap \top_{\mathsf{FWC,2010,Africa}}$. Intuitively, to embed $\mathsf{NationalTeam}_{\mathsf{FB}}$ from $\mathcal{C}_{\langle \mathsf{FWC,2010,Africa} \rangle}$ into a broader context one must restrict it to $\top_{\mathsf{FWC,2010,Africa}}$ because its interpretation in the broader context may be broader.

A ND system for a CKR $\mathfrak{K} = \langle \mathfrak{C}, \mathfrak{M} \rangle$ over $\langle \Gamma, \Sigma \rangle$ is shown in Table 2. Here α_i are either object-formulae of the form $\mathbf{d} : \phi$ ($\mathbf{d} \in \mathfrak{D}_\Gamma$, ϕ is a DL formula over Σ) or meta-formulae μ over Γ, while α and β_i are always object-formulae. A formula $\mathbf{d} : \phi$ is derivable from $\mathfrak{K}$ and Φ (denoted $\mathfrak{K}, \Phi \vdash \mathbf{d} : \phi$) if it is derivable from $\Phi \cup \{\mathbf{d} : \phi \mid \phi \in \mathcal{C}_{\mathbf{d}}, \mathbf{d} \in \mathfrak{D}_\Gamma\} \cup \{\mu \mid \mathfrak{M} \models \mu\}$ using the ND rules of Table 2. A shorthand $\mathfrak{K} \vdash \mathbf{d} : \phi$ is used for $\mathfrak{K}, \emptyset \vdash \mathbf{d} : \phi$.

Theorem 1 (Soundness and Completeness). *$\mathfrak{K} \vdash \mathbf{d} : \phi$ if and only if $\mathfrak{K} \models \mathbf{d} : \phi$.*

Let us show some example deductions in the CKR $\mathfrak{K}$ with structure depicted in Fig 2. Example 1 shows how knowledge is propagated from $\mathcal{C}_{\mathbf{wc}}$ to $\mathcal{C}_{\mathbf{i}}$ via the common super-context $\mathcal{C}_{\mathbf{f}}$, and Example 2 shows how knowledge is propagated from $\mathcal{C}_{\mathbf{wn}}$ to $\mathcal{C}_{\mathbf{f}}$ via the common sub-context $\mathcal{C}_{\mathbf{wc}}$. Finally Example 3 shows how contradicting knowledge can coexist in different separated context.

Example 1. The following deduction shows how the subsumption $\mathbf{wc} : \mathsf{WChamp} \sqsubseteq \mathsf{Player}$ propagates from the FWC context $\mathcal{C}_{\mathbf{wc}}$ to the Italian NFL context $\mathcal{C}_{\mathbf{i}}$. Notice that the result of this deduction, i.e., $\mathbf{i} : \mathsf{WChamp}_{\mathbf{wc}} \sqsubseteq \mathsf{Player}_{\mathbf{wc}}$, in the context $\mathcal{C}_{\mathbf{i}}$ is weaker than the premise as it holds only on the set of players of the Italian National League. In other words, the knowledge shifting from $\mathcal{C}_{\mathbf{wc}}$ to $\mathcal{C}_{\mathbf{i}}$ is limited by the domain of interpretation of $\mathcal{C}_{\mathbf{i}}$.

(1)	$\mathbf{wc} : \mathsf{WChamp} \sqsubseteq \mathsf{Player}$	premise
(2)	$\mathbf{f} : (\mathsf{WChamp} \sqsubseteq \mathsf{Player})@\mathbf{wc}$	Pop, $\mathbf{wc} \preceq \mathbf{f}$
(3)	$\mathbf{f} : \mathsf{WChamp}_{\mathbf{wc}} \sqsubseteq \mathsf{Player}_{\mathbf{wc}}$	by @
(4)	$\mathbf{f} : \mathsf{WChamp}_{\mathbf{wc}} \sqcap \top_{\mathbf{i}} \sqsubseteq \mathsf{Player}_{\mathbf{wc}} \sqcap \top_{\mathbf{i}}$	LReas
(5)	$\mathbf{f} : (\mathsf{WChamp}_{\mathbf{wc}} \sqsubseteq \mathsf{Player}_{\mathbf{wc}})@\mathbf{i}$	by @
(6)	$\mathbf{i} : \mathsf{WChamp}_{\mathbf{wc}} \sqsubseteq \mathsf{Player}_{\mathbf{wc}}$	Push, $\mathbf{i} \preceq \mathbf{f}$

$$\frac{\mathbf{d}:\phi_1 \ \ldots \ \mathbf{d}:\phi_n \quad \{\phi_1 \ldots \phi_n\} \models \phi}{\mathbf{d}:\phi}\ \mathsf{LReas} \qquad \frac{\mathbf{d}:\bot(a)}{\mathbf{e}:\top \sqsubseteq \bot}\ \mathsf{Bot}$$

$$\frac{\mathbf{d} \preceq \mathbf{e}}{\mathbf{f}:A_{\mathbf{d}} \sqsubseteq \top_{\mathbf{e}}} \quad \frac{-}{\mathbf{f}:\exists R_{\mathbf{d}}\top \sqsubseteq \top_{\mathbf{d}}} \quad \frac{-}{\mathbf{f}:\top \sqsubseteq \forall R_{\mathbf{d}}\top_{\mathbf{d}}}\ \mathsf{Top} \qquad \frac{\begin{array}{c}[\mathbf{d}:\top(a)]\\ \mathbf{d}:\top \sqsubseteq \bot\end{array}}{\mathbf{d}:\top \sqsubseteq \bot}\ a\mathsf{E}$$

$$\frac{\mathbf{e}:\phi@\mathbf{d} \quad \mathbf{e}:\top_{\mathbf{d}}(a_1) \ \cdots \ \mathbf{e}:\top_{\mathbf{d}}(a_n) \quad \mathbf{d} \preceq \mathbf{e}}{\mathbf{d}:\phi}\ \mathsf{Push} \qquad \frac{\mathbf{d}:\phi \quad \mathbf{d} \preceq \mathbf{e}}{\mathbf{e}:\phi@\mathbf{d}}\ \mathsf{Pop}$$

$$\frac{\mathbf{d}:A \sqcup B(x) \quad \begin{array}{c}[\mathbf{d}:A(x)]\\ \mathbf{e}:\phi\end{array} \quad \begin{array}{c}[\mathbf{d}:B(x)]\\ \mathbf{e}:\phi\end{array}}{\mathbf{e}:\phi}\ \sqcup\mathsf{E} \qquad \frac{\mathbf{d}:\exists R.A(x) \quad \begin{array}{c}[\mathbf{d}:R(x,y),\ \mathbf{d}:A(y)]\\ \mathbf{e}:\phi\end{array}}{\mathbf{e}:\phi}\ \exists\mathsf{E}$$

$$\frac{\mathbf{d}:\geqslant nR.A(x) \quad \begin{array}{c}[\mathbf{d}:y_i \neq y_j,\ \mathbf{d}:R(x,y_i),\ \mathbf{d}:A(y_i)]_{1 \leq i \neq j \leq n}\\ \mathbf{e}:\phi\end{array}}{\mathbf{e}:\phi}\ (\geqslant n)\mathsf{E}$$

Restrictions: **1)** LReas can be applied if every individual occurring in ϕ occurs in a ϕ_i for some $1 \leq i \leq n$; **2)** in the Push rule $a_1, \ldots, a_n$ are assumed to be all individuals occuring in ϕ; **3)** the individuals a, y, and y_i, $1 \leq i \leq n$, occuring in aE, $\exists$E, and $(\geqslant n)$E are new, not occuring elsewhere in $\mathfrak{K}$ and the proof apart from the assumptions discharded by these rules.

Table 2. CKR inference rules

Example 2. The following deduction shows how $\mathbf{wn}$: $\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro}$ (i.e., every football player mentioned in the world news is a professional) propagates from $\mathcal{C}_{\mathbf{wn}}$ to $\mathcal{C}_{\mathbf{f}}$, trough the common sub-context $\mathcal{C}_{\mathbf{wc}}$.

(1)	$\mathbf{wn}$: $\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro}$	premise
(2)	$\mathbf{wn}$: $(\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro})@\mathbf{wn}$	Pop, $\mathbf{wn} \preceq \mathbf{wn}$
(3)	$\mathbf{wn}$: $\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}}$	by @
(4)	$\mathbf{wn}$: $\mathsf{Player}_{\mathbf{f}} \sqcap \top_{\mathbf{wc}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}} \sqcap \top_{\mathbf{wc}}$	by LReas
(5)	$\mathbf{wc}$: $\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}}$	Push, $\mathbf{wc} \preceq \mathbf{wn}$
(6)	$\mathbf{f}$: $\mathsf{Player}_{\mathbf{f}} \sqcap \top_{\mathbf{wc}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}} \sqcap \top_{\mathbf{wc}}$	Pop, $\mathbf{wc} \preceq \mathbf{f}$
(7)	$\mathbf{f}$: $\mathsf{Player}_{\mathbf{f}} \sqcap \top_{\mathbf{wc}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}}$	LReas

Notice that we did not infer that $\mathbf{f}$: $\mathsf{Player}_{\mathbf{f}} \sqsubseteq \mathsf{Pro}_{\mathbf{wn}}$, i.e., that every Player of football is a professional player in the world news, but the fact that this subsumption holds only on the players of the FWC domain.

Example 3. Suppose that the Italian News context $\mathcal{C}_{\mathbf{in}}$ contains the facts that Rooney does not take part to the Italian league in 2010, i.e., $\neg\top_{\mathbf{i}}(\mathsf{Rooney})$, and that he is not considered a good football player, i.e., $\neg\mathsf{GoodPlayer}_{\mathbf{f}}(\mathsf{Rooney})$. Suppose also that the world news context $\mathcal{C}_{\mathbf{wn}}$ contains the opposite evaluation, i.e. $\mathsf{GoodPlayer}_{\mathbf{f}}(\mathsf{Rooney})$. In the CKR of Fig. 2, these two contradicting statements do not necessarily lead to inconsistency. Indeed, to derive inconsistency one has to find a context where to combine the two contradicting facts. However, to transfer the facts $\mathbf{wn}$: $\mathsf{GoodPlayer}_{\mathbf{f}}(\mathsf{Rooney})$ and $\mathbf{in}$: $\neg\mathsf{GoodPlayer}_{\mathbf{f}}(\mathsf{Rooney})$ into a common context, one have to pass through $\mathcal{C}_{\mathbf{i}}$. But the fact that Rooney is not an individual of $\mathcal{C}_{\mathbf{i}}$ disables any inference about Rooney in $\mathcal{C}_{\mathbf{i}}$. Model-theoretically we admit CKR models where $\mathsf{Rooney}^{\mathcal{I}_{\mathbf{wn}}} \neq \mathsf{Rooney}^{\mathcal{I}_{\mathbf{in}}}$.

4 Decidability and Complexity

Decidability of CKR entailment is proved indirectly by embedding a CKR into a single DL knowledge base, we will again use DL-embeddings. Given a meta-vocabulary Γ and an object-vocabulary $\Sigma = N_{\mathrm{C}} \uplus N_{\mathrm{R}} \uplus N_{\mathrm{I}}$, a DL-vocabulary $\#(\Gamma, \Sigma) = \#N_{\mathrm{C}} \uplus \#N_{\mathrm{R}} \uplus \#N_{\mathrm{I}}$ is defined as follows: $\#N_{\mathrm{C}} = \{A^{\mathbf{e}}_{\mathbf{d}} \mid A \in N_{\mathrm{C}} \wedge \mathbf{d}, \mathbf{e} \in \mathfrak{D}_{\Gamma}\}$; $\#N_{\mathrm{R}} = \{R^{\mathbf{e}}_{\mathbf{d}} \mid R \in N_{\mathrm{R}} \wedge \mathbf{d}, \mathbf{e} \in \mathfrak{D}_{\Gamma}\}$; $\#N_{\mathrm{I}} = \{a^{\mathbf{e}} \mid a \in N_{\mathrm{I}} \wedge \mathbf{e} \in \mathfrak{D}_{\Gamma}\}$. An embedding of $\mathcal{C}_{\mathbf{d}}$ into $\#(\Gamma, \Sigma)$ is now done by the $\#\mathbf{d}$ operator:

Definition 5 ($\#\mathbf{d}$ operator). *Given $\mathfrak{K} = \langle \mathfrak{C}, \mathfrak{M} \rangle$ over $\langle \Gamma, \Sigma \rangle$ and $\mathbf{d} \in \mathfrak{D}_{\Gamma}$, $(\cdot)\#\mathbf{d}$ is defined as $g^{*}_{\mathbf{d}}(\cdot)$, where $g_{\mathbf{d}} : \Sigma \rightarrow \#(\Gamma, \Sigma)$ is a DL-embedding defined as follows: (a) $g_{\mathbf{d}}(a) = a^{\mathbf{d}}$ for every individual a; (b) $g_{\mathbf{d}}(X_{\mathbf{d}'_{\mathrm{B}}}) = X^{\mathbf{d}}_{\mathbf{d}'_{\mathrm{B}}+\mathbf{d}}$ for every concept/role $X_{\mathbf{d}'_{\mathrm{B}}}$; (c) $\Sigma_c = \Sigma$; (d) $\Sigma_e = \emptyset$.*

Using the $\#\mathbf{d}$ operator we now transform a CKR $\mathfrak{K}$ over $\langle \Gamma, \Sigma \rangle$ into a DL theory $\#(\mathfrak{K})$ over $\#(\Gamma, \Sigma)$. For every individual a, concept C, role R, concept/role X, and for every $\mathbf{d}, \mathbf{e}, \mathbf{f} \in \mathfrak{D}_{\Gamma}$, $\#(\mathfrak{K})$ contains the following axioms (the gap in the numbering is to maintain the correspondence with Definition 3):

1. $\top^{\mathbf{f}}_{\mathbf{d}} \sqsubseteq \top^{\mathbf{f}}_{\mathbf{e}}$ for $\mathbf{d} \prec \mathbf{e}$;
2. $C^{\mathbf{d}}_{\mathbf{e}} \sqsubseteq \top^{\mathbf{d}}_{\mathbf{e}}$;
3. $\exists R^{\mathbf{d}}_{\mathbf{e}}.\top \sqsubseteq \top^{\mathbf{d}}_{\mathbf{e}}$ and $\top \sqsubseteq \forall R^{\mathbf{d}}_{\mathbf{e}}.\top^{\mathbf{d}}_{\mathbf{e}}$;
4. $a^{\mathbf{d}} = a^{\mathbf{e}}$, if $\mathbf{d} \prec \mathbf{e}$;
6. $X^{\mathbf{d}}_{\mathbf{d}} \equiv X^{\mathbf{e}}_{\mathbf{d}}$, if $\mathbf{d} \prec \mathbf{e}$;
7. $C^{\mathbf{d}}_{\mathbf{f}} \equiv C^{\mathbf{e}}_{\mathbf{f}} \sqcap \top^{\mathbf{d}}_{\mathbf{d}}$, if $\mathbf{d} \prec \mathbf{e}$;
8. $I^{\mathbf{d}}_{\mathbf{d}} \circ R^{\mathbf{e}}_{\mathbf{f}} \circ I^{\mathbf{d}}_{\mathbf{d}} \sqsubseteq R^{\mathbf{d}}_{\mathbf{f}}$ and $R^{\mathbf{d}}_{\mathbf{f}} \sqsubseteq R^{\mathbf{e}}_{\mathbf{f}}$, if $\mathbf{d} \prec \mathbf{e}$;
9. $\phi\#\mathbf{d}$ for all $\phi \in \mathrm{K}(\mathcal{C})$ and $\mathbf{d} = \dim(\mathcal{C})$.

Lemma 2. *Given a CKR $\mathfrak{K}$, (a) if $\mathfrak{K}$ is $\mathbf{d}$-satisfiable then $\#(\mathfrak{K})$ is satisfiable; (b) if there is a $\mathbf{d}$ such that $\#(\mathfrak{K}) \not\models \top^{\mathbf{d}}_{\mathbf{d}} \sqsubseteq \bot$, then $\mathfrak{K}$ is $\mathbf{d}$-satisfiable.*

Reasoning in CKR is now reduced into reasoning in $\mathcal{SROIQ}$. Subsumption is decidable for $\mathcal{SROIQ}$ KB that are $\precsim$-stratified [12]. Hence we can prove decidability only for CKRs that are transformed into $\precsim$-stratified KBs. We say that a CKR is $\precsim$-stratified if the set of RIA $\bigcup_{\mathbf{d} \in \mathfrak{D}_{\Gamma}} \{(R \sqsubseteq S)\#\mathbf{d} \mid R \sqsubseteq S \in \mathrm{K}(\mathcal{C}_{\mathbf{d}})\}$ is $\precsim$-stratified. The RIA introduced in step 8 are not $\precsim$-stratified, but it suffices to add $I^{\mathbf{d}}_{\mathbf{d}} \circ R^{\mathbf{e}}_{\mathbf{f}} \sqsubseteq S_1$, $S_1 \circ I^{\mathbf{d}}_{\mathbf{d}} \sqsubseteq R^{\mathbf{d}}_{\mathbf{f}}$, where S_1 is a new role w.r.t. each pair $R^{\mathbf{d}}_{\mathbf{f}}$ and $R^{\mathbf{e}}_{\mathbf{f}}$. Hence if a $\mathfrak{K}$ is $\precsim$-stratified, there is a $\precsim$-stratified $\mathcal{SROIQ}$ KB equivalent to $\#(\mathfrak{K})$, and hence subsumption is decidable.

Theorem 2. *If $\mathfrak{K}$ is $\precsim$-stratified, then checking if $\mathfrak{K} \models \mathbf{d} : C \sqsubseteq D$ is decidable with the complexity upper bound of* 2NEXPTIME.

The complexity upper bound is established by the fact that the number of dimensions (a fixed constant) and also the number of contexts are bounded. The number of contexts n is always smaller than the size m of the knowledge base $\mathfrak{K}$ because in order to initialize a context we must add several axioms into $\mathfrak{M}$. Consecutive analysis of the construction of $\#(\mathfrak{K})$ shows that its size is bounded by $k \times m \times n^2$ for some constant k, and therefore under $O(m^3)$. So the size of $\#(\mathfrak{K})$ and the time required to generate it is polynomial in the size of $\mathfrak{K}$.

5 Related Work

Both aRDF [25] and Context Description Framework [13] extend RDF triples by an n-tuple of qualification attributes with partially ordered domains. Apart from CKR being based on $\mathcal{SROIQ}$ it differs from these approaches by qualifying whole theories and not each formula separately. This approach is more compact as usually the context is shared by a group of formulae. An extension of RDFS to cope with context was proposed by [8] and further developed in [1]. A new predicate $\mathsf{isin}(c, \phi)$ is used to assert that the triple ϕ occurs in the context c. A set of operators to combine contexts ($c_1 \wedge c_1$, $c_1 \vee c_2$, $\neg c$) and to relate contexts ($c \Rightarrow c_2, c \rightarrow c_2$) is defined, making the approach particularly suited for manipulating contexts. Unfortunately, no sound and complete axiomatization or decision procedure was provided so far.

The contextual DL $\mathcal{ALC}_{\mathcal{ALC}}$ [14] is a multi-modal extension of the $\mathcal{ALC}$ DL with the contextual modal operator $[C]_r A$ representing "all objects of type A in all contexts of type C reachable from the current context via relation r." In both $\mathcal{ALC}_{\mathcal{ALC}}$ and CKR contextual structure is formalized in a meta-language separated from the domain language used to describe the domain. The main difference is that CKR is more expressive in the object-language ($\mathcal{SROIQ}$ vs. $\mathcal{ALC}$) but less expressive in the contextual assertions, allowing qualification of knowledge only w.r.t. individual contexts rather than context classes as in $\mathcal{ALC}_{\mathcal{ALC}}$.

The Metaview approach [24] enriches OWL ontologies with logically treated annotations and it can be used to model contextual metadata similarly to CKR albeit on per-axiom basis. The main difference is that in the Metaview approach the contextual level has no implications on ontology reasoning. Instead, a contextually sensitive query language MQL is provided.

CKR is also logically related to approaches such as multi-context systems [7], distributed description logics [4], and especially to package-based description logics [2] and semantic imports [19]. While similar techniques are employed in CKR in order to facilitate information reuse in between contexts, they are used to meet different goals. The amount of information that is possibly "imported" from one context to another by qualified symbols depends on the relation of these context in the CKR's coverage hierarchy, thus reflecting the underlaying ideas of the AI theories of context.

6 Conclusion

CKR is a novel framework for representing contextual knowledge in the SW. We have provided a sound and complete axiomatization and we have shown that reasoning in CKR is decidable at no additional complexity costs. After the recent introduction of a tractable version of CKR built on top of RDFS [11] we plan to investigate on other tractable local languages, e.g., OWL-Horst [23]. For the tractable version we have developed a prototype [9,11] on top of the Sesame 2 RDF triple store, where contexts have been naturally implemented with named graphs [5]. We also want to study a distributed tableaux based reasoning technique for CKR.

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