

# Embedding Defeasible Logic Programs into Generalized Logic Programs

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**Abstract.** A novel argumentation semantics of defeasible logic programs (DeLP) is presented. Our goal is to build a semantics, which respects existing semantics and intuitions of “classical” logic programming. Generalized logic programs (GLP) are selected as an appropriate formalism for studying both undermining and rebutting. Our argumentation semantics is based on a notion of conflict resolution strategy (CRS), in order to achieve an extended flexibility and generality. Our argumentation semantics is defined in the frame of assumption-based framework (ABF), which enables a unified view on different non-monotonic formalisms. We present an embedding of DeLP into an instance of ABF. Consequently, argumentation semantics defined for ABF are applicable to DeLP. Finally, DeLP with CRS is embedded into GLP. This transformation enables to commute argumentation semantics of a DeLP via semantics of the corresponding GLP.

## 1 Introduction

Defeasible Logic Programs (DeLPs) [8] combine ideas from Defeasible Logic [13,12] and Logic Programming. While classically, logic programs (LPs) feature default negation, which enables to express defeasible assumptions (i.e., propositions which are supposed to hold unless we have some hard evidence against them), DeLPs additionally introduce defeasible rules (i.e., rules which are supposedly applicable unless we have some hard evidence opposing them). Strict rules (i.e., regular LP rules) are denoted by  $\rightarrow$  and defeasible rules by  $\Rightarrow$ . Let us illustrate this with an example.

*Example 1.* Brazil is the home team, and has a key player injured. Home teams tend to work the hardest, and who works the hardest usually wins. The team who has a key player injured does not usually win. The program is formalized into the DeLP:

$$\begin{aligned} r_1: & \rightarrow \textit{home} & r_2: & \textit{home} \Rightarrow \textit{works\_hard} & r_3: & \textit{works\_hard} \Rightarrow \textit{wins} \\ r_4: & \rightarrow \textit{key\_player\_injured} & r_5: & \textit{key\_player\_injured} \Rightarrow \textit{not\_wins} \end{aligned}$$

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This is an extended version of our paper submitted to WLP 2014 which includes proofs. Some technical issues have been fixed w.r.t. the WLP submission which will be corrected also in the final paper.

In the program from Example 1 there are two strict and three defeasible rules. The strict rules are facts, hence *home* and *key\_player\_injured* should always be true. Based on the first fact we are able to derive that Brazil should win, using the two defeasible rules  $r_2$  and  $r_3$ , while based on the second fact we are able to derive that Brazil should not win, again relying on a defeasible rule, in this case  $r_5$ . Hence there is a conflict which somehow should be resolved.

Various approaches to DeLP typically rely on argumentation theory in order to determine which rules should be upheld and which should be defeated. However, as it can be perceived from Example 1, it is not always immediately apparent how this should be decided.

According to García and Simari [8], both rules immediately causing the conflict ( $r_3$  and  $r_5$ ) would be defeated, accepting *home*, *key\_player\_injured* and *works\_hard* as valid derivations while taking both *wins* and not *wins* as undecided. ASPIC<sup>+</sup> [14,11], on the other hand, allows two additional solutions, one with *wins* valid and the other one with not *wins* valid (if no other information is provided).

Some approaches, like ASPIC<sup>+</sup>, allow to specify a preference relation on rules. In such a case conflict resolution may take this into account. Specifically, ASPIC<sup>+</sup> has two built in conflict resolution strategies, *weakest link principle* by which the rule with smallest preference is defeated among those involved in each conflict, and *last link principle* by which only the rules immediately causing the conflict are considered and the least preferred is defeated.

We observe that more ways to resolve conflicts may be needed. This is due to the fact that defeasible rules are *domain specific*, a different conflict resolution strategy may be needed for a different domain, or in distinct application. We therefore argue that the conflict resolution strategy should be a user-specified parameter of the framework, and any DeLP framework should allow a generic way how to specify it (alongside some predefined strategies).

Some of the semantics proposed for DeLP satisfy the well accepted rationality properties for defeasible reasoning, such as *consistency* (extensions should be conflict-free) and closure (extensions should be closed w.r.t. the strict rules), as defined by Caminada and Amgoud [5]. While these properties are important, DeLP is an extension of LP, and some attention should be also devoted to keeping it in line with it.

Specifically, we would like to have the semantics backward-compatible with the underlying language of logic programs – if no defeasible rules are present, the extensions should be in line with the respective class of models.

In our previous work [2] we have formalized the notion of conflict resolution strategy (CRS) and we have proposed a DeLP framework which allows to use any such strategy. The relationship with the underlying class of LPs was not investigated though. In the current paper we extend this work as follows:

- We rebuild the argumentation semantics (including the notion of conflict resolution strategy) using the Assumption Based Framework (ABF), an argumentation formalism very close in spirit to logic programming.

- We show that the semantics satisfies the closure and consistency properties [5], and we also show two additional properties which govern the handling of default assumptions.
- We provide an alternative transformational semantics, which translates the DeLP and the given CRS into a regular logic program. We show that both semantics are equivalent. Thanks to the transformational semantics we also show full backward compatibility with the underlying class of generalized logic programs. What is more, the semantics of DeLP can now be computed using existing LP solvers.

## 2 Preliminaries

Generalized logic programs and assumption-based frameworks provide a background for our investigation. We are aiming at a computation of our argumentation semantics of DeLP in the frame of classical logic programs. Generalized logic programs (with default negations in the heads of rules) are selected as a simplest LP-formalism, which enables to consider both undermining and rebutting.

Assumption-based frameworks are used in our paper as a basis for building a semantics of DeLP. ABF is a general and powerful formalism providing a unified view on different non-monotonic formalisms using argumentation semantics.

### 2.1 Generalized Logic Programs

We will consider propositional generalized logic programs (GLPs) in this paper.

Let  $At$  be a set of atoms and  $\text{not } At$  be  $\{\text{not } a \mid a \in At\}$  a set of default literals. A literal is an atom or a default literal. The set of all literals is denoted by  $\mathcal{L}_{At}$ . If  $L = \text{not } a$  and  $a \in At$ , then by  $\text{not } L$  is denoted  $a$ . If  $S \subseteq \mathcal{L}_{At}$ , then  $\text{not } S = \{\text{not } a \mid a \in S\}$ . For example, if  $S = \{a, b, \text{not } c\}$ , then  $\text{not } S = \{\text{not } a, \text{not } b, c\}$ . In addition, for any set of literals  $S$  we introduce a notation as follows:  $S^+ = S \cap At, S^- = S \setminus S^+$ .

A rule  $r$  is an expression of the form

$$L_1, \dots, L_k \rightarrow L, \text{ where } k \geq 0, L, L_i \in \mathcal{L}_{At}$$

$L$  is called the head of the rule and denoted by  $\text{head}(r)$ . The set of literals  $\{L_1, \dots, L_k\}$  is called the body of  $r$ , denoted by  $\text{body}(r)$ .

A generalized logic program is a finite set of rules. We will often use only the term program. If  $At$  is the set of all atoms used in a program  $P$ , it is said that  $P$  is over  $At$ . If heads of all rules of a program  $P$  are atoms, it is said that the program is *normal*. A program  $P$  is called *positive* if for each literal  $L$  occurring in  $P$  in any rule (either in the head or in the body) we have  $L \in At \cup \{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$  where  $\mathbf{t}, \mathbf{u}, \mathbf{f}$  are special propositional constants.

Note that a GLP  $P$  can be viewed as consisting of two parts, a normal logic program  $P^+ = \{r \in P \mid \text{head}(r) \in At\}$  (also called the normal part of  $P$ ) and a set of “constraints”  $P^- = P \setminus P^+$  (also called the negative part of  $P$ ).

Our definitions of some basic semantic notions follow the ideas of Przymusiński [17]. See also [6]. However, an adaptation to the case of rules with default negations in head is needed. In our approach we will use the normal part of the program,  $P^+$ , as a generator to generate a broad set of candidate models and consecutively we will use the negative part,  $P^-$ , to filter out some of the models.

**Definition 1 (Partial and Total Interpretation).** *A set of literals  $S$  is consistent, if it does not contain a pair  $a, \text{not } a$ , where  $a \in \text{At}$ . A partial interpretation is a consistent set of literals. A total interpretation is a partial interpretation  $I$  s.t. for every  $a \in \text{At}$  either  $a \in I$  or  $\text{not } a \in I$ .*

Each interpretation can be viewed as a mapping  $I: \text{At} \mapsto \{0, \frac{1}{2}, 1\}$  where  $I(A) = 0$  if  $\text{not } A \in I$ ,  $I(A) = \frac{1}{2}$  if  $A \notin I$  and  $\text{not } A \notin I$ , and  $I(A) = 1$  if  $A \in I$ . A valuation given by an interpretation  $I$  is a mapping  $\hat{I}: \mathcal{L}_{\text{At}} \mapsto \{0, \frac{1}{2}, 1\}$  where  $\hat{I}(A) = I(A)$  and  $\hat{I}(\text{not } A) = 1 - I(A)$  for each atom  $A \in \text{At}$ , and  $\hat{I}(\mathbf{t}) = 1$ ,  $\hat{I}(\mathbf{u}) = \frac{1}{2}$ ,  $\hat{I}(\mathbf{f}) = 0$ . An interpretation  $I$  satisfies a rule  $r$  (denoted  $I \models r$ ) iff  $\hat{I}(\text{head}(r)) \geq \hat{I}(\text{body}(r)) = \min\{\hat{I}(L) \mid L \in \text{body}(r)\}$ .

**Definition 2 (Model).** *An interpretation  $I$  is a model of a generalized logic program  $\mathcal{P}$  iff  $I$  satisfies each rule in  $\mathcal{P}$ .*

As usual in logic programming, “classical” model, as defined above, are too broad and a number of more fine-grained semantics, based on certain notion of minimality are used. We proceed by defining these semantics summarily for GLPs. Not all of them were thoroughly investigated in literature, however we use analogy with other classes of logic programs, especially normal logic programs.

The notions of truth ordering and knowledge ordering on partial interpretations will be needed. For a partial interpretation  $I$ , let  $T(I) = \{A \in \text{At} \mid I(A) = 1\}$ ,  $F(I) = \{A \in \text{At} \mid I(A) = 0\}$ , and  $\mathcal{U}(I) = \{A \in \text{At} \mid I(A) = \frac{1}{2}\}$ .

**Definition 3 (Truth and Knowledge Ordering).** *If  $I, J$  are partial interpretations, then*

- $I \leq_t J$  iff  $T(I) \subseteq T(J)$  and  $F(I) \supseteq F(J)$ ,
- $I \leq_k J$  iff  $T(I) \subseteq T(J)$  and  $F(I) \subseteq F(J)$ ,

We will also need an operator to iterate the logic program on a given interpretation ( $\Psi_{\mathcal{P}}$ ). This operator is defined on positive programs only, hence we will also need the notion of a reduced program.

**Definition 4 (Operator  $\Psi_{\mathcal{P}}$ ).** *Let  $\mathcal{P}$  be a positive logic program and  $I$  be an interpretation. By  $\Psi_{\mathcal{P}}(I)$  we denote the interpretation given by*

$$\Psi_{\mathcal{P}}(I)(A) = \max\{\hat{I}(\text{body}(r)) \mid r \in \mathcal{P}, \text{head}(r) = A\}$$

for each atom  $A$ . The  $\alpha$ -iteration of  $\Psi_{\mathcal{P}}$  is defined as follows:

$$\Psi_{\mathcal{P}} \uparrow \alpha = \begin{cases} \text{not } \text{At} & \text{if } \alpha \text{ is the zero ordinal} \\ \Psi_{\mathcal{P}}(\Psi_{\mathcal{P}} \uparrow \beta) & \text{if } \alpha \text{ is the successor of an ordinal } \beta \\ \sup_k \{\Psi_{\mathcal{P}} \uparrow \beta \mid \beta < \alpha\} & \text{if } \alpha \text{ is a limit ordinal} \end{cases}$$

The  $t$ -least model of a positive logic program  $\mathcal{P}$  can be obtained by iterating  $\omega$  times the operator  $\Psi_{\mathcal{P}}$  starting with the  $t$ -least interpretation  $\text{not } At$ , i.e.  $\Psi_{\mathcal{P}} \uparrow \omega$  is the  $t$ -least model of  $\mathcal{P}$  [18].

**Definition 5 (Program Reduct).** *Let  $I$  be an interpretation. The reduct of a normal logic program  $\mathcal{P}$  is a positive logic program  $\mathcal{P}^I$  obtained from  $\mathcal{P}$  by replacing in every rule of  $\mathcal{P}$  all default literals which are true (resp. unknown, resp. false) in  $I$  by propositional constant  $\mathbf{t}$  (resp.  $\mathbf{u}$ , resp.  $\mathbf{f}$ ).*

Finally a fixed-point condition is expressed on a reduced program, which is formally captured by the operator  $\Gamma_{\mathcal{P}}$ .

**Definition 6 (Operator  $\Gamma_{\mathcal{P}}$ ).** *Let  $\mathcal{P}$  be a normal logic program and  $I$  be an interpretation. By  $\Gamma_{\mathcal{P}}(I)$  we denote the  $t$ -least model of  $\mathcal{P}^I$ .*

**Definition 7 (Semantics Family for GLPs).** *Let  $\mathcal{P}$  be a generalized logic program and  $I$  be a model of  $\mathcal{P}$ . Then*

- $I$  is a partial stable model of  $\mathcal{P}$  iff  $\Gamma_{\mathcal{P}^+}(I) = I$
- $I$  is a total stable model of  $\mathcal{P}$  iff  $I$  is a partial stable model of  $\mathcal{P}$  which is total
- $I$  is a well-founded model of  $\mathcal{P}$  iff  $I$  is a  $k$ -minimal partial stable model of  $\mathcal{P}$
- $I$  is a maximal stable model of  $\mathcal{P}$  iff  $I$  is a  $k$ -maximal partial stable model of  $\mathcal{P}$
- $I$  is a least-undefined stable model of  $\mathcal{P}$  iff  $I$  is a partial stable model of  $\mathcal{P}$  with subset-minimal  $\{A \in At \mid I(A) = \frac{1}{2}\}$ .

The produced semantics properly generalize existing semantics for normal logic programs.

**Proposition 1.** *If  $\mathcal{P}$  is a normal logic program, the notion of partial stable model in Definition 7 coincides with the definition of partial stable models in [17], the notion of total stable model in Definition 7 coincides with the definition of stable models in [10], the notion of well-founded model in Definition 7 coincides with the definition of well-founded model in [9], and the notions of maximal and least-undefined stable model in Definition 7 coincides with the definition of maximal and least-undefined stable models in [19].*

*If  $\mathcal{P}$  is a generalized logic program, the definition of stable models in Definition 7 coincides with the definition of stable models in [1].*

## 2.2 Assumption-based Framework

Assumption-based frameworks (ABF) [4] enable to view non-monotonic reasoning as a deduction from assumptions. Argumentation semantics of [7] were applied to sets of assumptions. As a consequence, a variety of semantic characterizations of non-monotonic reasoning has been provided.

An ABF is constructed over a deductive system. A *deductive system* is a pair  $(\mathcal{L}, \mathcal{R})$  where  $\mathcal{L}$  is a language, i.e. a set of well-formed sentences, and  $\mathcal{R}$  is a set of inference rules over  $\mathcal{L}$ , each rule is of the form  $\alpha_1, \dots, \alpha_n \rightarrow \gamma$ , where  $\alpha_i, \gamma \in \mathcal{L}, n \geq 0$ ,  $\alpha_i$  are called premises,  $\gamma$  is the consequence.

A *theory* is a set  $S \subseteq \mathcal{L}$  of sentences. A sentence  $\varphi$  is a *consequence* of a theory  $S$  iff there is a sequence  $\{\varphi_1, \dots, \varphi_n\}$ ,  $1 \leq n$ , of sentences such that  $\varphi = \varphi_n$  and for each  $1 \leq i \leq n$

- $\varphi_i \in S$ , or
- there exists an inference rule  $r \in \mathcal{R}$  such that  $head(r) = \varphi_i$  and  $body(r) \subseteq \{\varphi_1, \dots, \varphi_i\}$ .

By  $Cn_{\mathcal{R}}(S)$  we will denote the set of all consequences of  $S$ .

An *assumption-based framework* is a tuple  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  where  $(\mathcal{L}, \mathcal{R})$  is a deductive system,  $\mathcal{A} \subseteq \mathcal{L}$  is a set of *assumptions*, and  $\neg: \mathcal{A} \mapsto \mathcal{L}$  is a mapping called *contrariness function*. We say that the sentence  $\bar{\alpha}$  is the *contrary* of an assumption  $\alpha$ .

A *context* is a set  $\Delta$  of assumptions. We say that  $\Delta$  is *conflict-free* iff  $\{\alpha, \bar{\alpha}\} \not\subseteq Cn_{\mathcal{R}}(\Delta)$  for each assumption  $\alpha$ . A context  $\Delta$  is closed iff  $\Delta = \{\alpha \in \mathcal{A} \mid \alpha \in Cn_{\mathcal{R}}(\Delta)\}$ , i.e., only such assumptions, which are members of  $\Delta$  are derivable from  $\Delta$ .

ABFs enable to apply argumentation semantics to sets of assumptions and, consequently, subtle and rich semantic characterizations of sets of assumptions (and of their consequences) can be specified.

A context  $\Delta$  *attacks* an assumption  $\alpha$  iff  $\bar{\alpha} \in Cn_{\mathcal{R}}(\Delta)$ . A context  $\Delta$  *defends* an assumption  $\alpha$  iff each closed context attacking  $\alpha$  contains an assumption attacked by  $\Delta$ . A closed context  $\Delta$  is

- *attack-free* iff  $\Delta$  does not attack an assumption in  $\Delta$ ,
- an attack-free context  $\Delta$  is *admissible* iff  $\Delta$  defends each assumption in  $\Delta$ ,
- a closed context  $\Delta$  is *complete* iff  $\Delta$  is admissible and contains all assumptions defended by  $\Delta$ ;
- *grounded* iff  $\Delta$  is a subset-minimal complete context;
- *preferred* iff  $\Delta$  is a subset-maximal admissible context;
- *stable* iff  $\Delta$  is an attack-free context attacking each assumption which does not belong to  $\Delta$ .
- *semi-stable* iff  $\Delta$  is a complete context such that  $\Delta \cup \{\alpha \in \mathcal{A} \mid \Delta \text{ attacks } \alpha\}$  is subset maximal.

### 3 Defeasible Logic Programs

Our knowledge can be divided according to its epistemological status into two categories: on the one hand, one that is gained by deductively valid reasoning and on the other hand, knowledge that is reached by *defeasible reasoning* [13]. Defeasible logic programs (DeLPs) [15,8,14,11] consider two kinds of rules: *strict* and *defeasible*. Strict rules represent deductive reasoning: whenever their preconditions hold, we accept the conclusion. Defeasible rules formalize tentative

knowledge that can be defeated and validity of preconditions of a defeasible rule does not necessarily imply the conclusion. Given a set of literals  $\mathcal{L}_{At}$ , a strict (defeasible) rule is an expression  $L_1, \dots, L_n \rightarrow (\Rightarrow)L_0$ , where  $0 \leq i \leq n$  and  $L_i \in \mathcal{L}_{At}$ . We will use  $\rightsquigarrow$  to denote either a strict or a defeasible rule.

**Definition 8 (Defeasible Logic Program).** *Let  $At$  be a set of atoms,  $\mathcal{N}$  be a set of names, and  $At \cap \mathcal{N} = \emptyset$ . A defeasible logic program is a tuple  $\mathcal{P} = (\mathcal{S}, \mathcal{D}, name)$  where  $\mathcal{S}$  is a set of strict rules over  $\mathcal{L}_{At}$ ,  $\mathcal{D}$  is a set of defeasible rules over  $\mathcal{L}_{At}$ , and  $name: \mathcal{D} \mapsto \mathcal{N}$  is an injective naming function.*

Naming function will be used for transformation of defeasible rules into strict rules. We will treat  $name(r)$ , where  $r$  is a defeasible rule, as a new atom and interpret it as “the defeasible rule  $r$  can not be used” and consequently the meaning of not  $name(r)$  is “by default, the defeasible rule  $r$  can be used”.

### 3.1 From Arguments to Conflict Resolutions

The argumentation process usually consists of five steps [15,16,8,14,11]. At the beginning, a knowledge base is described in some logical language. The notion of an argument is then defined within this language. Then conflicts between arguments are identified. The resolution of conflicts is captured by an attack relation (also called “defeat relation” in some literature) among conflicting arguments. The status of an argument is then determined by the attack relation. In this paper, conflicts are not resolved by attacking some of the conflicting arguments, but by attacking some of the weak parts of an argument called *vulnerabilities*. This help us to satisfy argumentation rationality postulates [5] and keep the semantics aligned with LP intuitions.

Two kinds of arguments can usually be constructed in the language of defeasible logic programs. Default arguments correspond to default literals. Deductive arguments are constructed by chaining of rules.

We define several functions *prems*, *rules* and *vuls* denoting premises (i.e. default literals) and rules occurring in an argument. Intended meaning of *vuls*( $A$ ) is a set of vulnerabilities of an argument  $A$  (i.e. weak parts which can be defeated) consisting of premises and default negated names of defeasible rules of an argument  $A$ .

**Definition 9 (Argument).** *Let  $\mathcal{P} = (\mathcal{S}, \mathcal{D}, name)$  be a defeasible logic program. An argument  $A$  for a literal  $L$  is*

1. a default argument  $L$  where  $L$  is a default literal.

$$\begin{aligned} prems(A) &= \{L\} \\ rules(A) &= \emptyset \end{aligned}$$

2. a deductive argument  $[A_1, \dots, A_n \rightsquigarrow L]$  where each  $A_i$ ,  $1 \leq i \leq n$ ,  $n \geq 0$ , is an argument for a literal  $L_i$  and  $r: L_1, \dots, L_n \rightsquigarrow L$  is a rule in  $\mathcal{P}$ .

$$\begin{aligned} prems(A) &= prems(A_1) \cup \dots \cup prems(A_n) \\ rules(A) &= rules(A_1) \cup \dots \cup rules(A_n) \cup \{r\} \end{aligned}$$

For both kinds of an argument  $A$

$$vuls(A) = prems(A) \cup \text{not } name(rules(A) \cap \mathcal{D})$$

*Example 2.* Consider a defeasible logic program consisting of the only defeasible rule  $r: \text{not } b \Rightarrow a$ . Two default arguments  $A_1 = [\text{not } a]$ ,  $A_2 = [\text{not } b]$  and one deductive argument  $A_3 = [A_2 \Rightarrow a]$  can be constructed. We can see that  $vuls(A_1) = \{\text{not } a\}$ ,  $vuls(A_2) = \{\text{not } b\}$ ,  $vuls(A_3) = \{\text{not } b, \text{not } name(r)\}$ .

The difference between a default and a deductive argument for a literal  $\text{not } A$  is in policy how the conflict is resolved. Syntactical conflict between arguments is formalized in the following definition. As usual in the literature [14], we distinguish two kinds of conflicts: *undermining*<sup>3</sup> and *rebutting*. While an undermining conflict is about a falsification of a hypothesis (assumed by default), a rebutting conflict identifies situation where opposite claims can be derived.

**Definition 10 (Conflict).** *Let  $\mathcal{P}$  be a defeasible logic program. The pair of arguments  $(A, B)$  is called a conflict iff*

- $A$  is a deductive argument for a default literal  $\text{not } L$  and  $B$  is a deductive argument for the literal  $L$ ; or
- $A$  is a default argument for a default literal  $\text{not } L$  and  $B$  is a deductive argument for the literal  $L$ .

*The first kind is called a rebutting conflict and the second kind is called an undermining conflict.*

Previous definition just identifies the conflict and does not say how to resolve it. Now we formalize a notion of *conflict resolution* which captures a possible way how to resolve each conflict. In our paper, conflicts are not resolved through attack between arguments as in [8,15,14,11], but by attacking some of the vulnerabilities in the conflicting arguments. Since our goal is to define semantics for DeLP respecting existing semantics and intuitions in LP, we assume that all undermining conflicts are resolved in a fixed way as in LP: that is, by attacking a default argument. On the other hand, rebutting conflict is resolved by attacking some defeasible rule. Since, in general, there can be more reasonable ways how to choose which defeasible rules to attack, resolving of all rebutting conflicts is left as domain dependent for the user as an input. Note, that an attack on a defeasible rule  $r$  is formalized as an attack on a default literal  $\text{not } name(r)$  which is interpreted as “a defeasible rule  $r$  can not be used”.

**Definition 11 (Conflict Resolution).** *Let  $\mathcal{P}$  be a defeasible logic program. A conflict resolution is a pair  $\rho = (C, V)$ , where  $C = (A, B)$  is a conflict,  $A$  is an argument for  $\text{not } L$  and*

- $V$  is a default literal  $\text{not } L$ , if  $C$  is an undermining conflict; or

<sup>3</sup> Also called *undercutting* in [15].



- $V$  is a default literal  $\text{not name}(r)$ , where  $r$  is a defeasible rule in  $\text{rules}(A) \cup \text{rules}(B)$ , if  $C$  is a rebutting conflict.

Conflict resolution strategy  $R$  of  $\mathcal{P}$  is a set of conflict resolutions.

Let  $\rho = ((A, B), V)$  be a conflict resolution.  $V$  is called the *resolution* of  $\rho$ , and denoted by  $\text{res}(\rho)$ . The set of vulnerabilities of a conflict resolution, denoted by  $\text{vuls}(\rho)$ , is defined as:

$$\text{vuls}(\rho) = \begin{cases} \text{vuls}(A) \cup \text{vuls}(B) & \text{whenever } V \in \text{vuls}(A) \cap \text{vuls}(B) \\ (\text{vuls}(A) \cup \text{vuls}(B)) \setminus \{V\} & \text{otherwise} \end{cases}$$

Usually, there may be more ways how to resolve a conflict and a conflict resolution may resolve other conflicts as well, thus causing other conflict resolutions to be irrelevant or inapplicable. Intuitively, if all vulnerabilities in  $\text{vuls}(\rho)$  are undefeated (i.e. true), then in order to resolve the conflict in  $\rho$ , vulnerability  $\text{res}(\rho)$  should be defeated (i.e. false). Notions of  $\text{vuls}(\rho)$  and  $\text{res}(\rho)$  will be used for definition of the argumentation semantics in the next subsection.

*Example 3.* Consider the conflict  $C = ([\Rightarrow b] \rightarrow \text{not } c, [[\Rightarrow a] \rightarrow c])$ . Then  $\rho_1 = (C, \text{not name}(\Rightarrow a))$ ,  $\rho_2 = (C, \text{not name}(\Rightarrow b))$  are two conflict resolutions and  $\text{vuls}(\rho_1) = \{\text{not name}(\Rightarrow b)\}$ ,  $\text{vuls}(\rho_2) = \{\text{not name}(\Rightarrow a)\}$ .

*Example 4.* Let  $C = ([\text{not } a], [[\text{not } a] \rightarrow a])$  Then  $\rho = (C, \text{not } a)$  is a conflict resolution and  $\text{vuls}(\rho) = \{\text{not } a\}$ .

Following examples show that there more reasonable ways how to resolve rebutting conflicts. Note that following weakest-link and last-link strategies are inspired from ASPIC<sup>+</sup> [14], where however arguments instead of defeasible rules are defeated. In both examples we assume existence of an user specified preference order  $\prec$  on defeasible rules.

In the following strategy, we will compare last defeasible rules of conflicting arguments and choose a  $\prec$ -minimal one as a resolution of a conflict.

*Example 5.* Given the defeasible logic program

$$\begin{aligned} r_1: & \Rightarrow b \\ r_2: & b \Rightarrow a \\ r_3: & \Rightarrow \text{not } a \end{aligned}$$

and order  $r_1 \prec r_3$ ,  $r_1 \prec r_2$ ,  $r_2 \prec r_3$ , deductive arguments are

$$A_1 = [\Rightarrow b] \quad A_2 = [A_1 \Rightarrow a] \quad A_3 = [\Rightarrow \text{not } a]$$

Then  $R = \{((A_3, A_2), \text{not name}(r_3))\}$  is the last link strategy.

The next strategy compares all defeasible rules of conflicting arguments and chooses a  $\prec$ -minimal one as a resolution of a conflict.

*Example 6.* Consider the defeasible logic program and the order on rules from Example 5. The weakest link strategy is  $R = \{((A_3, A_2), \text{not name}(r_1))\}$ .

Note that in [15,8,14,11] conflict resolution strategy from Example 6, where a non-last defeasible rule (i.e.  $r_1$  is not the last defeasible rule used in an argument  $A_2$ ) is used as a resolution of the conflict, is not possible, what makes our approach in this respect more flexible and general.

### 3.2 Argumentation Semantics

In the previous subsection, definition of an argument structure, conflicts identification and examples of various conflict resolutions were discussed. However, the status of literals and the actual semantics has not been stated.

Argumentation semantics for defeasible logic programs will be formalized within ABF – a general framework, where several existing non-monotonic formalisms have been embedded [4]. In order to use some of the existing argumentation semantics, we need to specify ABF’s language  $\mathcal{L}$ , set of inference rules  $\mathcal{R}$ , set of assumptions  $\mathcal{A}$  and the contrariness function  $\bar{\cdot}$ . Since ABF provides only one kind of inference rules (i.e. strict), we need to transform defeasible rules into strict. We transform defeasible rule  $r$  by adding a new assumption  $\text{not } \text{name}(r)$  into the preconditions of  $r$ . Furthermore, chosen conflict resolutions  $R$  determining how rebutting conflicts will be resolved are transformed into new inference rules. Intuitively, given a conflict resolution  $\rho$ , if all assumptions in  $\text{vuls}(\rho)$  are accepted, then, in order to resolve the conflict in  $\rho$ , the assumption  $\text{res}(\rho)$  should be defeated. To achieve this, an inference rule  $\text{vuls}(\rho) \rightarrow \overline{\text{res}(\rho)}$  for each conflict resolution  $\rho \in R$  is added to the set of inference rules  $\mathcal{R}$ .

**Definition 12 (Instantiation).** *Let  $\mathcal{P} = (\mathcal{S}, \mathcal{D}, \text{name})$  be a defeasible logic program built over the language  $\mathcal{L}_{At}$  and  $R$  a set of conflict resolutions. An assumption based framework respective to  $\mathcal{P}$  and  $R$  is  $(\mathcal{L}, \mathcal{R}, \mathcal{A}, \bar{\cdot})$ , where:*

- $\mathcal{L} = \mathcal{L}_{At} \cup \mathcal{L}_{\mathcal{N}}$ ,
- $\mathcal{R} = \mathcal{S} \cup \{\text{body}(r) \cup \{\text{not } \text{name}(r)\} \rightarrow \text{head}(r) \mid r \in \mathcal{D}\} \cup \{\text{vuls}(\rho) \rightarrow \overline{\text{res}(\rho)} \mid \rho \in R\}$ ,
- $\mathcal{A} = \overline{\text{not } At} \cup \text{not } \mathcal{N}$ ,
- $\overline{\text{not } A} = A$  for each atom  $A \in At \cup \mathcal{N}$ .

*Example 7.* Consider a conflict resolution strategy  $R = \{\rho_1, \rho_2\}$  and a defeasible logic program  $\mathcal{P}$  consisting of rules in the conflict  $C$  from Example 3, where  $\text{name}(\Rightarrow b) = r_1$ ,  $\text{name}(\Rightarrow a) = r_2$ . Assumption-based framework respective to  $\mathcal{P}$  and  $R$  is following:

- $\mathcal{L} = \{a, \text{not } a, b, \text{not } b, c, \text{not } c\} \cup \{r_1, \text{not } r_1, r_2, \text{not } r_2\}$
- $R = \{b \rightarrow \text{not } c, a \rightarrow c\} \cup \{\text{not } r_1 \rightarrow b, \text{not } r_2 \rightarrow a\} \cup \{\text{not } r_1 \rightarrow r_2, \text{not } r_2 \rightarrow r_1\}$
- $\mathcal{A} = \{\text{not } a, \text{not } b, \text{not } c\} \cup \{\text{not } r_1, \text{not } r_2\}$
- $\overline{\text{not } A} = A$  for each  $A \in \{a, b, c\} \cup \{r_1, r_2\}$

Now we define the actual semantics for defeasible logic programs.

Given an ABF  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \bar{\cdot})$ , by  $\mathcal{F}^+$  we denote its flattening – that is,  $\mathcal{F}^+$  is the ABF  $(\mathcal{L}, \{r \in \mathcal{R} \mid \text{head}(r) \notin \mathcal{A}\}, \mathcal{A}, \bar{\cdot})$ .

**Definition 13 (Extension).** Let  $\mathcal{P} = (\mathcal{S}, \mathcal{D}, name)$  be a defeasible logic program built over the language  $\mathcal{L}_{At}$ ,  $R$  a set of conflict resolutions and  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  an assumption-based framework respective to  $\mathcal{P}$  and  $R$ . A set of literals  $\mathcal{E} \subseteq \mathcal{L}$  is

1. a complete extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$  with  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$  and  $Cn_{\mathcal{R}}(\mathcal{E}') \subseteq \mathcal{E}'$ ;
2. a grounded extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a subset-minimal complete extension of  $\mathcal{P}$  with respect to  $R$ ;
3. a preferred extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a subset-maximal complete extension of  $\mathcal{P}$  with respect to  $R$ ;
4. a semi-stable extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a complete extension of  $\mathcal{P}$  with respect to  $R$  with subset-minimal  $\mathcal{E}' \setminus \mathcal{E}$ ;
5. a stable extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a complete extension of  $\mathcal{P}$  with respect to  $R$  with  $\mathcal{E}' = \mathcal{E}$ .

where  $\mathcal{E}' = \mathcal{L} \setminus \text{not } \mathcal{E}$ .

*Example 8.* Consider an assumption-based framework from Example 7. Then  $\mathcal{E}_1 = \{\text{not } r_1, \text{not } c, \text{not } a, b, r_2\}$ ,  $\mathcal{E}_2 = \emptyset$  and  $\mathcal{E}_3 = \{\text{not } r_2, r_1, a, c, \text{not } b\}$  are complete extensions of  $\mathcal{F}^+$  and  $Cn_{\mathcal{R}}(\mathcal{E}_i) \subseteq \mathcal{E}_i$ , for  $1 \leq i \leq 3$ . Furthermore,  $\mathcal{E}_2$  is the grounded extension and  $\mathcal{E}_1$ ,  $\mathcal{E}_3$  are preferred, semi-stable and stable extensions of  $\mathcal{F}^+$ .

### 3.3 Transformational Semantics

The argumentation semantics defined above allows us to deal with conflicting rules and to identify the extensions of a DeLP, provided a given CRS, and hence it constitutes a reference semantics. This semantics is comparable to existing argumentation-based semantics for DeLP, and, as we show below, it satisfies the expected desired properties of defeasible reasoning. In this section we investigate on the relation of the argumentation-based semantics and classical logic programming. As we show, an equivalent semantics can be obtained by transforming the DeLP and the given CRS into a classical logic program, and computing the respective class of models.

In fact the transformation that is required is essentially the same which we used to embed DeLPs with CRS into ABFs. The names of rules become new literals in the language, intuitively if  $name(r)$  becomes true it means that the respective defeasible rule is defeated. By default  $\text{not } name(r)$  holds and so all defeasible rules can be used unless the program proves otherwise. The conflict resolution strategy  $R$  that is to be used is encoded by adding a set of rules of the form  $vuls(\rho) \rightarrow \text{not } res(\rho)$  for each conflict resolution  $\rho \in R$ , where the head of any such rules is always an atom (i.e.  $res(\rho)$  is a default literal, so the double negation is removed) and the body is a set of default literals.

Formally the transformation is defined as follows:

**Definition 14 (Transformation).** Let  $\mathcal{P} = (\mathcal{S}, \mathcal{D}, \text{name})$  be a defeasible logic program and  $R$  be a set of conflict resolutions. Transformation of  $\mathcal{P}$  with respect to  $R$  into a generalized logic program, denoted as  $T(\mathcal{P}, R)$ , is defined as

$$T(\mathcal{P}, R) = \mathcal{S} \cup \{ \text{body}(r) \cup \{ \text{not name}(r) \} \rightarrow \text{head}(r) \mid r \in \mathcal{D} \} \cup \{ \text{vuls}(\rho) \rightarrow \text{not res}(\rho) \mid \rho \in R \}$$

Thanks to the transformation, we can now compute the semantics of each DeLPs relying on the semantics of generalized logic programs. Given a DeLP  $\mathcal{P}$  and the assumed CRS  $R$ , the extensions of  $\mathcal{P}$  w.r.t.  $R$  corresponds to the respective class of models. Complete extensions correspond to partial stable models, the grounded extension to the well-founded model, stable extensions to total stable models, and so on.

**Lemma 1.** Let  $\mathcal{P}$  be a normal logic program over a set of atoms  $At$  and  $\mathcal{F} = (\mathcal{L}_{At}, \mathcal{P}, \text{not } At, \neg)$  be an assumption-based framework where  $\overline{\text{not } A} = A$  for each atom  $A \in At$ . Then  $\mathcal{E}$  is a partial stable model of  $\mathcal{P}$  iff  $\mathcal{E}$  is a complete extension of  $\mathcal{F}$ .

*Proof (Sketch).* The correspondence between partial stable models of normal logic programs and complete extensions of abstract argumentation frameworks [7] was proved by Wu et al. [22]. The instantiation of Dung's abstract argumentation framework for normal logic programs is analogous to the instantiation of assumption-based framework in our approach. Therefore this result carries over.  $\square$

**Lemma 2.** Let  $\mathcal{P}$  be a generalized logic program over a set of atoms  $At$  and  $\mathcal{F} = (\mathcal{L}_{At}, \mathcal{P}, \text{not } At, \neg)$  be an assumption-based framework where  $\overline{\text{not } A} = A$  for each atom  $A \in At$ . Then  $\mathcal{E}$  is a partial stable model of  $\mathcal{P}$  iff  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$ ,  $Cn(\mathcal{E}) \subseteq \mathcal{E}$ , and  $Cn(\mathcal{E}') \subseteq \mathcal{E}'$  where  $\mathcal{E}' = \mathcal{L} \setminus \text{not } \mathcal{E}$ .

*Proof.* Please note, that for each consistent set  $\mathcal{E}$  of literals,  $\mathcal{E} = \{L \in \mathcal{L}_{At} \mid \hat{\mathcal{E}}(L) = 1\} = \{L \in \mathcal{L}_{At} \mid \hat{\mathcal{E}}(L) \geq 1\}$ ,  $\text{not } \mathcal{E} = \{L \in \mathcal{L}_{At} \mid \hat{\mathcal{E}}(L) = 0\} = \{L \in \mathcal{L}_{At} \mid \hat{\mathcal{E}}(L) < \frac{1}{2}\}$ , and  $\mathcal{E}' = \mathcal{L}_{At} \setminus \text{not } \mathcal{E} = \{L \in \mathcal{L}_{At} \mid \hat{\mathcal{E}}(L) \geq \frac{1}{2}\}$ .

( $\Rightarrow$ ) Let  $\mathcal{E}$  be a partial stable model of  $\mathcal{P}$ . Then, according to Definition 7,  $\mathcal{E}$  is a model of  $\mathcal{P}$  and  $\mathcal{E}$  is a partial stable model of  $\mathcal{P}^+$ . According to Lemma 1,  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$ . Since  $\mathcal{E}$  is a model of  $\mathcal{P}$ , for each rule  $r \in \mathcal{P}$  with  $\hat{\mathcal{E}}(\text{body}(r)) \geq 1$  holds  $\hat{\mathcal{E}}(\text{head}(r)) \geq 1$ , i.e.  $Cn(\mathcal{E}) \subseteq \mathcal{E}$ . Similarly, for each rule  $r \in \mathcal{P}$  with  $\hat{\mathcal{E}}(\text{body}(r)) \geq \frac{1}{2}$  holds  $\hat{\mathcal{E}}(\text{head}(r)) \geq \frac{1}{2}$ , i.e.  $Cn(\mathcal{E}') \subseteq \mathcal{E}'$ .

( $\Leftarrow$ ) Let  $\mathcal{E}$  be a complete extension of  $\mathcal{F}^+$ ,  $Cn(\mathcal{E}) \subseteq \mathcal{E}$ , and  $Cn(\mathcal{E}') \subseteq \mathcal{E}'$ . Since  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$ , according to Lemma 1,  $\mathcal{E}$  is a partial stable model of  $\mathcal{P}^+$ . Since  $Cn(\mathcal{E}) \subseteq \mathcal{E}$ , for all rules  $r \in \mathcal{P}$  with  $\hat{\mathcal{E}}(\text{body}(r)) \geq 1$  holds  $\hat{\mathcal{E}}(\text{body}(r)) \geq 1$ . Since  $Cn(\mathcal{E}') \subseteq \mathcal{E}'$ , for all rules  $r \in \mathcal{P}$  with  $\hat{\mathcal{E}}(\text{body}(r)) \geq \frac{1}{2}$  holds  $\hat{\mathcal{E}}(\text{body}(r)) \geq \frac{1}{2}$ . Therefore  $\mathcal{E}$  is a model of  $\mathcal{P}$ , and according to Definition 7,  $\mathcal{E}$  is a partial stable model of  $\mathcal{P}$ .  $\square$

**Proposition 2.** Let  $\mathcal{P}$  be a defeasible logic program and  $R$  be a set of conflict resolutions. Then

1.  $\mathcal{E}$  is a complete extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a partial stable model of  $T(\mathcal{P}, R)$ .
2.  $\mathcal{E}$  is a grounded extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a well-founded model of  $T(\mathcal{P}, R)$ .
3.  $\mathcal{E}$  is a preferred extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a maximal partial stable model of  $T(\mathcal{P}, R)$ .
4.  $\mathcal{E}$  is a semi-stable extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a least-undefined stable model of  $T(\mathcal{P}, R)$ .
5.  $\mathcal{E}$  is a stable extension of  $\mathcal{P}$  with respect to  $R$  iff  $\mathcal{E}$  is a total stable model of  $T(\mathcal{P}, R)$ .

*Proof.* Since the set of inference rules of the assumption-based framework respective to  $\mathcal{P}$  and  $R$  is  $T(\mathcal{P}, R)$ , the correspondence of complete extensions and partial stable models directly follows from Lemma 2. Because subset-ordering and  $k$ -ordering on interpretations coincide, so do grounded extensions and well-founded models; and preferred extensions and maximal stable models. Let  $\mathcal{E}$  be a complete extension from Definition 13 and  $\mathcal{E}' = \mathcal{L} \setminus \text{not } \mathcal{E}$ . Since  $\mathcal{E} = \{L \in \mathcal{L} \mid \hat{\mathcal{E}}(L) \geq 1\}$  and  $\mathcal{E}' = \{L \in \mathcal{L} \mid \hat{\mathcal{E}}(L) \geq \frac{1}{2}\}$ , we have  $\mathcal{E}' \setminus \mathcal{E} = \{L \in \mathcal{L} \mid \hat{\mathcal{E}} = \frac{1}{2}\}$ . Therefore semi-stable extensions and least-undefined stable models coincide. If  $\mathcal{E}' = \mathcal{E}$ , then there does not exist a literal  $L \in \mathcal{L}$  with  $\hat{\mathcal{E}}(L) = \frac{1}{2}$ , i.e. stable extensions and total stable models coincide.  $\square$

A remarkable special case happens when the input program  $\mathcal{P}$  has not defeasible rules, and hence in fact it is a regular GLP. In such a case our argumentation-based semantics exactly corresponds to the respective class of models for the GLP. This shows complete backward compatibility of our semantics with the underlying class of logic programs.

**Corollary 1.** *Let  $\mathcal{S}$  be a generalized logic program and  $\mathcal{P} = (\mathcal{S}, \emptyset, \emptyset)$  a defeasible logic program with the empty set of conflict resolutions. Then*

1.  $\mathcal{E}$  is a complete extension of  $\mathcal{P}$  iff  $\mathcal{E}$  is a partial stable model of  $\mathcal{S}$ .
2.  $\mathcal{E}$  is a grounded extension of  $\mathcal{P}$  iff  $\mathcal{E}$  is a well-founded model of  $\mathcal{S}$ .
3.  $\mathcal{E}$  is a preferred extension of  $\mathcal{P}$  iff  $\mathcal{E}$  is a maximal partial stable model of  $\mathcal{S}$ .
4.  $\mathcal{E}$  is a semi-stable extension of  $\mathcal{P}$  iff  $\mathcal{E}$  is a least-undefined partial stable model of  $\mathcal{S}$ .
5.  $\mathcal{E}$  is a stable extension of  $\mathcal{P}$  iff  $\mathcal{E}$  is a total stable model of  $\mathcal{S}$ .

## 4 Properties

In the previous section we have proposed a new semantics for DeLPs and we have showed that it can be characterized in two ways, based on argumentation-theory, by an embedding into ABF, and by a transformation to a GLP. In this section we will have a look on a desired properties for defeasible reasoning, and show that our semantics satisfies these properties.

The properties are formulated in general, that is, they should be satisfied for any a defeasible logic program  $\mathcal{P}$ , any set of conflict resolutions  $R$ , and any extension  $\mathcal{E}$  of  $\mathcal{P}$  w.r.t.  $R$ .

The first two properties formulated below are well known, they were proposed by Caminada and Amgoud [5]. The closure property originally requires that nothing new can be derived from the extension using strict rules. We use a slightly more general formulation, nothing should be derived using the consequence operator  $Cn_{\mathcal{P}}()$  which applies all the strict rules and in addition also all the defeasible rules which are not defeated according to  $R$ . The original property [5] is a straightforward consequence of this.

*Property 1 (Closure).* Let  $\mathcal{R}' = \mathcal{S} \cup \{body(r) \cup \{\text{not } name(r)\} \rightarrow head(r) \mid r \in \mathcal{D}\}$ . Then  $Cn_{\mathcal{R}'}(\mathcal{E}) \subseteq \mathcal{E}$ .

The consistency property [5] formally requires that all conflicts must be resolved in any extension.

*Property 2 (Consistency).*  $\mathcal{E}$  is consistent.

In addition we propose two new desired properties which are concerned with handling of the default assumptions. Reasoning with default assumptions is a fixed part of the semantics of GLPs (and most other classes of logic programs), and therefore in DeLPs it should be governed by similar principles. The first property (dubbed *positive defeat*) requires that for each default literal  $\text{not } A$ , this literal should be always defeated in any extension  $\mathcal{E}$  such that there is a conflict resolution  $\rho \in R$  whose assumptions are all upheld by  $\mathcal{E}$ ; and, in such a case the opposite literal  $A$  should be part of the extension  $\mathcal{E}$ .

*Property 3 (Positive Defeat).* For each default literal  $\text{not } A \in \mathcal{L}_{\mathcal{N}}$ , if there exists a conflict resolution  $\rho \in R$  with  $res(\rho) = \text{not } A$  and  $vuls(\rho) \subseteq \mathcal{E}$  then  $A \in \mathcal{E}$ .

The previous property handles all cases when there is an undefeated evidence against  $\text{not } A$  and requires that  $A$  should hold. The next property (dubbed *negative defeat*) handles the opposite case. If there is no undefeated evidence against  $\text{not } A$ , in terms of a conflict resolution  $\rho \in R$  whose assumptions are all upheld by  $\mathcal{E}$ , then  $\text{not } A$  should be part of the extension  $\mathcal{E}$ .

*Property 4 (Negative Defeat).* For each default literal  $\text{not } A \in \mathcal{L}_{\mathcal{N}}$ , if for each conflict resolution  $\rho \in R$  with  $res(\rho) = \text{not } A$  we have  $vuls(\rho) \not\subseteq \mathcal{E}$  then  $A \notin \mathcal{E}$ .

Closure and consistency trivially hold for our semantics, as the semantics was designed with these properties in mind. They are assured by the definition of complete extension of a DeLP (Definition 13).

**Proposition 3.** *Each complete extension  $\mathcal{E}$  of a defeasible logic program  $\mathcal{P}$  with respect to a set of conflict resolutions  $R$  satisfies the Property 1.*

*Proof.* Let  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  be an assumption-based framework respective to a defeasible logic program  $\mathcal{P}$  and a set of conflict resolutions  $R$ . According to Definition 13, for each complete extension  $\mathcal{E}$  holds  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$ . According to Definition 12,  $\mathcal{R}' \subseteq \mathcal{R}$  and thus  $Cn_{\mathcal{R}'}(\mathcal{E}) \subseteq Cn_{\mathcal{R}}(\mathcal{E})$ . Therefore  $Cn_{\mathcal{R}'}(\mathcal{E}) \subseteq \mathcal{E}$ .  $\square$

**Proposition 4.** *Each complete extension  $\mathcal{E}$  of a defeasible logic program  $\mathcal{P}$  with respect to a set of conflict resolutions  $R$  is consistent.*

*Proof.* Let  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  be an assumption-based framework respective to a defeasible logic program  $\mathcal{P}$  and a set of conflict resolutions  $R$ . According to Definition 13,  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$  and  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$ . Let  $\Delta = \mathcal{E} \cap \mathcal{A}$ . Since  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$ ,  $\Delta$  is closed and attack-free in  $\mathcal{F}$ . Therefore  $\Delta$  is also conflict-free and  $\mathcal{E} = Cn_{\mathcal{R}}(\Delta)$  is consistent.  $\square$

Satisfaction of a positive and a negative defeat properties follow from the instantiation of an ABF (Definition 12), where an inference rule  $vuls(\rho) \rightarrow res(\rho)$  is added for every conflict resolution  $\rho \in R$ .

**Proposition 5.** *Each complete extension  $\mathcal{E}$  of a defeasible logic program  $\mathcal{P}$  with respect to a set of conflict resolutions  $R$  has the property of positive defeat.*

*Proof.* Let  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  be an assumption-based framework respective to a defeasible logic program  $\mathcal{P}$  and a set of conflict resolutions  $R$ . According to Definition 13,  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$  and  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$ . According to Definition 12,  $\mathcal{R} \supseteq \{vuls(\rho) \rightarrow res(\rho) \mid \rho \in R\}$ . Let not  $A \in \mathcal{L}_{\mathcal{N}}$  and  $\rho \in R$  be a resolution with  $res(\rho) = \text{not } A$  and  $vuls(\rho) \subseteq \mathcal{E}$ . Since  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$  and  $\mathcal{R} \supseteq \{vuls(\rho) \rightarrow res(\rho) \mid \rho \in R\}$ , we have  $A \in \mathcal{E}$ .  $\square$

**Proposition 6.** *Each complete extension  $\mathcal{E}$  of a defeasible logic program  $\mathcal{P}$  with respect to a set of conflict resolutions  $R$  has the property of negative defeat.*

*Proof.* Let  $\mathcal{F} = (\mathcal{L}, \mathcal{R}, \mathcal{A}, \neg)$  be an assumption-based framework respective to a defeasible logic program  $\mathcal{P}$  and a set of conflict resolutions  $R$ . According to Definition 13,  $\mathcal{E}$  is a complete extension of  $\mathcal{F}^+$  and  $Cn_{\mathcal{R}}(\mathcal{E}) \subseteq \mathcal{E}$ . According to Definition 12,  $\mathcal{R} \supseteq \{vuls(\rho) \rightarrow res(\rho) \mid \rho \in R\}$ . Let not  $A \in \mathcal{L}_{\mathcal{N}}$  such that for each  $\rho \in R$  with  $res(\rho) = \text{not } A$  holds  $vuls(\rho) \not\subseteq \mathcal{E}$ . Since  $\mathcal{E} = Cn_{\mathcal{R}}(\Delta)$  for some context  $\Delta \subseteq \mathcal{A}$ ,  $A \notin Cn_{\mathcal{R}}(\Delta) = \mathcal{E}$ .  $\square$

## 5 Related Work

There are two well-known argumentation-based formalisms with defeasible inference rules – defeasible logic programs [8] and ASPIC<sup>+</sup> [14,11]. It is known that the semantics in [8] does not satisfy rationality postulates formalized in [5], especially the closure property. Although ASPIC<sup>+</sup> satisfies all postulates in [5], it uses transposition or contraposition which violate directionality of inference rules [3] and thus violating LP intuitions.

Francesca Toni's paper [20] describes a mapping of defeasible reasoning into assumption-based argumentation framework. The work takes into account the properties [5] that we also consider (closedness and consistency), and it is formally proven that the constructed assumption-based argumentation framework's semantics is closed and consistent. However no explicit connection with existing LP semantics is discussed in [20].

The paper [21] does not deal with DeLP, but on how to encode defeasible semantics inside logic programs. The main objective is on explicating a preference ordering on defeasible rules inside a logic program, so that defeats (between defeasible logic rules) are properly encoded in LP. This is achieved with a special predicate *defeated* with special semantics.

Caminada et al. [6] investigated how abstract argumentation semantics and semantics for normal logic programs are related. Authors found out that abstract argumentation is about minimizing/maximizing argument labellings, whereas logic programming is about minimizing/maximizing conclusion labellings. Further, they proved that abstract argumentation semantics cannot capture the L-stable semantics for normal logic programs.

## 6 Conclusion

In this paper we investigated with a question how to define semantics for defeasible logic programs, which satisfies both the existing rationality postulates from the area of structured argumentation and is also aligned with LP semantics and intuitions. To achieve these objectives, we diverged from the usual argumentation process methodology. Most importantly, conflicts are not resolved by attacking some of the conflicting arguments, but by attacking some of the weak parts of an argument called vulnerabilities. Main contributions are as follows:

- We presented an argumentation semantics of defeasible logic programs, based on conflict resolution strategies within assumption-based frameworks.
- The semantics satisfies desired properties like consistency and closedness under the set of strict rules.
- Further, a transformational semantics is constructed. A transformation, which takes a defeasible logic program and a conflict resolution strategy as an input, generates a corresponding generalized logic program. As a consequence, a computation of argumentation semantics of DeLP in the frame of GLP is enabled.
- Equivalence of a transformational and an argumentation semantics is provided. Specifically, complete extensions correspond to partial stable models, the grounded extension to the well-founded model, stable extensions to total stable models, preferred extensions to maximal partial stable models and semi-stable extensions to least-undefined partial stable models.

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