Expressive Description Logic with Instantiation Metamodelling

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Abstract

We investigate a higher-order extension of the description logic (DL) $\mathcal{SROIQ}$ that provides a fixedly interpreted role semantically coupled with instantiation. It is useful to express interesting meta-level constraints on the modelled ontology. We provide a model-theoretic characterization of the semantics, and we show the decidability by means of reduction.

Introduction

Metamodelling allows to model with universal entities (concepts and roles) as if they were individuals. This is useful in some complex domains with a high number of universal entities where it makes sense to further categorize them into meta concepts, and use meta roles to express relations among them. In the biological taxonomy, organisms are classified into taxa. The giraffes are classified as the species $\mathsf{G.camelopardalis}$. One well known preserved specimen of giraffe is called Zarafa: $\mathsf{Zarafa} : \mathsf{G.camelopardalis}$. The giraffes are part of larger taxa: $\mathsf{G.camelopardalis} \sqsubseteq \mathsf{Giraffa}$ and in turn $\mathsf{Giraffa} \sqsubseteq \mathsf{Giraffidae}$.

Biological taxonomy is complex and hence taxa are sorted into a hierarchy of ranks. In a DL-like syntax we could write: $\mathsf{G.camelopardalis} : \mathsf{Species}$, $\mathsf{Giraffa} : \mathsf{Genus}$, and $\mathsf{Giraffidae} : \mathsf{Family}$. All taxa belong to the concept Taxon (i.e., $\mathsf{G.camelopardalis} : \mathsf{Taxon}$, etc.) and all ranks into concept Rank (i.e., $\mathsf{Species} : \mathsf{Rank}$, etc.).

Metamodelling allows to express complex constraints on top of such a meta classification. We may want to say that different ranks are disjoint, e.g., $\mathsf{Species} \sqcap \mathsf{Genus} \sqsubseteq \bot$, etc. Similarly, taxa and ranks should be disjoint: $\mathsf{Taxon} \sqcap \mathsf{Rank} \sqsubseteq \bot$. Using meta roles we may define $\mathsf{LinnaeanSpecies} \equiv \mathsf{Species} \sqcap \exists \mathsf{definedBy}.\{\mathsf{vonLinné}\}$ where $\mathsf{definedBy}$ relates concepts with individuals.

Meta classification cannot be expressed in regular DLs. It is expressible in RDF(S), which has higher-order semantics but also limited expressivity of axioms (e.g., the constraints above cannot be expressed in it). This could be overcome in OWL 2 with punning (Cuenca Grau et al. 2008), but punning is effectively equivalent to Motik's π-semantics which lacks desired semantic properties (Motik 2007).

A number of higher-order DLs have been proposed with the intent to facilitate metamodelling. The first group relies on HiLog-style semantics (Chen, Kifer, and Warren 1993), used also for RDF: Each entity name is interpreted as an *intension* – an unstructured object representing the entity's internal meaning. Concept and role *extensions* are then assigned to intensions. Works in this group include those of Motik (2007), De Giacomo, Lenzerini, and Rosati (2011), and Homola et al. (2014). The second group relies on Henkin's (1950) general semantics for higher-order logic, in which concepts are directly interpreted as sets, meta concepts as sets of sets, etc. This semantics is stronger and has distinct properties. This category includes the works of Pan and Horrocks (2007) and Motz, Rohrer, and Severi (2015).

In metamodelling we would like to constraint also on the ontological structure of the modelled domain. For example, the biological taxonomy requires that each taxon must be classified within some rank. To achieve this we would like to have a fixedly interpreted role, which we will call $\mathsf{instanceOf}$, that associates instances with the classes they belong to, and we would like to be able to model with this role (i.e., this is alike to modelling with $\mathsf{rdf{:}type}$ in the undecidable OWL Full). Thus we could assert the constraint from above: $\mathsf{Taxon} \sqsubseteq \exists \mathsf{instanceOf}.\mathsf{Rank}$. Or as a slightly more involved example, we could require that for each Linnaean species, there exists a specimen (instance of the species) located in British Musem: $\mathsf{LinnaeanSpecies} \sqsubseteq \exists \mathsf{instanceOf}^{-}.(\mathsf{Specimen} \sqcap \exists \mathsf{locatedIn}.\{\mathsf{britishMusem}\})$.

This problem was previously addressed by Glimm, Rudolph, and Völker (2010) who proposed an axiomatization for $\mathsf{instanceOf}$ (called type) for metamodelling within OWL 2. They were not interested in the development of DL framework with a higher-order model-theoretic semantics and investigating its properties, as we do in this paper.

We propose $\mathcal{HIR}(\mathcal{SROIQ})$ (where $\mathcal{HIR}$ is read as "higher") with the following features suitable for metamodelling: (1) It sports HiLog-style semantics, which has desirable properties when it comes to metamodelling, as discussed later on. (2) It maintains basic separation between individuals, concepts, and roles, conforming to established practice in ontologies. (3) It allows meta concepts and meta roles which are promiscuous (they can classify/relate any entities), but a type hierarchy can be axiomatized if needed. (4) It features a fixedly interpreted $\mathsf{instanceOf}$ role, repre-

Syntax	Extension	Syntax	Extension
R_0	$R_0^{\mathcal{IE}}$	U	$\Delta_\mathrm{I} \times \Delta_\mathrm{I}$
instanceOf	$\{ (x,y) \mid x \in \Delta^\mathcal{I} \wedge y \in \Delta^\mathcal{I}_\mathrm{C} \wedge x \in y^\mathcal{E} \}$		
R^-	$\{ (y,x) \mid (x,y) \in R^\mathcal{E} \}$	$S \cdot Q$	$S^\mathcal{E} \circ Q^\mathcal{E}$
A	$A^{\mathcal{IE}}$	$\{B\}$	$\{B^\mathcal{I}\}$
$\neg C$	$\Delta^\mathcal{I} \setminus C^\mathcal{E}$	$C \sqcap D$	$C^\mathcal{E} \cap D^\mathcal{E}$
$\exists R.C$	$\{ x \mid \exists y.(x,y) \in R^\mathcal{E} \wedge y \in C^\mathcal{E} \}$		
$\geqslant n R.C$	$\{ x \mid \#\{ y \mid (x,y) \in R^\mathcal{E} \wedge y \in C^\mathcal{E} \} \geq n \}$		
$\exists R.$Self	$\{ x \mid (x,x) \in R^\mathcal{E} \}$		
Syntax	**Semantics**	**Syntax**	**Semantics**
$C \sqsubseteq D$	$C^\mathcal{E} \subseteq D^\mathcal{E}$	$B : C$	$B^\mathcal{I} \in C^\mathcal{E}$
$w \sqsubseteq R$	$w^\mathcal{E} \subseteq R^\mathcal{E}$	$B_1, B_2 : R$	$(B_1^\mathcal{I}, B_2^\mathcal{I}) \in R^\mathcal{E}$
$\mathrm{Dis}(P,R)$	$P^\mathcal{E} \cap R^\mathcal{E} = \emptyset$	$B_1, B_2 : \neg R$	$(B_1^\mathcal{I}, B_2^\mathcal{I}) \notin R^\mathcal{E}$

Table 1: Syntax and semantics of $\mathcal{HIR}(\mathcal{SROIQ})$

senting instantiation, which can be freely modeled with.

For an extended technical report with proofs please refer to `http://kedrigern.dcs.fmph.uniba.sk/reports/download.php?id=60`.

$\mathcal{HIR}(\mathcal{L})$ Description Logics

We will now introduce a higher-order DL $\mathcal{HIR}(\mathcal{SROIQ})$ which extends $\mathcal{SROIQ}$ (Horrocks, Kutz, and Sattler 2006) with metamodelling capabilities. It provides meta concepts and meta roles by allowing concepts and roles to classify/relate not only individuals, but also concepts (designated by the letter $\mathcal{H}$) and roles (designated by the letter $\mathcal{R}$). Concepts, role domains, and role ranges are *promiscuous*, that is, they may contain a mix of different entities. This is useful to allow concepts such as Deprecated and roles such as definedBy. If desired, typed concepts may be introduced into $\mathcal{HIR}(\mathcal{SROIQ})$ by additional axiomatization.

$\mathcal{HIR}(\mathcal{SROIQ})$ employs HiLog-based semantics, which has desired properties for metamodelling, as discussed later on. Unlike some other HiLog-based DLs, it distinguishes among individuals, concepts, and roles. Individuals have no extensions, concepts' extensions are sets of entities they classify, and roles' extensions are sets of pairs of entities they relate. Such distinction is fundamental from the ontological standpoint, it saves users accustomed to first-order DLs from surprises, and provides basic sanity checks.

Our logic features a fixed instanceOf role (designated by the letter $\mathcal{I}$ in $\mathcal{HIR}$). This role metamodels the instantiation relation, i.e., it connects exactly those pairs (X, Y) where X belongs to the concept extension of Y. The instanceOf role is freely usable in axioms like any other role.

Syntax and Semantics

Definition 1. Let $N = N_\mathrm{I} \uplus N_\mathrm{C} \uplus N_\mathrm{R}$ be a DL vocabulary such that instanceOf $\in N_\mathrm{R}$. $\mathcal{HIR}(\mathcal{SROIQ})$ *role expressions* are inductively defined as the smallest set containing the expressions listed in Table 1 (upper part), where $R_0 \in N_\mathrm{R} \setminus \{\text{instanceOf}, \text{U}\}$, R is an atomic or inverse role, S and Q are role expressions. $\mathcal{HIR}(\mathcal{SROIQ})$ *descriptions* are inductively defined as the smallest set containing the expressions listed in Table 1 (middle part), where $A \in N_\mathrm{C}$, $B \in N$, C and D are descriptions, and R is an atomic or inverse role. A $\mathcal{HIR}(\mathcal{SROIQ})$ *knowledge base* $\mathcal{K}$ is a finite set of *axioms* of the forms listed in Table 1 (bottom part), where $B, B_1, B_2 \in N$, C and D are descriptions, P and R are atomic or inverse roles, and w is a role chain.

$\mathcal{HIR}(\mathcal{L})$ for a fragment $\mathcal{L}$ of $\mathcal{SROIQ}$ is the corresponding fragment of $\mathcal{HIR}(\mathcal{SROIQ})$.

$\mathcal{HIR}(\mathcal{SROIQ})$ easily models the taxonomic example from the Introduction. Taxa are classified to meta concepts of ranks (1), ranks to the meta meta concept Rank (2). Meta concepts can be used just as concepts (3).

$$\text{G. camelopardalis} : \text{Species} \quad \text{Giraffa} : \text{Genus} \tag{1}$$

$$\text{Species} : \text{Rank} \quad \text{Genus} : \text{Rank} \tag{2}$$

$$\text{Species} \sqcup \text{Genus} \sqcup \cdots \sqsubseteq \text{Taxon} \quad \text{Species} \sqcap \text{Genus} \sqsubseteq \bot \tag{3}$$

Meta roles can connect concepts with individuals, e.g., a taxon or rank with a person (4), but also with other concepts, e.g., one species with its evolutionary successor species (5). We can then express complex meta concepts such as LinnaeanSpecies (6). Since $\mathcal{HIR}$ allows roles as instances, we can also classify different types of animal behaviour (7):

$$\exists \text{definedBy}.\top \sqsubseteq \text{Taxon} \sqcup \text{Rank} \quad \top \sqsubseteq \forall \text{definedBy}.\text{Person} \\ \text{Giraffa}, \text{M. T. Brünnich} : \text{definedBy} \tag{4}$$

$$\exists \text{successorOf}.\top \sqsubseteq \text{Species} \quad \top \sqsubseteq \forall \text{successorOf}.\text{Species} \tag{5}$$

$$\text{LinnaeanSpecies} \equiv \text{Species} \sqcap \exists \text{definedBy}.\{\text{vonLinné}\} \tag{6}$$

$$\text{migratesTo} : \text{Behaviour} \quad \text{imitates} : \text{LearningBehaviour} \\ \text{LearningBehaviour} \sqsubseteq \text{Behaviour} \tag{7}$$

$\mathcal{HIR}$ semantics is HiLog-based, with a denotation function $\cdot^\mathcal{I}$ from names to intensions and an extension function $\cdot^\mathcal{E}$. When treated as a concept instance or a role actor, a name's semantics is its intension. When treated as a concept or a role, the extension of the name's intension is considered. The instanceOf role, as mentioned before, has fixed semantics, defined in Table 1, just like rdf:type in RDF.

Definition 2. A $\mathcal{HIR}$ *interpretation* of a DL vocabulary N with instanceOf $\in N_\mathrm{R}$ is a triple $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I}, \cdot^\mathcal{E})$ such that:

1. $\Delta^\mathcal{I} = \Delta^\mathcal{I}_\mathrm{I} \uplus \Delta^\mathcal{I}_\mathrm{C} \uplus \Delta^\mathcal{I}_\mathrm{R}$ where $\Delta^\mathcal{I}_\mathrm{I}, \Delta^\mathcal{I}_\mathrm{C}, \Delta^\mathcal{I}_\mathrm{R}$ are pairwise disjoint,
2. $a^\mathcal{I} \in \Delta^\mathcal{I}_\mathrm{I}$ for each $a \in N_\mathrm{I}$, $A^\mathcal{I} \in \Delta^\mathcal{I}_\mathrm{C}$ for each $A \in N_\mathrm{C}$, $R^\mathcal{I} \in \Delta^\mathcal{I}_\mathrm{R}$ for each $R \in N_\mathrm{R}$,
3. $R^\mathcal{I} \neq S^\mathcal{I}$ whenever $R, S \in N_\mathrm{R}$ and $R \neq S$ (unique role assumption),
4. $c^\mathcal{E} \subseteq \Delta^\mathcal{I}$ for each $c \in \Delta^\mathcal{I}_\mathrm{C}$, $r^\mathcal{E} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$ for each $r \in \Delta^\mathcal{I}_\mathrm{R}$.

Extensions of role expressions $R^\mathcal{E}$ and of descriptions $C^\mathcal{E}$ are inductively defined according to Table 1.

Note that without the unique role assumption (URA) it is easy to cause two role names to have the same intension (e.g. by an axiom $\{R\} \equiv \{S\}$), and thus also the same extension. That, as Motik (2007) has shown, leads to undecidability in logics admitting transitive roles and cardinality restrictions.

Definition 3. An axiom φ is *satisfied* by a $\mathcal{HIR}$ interpretation $\mathcal{I}$ ($\mathcal{I} \models \varphi$) if $\mathcal{I}$ satisfies the respective semantic constraints from the lower part of Table 1. A $\mathcal{HIR}$ interpretation $\mathcal{I}$ is a *model* of $\mathcal{K}$ ($\mathcal{I} \models \mathcal{K}$) if $\mathcal{I}$ satisfies every axiom

$\varphi \in \mathcal{K}$. A concept C is *satisfiable in* $\mathcal{K}$ if there exists a model $\mathcal{I}$ of $\mathcal{K}$ such that $C^{\mathcal{I}} \neq \emptyset$. An axiom φ is *entailed* by $\mathcal{K}$ ($\mathcal{K} \models \varphi$) if $\mathcal{I} \models \varphi$ holds for each $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{K}$.

The instanceOf role allows to "move across" meta layers in modelling: Restrictions on instanceOf can select instances of concepts satisfying various meta criteria, e.g., specimens of Linnaean species described by someone else than von Linné (8). Conversely, restrictions on instanceOf$^-$ select concepts whose instances satisfy complex criteria, e.g., species with specimens located in the British Museum (9). We can thus express that every instance of any taxon is an organism (10). Assuming that every taxon is an instance of some rank and all ranks are instances of Rank, an equivalent statement is possible via the meta meta level (11). We can also express mutual disjointness of Species by asserting that each Organism is classified as at most one species (12).

$$\begin{aligned}&\mathsf{Specimen} \sqcap \exists \mathsf{describedBy}.\neg\{\mathsf{vonLinn\acute{e}}\}\\ &\quad\sqcap \exists \mathsf{instanceOf}.(\mathsf{Species} \sqcap \exists \mathsf{definedBy}.\{\mathsf{vonLinn\acute{e}}\})\end{aligned} \tag{8}$$

$$\begin{aligned}&\mathsf{Species} \sqcap \exists \mathsf{instanceOf}^-.(\mathsf{Specimen} \sqcap\\ &\qquad \exists \mathsf{locatedIn}.\{\mathsf{britishMusem}\})\end{aligned} \tag{9}$$

$$\exists \mathsf{instanceOf}.\mathsf{Taxon} \sqsubseteq \mathsf{Organism} \tag{10}$$

$$\exists \mathsf{instanceOf}.\exists \mathsf{instanceOf}.\mathsf{Rank} \sqsubseteq \mathsf{Organism} \tag{11}$$

$$\mathsf{Organism} \sqsubseteq \leqslant 1\, \mathsf{instanceOf}.\mathsf{Species} \tag{12}$$

We can also create subroles of the instanceOf role, e.g., hasType assigning a prototypical specimen to each species (13), and use them also in number restrictions, e.g., to assert that each species has exactly one holotype (the "most notable" specimen) and it is located in a major museum (14). While we could have created the meta role hasType anyway, without using instanceOf we could not assure that it connects each species with one of its instances.

$$\begin{aligned}&\mathsf{hasType} \sqsubseteq \mathsf{instanceOf}^-\\ &\exists \mathsf{hasType}.\top \sqsubseteq \mathsf{Species} \qquad \top \sqsubseteq \forall \mathsf{hasType}.\mathsf{Specimen}\end{aligned} \tag{13}$$

$$\begin{aligned}&\mathsf{Species} \sqsubseteq \leqslant 1 \mathsf{hasType}.\mathsf{Holotype} \sqcap\\ &\quad =1 \mathsf{hasType}.(\mathsf{Holotype} \sqcap \exists \mathsf{locatedIn}.\mathsf{MajorMusem})\end{aligned} \tag{14}$$

Decidability

We will now show how $\mathcal{HIR}(\mathcal{SROIQ})$ can be reduced to first-order $\mathcal{SROIQ}$. The reduction is based on the ideas of Glimm, Rudolph, and Völker (2010). For each concept A, a new individual name i_A is introduced to represent A's intension. These new names are instances of a new concept $\top_{\mathrm{C}}$ of concept intensions. The relationship between A and i_A is expressed through the instanceOf role. In the reduced knowledge base, instanceOf is an ordinary, axiomatized role.

Definition 4 (First-Order Reduction). A DL vocabulary N with instanceOf $\in N_{\mathrm{R}}$ is reduced into a DL vocabulary $N^1 := (N^1_{\mathrm{C}}, N^1_{\mathrm{R}}, N^1_{\mathrm{I}})$ where $N^1_{\mathrm{C}} = N_{\mathrm{C}} \uplus \{\top_{\mathrm{C}}, \top_{\mathrm{R}}\}$, $N^1_{\mathrm{R}} = N_{\mathrm{R}}$, and $N^1_{\mathrm{I}} = N_{\mathrm{I}} \uplus \{ \mathsf{i}_A \mid A \in N_{\mathrm{C}} \} \uplus \{ \mathsf{i}_R \mid R \in N_{\mathrm{R}} \}$ for fresh names $\top_{\mathrm{C}}$, $\top_{\mathrm{R}}$, i_A and i_R for all $A \in N_{\mathrm{C}}, R \in N_{\mathrm{R}}$.

A given $\mathcal{HIR}(\mathcal{SROIQ})$ KB $\mathcal{K}$ in N is reduced into a $\mathcal{SROIQ}$ KB $\mathcal{K}^1 := \mathrm{Int}(\mathcal{K}) \cup \mathrm{InstSync}(N) \cup \mathrm{Typing}(N) \cup \mathrm{URA}(N)$ in N^1 where: $\mathrm{Int}(\mathcal{K})$ is obtained from $\mathcal{K}$ by replacing each occurence of $A \in N_{\mathrm{C}}$ and $R \in N_{\mathrm{R}}$ in a nominal or in the left-hand side of a concept or (negative) role assertion by i_A and i_R, respectively. $\mathrm{InstSync}(N)$ consists of axioms $A \equiv \exists \mathsf{instanceOf}.\{\mathsf{i}_A\}$ for all $A \in N_{\mathrm{C}}$. $\mathrm{Typing}(N)$ consists of axioms $\top \sqsubseteq \forall \mathsf{instanceOf}.\top_{\mathrm{C}}$, $\top_{\mathrm{R}} \sqsubseteq \neg\top_{\mathrm{C}}$, $a\colon \neg\top_{\mathrm{C}} \sqcap \neg\top_{\mathrm{R}}$, $\mathsf{i}_R\colon \top_{\mathrm{R}}$, and $\mathsf{i}_A\colon \top_{\mathrm{C}}$ for all $a \in N_{\mathrm{I}}$, $A \in N_{\mathrm{C}}$, and $R \in N_{\mathrm{R}}$. $\mathrm{URA}(N)$ consists of axioms $R\colon \neg\{S\}$ for all pairs of distinct role names $R, S \in N_{\mathrm{R}}$.

The following theorem asserts that $\mathcal{K}^1$ is just as strong as $\mathcal{K}$. The more involved part of its proof is finding a $\mathcal{HIR}(\mathcal{SROIQ})$ model $\mathcal{I}$ of $\mathcal{K}$ for a first-order model $\mathcal{J}$ of $\mathcal{K}^1$. We define the set of concept intensions $\Delta^{\mathcal{I}}_{\mathrm{C}}$ as $\top^{\mathcal{J}}_{\mathrm{C}}$, and let the intension of each atomic concept A be $\mathsf{i}^{\mathcal{J}}_A$. We then define for each $c \in \Delta^{\mathcal{I}}_{\mathrm{C}}$ the extension $c^{\mathcal{E}}$ as the set of all xs related to it by $\mathsf{instanceOf}^{\mathcal{J}}$. This ensures $\mathsf{instanceOf}^{\mathcal{E}} = \mathsf{instanceOf}^{\mathcal{J}}$. Since the interpretation of instanceOf is constrained by the InstSync axioms, extension of each atomic concept $A^{\mathcal{IE}}$ is equal to its first-order interpretation $A^{\mathcal{J}}$.

Theorem 1. *For any $\mathcal{HIR}(\mathcal{SROIQ})$ KB $\mathcal{K}$ and any axiom φ in a common vocabulary N, $\mathcal{K} \models \varphi$ iff $\mathcal{K}^1 \models \mathrm{Int}(\varphi)$.*

Observe that (a) the size of $\mathcal{K}^1$ (string-length-wise) is at most quadratic in the size of $\mathcal{K}$ (if N consists exactly of symbols occurring in $\mathcal{K}$); (b) if $\mathcal{K}$ satisfies, for all roles including instanceOf, the $\mathcal{SROIQ}$ restrictions (Horrocks, Kutz, and Sattler 2006), so does $\mathcal{K}^1$; (c) $\mathcal{SROIQ}$ concept satisfiability is decidable in N2ExpTime (Kazakov 2008). Hence the following corollary.

Corollary 1. *Let a $\mathcal{HIR}(\mathcal{SROIQ})$ KB $\mathcal{K}$ be such that only simple roles occur in cardinality restrictions. Concept satisfiability and entailment in a $\mathcal{HIR}(\mathcal{SROIQ})$ KB are then decidable in* N2ExpTime*.*

In general, $\mathcal{HIR}(\mathcal{L})$ reduces to $\mathcal{LO}$ if the DL $\mathcal{L}$ admits GCIs, existential restriction, and complement. The decidability and complexity of standard reasoning tasks for $\mathcal{HIR}(\mathcal{L})$ are then the same as for $\mathcal{LO}$.

Type Hierarchy Axiomatization

$\mathcal{HIR}(\mathcal{L})$ concepts are fully promiscuous, i.e., they can simultaneously have as instances entities of all kinds – individuals, roles, or concepts. This is useful for concepts such as Deprecated (any entity may become deprecated) and roles such as definedBy (whose domain may contain any entity). However, promiscuity of many concepts is undesired, e.g., Person and Museum should classify only individuals, Species only concepts with individual instances. For such cases, we introduce a typing framework axiomatization:

Definition 5 (Typing framework). Given $n > 0$, a $\mathcal{HIR}$ *KB with n types* contains concept names $\top^{X(i)}$ for each i, $0 < i \leq n$, and each $X \in \{\mathrm{I}, \mathrm{R}, \mathrm{IR}\}$, and axioms listed in Table 2 for all $X, Y \in \{\mathrm{I}, \mathrm{R}, \mathrm{IR}\}$ and $Z \in \{\mathrm{I}, \mathrm{R}\}$ s.t. $X \neq Z$.

At the first level, the $\top^{\mathrm{I}(1)}$ concept classifies precisely all individuals, $\top^{\mathrm{R}(1)}$ precisely all roles, and $\top^{\mathrm{IR}(1)}$ all individuals or roles. At the second level, $\top^{\mathrm{I}(2)}$ classifies precisely all concepts with only individual instances, $\top^{\mathrm{R}(2)}$ concepts of roles, and $\top^{\mathrm{IR}(2)}$ concepts of individuals or roles, etc.

The typing desired in our example is asserted by axioms $\mathsf{Organism} \sqcup \mathsf{Person} \sqcup \mathsf{Museum} \sqsubseteq \top^{\mathrm{I}(1)}$, $\mathsf{Taxon} \sqsubseteq \top^{\mathrm{I}(2)}$,

$\top^{X(t)} \sqsubseteq \forall \mathsf{instanceOf}^{-}.\top^{X(t-1)}$	for each t s. t. $0 < t \leq n$
$a\colon \top^{\mathrm{I}(1)},\ R\colon \top^{\mathrm{R}(1)}$	for each $a \in N_{\mathrm{I}}, R \in N_{\mathrm{R}}$
$\top^{X(t)} \sqsubseteq \neg\top^{Y(u)}$	for each $t \neq u$, $0 < t, u \leq n$
$\top^{X(t)} \sqsubseteq \neg\top^{Z(t)}$	for each t s. t. $0 < t \leq n$
$\top^{\mathrm{IR}(1)} \equiv \top^{\mathrm{I}(1)} \sqcup \top^{\mathrm{R}(1)}$	

Table 2: Knowledge base with n types

$\mathsf{Rank} \sqsubseteq \top^{\mathrm{I}(3)}$. Typing propagates to subconcepts and instances: $\mathsf{G.camelopardalis} \sqsubseteq \top^{\mathrm{I}(1)}$ and $\mathsf{Species} \sqsubseteq \top^{\mathrm{I}(2)}$ are now entailed, and so for other taxa and ranks. Domains and ranges of roles are typed similarly, e.g., $\exists\mathsf{successorOf}.\top \sqsubseteq \top^{\mathrm{I}(2)}$ and $\top \sqsubseteq \forall\mathsf{successorOf}.\top^{\mathrm{I}(2)}$.

Some Properties of $\mathcal{HIR}(\mathcal{L})$ Logics

$\mathcal{HIR}(\mathcal{L})$ has the basic properties of HiLog-based logics: *intensional regularity* $\mathcal{K} \models X = Y \implies \mathcal{K} \models X \equiv Y$ and the lack of *extensionality* $\mathcal{K} \models X \equiv Y \implies \mathcal{K} \models X = Y$.

If $\mathcal{K} \models A = B$ for concept names A and B, then in every model $\mathcal{I}$ we have $A^{\mathcal{I}} = B^{\mathcal{I}}$ and therefore $A^{\mathcal{I}\mathcal{E}} = B^{\mathcal{I}\mathcal{E}}$; hence $\mathcal{K} \models A \equiv B$. $\mathcal{HIR}(\mathcal{L})$ is thus intensionally regular for concepts. This is quite a natural requirement for metamodelling (cf. Motik (2007)). If we assert that an international and a Slovak name denote the same species ($\mathsf{G.camelopardalis} = $ Žirafa štihla), we expect their extensions to also be equal.

A $\mathcal{HIR}$ model of a KB $\mathcal{K} \models A \equiv B$ can assign A and B distinct intensions $A^{\mathcal{I}} = a \neq b = B^{\mathcal{I}}$ sharing the extension $a^{\mathcal{E}} = b^{\mathcal{E}} \subseteq \Delta^{\mathcal{I}}$. $\mathcal{HIR}(\mathcal{L})$ thus lacks extensionality. This enables deprecating the old binomial name of the species Cervus camelopardalis without deprecating its newer name Giraffa camelopardalis although they classify the same organisms (Giraffa c. $\equiv$ Cervus c.), or modelling of single-species genera (e.g., Sommeromys: Genus, S. macrorhinos: Species, Sommeromys $\equiv$ S. macrorhinos) without contradicting $\mathsf{Species} \sqcap \mathsf{Genus} \sqsubseteq \bot$.

$\mathcal{HIR}$ shares intensional regularity and the lack of extensionality with other DL descendants of HiLog (cf. Introduction). We have argued against extensionality in metamodelling applications. Use cases where extensionality is needed were demonstrated by Motz, Rohrer, and Severi (2015), and can be covered by one of the logics with Henkin's general semantics (ibid.; Pan and Horrocks 2007), though they lack metamodelling of instantiation. As showed by Motik (2007), punning in OWL 2 (Cuenca Grau et al. 2008), semantically equivalent to his π-semantics, is neither intensionally regular nor extensional; it supports only very basic metamodelling.

$\mathcal{HIR}(\mathcal{SROIQ})$'s expressivity makes it vulnerable to Russel's paradox: There is no model of a KB with a concept of concepts which are not instances of themselves, e.g., $\mathsf{Barber} \equiv \neg\exists\mathsf{instanceOf}.\mathsf{Self}$. However, due to reducibility, the underlying inconsistency can already be produced in $\mathcal{SROIQ}$.

Instantiation in $\mathcal{HIR}(\mathcal{L})$ is not necessarily well founded, e.g., instanceOf can be a super-role of a role with an infinite descending chain. Some approaches to metamodelling (Pan and Horrocks 2007; Motz, Rohrer, and Severi 2015) avoid or ban non-well-founded instantiation. However, it does not impact decidability (due to reducibility, Thm. 1), and can be prevented by using the typing framework from Definition 5.

Conclusions

We have introduced a higher-order DL framework $\mathcal{HIR}(\mathcal{L})$ which enriches a DL $\mathcal{L}$ with promiscuous higher-order concepts and roles, and a fixedly interpreted role instanceOf, whose semantics is akin to rdf:type in RDF/OWL. This is useful for metamodelling, specifically to traverse the levels of higher orders, to express complex properties of concepts based on their membership in extensions of meta concepts and vice versa, and to constrain the ontological structure of the modelled domain by asserting constraints on instanceOf. We have showed a polynomial reduction from $\mathcal{HIR}(\mathcal{L})$ to the underlaying DL $\mathcal{L}$. This enables to use off-the-shelf reasoners and to maintain the same computational complexity as the underlaying logic.

Our work is most closely related to that of Homola et al. (2014) which is here extended by promiscuity and modelling with instantiation. We base many of our constructions on the work of Glimm, Rudolph, and Völker (2010) who, however, do not enable orders higher than the second, meta roles, nor promiscuity. They also do not provide any higher-order model-theoretic characterization, which is instrumental to study the semantic properties of the logic.

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