A Distributed Assumption-Based Framework for a Set of Heterogeneous Knowledge Bases*

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Abstract. We are focused on a multi-agent environment with heterogeneous representation languages used by different agents. Our goal is to develop a framework, which enables representation of distributed knowledge, and to provide unifying views (partial or global) on the shared knowledge within this distributed environment.
Distributed assumption-based framework (DABF) is proposed as a tool for this goal. The assumption-based framework (ABF) has a potential for a unifying representation of heterogeneous knowledge bases, written in different languages.
A DABF consists of local ABFs of agents. A visibility relation on a set of agents is defined. Unifying views on DABF are specified. Argumentation semantics are used for a semantic characterization of shared knowledge contained in a DABF. Finally, a procedural semantics is presented – argumentation games, which compute complete sets of assumptions.

1 Introduction

Motivation. We present a framework, which enables to collect knowledge from different heterogeneous knowledge bases and, most importantly, to provide a unifying view (partial or global) on this distributed knowledge. The following motivating example illustrates our goals.

Example 1. Imagine an extensive investigation of a complicated crime. A team of professionals is gathered for the investigation. The team consists of different types of persons – some detectives, forensic specialists and information gained from some witnesses is used. The investigation is coordinated and evaluated by the chair of the team.

Some findings may be considered as strong facts, some as speculations, hypotheses, based on some assumptions, also some general claims are included.

The chair collects the given knowledge obtained from different sources, derives consequences and resolves conflicts, in order to create a unifying and justified view on the distributed knowledge.

* This is an extended version of our eponymous submission to EUMAS 2014 which in addition includes proofs (see Appendix A).

We add a remark to this scenario. Our framework enables to specify more interesting structures on sets of agents, e.g., no agent has a global view on the distributed knowledge, only different partial views of agents are possible. □

Let us shift from the illustrative scenario to a multi-agent environment, where different (software) agents represent their knowledge using different languages. This is the frame for which we are going to build a formal description.

Problem. The formal description should be able to handle:

1. a collection of the (shared) knowledge from different sources into one framework;
2. a derivation of consequences; and most importantly,
3. conflicts solving and a construction of unifying and well justified views on the represented domain.

From the formal side, our aim is the development of a *distributed assumption-based framework.* in order to gain a theoretical framework suitable for study of shared knowledge in multi-agent scenarios.

Background. As the basic formal tool for our goals we chose the assumption-based framework (ABF) [2, 8].[3] ABF provides a potential for a unifying representation of heterogeneous knowledge bases, written in different languages. Different non-monotonic formalisms are instances of ABF, e.g., logic programs, default logic, autoepistemic logic, non-monotonic modal logic, defeasible logic. Thus, ABF can be used as a formal tool for a representation of knowledge, recorded in different languages and distributed within an heterogeneous multi-agent environment.

ABF understands non-monotonic reasoning as a deduction from assumptions. A set of assumptions, a knowledge base and a contrariness function, which assigns contraries to assumptions, are in the core of an ABF. Argumentation semantics of Dung [7] were applied to (sub)sets of assumptions. As a consequence, a variety of semantic characterizations of non-monotonic reasoning has been enabled.

Proposed solution We propose a *distributed* extension of ABF (DABF). A set of agents is supposed, they may use different representation languages, translatable into an ABF. DABF consists of the set of all ABFs of all agents.

A visibility relation on a set of agents is defined. If an agent α_2 is visible to an agent α_1, then α_1 can read the knowledge base, assumptions, rules and use the contrariness function of the ABF assigned to the visible agent α_2. The visibility relation is very general, without any constraints placed on it. Various resulting graph structures enable to capture manifold and diverse descriptions of different relations between agents and types of shared knowledge within a set of agents.

[3] A discussion of other formalisms related to our approach can be find in Section 6.

An agent shares and is able to access all (local) ABFs of agents, visible by him. The agent collects all shared local ABFs into one ABF, composed by component-wise union of all "visible" local ABFs.

Unions of local ABFs represent a simple, initial step to building unifying views on the distributed knowledge. We plan to extend our framework in the future by more subtle constructions, as hinted in Conclusions, Section 7.

Collected/shared knowledge is processed using various argumentation semantics. Argumentation point of view is an inherent feature of ABF and it is naturally used also in DABF as a tool of conflicts solving (i.e., semantic specification of what part of the knowledge is acceptable). Also a procedural semantics and a computational tool, argumentation game, is introduced.

Main contributions of our paper are as follows.

A construct of distributed ABF (DABF) is introduced, which enables a representation of knowledge contained in a multi-agent environment with heterogeneous representation languages .

Unifying views on DABF are specified. If all agents are visible, we can speak about a global view, otherwise about a partial view on the given DABF. In both cases notions of ABF may be applied to a semantic characterization of shared knowledge contained in DABF.

Finally, a procedural semantics for our framework is presented. Argumentation games, which compute skeptical and credulous versions of complete argumentation semantics, are introduced. Complete semantics is selected because of its generality – e.g., grounded, preferred and stable semantics are special cases of complete semantics.

Roadmap After a recapitulation of ABF in Section 2, the core of our paper starts in Section 3 with a description of DABF and some unifying views on knowledge contained in the DABF. Next in Section 4, properties of the unifying views on DABF and knowledge contained in DABF are studied. Section 5 presents the procedural semantics. Finally, related work is discussed, along with a summary of contributions and a description of open problems. Proofs are included in Appendix A.

2 Assumption-based framework

An assumption-based framework [2] is constructed over a deductive system. A *deductive system* is a pair $(\mathcal{L}, \mathcal{R})$, where $\mathcal{L}$ is a language, i.e., a countable set of sentences (formulae)[4] and $\mathcal{R}$ is a set of rules of the form

$$\frac{\alpha_1; \ldots; \alpha_n}{\gamma},$$

[4] No details are specified for the language. It is a matter of various instances of ABF, how sentences are specified.

where $\alpha_i, \gamma \in \mathcal{L}, n \geq 0$, α_i are called premises, γ is the consequence. Sometimes also notation of the form $\alpha_1; \ldots; \alpha_n / \gamma$ is used in the following.

A theory T is a subset of $\mathcal{L}$. A *deduction* from a theory T is a sequence $\langle f_1, \ldots, f_n \rangle$, where $n \geq 1$, $f_i \in T$ or f_i is the consequence of a rule $r \in \mathcal{R}$, where the set of premises of r is a subset of $\{f_1, \ldots, f_{i-1}\}$. The deduction of a sentence f is a deduction with the last member f. We will denote by $T \vdash_{\mathcal{R}} f$ that there is a deduction of f from the theory T using rules $\mathcal{R}$ (also $T \vdash f$ is used, if $\mathcal{R}$ is clear from the context). The symbol $T^{\leadsto \mathcal{R}}$, borrowed from [6], is used as a shorthand for $\{f \in \mathcal{L} \mid T \vdash_{\mathcal{R}} f\}$.

An *assumption-based framework* (ABF) is (in this paper) a tuple $(\mathcal{L}, \mathcal{R}, Ab, c)$, where $(\mathcal{L}, \mathcal{R})$ is a deductive system, $Ab \subseteq \mathcal{L}$ is a set of *assumptions*, $c \colon Ab \longrightarrow \mathcal{L}$ is a mapping called *contrariness function*. The formula $c(a)$ is called the *contrary* of an assumption a. Notice that c is not defined for $\mathcal{L} \setminus Ab$. In all instances of ABF considered in [2] it holds: for each a, $c(a) \in \mathcal{L} \setminus Ab$, hence $c(c(a))$ is not defined.

Originally, ABF has been defined as a triple (K, Ab, c) over a deductive system $(\mathcal{L}, \mathcal{R})$, where $K \subseteq \mathcal{L}$ is a knowledge base. Notice that a knowledge base is a theory, as defined above. K is omitted from our definition of ABF, since every formula $f \in K$ can be formalized as a rule r of the form $/f$.

"Classical" abstract argumentation semantics of [7] were applied to sets of assumptions in [2]. A mapping $\sigma : ABF \longrightarrow 2^{2^{Ab}}$ is called a semantics in the following. A semantics assigns a set $\mathcal{S}$ of sets of assumptions to an ABF. The semantics' are based on two features:

- conflict-freeness; a set of assumptions Δ is conflict-free, if an assumption a together with its contrary $c(a)$ is not deducible from Δ,
- an ability of a set of assumptions to counterattack attacks against its members (i.e., to defend each assumption in the set).

A set of assumptions Δ_1 attacks another set of assumptions Δ_2, if a contrary of an assumption from Δ_2 is deducible from Δ_1. Formally:

Definition 1. *In an ABF, a set of assumptions $\Delta_1 \subseteq Ab$* attacks *a set of assumptions $\Delta_2 \subseteq Ab$, if for some $a \in \Delta_2$ holds that $\Delta_1 \vdash c(a)$.*

A set of assumptions Δ is *closed* iff $\Delta = \{a \in A \mid \Delta \vdash a\}$, i.e., no other assumptions, only members of Δ are deducible from Δ.

Definition 2. *A closed set of assumptions Δ* defends *an assumption $a \in Ab$ iff for each closed set of assumptions Δ' holds: if Δ' attacks a then Δ attacks $\Delta' \setminus \Delta$. The set $\{a \in A \mid \Delta$ defends $a\}$ is denoted by $Def(\Delta)$.*

Different semantics are defined for ABFs. However, we will define here only conflict-free, maximal conflict-free (also known as naive semantics), admissible and complete sets of assumptions.

Definition 3 – *A set of assumptions Δ is* conflict-free *in an ABF iff for all $a \in \Delta$ holds $\{a, c(a)\} \not\subseteq \Delta^{\leadsto \mathcal{R}}$.*

- *It is said that a* naive *semantics assigns to an ABF the set of all maximal (w.r.t. $\subseteq$) conflict-free sets of assumptions.*
- *A set of assumptions is admissible iff it is closed and $\Delta \subseteq Def(\Delta)$.*
- *A set of assumptions Δ is* complete *iff Δ is closed and $\Delta = Def(\Delta)$. If Δ is a complete set of assumptions, $\Delta^{\leadsto \mathcal{R}}$ is called a* complete extension.

3 Distributed assumption-based framework

Distributed assumption-based framework is introduced in our work as a tool for a representation of knowledge, distributed within a multi-agent environment and represented by heterogeneous languages compilable into ABF.

Let a set of agents AG be given. For each agent $\alpha_i \in AG$ there is a knowledge base K_i written in a language L_i and compilable into ABF. Let the corresponding ABF be $\mathcal{A}_i = (\mathcal{L}_i, \mathcal{R}_i, Ab_i, c_i)$.

Definition 4 (Distributed assumption-based framework). *Let a set of agents AG be given. For each $\alpha_i \in AG$ there is an ABF $\mathcal{A}_i = (\mathcal{L}_i, \mathcal{R}_i, Ab_i, c_i)$.*

The set $\{\mathcal{A}_i \mid \exists i\ \alpha_i \in AG\}$ is called a distributed *assumption-based framework (DABF).*

We assume in this paper logical languages without function symbols. There are different possible options how to make a design decision, concerning a specification of other features of languages $\mathcal{L}_i$ in different $\mathcal{A}_i$. It is possible to suppose translations between different languages, even disjoint languages are an option. We believe that for a set of cooperating agents, aiming at knowledge sharing, it is appropriate to assume common parts of vocabularies, i.e., each pair $\mathcal{L}_i$, $\mathcal{L}_j$, $i \neq j$, may share a non-empty common set of predicate symbols and individual constants.

On the other hand, different formation rules for sentences of different $\mathcal{L}_i$ are possible. Also different sets of assumptions for different $\mathcal{A}_i$ are supposed.[5]

Thus summing up, we accept a general and abstract approach of [2] to a specification of $\mathcal{L}_i$.

We are going to describe some ways, how knowledge distributed among agents in a DABF may be shared and processed within a group of agents.

Peer-to-peer architecture. Knowledge sharing in our framework is based on a visibility relation, which determines a structure on a set of agents.

Definition 5. *Consider a directed graph $\mathcal{G} = (AG, E)$. If $\alpha_i, \alpha_j \in AG, (\alpha_i, \alpha_j) \in E$, it is said that the assumption-based framework $\mathcal{A}_j$ of the agent α_j is visible by α_i.*

[5] Compare, e.g., languages of normal, extended, generalized, disjunctive logic programs, defeasible logic or default logic and different types of assumptions acceepted in those languages. For details see [2].

No constraints are a priori placed on the visibility relation, in order to enable flexible options for specifying a flow of knowledge within a set of agents. Particularly:

- it is not assumed that E is a reflexive relation;
- it is possible that there is an agent α_1 s.t. for no α_2 it holds $(\alpha_1, \alpha_2) \in E$ or $(\alpha_2, \alpha_1) \in E$;
- complete graphs may be considered too;
- the visibility relation may be acyclic or cyclic for different sets of agent is based ts, in the case of a cycle the agents mutually share their knowledge.

Intuitively, if $(\alpha_i, \alpha_j) \in E$, then α_i can read the content of $\mathcal{A}_j$ and provide corresponding deductions. Each agent α_i creates its own view on the content of all visible ABFs. This view is represented by an ABF $\mathcal{V}_i$.

A preliminary remark before defining a view $\mathcal{V}_i$ follows: we accept a convention in order to ensure that the contrariness function c, defined as union of some "local" c_j is indeed a function: $a \in Ab_i \cap Ab_j$ implies $c_i(a) = c_j(a)$. This convention is satisfied in all known instances of ABF.

Definition 6. *The view $\mathcal{V}_i$ of an agent $\alpha_i \in AG$ is defined as follows.*
$\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$*, where* $\mathcal{L} = \bigcup_j \mathcal{L}_j, \mathcal{R} = \bigcup_j \mathcal{R}_j, Ab = \bigcup_j Ab_j, c = \bigcup_j c_j$ *for each j s.t. $(\alpha_i, \alpha_j) \in E$.*

Let us illustrate the idea of a view by a very simple toy example.

Example 2. Let be $AG = \{\alpha_0, \alpha_1, \alpha_2\}$ and $E = \{(\alpha_0, \alpha_1), (\alpha_0, \alpha_2)\}$. For simplicity, let $\mathcal{A}_0$, the ABF assigned to α_0 be $(\emptyset, \emptyset, \emptyset, \emptyset)$.

Agent α_1 uses a language of normal logic programs. Its knowledge base K_1 is the logic program $\{a \leftarrow \mathit{not}\ d\}$, an intuitive meaning of the rule is: if it is not known d, then a.

Agent α_2 uses a language of default logic, K_2 is the default theory $(\{a : Mc/b\}, \emptyset)$, i.e. the empty set of propositional formulae and a default rule with an intuitive meaning: if a holds and if c is possible, then b.

The ABF $\mathcal{A}_1$ of the agent α_1 is here constructed as follows: $Ab_1 = \{\mathit{not}\ a, \mathit{not}\ d\}$, $\mathcal{L}_1 = \{a, d\} \cup Ab_1$, $\mathcal{R}_1 =$

$$\left\{ \frac{}{a \leftarrow \mathit{not}\ d}, \frac{a \leftarrow \mathit{not}\ d; \quad \mathit{not}\ d}{a} \right\}.$$

The contrariness function c_1 assigns a to $\mathit{not}\ a$ and d to $\mathit{not}\ d$.

$\mathcal{A}_2$ is constructed as follows. $Ab_2 = \{Ma, Mb, Mc, M\neg a, M\neg b, M\neg c\}$, $\mathcal{L}_2 = \{a, b, c, \neg a, \neg b, \neg c\} \cup Ab_2$, c_2 assigns $\neg\phi$ to $M\phi$, where $\phi \in \mathcal{L}_2 \setminus Ab_2$, a convention is accepted: if ϕ is a negation of an atom, then $\neg\phi$ is the corresponding atom. $\mathcal{R}_2$ is a singleton

$$\left\{ \frac{a; \quad Mc}{b} \right\}.$$

Finally, the view $\mathcal{V}_0$ of α_0 on the visible ABFs is $(\mathcal{L}_1 \cup \mathcal{L}_2, \mathcal{R}_1 \cup \mathcal{R}_2, Ab_1 \cup Ab_2, c_1 \cup c_2)$. A derivation of $\{a, b\}$ from $\{not\ d, Mc\}$ is enabled in this environment. There is no conflict in our example, thus, we will not demosnstrate the role of argumentation semantics. □

Notice that different agents may use different kinds of assumptions. However, when a unifying view is created, assumptions are processed correctly. Consider an assumption $a \in Ab$. Obviously, if a occurs in $r \in \mathcal{R}_i$, then $a \in Ab_i$ (r contains only sentences of $\mathcal{L}_i$, and $Ab_i \subseteq \mathcal{L}_i$).

We are going to define distributed deduction provided by an agent α_i, i.e. a deduction using rules and assumptions contained in ABFs of all agents α_j s.t. $(\alpha_i, \alpha_j) \in E$.

Definition 7 (Distributed deduction) *Let a set of agents* $AG = \{\alpha_1, \dots, \alpha_r\}$, $r \geq 2$, *and an agent* $\alpha_i \in AG$ *be given. Consider a sequence* $s = \langle f_1, \dots, f_n \rangle$, $n \geq 1$ *and a set of assumptions* $\Delta = \{f_1, \dots, f_n\} \cap \bigcup_j Ab_j$, *where where* $(\alpha_i, \alpha_j) \in E$.

Then s *is called a* distributed deduction of f_n from Δ from the viewpoint of α_i *iff for each* $m \in \{1, \dots, n\}$

- *either* $f_m \in Ab_j$ *for some* j *s.t.* $(\alpha_i, \alpha_j) \in E$;
- *or there is a rule* $g_1, \dots, g_p / f_m \in R_j$ *s.t.* $\{g_1, \dots, g_p\} \subseteq \{f_1, \dots, f_{m-1}\}$ *for some* j *s.t.* $(\alpha_i, \alpha_j) \in E$.

A view $\mathcal{V}_i$ coincides with the set of all distributed deductions from the viewpoint of the agent α_i.

Fact 1 *Consider* $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$.

1. *Suppose that* $(\alpha_i, \alpha_j) \in E$, *then* $\vdash_{\mathcal{R}_j} \subseteq \vdash_{\mathcal{R}}$,
2. $\langle f_1, \dots, f_n \rangle$, *is a distributed deduction of* f_n *from* Δ *from the viewpoint of* α_i *iff* $\Delta \vdash_{\mathcal{R}} f_n$,
3. $\vdash_{\mathcal{R}}$ *is the smallest relation satisfying conditions 1 and 2.*

Recursive peer-to-per architecture. An agent α_i can use knowledge of an agent α_j directly, if $(\alpha_i, \alpha_j) \in E$. However, α_i may also indirectly use knowledge visible by α_j, and so on.

Definition 8. *Let a directed graph* $\mathcal{G} = (AG, E)$ *be given. It is said, that an agent* α_n *is accessible by the agent* α_i, *if there is a sequence* $\langle r_1, \dots, r_k \rangle$, $k \geq 2$ *s.t.* $r_1 = i, r_k = n$ *and* $(\alpha_{r_j}, \alpha_{r_{j+1}}) \in E$, $j \in \{1, \dots, k-1\}$.

Definitions of a view of an agent on the ABFs of all accessible agents and of distributed deduction w.r.t. the accessibility relation follow. Both definitions use the same pattern as in the case of peer-to-peer architecture, but we prefer explicit definition over a reference to a pattern.

Definition 9. $\mathcal{V}_i^* = (\mathcal{L}, \mathcal{R}, Ab, c)$, *where for each* j *s.t.* α_j *is* accessible *by* α_i *we have* $\mathcal{L} = \bigcup_j \mathcal{L}_j, \mathcal{R} = \bigcup_j \mathcal{R}_j, Ab = \bigcup_j Ab_j, c = \bigcup_j c_j$.

Definition 10 *Let a set of agents* $AG = \{\alpha_1, \ldots, \alpha_r\}$*,* $r \geq 2$*, and an agent* $\alpha_i \in AG$ *be given. Consider a sequence* $s = \langle f_1, \ldots, f_n \rangle$*,* $n \geq 1$ *and a set of assumptions* $\Delta = \{f_1, \ldots, f_n\} \cap \bigcup_j Ab_j$*, where* α_j *is accessible by* α_i*.*

Then s *is called a* distributed deduction of f_n from Δ from the viewpoint of α_i via accessibility relation *iff for each* $m \in \{1, \ldots, n\}$

- *either* $f_m \in Ab_j$ *for some* j *s.t.* α_j *is accessible by* α_i*;*
- *or there is a rule* $g_1, \ldots, g_p/f_m \in R_j$ *s.t.* $\{g_1, \ldots, g_p\} \subseteq \{f_1, \ldots, f_{m-1}\}$ *for some* j *s.t.* α_j *is accessible by* α_i*.*

Fact 2 *Consider* $\mathcal{V}_i^* = (\mathcal{L}, \mathcal{R}, Ab, c)$*.*

1. *Suppose that* α_j *is accessible from* α_i*. Then* $\vdash_{\mathcal{R}_j} \subseteq \vdash_{\mathcal{R}}$*,*
2. *A sequence* $\langle f_1, \ldots, f_n \rangle$ *is a distributed deduction of* f_n *from* Δ *via accessibility relation from the viewpoint of* α_i *iff* $\Delta \vdash_{\mathcal{R}} f_n$*,*
3. $\vdash_{\mathcal{R}}$ *is the smallest relation satisfying conditions 1 and 2.*

Now, we are going to define two basic kinds of unifying views on the DABF.

Definition 11 (Global and partial view). *If there is an agent* α_0 *s.t. for each agent* $\alpha_i \in AG$ *we have* $(\alpha_0, \alpha_i) \in E$ *(or if each* α_i *is accessible by* α_0*), then* $\mathcal{V}_0$ *(or* $\mathcal{V}_0^*$*) provides a* global view *on the given DABF.*

Otherwise, $\mathcal{V}_0$ *(or* $\mathcal{V}_0^*$*) provides a* partial view *on the given DABF.*

Given a partial or a global view on a DABF, argumentation semantics can be applied to $\mathcal{V}_i$ or $\mathcal{V}_i^*$. We emphasize that argumentation semantics are an inherent feature of the original ABF [2], not our arbitrary/unjustified decision. Use of argumenation semnics in [2] enables a rich variety of semantic specifications of non-monotonic languages.

4 Properties

An analysis of unifying views on knowledge contained in a DABF is presented in this section. Two cases are discussed separately – a global view on a DABF and a partial view on a DABF. Both cases are described for the recursive peer-to-peer architecture. Remind that in this architecture a view of an agent α_i is represented by $\mathcal{V}_i^* = (\mathcal{L}, \mathcal{R}, Ab, c)$, where all components of this tuple are unions of the corresponding components of ABFs of all agents accessible by α_i.

Intuitively, a unifying view $\mathcal{V}_i^*$ contains *shared* knowledge of all agents visible by an agent α_i. If $\mathcal{V}_i^*$ is a global view, we can speak about *group* knowledge in the sense that $\mathcal{V}_i^*$ contains total knowledge of *all* agents $\alpha \in AG$ (a union, in a sense). *Common* knowledge of a set of agents $M \subseteq AG$ is the knowledge shared by each agent in M (an intersection, in a sense).

Global view on DABF. Suppose a DABF, i.e., all local ABFs of all agents in AG. A global view on the DABF is represented by a single $\mathcal{V}_0^* = (\mathcal{L}, \mathcal{R}, Ab, c)$. For the case of recursive peer-to-peer architecture it means that each agent $\alpha_i \in AG$ is accessible by an agent $\alpha_0 \in AG$.

We are going to define a meta-predicate *it is known*. Actually, we use this meta-predicate only as a syntactic sugar, which enables an intuitive expression of a fact that something is known.

Given a unifying view $\mathcal{V}_0^*$, the knowledge collected from all agents may be conflicting. Argumentation semantics' provide a suitable characterization of knowledge contained in the unifying views. They allow to identify subsets of assumptions Ab which are acceptable in a sense of a given semantics.[6]

Consider an arbitrary argumentation semantics σ. We refer to a general definition of an argumentation semantics σ in Definition 3, Section 2. First, we introduce a shortcut: let an ABF be given and $\sigma : ABF \longrightarrow 2^{2^{Ab}}$ be a semantics. If $\Delta \in \sigma(ABF)$, then it is said that Δ is a σ-set of assumptions.

Definition 12 *Suppose that there is a global view $\mathcal{V}_0^* = (\mathcal{L}, \mathcal{R}, Ab, c)$ on a DABF. Suppose that K is not an expression of $\mathcal{L}$. Let $\Delta \subseteq Ab$ be a σ-set of assumptions.*

If $f \in \Delta^{\leadsto_\mathcal{R}}$, we will write $K^\sigma(\Delta/f)$.

An intuitive meaning of $K^\sigma(\Delta/f)$ is that f is known, if a σ-set Δ is assumed. There is an analogy between our meta-predicate K^σ and the epistemic operator K. However, if ϕ is a formula of an epistemic logic, then $K\phi$ is a formula of the same language, but $K^\sigma(\Delta/f)$ is not a sentence of $\mathcal{V}_0^*$. As a consequence, iterated applications of our meta-predicate are not defined and introspection is impossible in our framework.

The following consequence of Fact 2 expresses $K^\sigma(\Delta/f)$ in terms of distributed deduction.

Fact 3 *Suppose that there is $\alpha_0 \in AG$ s.t. each agent $\alpha \in AG$ is accessible by α_0. Let Δ be a σ-set of assumptions.*

$K^\sigma(\Delta/f)$ iff there is a distributed deduction of f from Δ from the viewpoint of α_0.

We are proceeding to a specification of group knowledge. If there is an agent with a global view $\mathcal{V}^*$ on a DABF, then group knowledge of AG can be defined in the frame of this $\mathcal{V}^*$. We will specify group knowledge according to a semantics σ. If the semantics σ is fixed, the σ-group knowledge consists of all possible "pieces of knowledge" $K^\sigma(\Delta/f)$, where Δ is any σ-set of assumptions.

Definition 13. *Suppose that there is a global view $\mathcal{V}^* = (\mathcal{L}, \mathcal{R}, Ab, c)$ on a DABF.*

Then σ-group knowledge of AG contained in $\mathcal{V}^*$ *is $G = \{K^\sigma(\Delta/f) \mid \Delta \subseteq Ab$ is a σ-set of assumptions$\}$.*

[6] Section 5 is devoted to a computational method of identifying such sets w.r.t. the complete semantics.

Notice that the group knowledge was defined relative to an argumentation semantics. It means that we accept different semantic specifications (different understandings) of group knowledge. The same holds in the following, if the parameter σ is used.

The following fact specifies two other viewpoints on σ-group knowledge. The first one follows directly from th Definitions 12 and 13. The second one from Fact 2.

Fact 4 *Suppose that there is a global view $\mathcal{V}_0^* = (\mathcal{L}, \mathcal{R}, Ab, c)$ on a DABF. Let G be the σ-group knowledge of AG. Then*

- $K^\sigma(\Delta/f) \in G$ *iff* $f \in \{\Delta^{\rightsquigarrow_\mathcal{R}} \mid \Delta$ *is a σ-set of assumptions* $\}$,
- $K^\sigma(\Delta/f) \in G$ *iff Δ is a σ-set of assumptions and there is a distributed deduction of f from Δ from the viewpoint of α_0.*

Partial views on a DABF. Consider an agent α_i, for which there is a non-empty set of agents α_j s.t. α_j is not accessible by α_i. The partial view $\mathcal{V}_i^*$ alone cannot serve as a basis for defining group knowledge within AG. Let us discuss the problem how the group knowledge of AG can be approximated, if only partial views are given.

Knowledge contained in one $\mathcal{V}_i^* = (\mathcal{L}, \mathcal{R}, Ab, c)$ is specified similarly as for the case of global views. A set of meta-predicates K_i^σ is assigned to the agent α_i and to respective $\mathcal{V}_i^*$.

Definition 14. *Let a partial view $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$ be given. Suppose that K_i^σ are not expressions of $\mathcal{L}$. Let σ be an argumentation semantics.*

We will write $K_i^\sigma(\Delta/f)$ iff $\Delta \subset Ab$ is a σ-set of assumptions and $f \in \Delta^{\rightsquigarrow_R}$.

Definition 15. *Knowledge available to α_i, contained in $\mathcal{V}_i^*$ (shared knowledge) is $Sh_i^\sigma = \{K_i^\sigma(\Delta/f) \mid \Delta$ is a σ-set of assumptions $\}$.*

Sh_i^σ represents shared knowledge, collected by an agent α_i from the set M of accessible agents and justified by an argumentation semantics σ. Actually, Sh_i^σ is group knowledge of M. However, we are interested only in group knowledge of AG.

A naive definition of group knowledge within AG, if there is no $\alpha \in AG$ with a global view on DABF, could be imagined, e.g. as $G^\sigma = \{K^\sigma(\Delta/f) \mid \exists i\ K_i^\sigma(\Delta/f) \in Sh_i^\sigma\}$. It is shown in the next example, that this definition has counterintuitive consequences.

Example 5 *Assume that there are only two agents, α_1 and α_2, the visibility relation E is defined as $\{(\alpha_1, \alpha_1), (\alpha_2, \alpha_2)\}$.*

Let be $K^\sigma(\Delta/a) \in G^\sigma$ because of $K_1^\sigma(\Delta/a)$, $K^\sigma(\Delta/b) \in G^\sigma$ because of $K_2^\sigma(\Delta/b)$. We expect that $K^\sigma(\Delta/a \wedge b)$ is a consequence of $K^\sigma(\Delta/a)$ and $K^\sigma(\Delta/b)$.

But $K^\sigma(\Delta/a \wedge b) \notin G^\sigma$, because of $b \notin \Delta^{\rightsquigarrow_{\mathcal{R}_1}}$ and $a \notin \Delta^{\rightsquigarrow_{\mathcal{R}_2}}$.

Next we investigate a question, how can be the group knowledge of AG approximated by partial views $\mathcal{V}_i^*$ of some agents. A notion of cover is introduced to this end. Its aim is to express how shared views of some agents together contain the group knowledge.

Definition 16. *Consider a subset of agents $M \subseteq AG$ and the set P of corresponding partial views $\mathcal{V}_j^*$ of all $\alpha_j \in M$. Let be $C = (\mathcal{L}, \mathcal{R}, Ab, c)$ the componentwise union of partial views $\mathcal{V}_j^*$ of P.*[7]

Let be $G = (\bigcup_i \mathcal{L}, \bigcup_i \mathcal{R}, \bigcup_i Ab, \bigcup_i c)$, for each agent $\alpha_i \in AG$.

It is said that P covers *the group knowledge of AG iff $C = G$.*

If M is a minimal (w.r.t. $\subseteq$) set of agents s.t. $C = G$, it is said that P covers minimally the group knowledge of AG.

Proposition 6 *Let be $M \subseteq AG$ and P be the set of corresponding partial views.*

If P minimally covers the group knowledge of AG, then for no pair of agents $\alpha_i, \alpha_j \in M$ holds that α_i is accessible by α_j.

If for cooperation of all agents, also agents not connected by the visibility relation in AG is needed, the visibility relation should be replaced or reinforced somehow. A transfer of knowledge between agents not connected by the visibility relation can be enabled by a communication on demand between agents in $\mathcal{G}$. Details of a communication language, enabling a transfer of knowledge (a transfer of queries and responses) between agents not connected by the visibility relation will be specified in a next paper, based on [10].

Finally, we add some remarks to the problem of common knowledge in AG. In general, it is possible that there is no common knowledge in AG. It is in the case if there are agents, which are not visible, i.e. $\exists \alpha_i \in AG\ \forall \alpha_j \in AG,\ (\alpha_j, \alpha_i) \notin E$.

Anyway, common knowledge in AG may be understood as as the knowledge of those agents that are visible by all $\alpha \in AG$. If an agent is visible by all agents, its knowledge is common to all gents.

Let M be the set of all agents visible by all $\alpha \in AG$ and U be the componentwise union of the ABFs of all agents in M (even if sometimes this union is $(\emptyset, \emptyset, \emptyset, \emptyset)$ and there is no common knowledge). Knowledge contained in U can be defined in the style used in Definition 15.

5 Argument games

In this section we define a procedural semantics based on skeptical and credulous argument games for complete semantics. We assume a *peer-to-peer architecture* of DABF represented as an oriented graph (AG, E), with E referring to the visibility relation between agents in AG as described above. Proof theory of argumentation is well studied area and argument games for various semantics were proposed [16], [18]. The process of proving a formula f in an ABF $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$ visible by an agent α_i takes two steps: i) Find a set of assumptions $\Delta \subseteq Ab$ deriving f

[7] $\mathcal{L}$ is the union of languages in $\mathcal{V}_j^*$ of all $\alpha_j \in M$ etc.

with respect to $\mathcal{R}$ and ii) Justify all assumptions in Δ. The second step can be accomplished by argument games described in this section.

Essentially, a dialogue is a switching process between two players resulting to a sequence of moves, with each move attacking to the previous stated. The aim of the first player, called *proponent* (PRO) is to prove an initial argument, while the second' s, called *opponent* (OPP), to prevent it. Intuitively, a move (pl, Δ) is a pair denoting: player pl claims that the set of assumptions Δ is true.

Definition 17 (Move). *Let* $(\mathcal{L}, \mathcal{R}, Ab, c)$ *be an ABF visible by an agent* α_i. *A* move *is a pair* $\mu = (pl, \Delta)$, *where* $pl \in \{\text{OPP}, \text{PRO}\}$ *denotes the player and* $\Delta \subseteq Ab$ *is a set of assumptions.*

A move (pl, Δ) attacks a move (pl', Δ') iff Δ attacks Δ.

Definition 18 (Argument Dialogue). *A* dialogue *is a finite nonempty sequence of moves* $\mu_1, \ldots, \mu_n$, $1 \leq i < n$ *where:*

- $pl_i = \text{PRO}$ *(*OPP*) iff* i *is odd (even)*
- μ_{i+1} *attacks* μ_i

For a given move, there can be more than one counterarguments. This leads to a tree representation of discussion, where each node corresponds to a move, and its children to all counter moves played by the opposite player. This is formally described with the following definition:

Definition 19 (Argument Game). *Let* $(\mathcal{L}, \mathcal{R}, Ab, c)$ *be an ABF visible by an agent* α_i. *An* argument game *for a set of assumptions* $\Delta \subseteq Ab$ *is a finite tree such that:*

- (PRO, Δ) *is the root,*
- *all branches are dialogues,*
- *if move* μ *played by* PRO *is a node in the tree, then every move* (OPP, Δ) *attacking* μ *is a child of* μ.
- *if* μ, μ' *are moves played by* PRO *in* T *(not necessarily in the same branch) then* μ *does not attack* μ'.

Note that the fourth bullet above is necessary to guarantee the soundness result for argument games. In general, a player wins a dialogue if the counterpart does not have any more moves to play, which can roughly be paraphrased as "the one who has the last word laughs best". The burden of proof is on the player PRO, so we assume that she proves the initial argument if she wins all branches of the dialogue. On the other hand, OPP has the burden of attack, meaning that she will state all possible counterarguments, against PRO's arguments, forming new branches that PRO will have to win. For her, it is sufficient to win at least one branch of the game. The following definition also explains why it is necessary for OPP to play all possible moves and for PRO only one.

Definition 20 (Winner). *A player pl* wins a dialogue *iff she plays the last move in it. Player* PRO *(resp.* OPP*)* wins an argument game T *iff she wins all (resp. at least one of the) branches in the argument game T. An argument game is* successful *iff it is won by* PRO.

Definition 21 (Proved formula). *Let $(\mathcal{L}, \mathcal{R}, Ab, c)$ be an ABF visible by an agent α_i. A formula f is* proved *by an agent α_i iff there is a successful argument game for a set of assumptions $\Delta \subseteq Ab$ such that $\Delta \vdash_{\mathcal{R}} f$.*

Recall that the root of the game T is the move (PRO, Δ), where Δ (resp. $\mathcal{R}$) is a set of assumptions (resp. inference rules) visible by an agent α_i and f is a consequence of Δ w.r.t. $\mathcal{R}$.

Definition 22 (Skeptical and Credulous Game). *An argument game T is called* skeptical *(resp.* credulous*) iff in each branch D of T holds: if (pl, Δ), $(pl, \Delta') \in D$ are two moves played by* PRO *(resp.* OPP*), then $\Delta \neq \Delta'$.*

ABF $\mathcal{V}_i$ *skeptically* (resp. *credulously*) *entails* a formula f with respect to semantics σ, $\mathcal{V}_i \models^{sk}_{\sigma} f$ (resp. $\mathcal{V}_i \models^{cr}_{\sigma} f$) iff for each (resp. at least one) σ-extension $\mathcal{E}$ of $\mathcal{V}_i$, $f \in \mathcal{E}$.[8] Note that, as following proposition shows, argument games can be applied in finite ABFs (i.e. the set of assumptions and the set of inference rules is finite) only.

Proposition 7 *Let $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$ be a finite ABF visible by an agent α_i and $f \in \mathcal{L}$ a formula. Then*

1. *$\mathcal{V}_i \models^{sk}_{complete} f$ iff f is skeptically provable by α_i.*
2. *$\mathcal{V}_i \models^{cr}_{complete} f$ iff f is credulously provable by α_i.*

6 Related work

Agent's knowledge modelled by ABFs with belief rules and desire rules is investigated in [12]. In order to resolve conflict between the goals of two agents, their respective ABFs are extended by rules of the other agent. Intuitively, the resolution is to create a win-win situation by satisfying both agents desires at the same time. The extended ABFs are merged. A merge operator is constructed to this end. Problems of knowledge sharing, unifying views on the distributed knowledge and group knowledge are not considered in this paper.

Multi-context systems (MCS) [13, 4] enable to combine multiple knowledge based in terms of (possibly non-monotonic [4]) bridge rules. Bridge rules enable to import parts of knowledge from one knowledge base to another. However, only consequences of local reasoning are imported, and it is up to each agent how it interprets the imported knowledge. Thus MCS do not provide any form of unified view over the distributed knowledge.

[8] Remind that an extension was defined in Section 2 as $\Delta^{\rightsquigarrow_{\mathcal{R}}}$.

Argumentation context systems (ACS) of [5] were developed as a framework for distributed argumentation. Arbitrary directed graphs of argumentation modules are considered. An argumentation module is a pair (A, Med), where A is an abstract argumentation framework and Med is a mediator. Mediators enable to combine knowledge imported from distinct context before it is imported into the source context. Since the contexts themselves are argumentation formalisms, the function of mediators can be seen as meta-argumentation. In the future we may consider incorporating a certain form of meta-argumentation into DABF for the construction of the unifying view for a group of agents. It is important to stress that this not the goal of ACS, where each context has its own interpretation of the imported knowledge form of its dedicated mediator.

There is only a superficial similarity between belief merging and DABF. Individual belief sets are an input of belief merging operators [14] and their outcome is a collective belief set (a consensus between individuals' views). On the other hand, we do not try to generate a collective "belief set". Al local ABFs are preserved and individual views (from the viewpoint of different argumentation semantics) on all visible "belief sets" are created.

Judgment aggregation [15] and computational social choice [3] are similar to belief merge, a collective outcome is required, in particular, voting methods are used. As was noticed, we are not aimming at a collective outcome.

We are closer to epistemic logic [11], where group knowledge or common knowledge are defined. However, no introspection or knowledge abut knowledge of other agents is enabled in our framework.

A number of argument games for various semantics have been proposed [17, 1]. A dialogue game, which is based on assumptions and gives both credulous and skeptical proofs for propositions is cited in [19]. A set of non-assumption moves are proposed in order to transform from one state of the dialogue to the other, while the final state of the dialogue is evaluated by an annotation process. Although this approach gives a good formalization for the game-theoretical semantics of a dialogue, in our paper the extra non-assumption locutions do not contribute essentially to the procedure of proof.

Our algorithm could also be compared to the construction of an abstract dispute tree described in [9, 8]. In this paper, assumptions are used as premises to construct backward and forward arguments and based on them, admissible trees are defined, for which all arguments of the proponent are defended against opponent' s. However, in [9] there is no restriction on moves repetition, which is in our approach necessary to compute the credulous and skeptical justification.

7 Conclusions and Future Work

This paper presented a distributed assumption-based framework. The framework enables to create unifying views on knowledge distributed within an heterogeneous multi-agent environment.

Among our future goals is to specify more subtle structures, e.g. distinguishing public and private parts of knowledge, introduction of different grades of credibility assigned to sentences and justifications/derivations, dynamic priorities on a set of agents.

Finally, an inclusion of actions into our framework is a goal of a future research. As a consequence, a communication language as a tool for a collaboration of agents not connected by a visibility relation should be developed.

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A Proofs

Fact 1 *Consider* $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$.

1. *Suppose that* $(\alpha_i, \alpha_j) \in E$, *then* $\vdash_{\mathcal{R}_j} \subseteq \vdash_{\mathcal{R}}$,
2. $\langle f_1, \ldots, f_n \rangle$ *is a distributed deduction of* f_n *from* Δ *from the viewpoint of* α_i *iff* $\Delta \vdash_{\mathcal{R}} f$,
3. $\vdash_{\mathcal{R}}$ *is the smallest relation satisfying conditions 1 and 2.*

Proof. – Suppose that $\Delta \vdash_{\mathcal{R}_j} f$. It holds that $\mathcal{R}_j \subseteq \mathcal{R}$, hence $\Delta \vdash_{\mathcal{R}} f$.

– $\Rightarrow$: consider a distributed deduction $s = \langle f_1, \ldots, f_n \rangle$ of f_n from Δ from the viewpoint of α_i. For each rule r applied for a derivation of a $f_i \in s$ holds that $r \in \mathcal{R}$.
  $\Leftarrow$: Suppose $\Delta \vdash_{\mathcal{R}} f_n$, i.e., there is a prof of f_n from Δ and each rule is "visible" by α_i. Hence, the proof is a distribted deduction from the viewpoint of α_i.
– Finally, suppose that there is a set of rules $\mathcal{Q} \subset \mathcal{R}$ satisfying conditions 1 and 2. Thus, there is a rule $r \in \mathcal{R} \setminus \mathcal{Q}$. If there is a distributed deduction of a formula f from a set of assumptions Δ from the viewpoint of α_i containing r. then $\Delta \not\vdash_Q f$. Contradiction.

The proof of Fact 2 is analogical.

Fact 3 *Suppose that there is* $\alpha_0 \in AG$ *s.t. each agent* $\alpha \in AG$ *is accessible by* α_0. *Let* Δ *be a* σ*-set of assumptions.*

$K^\sigma(\Delta/f)$ *iff there is a distributed deduction of* f *from* Δ *from the viewpoint of* α_0. □

Proof. $K^\sigma(\Delta/f)$ iff $f \in \Delta^{\leadsto_{\mathcal{R}}}$ iff $\Delta \vdash_{\mathcal{R}} f$ iff there is a distributed deduction of f from Δ from the viewpoint of α_0.

Proposition 6 *Let be* $M \subseteq AG$ *and* P *be the set of corresponding partial views.*

If P *minimally covers the group knowledge of* AG, *then for no pair of agents* $\alpha_i, \alpha_j \in M$ *holds that* α_i *is accessible by* α_j.

Proof. Suppose that α_i is accessible by α_j. Then $M \setminus \{\alpha_i\}$ covers the group knowledge of AG. Contradiction.

Let the powers of characteristic function Def be defined as following:

- $Def^0 = \emptyset$
- $Def^{i+1} = Def(Def^i)$

The grounded (the least complete with respect to $\subseteq$) set of assumptions will be denoted Δ_g. We will prove following lemma, which will be used in proofs of soundness and completeness of argument games. Its intuitive meaning is that an assumption α to be in Δ_g can not be defended only by itself.

Lemma 1. *Given an ABF $(\mathcal{L}, \mathcal{R}, Ab, c)$ and a finite ordinal i, an assumption $\alpha \in Def^{i+1}$ iff for each set of assumptions Δ attacking $\{\alpha\}$, there is a set of assumptions $\Delta' \subseteq Def^i$ such that Δ' attacks Δ and $\alpha \notin \Delta'$.*

Proof. Induction on i:

1. $i = 0$.
   Let $\alpha \in Def^{i+1}$. Then $\alpha \in Def(\emptyset)$ and consequent holds trivially.
   Let for each set of assumptions Δ attacking α, there is a set of assumptions $\Delta' \subseteq Def^i$ such that Δ' attacks Δ and $\alpha \notin \Delta'$. Since $Def^i = \emptyset$, previous holds only if α is not attacked by set of assumptions. So $\alpha \in Def^1 = Def^{i+1}$.
2. $i > 0$.
   Suppose that inductive hypothesis holds (IH). That is:
   Assumption $\alpha \in Def^{i+1}$ iff for each set of assumptions Δ attacking α, there is a set of assumptions $\Delta' \subseteq Def^i$ such that Δ' attacks Δ and $\alpha \notin \Delta'$.
   holds for $i \geq 0$. We want to prove it for $i + 1$.
   Let $\alpha \in Def^{i+2}$. Then $\alpha \in Def(Def^{i+1})$. If $\alpha \in Def^{i+1}$, the consequence follows from IH. Suppose that $\alpha \notin Def^{i+1}$. From $\alpha \in Def(Def^{i+1})$ we have: for each set of assumptions Δ attacking α there is a set of assumptions $\Delta' \subseteq Def^{i+1}$ attacking set of assumptions Δ. Now since $\alpha \notin Def^{i+1}$, it is the case that $\alpha \notin \Delta'$. So the consequence holds.
   Let for each set of assumptions Δ attacking α there is a set of assumptions $\Delta' \subseteq Def^{i+1}$ such that Δ' attacks Δ and $\alpha \notin \Delta'$. Then directly from definition of Def we have $\alpha \in Def(Def^{i+1}) = Def^{i+2}$.

Proposition 7 *Let $\mathcal{V}_i = (\mathcal{L}, \mathcal{R}, Ab, c)$ be a finite ABF visible by an agent α_i and $f \in \mathcal{L}$ a formula. Then*

1. $\mathcal{V}_i \models^{sk}_{complete} f$ iff f is skeptically provable by α_i.
2. $\mathcal{V}_i \models^{cr}_{complete} f$ iff f is credulously provable by α_i.

Proof. 1. Suppose $\mathcal{V}_i \models^{sk}_{complete} f$. Then there is a set of assumptions $\Delta \subseteq Ab$ such that $\Delta \vdash f$ and Δ is defended by every complete (and consequently also by the grounded) set of assumptions. Let Δ_g be the grounded set of assumptions. Now we will create the skeptical argument game T for Δ. The T will satisfy following specification:
   - if (Pro, Δ) is the root of T then $\Delta \subseteq \Delta_g$.
   - if $\mu = (\text{Pro}, \Delta) \in T$ is a move in the game such that $\Delta \subseteq \Delta_g$ and μ' is a child of μ, then there is a move $\mu_2 = (\text{Pro}, \Delta_2)$ such that $\Delta_2 \subseteq \Delta_g$ and μ_2 is a child of μ'.

   Procedure creating argument game T for the set of assumptions Δ deriving formula f is following:
   1 The root of T is (Pro, Δ), where $\Delta \vdash f$.
   2 If μ is a move in T played by the proponent, then every move (Opp, Δ') attacking μ will be added as a child of μ.

3 If μ' is a move in T played the opponent such that μ' is a child of a move $\mu = (\text{Pro}, \Delta)$ and $\Delta \subseteq \Delta_g$, then since Δ_g is a complete set of assumptions, it defends all its members and consequently there is a set of assumptions $\Delta_2 \subseteq \Delta_g$ such that move $\mu_2 = (\text{Pro}, \Delta_2)$ attacks μ' and μ_2 does not occur in the path from the root to μ'. Note that such set of assumptions Δ_2 is guaranteed to exist by Lemma 1 and an assumption that Δ_g is the grounded set of assumptions defending Δ. Add such move μ_2 as a child of μ'.

From the construction of T we have $\Delta' \subseteq \Delta_g$ for each $\mu = (\text{Pro}, \Delta')$ in T. It is not hard to see that these three steps induce a skeptical argument game T for the set of assumptions Δ and T satisfies the specification above. Note that the property that no moves played by Pro are attacking follows from the fact that $\Delta \subseteq \Delta_g$ for each move (Pro, Δ) in the tree T.

Now the fact that T is successful follows from the assumption that Δ_g is a complete set of assumptions.

Suppose f is skeptically proved by an agent α_i. Then there is a set of assumptions $\Delta \subseteq Ab$ such that $\Delta \vdash f$ and there is a successful skeptical argument game T for Δ. Consider the set of all assumptions E played by Pro in T,

$$E = \bigcup \{\Delta \mid (\text{Pro}, \Delta) \in T\}$$

We should now argue that E is attack-free and defends all its members. It is easy to see that attack-freeness of E directly follow from the definition of argument game.

Now we show admissibility of E. Let $a \in E$ and Δ' be a set of assumptions attacking $\{a\}$. Let $\mu = (\text{Pro}, \Delta) \in T$ be a move with $a \in \Delta$. Then (Opp, Δ') is a child of μ. Since T is successful, in every branch Pro plays the last move. Therefore there is a set of assumptions $\Delta_2 \in E$ attacking Δ'.

Now we will extend E in order it is complete set of assumptions. Recall that we assume that ABF $\mathcal{V}_i$ is finite. Consequently the power set of the set of assumptions $2^{Ab} = \{\Delta_1, \ldots, \Delta_n\}$ is finite as well, $n > 0$, which can be iterated. Let us create the following sequence E_i, $0 \leq i < n$, of sets of assumptions:

$$E_0 = E$$
$$E_{i+1} = \begin{cases} E_i \cup \Delta_{i+1} & \text{if } E_i \cup \Delta_{i+1} \text{ is admissible} \\ E_i & \text{otherwise} \end{cases}$$

It is easy to see that E_n is closed, maximal admissible and consequently complete. Now we prove that E_n is the least complete set of assumptions.

Assume by contradiction there is a complete set of assumptions $E' \subset E_n$. First we will argue that $E \subseteq E'$. Let $ch(\mu)$ denote the set of all children of μ in T. Realize that $\bigcup ch(ch(\mu))$ then denotes the set of all moves, played by the same player as μ, attacking some move in $ch(move)$. Now for each move $\mu \in T$ played by Pro we assign the number $n(\mu)$ defined as

$$n(\mu) \begin{cases} 1 & \text{if } \mu \text{ is a leaf in } T \\ 1 + max(\{n(\mu') \mid \mu' \in \bigcup ch(ch(\mu))\}) & \text{otherwise} \end{cases}$$

The fact that $E \subseteq E'$ now can be proved by induction on $n(\mu)$.

We argued that $E \subseteq E'$. From $E' \subset E_n$ it follows there is an assumption in $E_n \setminus E'$. Let i, $1 \leq i \leq n$, n is the cardinality of 2^{Ab}, be the least number such that $\Delta_i \subseteq E_n$ and $\Delta_i \nsubseteq E'$. Then $E' \supseteq E_{i-1}$, where E_{i-1} defends Δ_i. But from monotonicity of characteristic function *Def* we have E' defends Δ_i as well. Then since $\Delta_i \nsubseteq E'$ it must be the case that $E' \cup \Delta_i$ is not attack-free. Then E' attacks Δ_i. Since E' defends Δ_i, E' attacks itself. Contradiction that E' is the grounded set of assumptions! Hence E_n is the least (grounded) set of assumptions.

Finally, since the grounded set of assumptions defends Δ deriving f, $\mathcal{V}_i \models^{sk}_{complete} f$ holds.

2. Suppose $\mathcal{V}_i \models^{cr}_{complete} f$. Then there is a set of assumptions Δ such that $\Delta \vdash f$ and the set of assumptions Δ is defended by at least one complete set of assumptions Δ_c. Now we will create the credulous argument game T for Δ. The T will satisfy following specification:
   - if $(\textsc{Pro}, \Delta)$ is the root of T then $\Delta \subseteq \Delta_c$.
   - if $\mu = (\textsc{Pro}, \Delta)$ is a move in T such that $\Delta \subseteq \Delta_c$ and μ' is a child of μ, then there is a move $\mu_2 = (\textsc{Pro}, \Delta_2)$ with $\Delta_2 \subseteq \Delta_c$ such that μ_2 is a child of μ'.

   Procedure creating argument game T for the set of assumptions Δ is following:

   1 The root of T is $(\textsc{Pro}, \Delta)$, where $\Delta \vdash f$.

   2 If $\mu = (\textsc{Pro}, \Delta)$ is a node in T, then every move $(\textsc{Opp}, \Delta')$ attacking μ and not yet occurring in the path from the root to μ, will be added as a child of μ.

   3 If $\mu' = (\textsc{Opp}, \Delta)$ is a node in T such that μ' is a child of a move $\mu = (\textsc{Pro}, \Delta)$ and $\Delta \subseteq \Delta_c$, then Δ_c defends all assumptions in Δ and consequently there is a set of assumptions $\Delta_2 \subseteq \Delta_c$ such that move $\mu_2 = (\textsc{Pro}, \Delta_2)$ attacks μ'. Add such move μ_2 as a child of μ'.

   From the construction of T we have $\Delta \subseteq \Delta_c$ for each move $\mu = (\textsc{Pro}, \Delta)$. It is not hard to see that these three steps induce a credulous argument game T for the set of assumptions Δ and T satisfies the specification above. Note that the property that no moves played by Pro are attacking follows from the fact that $\Delta \subseteq \Delta_c$ for each move $\mu = (\textsc{Pro}, \Delta)$ in T.

   Now the fact that T is successful follows from the assumption that Δ_c is a complete extension.

   Suppose f is credulously proved by an agent α_i. Then there is a set of assumptions Δ such that $\Delta \vdash f$ and there is a successful credulous argument game T for Δ. Consider the set of all assumptions E played by Pro in T,

$$E = \bigcup \{\Delta \mid (\textsc{Pro}, \Delta) \in T\}$$

   We should now argue that E is attack-free and defends all its members. It is easy to see that attack-freeness of E directly follow from the definition of argument game.

Now we show admissibility of E. Let $a \in E$ and Δ' be a set of assumptions attacking a. Let $\mu = (\textsc{Pro}, \Delta) \in T$ be a move with $a \in \Delta$. There are two cases to consider:

- $(\textsc{Opp}, \Delta')$ is a child of μ. Since T is successful, in every branch Pro plays the last move. Therefore there is a set of assumptions $\Delta_2 \subseteq E$ attacking Δ'.
- $\mu' = (\textsc{Opp}, \Delta')$ is not a child of μ. Then since Δ' attacks Δ, it must the case that μ' was already played in the branch somewhere between the root's node and μ. From this directly follows existence of a set of assumptions $\Delta_2 \subseteq E$ attacking Δ'.

Now we will extend E in order it is complete set of assumptions. Recall that we assume that ABF $\mathcal{V}_i$ is finite. Consequently the power set of the set of assumptions $2^{Ab} = \{\Delta_1, \ldots, \Delta_n\}$ is finite as well, $n > 0$, which can be iterated. Let us create the following sequence E_i, $0 \leq i < n$, of sets of assumptions:

$$E_0 = E$$
$$E_{i+1} = \begin{cases} E_i \cup \Delta_{i+1} & \text{if } E_i \cup \Delta_{i+1} \text{ is admissible} \\ E_i & \text{otherwise} \end{cases}$$

It is easy to see that E_n is closed, maximal admissible and consequently complete.

Finally, since there is a complete set of assumptions defending Δ such that $\Delta \vdash f$, $\mathcal{V}_i \models^{cr}_{complete} f$ holds.