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ALTERNATIVE CHARACTERIZATION OF CONJUNCTIVE BRIDGE RULES IN DISTRIBUTED DESCRIPTION LOGICS (PRELIMINARY REPORT)

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Distributed description logics (DDLs) is a KR formalism that enables reasoning with multiple ontologies interconnected by directional semantic mapping (bridge rules). DDLs capture the idea of importing and reusing concepts between ontologies and thus combine well with intuitions behind the Semantic Web. In this paper, we present a preliminary consideration on the possible alternative characterisation of the semantics that handles so called conjunctive bridge rules. We believe that better understanding of this subject may help us to devise more efficient reasoning techniques. We report, that instead of verifying numerous conditions, we may restrict the mapping that is used in interpretations of DDL knowledge bases. However, it is subject to further investigation whether the semantics keeps all the desired properties.

K e y w o r d s: distributed description logics, conjunctive bridge rules

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1 INTRODUCTION

Devision of distributed ontology representation frameworks is an interesting research topic [1–7] with practical motivation. Among the approaches we find Distributed description logics (DDLs) (Serafini et al. [2]) of particular interest. DDLs are built upon the formal and well established framework of Description logics (DLs) [8,9]. They are intended especially to enable reasoning between multiple ontologies connected by directional semantic mapping (bridge rules), and thus they capture the idea of importing and reusing concepts between ontologies. This goes well with the basic assumption of the Semantic Web that no central ontology but rather many ontologies with redundant knowledge will exist [10].

Recently we have proposed an extension to DDLs in [11]. We have introduced so called conjunctive onto-bridge rules, in order to cope with a problem that has been pointed out in [4]. We have also described a transformational semantics, and hence a naïve decision procedure for the adjusted framework, that makes reuse of the decision procedure which is known for the original framework.

In this paper, we present a preliminary consideration on the possible alternative characterisation of the part of the semantics that handles conjunctive bridge rules. We believe that better understanding of this subject may help us to devise more efficient reasoning techniques. We report, that instead of verifying numerous conditions, we may restrict the mapping that is used in interpretations of DDL knowledge bases. This, however, rules out some interpretations, and it is subject to further investigation whether the semantics keeps all the desired properties.

2 DISTRIBUTED DESCRIPTION LOGICS

As given in [2], a DDL knowledge base $\mathfrak{T}$ (called distributed TBox) is a pair $\langle\{\mathcal{T}_i\}_{i\in I},\mathfrak{B}\rangle$ consisting of a set of local TBoxes $\{\mathcal{T}_i\}_{i\in I}$ and a set of bridge rules $\mathfrak{B}=\bigcup_{i,j\in I,i\neq j}\mathfrak{B}_{ij}$ between these local TBoxes, for some non-empty index-set I. Each local TBox is a knowledge base written in its own DL language $\mathcal{L}_i$ but $\mathcal{L}_i$ is always a sub-language of $\mathcal{SHIQ}$ [9]. For the lack of space we cannot summarize the syntax and semantics of $\mathcal{SHIQ}$ in full detail; let it suffice to say that each local TBox $\mathcal{T}_i$ is a collection of axioms of the form $i : C \sqsubseteq D$. Intuitively, this asserts that the concept D is more general than the concept C. We kindly refer the reader to [2] for more details. Each $\mathfrak{B}_{ij}$ is a set of directed bridge rules from $\mathcal{T}_i$ to $\mathcal{T}_j$. Intuitively, these are meant to “import” information from $\mathcal{T}_i$ to $\mathcal{T}_j$, and therefore $\mathfrak{B}_{ij}$ and $\mathfrak{B}_{ji}$ may possibly, and expectedly, differ. Bridge rules of $\mathfrak{B}_{ij}$ are of two basic kinds, *into*-bridge rules and *onto*-bridge rules (in the respective order):

$$i : A \xrightarrow{\sqsubseteq} j : G\,, \qquad i : B \xrightarrow{\sqsupseteq} j : H\,.$$

Moreover, in [11] we have introduced *conjunctive* onto-bridge rules with the following syntax:

$$i : B \overset{\sqsupseteq}{\twoheadrightarrow} j : H\,.$$

A distributed interpretation $\mathfrak{I}=\langle\{\mathcal{I}_i\}_{i\in I},\{r_{ij}\}_{i,j\in I,i\neq j}\rangle$ of a distributed TBox $\mathfrak{T}$ consists of a set of local interpretations $\{\mathcal{I}_i\}_{i\in I}$ and a set of domain relations $\{r_{ij}\}_{i,j\in I,i\neq j}$. Each $\mathcal{I}_i$ is either an interpretation in the respective DL $\mathcal{L}_i$, that is, a pair $\langle\Delta^{\mathcal{I}_i},\cdot^{\mathcal{I}_i}\rangle$, or it is a special interpretation $\mathcal{I}^\epsilon=\langle\emptyset,\cdot^\epsilon\rangle$ called a hole. The domain

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of each $\mathcal{I}_i$ is always required to be nonempty except for holes. Holes are used to fight propagation of inconsistency in the distributed knowledge base. Each domain relation r_{ij} in turn, is a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$, and it is used to interpret bridge rules in $\mathfrak{B}_{ij}$. We denote by $r_{ij}(d)$ the set $\{d' \mid \langle d, d' \rangle \in r_{ij}\}$, by $r_{ij}(D)$ the set $\bigcup_{d \in D} r_{ij}(d)$, and by r the union $\bigcup_{i,j \in I} r_{ij}$.

Definition 1. For every i and j, a distributed interpretation $\mathfrak{I}$ satisfies the elements of a distributed TBox $\mathfrak{T}$ (denoted by $\mathfrak{I} \models_\epsilon \cdot$) according to the following clauses:

1. $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$ if $\mathcal{I}_i \models C \sqsubseteq D$,
2. $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ if $\mathcal{I}_i$ is a model of $\mathcal{T}_i$,
3. $\mathfrak{I} \models_\epsilon i : C \xrightarrow{\sqsubseteq} j : G$ if $r_{ij}(C^{\mathcal{I}_i}) \subseteq G^{\mathcal{I}_j}$,
4. $\mathfrak{I} \models_\epsilon i : C \xrightarrow{\sqsupseteq} j : G$ if $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$,
5. $\mathfrak{I} \models_\epsilon i : C \overset{\sqsupseteq}{\twoheadrightarrow} j : G$ if for each $i : D \overset{\sqsupseteq}{\twoheadrightarrow} j : H \in \mathfrak{B}$, $r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j}$,
6. $\mathfrak{I} \models_\epsilon \mathfrak{B}$ if $\mathfrak{I}$ satisfies all bridge rules in $\mathfrak{B}$,
7. $\mathfrak{I} \models_\epsilon \mathfrak{T}$ if $\mathfrak{I} \models_\epsilon \mathfrak{B}$ and $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ for each i.

If $\mathfrak{I} \models_\epsilon \mathfrak{T}$ then we say that $\mathfrak{I}$ is a (distributed) model of $\mathfrak{T}$. Finally, given C and D of some local TBox $\mathcal{T}_i$ of $\mathfrak{T}$, C is subsumed by D in $\mathfrak{T}$ (denoted by $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$) whenever, for every distributed interpretation $\mathfrak{I}$, $\mathfrak{I} \models_\epsilon \mathfrak{T}$ implies $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$.

If one uses conjunctive bridge rules instead of the normal ones, the following characteristic holds [11].

Theorem 1. *Given a distributed TBox $\mathfrak{T}$ with a set of bridge rules $\mathfrak{B}$ and some local TBoxes $\mathcal{T}_i$ and $\mathcal{T}_j$ such that $i \neq j$, if for some $n > 0$ the bridge rules $i : C_1 \overset{\sqsupseteq}{\twoheadrightarrow} j : G_1, \dots, i : C_n \overset{\sqsupseteq}{\twoheadrightarrow} j : G_n$ are all part of $\mathfrak{B}$ then for every distributed interpretation $\mathfrak{I}$ such that $\mathfrak{I} \models_\epsilon \mathfrak{T}$*

$$(G_1 \sqcap \dots \sqcap G_n)^{\mathcal{I}_j} \subseteq r_{ij}\Big((C_1 \sqcap \dots \sqcap C_n)^{\mathcal{I}_i}\Big)$$

holds.

We have also established the following reduction in [11]. It says that the problem of deciding subsumption with respect to a distributed knowledge base that allows conjunctive bridge rules is reducible to the case with normal bridge rules only.

Theorem 2. *Given a distributed TBox $\mathfrak{T}$ with a set of bridge rules $\mathfrak{B}$ that admits conjunctive into-bridge rules, let $\mathfrak{T}'$ and $\mathfrak{B}'$ be obtained in two steps:*

1. *adding $i : C \sqcap D \xrightarrow{\sqsupseteq} j : G \sqcap H$ to $\mathfrak{B}$ for each pair of $i : C \overset{\sqsupseteq}{\twoheadrightarrow} j : G \in \mathfrak{B}$ and $i : D \overset{\sqsupseteq}{\twoheadrightarrow} j : H \in \mathfrak{B}$,*
2. *removing all conjunctive bridge rules from $\mathfrak{B}$.*

Then for every $i \in I$ and for every two concepts of $\mathcal{T}_i$, say C and D, $\mathfrak{T} \models_\epsilon i : C \sqcap D$ if and only if $\mathfrak{T}' \models_\epsilon i : C \sqcap D$.

As a tableaux reasoning algorithm is known for the original DDLs [2], this result provides us with a decision procedure for DDLs with conjunctive bridge-rules. Yet, the transformation leads to quadratic blow-up in the number of conjunctive bridge rules in the worst case, and so the computational properties of the overall procedure may not be satisfied. This suggests further investigation of reasoning in presence of conjunctive bridge rules.

3 ALTERNATIVE CHARACTERIZATION OF CONJUNCTIVE BRIDGE RULES

The most obstructive part of the reduction above is indeed that fact that it introduces one new normal onto-bridge rule for each pair of conjunctive onto-bridge rules that are between two local TBoxes $\mathcal{T}_i$ and $\mathcal{T}_j$, this basically copies the constraint asserted by Clause 5 of Definition 1 – the newly introduced normal bridge rule serves to guarantee the satisfaction of the constraint. Here the quadratic blow-up is introduced: we remove n conjunctive bridge-rules between $\mathcal{T}_i$ and $\mathcal{T}_j$ and replace them with $n^2/2+n$ normal onto-bridge rules (for each $i, j \in I$, $i \neq j$).

In this section we sketch an alternative approach. Instead of introducing one constraint for each pair of rules, we concentrate on the mapping r. Consider the constraints that are generated by Clause 5. In every distributed interpretation $\mathfrak{I}$ it requires the inclusion $G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i})$ to hold between the concepts C, D that we map from and G, H that we map to. In the following we discuss some possibilities how to alter this condition, aiming to amend the computational issue. The following theorem shows that we actually only need to consider the relation between the intersection of r-images $r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i})$ and the r-image of the intersection $r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i})$.

Theorem 3. *In every distributed interpretation $\mathfrak{I}$ that satisfies $G^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i})$ and $H^{\mathcal{I}_j} \subseteq r_{ij}(D^{\mathcal{I}_i})$ the following holds:*

$$r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i}) \subseteq r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \implies G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \ .$$

P r o o f . If we intersect $G^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i})$ and $H^{\mathcal{I}_j} \subseteq r_{ij}(D^{\mathcal{I}_i})$ we derive:

$$G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i}) \ .$$

Together with $r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i}) \subseteq r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i})$, which follows from the definition of the mapping r_{ij}, this yields $G^{\mathcal{I}_j} \cap H^{\mathcal{I}_j} \subseteq r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i})$.

Using this result we reformulate Clause 5. We simply exchange the constraints. If we do this, the semantics keeps the desired property (that of Theorem 1) that we want to retain.

Theorem 4. *If Clause* 5 *in Definition* 1 *is replaced by:*

5'. $\mathfrak{I} \models_\epsilon i : C \overset{\sqsupseteq}{\twoheadrightarrow} j : G$ *if* $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$ *and for each* $i : D \overset{\sqsupseteq}{\twoheadrightarrow} j : H \in \mathfrak{B}$, $r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \supseteq r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i})$,

then Theorem 1 *is still satisfied in the adjusted semantics.*

P r o o f . This is essentially a consequence of Theorem 3 which says that the constraints generated by Clause 5' imply those that would be generated if we used Clause 5 instead. This condition is then essential to establish Theorem 1 (see [12] for details).

Thus the adjusted semantics still satisfies the desired property as does the semantics that we have originally proposed for conjunctive bridge rules in [11]. However, Clause 5' generates conditions that are stronger and hence we have ruled out some models in the modified semantics. It is not yet clear whether this really affects the semantics or not. Elimination of several models need not necessarily affect whether $\mathfrak{T} \models_\epsilon i : C \sqcap D$ holds. To answer this question further investigation is needed.

If we reinspect Clause 5', we notice that by (locally) restricting the mapping r_{ij} the clause no longer needs to consider the intersection of each pair G and H of concepts that are targeted by bridge rules in $\mathcal{T}_j$. Taking this consideration one step further, we conjecture that it might be possible to ensure the desired behaviour of the semantics (in terms of Theorem 1) by globally restricting the mapping r in each distributed interpretation to certain class of mappings. In this respect we suggest mappings that we call strictly injective.

Definition 2. Given two sets S and T, let $m : S \mapsto 2^T$ be a mapping. We call m strictly injective, if there is no element $t \in T$ such that there are two elements $s_1, s_2 \in S$ with $s_1 \neq s_2$, $t \in m(s_1)$ and $t \in m(s_2)$.

Once we restrict mappings admissible to appear in distributed interpretations to strictly injective mappings, it makes no sense any more to distinguish between conjunctive and normal onto-bridge rules, since the desired behaviour is exhibited by both kinds, as we put it in the following theorem.

Theorem 5. *If we remove conjunctive bridge-rules from the language, and also Clause* 5 *from Definition* 1, *and we only allow strictly injective mappings in distributed interpretations, the language and the semantics that we end up with satisfies Theorem* 1 *in which conjunctive onto-bridge rules are replaced with normal ones.*

P r o o f . This follows immediately once we observe that of the two conditions generated by Clause 5', the condition $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$ is also generated for each normal onto-bridge rule by Clause 4, and the second condition – that is, for each $i : D \overset{\sqsupseteq}{\twoheadrightarrow} j : H \in \mathfrak{B}$, $r_{ij}(C^{\mathcal{I}_i} \cap D^{\mathcal{I}_i}) \supseteq r_{ij}(C^{\mathcal{I}_i}) \cap r_{ij}(D^{\mathcal{I}_i})$ – holds because r_{ij} is strictly injective.

Such a restriction, however, rules out even more distributed interpretations. This even conflicts with the intuition that is behind the mapping r: the mapping represents correspondence between individuals in the local models and it has been proposed as intuitive to allow situations in which one particular individual in some local model corresponds to two or more distinct individuals in another local model (e.g. a married couple is modeled by one individual in one local TBox but it is modeled as two distinct individuals in another one). The question whether and how much the semantics is affected by such a restriction is open and it is a part of our ongoing research.

4 CONCLUSION AND FUTURE WORK

We have discussed some alternative possibilities how to establish a semantics for DDLs [2] with so called conjunctive onto-bridge rules [11]. Our aim is to develop better understanding on what exactly is required (or, what are the possibilities) in order to obtain a semantics with desired behaviour. This might be of some use when developing reasoning procedures for the DDLs with conjunctive onto-bridge rules and implementing them. Our previous investigation [11] offers a naïve approach by reduction to normal onto-bridge rules which however yields significant blow-up in the number of bridge rules.

Our preliminary investigation shows that instead of verifying one condition of inclusion for each pair of conjunctive onto-bridge rules between two local TBoxes $\mathcal{T}_i$ and $\mathcal{T}_j$, we may choose to verify whether the mapping r used in the interpretation to relate individuals of local domains assigned to $\mathcal{T}_i$ and $\mathcal{T}_j$ meets certain conditions. This however rules out some admissible distributed models and requires further investigation. Also, this approach is of little direct use with the reasoning technique currently known for DDLs (see [2]) since the mapping is not directly constructed during the reasoning process, but rather conditions are asserted that imply existence of the mapping. This issues are subject to our ongoing investigation.

There are several interesting research directions that relate to DDLs:

- studying different kinds of bridge rules, such that for instance bridging between ABox individuals, as already suggested in [5]. Also, combining bridge-rules and link relations used in $\mathcal{E}$-connections in a unified framework may be of some interest,
- studying DDLs over more expressive DLs such as $\mathcal{SHOIQ}$ [13] and $\mathcal{SROIQ}$. [14] Our preliminary thoughts suggest that including nominals may not be trivial at all,
- studying the computational complexity of the distributed reasoning algorithm.

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