Subsumption Propagation between Remote Ontologies in Distributed Description Logic

Martin Homola

Comenius University, Bratislava, Slovakia,
Faculty of Mathematics, Physics and Informatics,
homola@fmph.uniba.sk

Abstract. Distributed Description Logics (DDL) is a KR formalism that enables reasoning with multiple ontologies interconnected by directional semantic mapping. Subsumption propagation in DDL from one ontology to another as a result of mappings has been studied, but only for a simplified case when only two ontologies are involved. In this paper we study subsumption propagation in more complex cases, when two ontologies are only connected indirectly, via several other ontologies. We characterize cases in which such subsumption propagation occurs. However, we also identify more complex situations in which subsumption propagation does not occur even if we would expect it. In addition, we propose an adjusted semantics for DDL. Under this semantics, subsumption propagates to remote ontologies to a far greater extent. Other desired properties that have been postulated for DDL, such as directionality and restrained inconsistency propagation are retained.

1 Introduction and Motivation

Distributed description logic (DDL) is a KR formalism introduced by Borgida and Serafini in [1] and later developed in [2,3]. It is intended especially to enable reasoning over systems of multiple ontologies connected by directional semantic mapping, built upon the formal, logical and well established framework of Description Logics (DLs). DDL captures the idea of importing and reusing concepts between several ontologies. This idea combines well with the basic assumption of the Semantic Web that no central ontology but rather many ontologies with redundant knowledge will exist [4].

In DDL semantic mapping between ontologies is specified with so called bridge rules. With bridge rules one is able to assert that some concept, say C, local to ontology $\mathcal{T}_1$, is mapped to an independent ontology $\mathcal{T}_2$ as a subconcept/superconcept of some $\mathcal{T}_2$-local concept, say D. Moreover, bridge rules are directed, and hence if there is a bridge rule with direction from $\mathcal{T}_1$ to $\mathcal{T}_2$, then $\mathcal{T}_2$ reuses knowledge from $\mathcal{T}_1$ but not necessarily the other way around. The mechanism of knowledge reuse is demonstrated by the example below.

Example 1. Consider an ontology $\mathcal{T}_1$ that deals with wine. In this ontology there are concepts like Wine_1 and $\mathsf{Beverage}_1$. Moreover, Wine_1 is a subconcept of

$\mathsf{Beverage}_1$. There is also another ontology $\mathcal{T}_2$ with concepts Drink_2 and its subconcept Milk_2. We also have a bridge rule, that maps from $\mathcal{T}_1$ to $\mathcal{T}_2$ and states that $\mathsf{Beverage}_1$ is a subconcept of Drink_2. Now we decide to add some wine to $\mathcal{T}_2$. We add a concept $\mathsf{Beaujolais}_2$ into $\mathcal{T}_2$ and one more bridge rule, that maps from $\mathcal{T}_1$ to $\mathcal{T}_2$ and states that Wine_1 is a superconcept of $\mathsf{Beaujolais}_2$. Thanks to these mappings, the knowledge that Wine_1 is a subconcept of $\mathsf{Beverage}_1$ propagates to $\mathcal{T}_2$ and we infer that $\mathsf{Beaujolais}_2$ is a subconcept of Drink_2.

Subsumption propagation, as seen above, has been described as desired and one of the main features of DDL and it has been studied (cf. [1,2,3,5]). However, almost exclusively in the simplified case, when only two ontologies are involved. The single exception known to us is the directionality property [3] which aims rather at non-occurrence of undesired knowledge propagation.

Subsumption propagation in more complex scenarios within the DDL framework, especially with more than two interacting ontologies – to our best knowledge – has not been studied. We find this issue interesting and important, as DDL is intended for such complex distributed systems of ontologies. And so, we focus on this subject in this paper. We first study the problem under the original semantics as introduced in [1,2,3]. Indeed in some cases subsumption is propagated by chains of bridge rules of arbitrary length, from one ontology to another that are not connected directly. However, we have also discovered patterns in which propagation of subsumption does not occur, even if we believe it intuitively should. Let us have a closer look on one such example.

Example 2. Consider a change in the situation from Example 1 as depicted in Fig. 2 (a). There is now no such concept $\mathsf{Beverage}_1$ in ontology $\mathcal{T}_1$, since it only deals with wine. The top concept here is Wine_1. Instead, there is another ontology $\mathcal{T}_0$ that deals with food and drinks; it contains $\mathsf{Beverage}_0$, and there is a mapping from $\mathcal{T}_0$ to $\mathcal{T}_1$ stating that Wine_1 is a subconcept of $\mathsf{Beverage}_0$. Concepts in $\mathcal{T}_2$ are unchanged, but the two bridge rules now map, first: from $\mathcal{T}_1$ to $\mathcal{T}_2$ asserting $\mathsf{Beaujolais}_2$ a subconcept of Wine_1, and second from $\mathcal{T}_0$ to $\mathcal{T}_2$: asserting Drink_2 a superconcept of $\mathsf{Beverage}_0$. In DDL we no longer infer that $\mathsf{Beaujolais}_2$ is a subconcept of Drink_2.

Further in this paper, inspired by Package-based DL (Bao et. al [6,7]) we explore an adjustment[1] of the original semantics of DDL that imposes so called compositional consistency condition on domain relations in DDL interpretations. Under this semantics subsumption propagates to remote ontologies to a far greater extent. The problem depicted by Example 2 no longer occurs under the adjusted semantics. Consequently, we characterize how subsumption propagates under the adjusted semantics, and we evaluate the semantics with respect to the desiderata postulated for DDL.

[1] In the first version of this paper we have explored so called progressive semantics for DDL. Thanks to helpful comment by an anonymous referee who have reviewed the paper pointing out the relation of this work with P-DL, we were able to come up with a better solution and the progressive semantics is now obsolete.

2 Distributed Description Logics

As introduced in [1,2,3] a DDL knowledge base consists of a distributed TBox $\mathfrak{T}$ – a set of local TBoxes $\{\mathcal{T}_i\}_{i\in I}$, and a set of bridge rules $\mathfrak{B} = \bigcup_{i,j\in I, i\neq j} \mathfrak{B}_{ij}$ between these local TBoxes, for some non-empty index-set I. Each of the local TBoxes $\mathcal{T}_i$ is a collection of axioms called general concept inclusions (GCIs) in its own local DL $\mathcal{L}_i$ of the form $i : C \sqsubseteq D$. It is assumed that each $\mathcal{L}_i$ is a sublanguage of $\mathcal{SHIQ}$ [8]. Each $\mathfrak{B}_{ij}$ is a set of directed bridge rules from $\mathcal{T}_i$ to $\mathcal{T}_j$. Intuitively, these are meant to "import" information from $\mathcal{T}_i$ to $\mathcal{T}_j$ and therefore $\mathfrak{B}_{ij}$ and $\mathfrak{B}_{ji}$ are possibly and expectedly distinct. Bridge rules of $\mathfrak{B}_{ij}$ are of two forms, *into*-bridge rules and *onto*-bridge rules (in the respective order):

$$i : A \xrightarrow{\sqsubseteq} j : G \ , \qquad\qquad i : B \xrightarrow{\sqsupseteq} j : H \ .$$

Given a TBox $\mathcal{T}$, a hole is an interpretation $\mathcal{I}^\epsilon = \langle \emptyset, \cdot^\epsilon \rangle$ with empty domain. Holes are used for fighting propagation of inconsistency. We use the most recent definition for holes, introduced in [3]. A distributed interpretation $\mathfrak{I} = \langle \{\mathcal{I}_i\}_{i\in I}, \{r_{ij}\}_{i\in I, i\neq j} \rangle$ of a distributed TBox $\mathfrak{T}$ consists of a set of local interpretations $\{\mathcal{I}_i\}_{i\in I}$ and a set of domain relations $\{r_{ij}\}_{i\in I, i\neq j}$. For each $i \in I$, either $\mathcal{I}_i = (\Delta^{\mathcal{I}_i}, \cdot^{\mathcal{I}_i})$ is an interpretation of local TBox $\mathcal{T}_i$ or $\mathcal{I}_i = \mathcal{I}^\epsilon$ is a hole, Each domain relation r_{ij} is a subset of $\Delta^{\mathcal{I}_i} \times \Delta^{\mathcal{I}_j}$. We denote by $r_{ij}(d)$ the set $\{d' \mid \langle d, d' \rangle \in r_{ij}\}$ and by $r_{ij}(D)$ the set $\bigcup_{d\in D} r_{ij}(d)$.

Definition 1. *For every i and j, a distributed interpretation $\mathfrak{I}$ satisfies the elements of a distributed TBox $\mathfrak{T}$ (denoted by $\mathfrak{I} \models_\epsilon \cdot$) according to the following clauses:*

1. $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$ *if* $\mathcal{I}_i \models C \sqsubseteq D$.
2. $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ *if* $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$ *for each* $C \sqsubseteq D \in \mathcal{T}_i$.
3. $\mathfrak{I} \models_\epsilon i : C \xrightarrow{\sqsubseteq} j : G$ *if* $r_{ij}(C^{\mathcal{I}_i}) \subseteq G^{\mathcal{I}_j}$.
4. $\mathfrak{I} \models_\epsilon i : C \xrightarrow{\sqsupseteq} j : G$ *if* $r_{ij}(C^{\mathcal{I}_i}) \supseteq G^{\mathcal{I}_j}$.
5. $\mathfrak{I} \models_\epsilon \mathfrak{B}$ *if* $\mathfrak{I}$ *satisfies all bridge rules in* $\mathfrak{B}$.
6. $\mathfrak{I} \models_\epsilon \mathfrak{T}$ *if* $\mathfrak{I} \models_\epsilon \mathfrak{B}$ *and* $\mathfrak{I} \models_\epsilon \mathcal{T}_i$ *for each* i.

If $\mathfrak{I} \models_\epsilon \mathfrak{T}$ then we say that $\mathfrak{I}$ is a (distributed) model of $\mathfrak{T}$. Finally, given C and D of some local TBox $\mathcal{T}_i$ of $\mathfrak{T}$, C is subsumed by D in $\mathfrak{T}$ (denoted by $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$) whenever, for every distributed interpretation $\mathfrak{I}$, $\mathfrak{I} \models_\epsilon \mathfrak{T}$ implies $\mathfrak{I} \models_\epsilon i : C \sqsubseteq D$.

Throughout [1,2,3] various desired properties have been postulated for DDL.

Property 1 (Monotonicity). The monotonicity property is satisfied whenever in every distributed TBox $\mathfrak{T}$ it holds that $\mathcal{T}_i \models C \sqsubseteq D \implies \mathfrak{T} \models_\epsilon i : C \sqsubseteq D$.

Property 2 (Directionality). The directionality property is satisfied whenever in every distributed TBox $\mathfrak{T}$ with index set I it holds that if there is no directed path of bridge rules from $i \in I$ to $j \in I$, then $\mathfrak{T} \models_\epsilon j : C \sqsubseteq D \iff \mathfrak{T}' \models_\epsilon j : C \sqsubseteq D$, where $\mathfrak{T}'$ is obtained by removing $\mathcal{T}_i$, $\mathfrak{B}_{ik}$ and $\mathfrak{B}_{li}$ from $\mathfrak{T}$ for each $k, l \in I$.

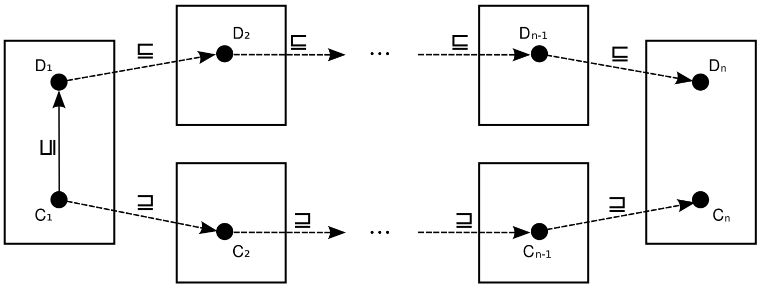

Fig. 1. Depiction of the distributed TBox from Theorem 2. Local subsumption is indicated by solid arrows and bridge rules are indicated by dashed arrows.

Let $\models_d$ be a kind of entailment just as $\models_\epsilon$ only that it does not allow holes as local interpretations. Distributed models with respect to $\models_d$ are called d-models. Given $J \subseteq I$, $\mathfrak{T}(\epsilon_J)$ is obtained from $\mathfrak{T}$ by removing each $\mathcal{T}_j$ such that $j \in J$, and adding $\{D \sqsubseteq \bot \mid j : C \stackrel{\sqsupseteq}{\rightarrow} i : D \in \mathfrak{B} \wedge j \in J\}$ to each $\mathcal{T}_i$, $i \in I \setminus J$.

Property 3 (Local inconsistency). The local inconsistency property is satisfied whenever in each distributed TBox $\mathfrak{T}$ the following holds: $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$ if and only if for any $J \subseteq I$, not containing i, $\mathfrak{T}(\epsilon_J) \models_d i : C \sqsubseteq D$.

Property 4 (Simple subsumption propagation). The simple subsumption propagation property is satisfied whenever for each distributed TBox $\mathfrak{T}$ the following holds: if $i : C \stackrel{\sqsupseteq}{\rightarrow} j : G \in \mathfrak{B}$ and $i : D \stackrel{\sqsubseteq}{\rightarrow} j : H \in \mathfrak{B}$ then $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D \implies \mathfrak{T} \models_\epsilon j : G \sqsubseteq H$.

3 Subsumption Propagation in DDL

Among the properties of DDL discussed in literature we find properties that deal with propagation of subsumption along bridge rules. The simplest case has been showed in [1,2,3]:

Theorem 1. *The simple subsumption propagation property is satisfied in DDL.*

While generalizations of this property are found in [1,2,3] and also [5], they always deal with cases when only two local TBoxes are involved. As DDL is designed to cope with complex systems of ontologies, naturally we are curious how subsumption propagates in such complex systems. Notably, whether consequences of local subsumption assertions are carried over to local TBoxes that are three and more bridge-rules away. Our main disclosure in this respect is presented in the theorem below. In certain cases the knowledge encoded in a local assertion is propagated to remote parts of the distributed ontology.

Theorem 2. *In every distributed TBox $\mathfrak{T}$, such as depicted in Fig. 1, with set of bridge rules $\mathfrak{B}$ that features n local TBoxes $\mathcal{T}_1, \ldots, \mathcal{T}_n$ and concepts $C_i, D_i \in \mathcal{T}_i$, for $1 \leq i \leq n$, such that*

1. $C_1 \sqsubseteq D_1$,
2. $i : C_i \xrightarrow{\sqsupseteq} i+1 : C_{i+1} \in \mathfrak{B}$, *for* $1 \leq i < n$,
3. $i : D_i \xrightarrow{\sqsubseteq} i+1 : D_{i+1} \in \mathfrak{B}$, *for* $1 \leq i < n$,

then the following holds: $\mathfrak{T} \models_\epsilon n : C_n \sqsubseteq D_n$.

Proof. By mathematical induction on n. Base case for $n = 1$ is exactly covered by Theorem 1.

Induction step. Let $n > 1$. We get $\mathfrak{T} \models_\epsilon n-1 : C_{n-1} \sqsubseteq D_{n-1}$ from induction hypothesis. Since we have two bridge rules $n-1 : C_{n-1} \xrightarrow{\sqsupseteq} n : C_n \in \mathfrak{B}$ and $n-1 : D_{n-1} \xrightarrow{\sqsubseteq} n : D_n \in \mathfrak{B}$, we apply Theorem 1 once again and thus we derive $\mathfrak{T} \models_\epsilon n : C_n \sqsubseteq D_n$. □

Unfortunately, the assumption of Theorem 2 that for each i, C_i and D_i both belong to the very same local TBox is strict. If this requirement is violated, subsumption propagation no longer occurs.

Example 3. Consider a distributed TBox $\mathfrak{T}$, as depicted in Fig. 2 (b), featuring local TBoxes $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_{2'}$, and $\mathcal{T}_3$, with concepts $C_1, D_1 \in \mathcal{T}_1$, $C_2 \in \mathcal{T}_2$, $D_2 \in \mathcal{T}_{2'}$, $C_3, D_3 \in \mathcal{T}_3$, such that:

1. $1 : C_1 \xrightarrow{\sqsupseteq} 2 : C_2 \in \mathfrak{B}$, $2 : C_2 \xrightarrow{\sqsupseteq} 3 : C_3 \in \mathfrak{B}$,
2. $1 : D_1 \xrightarrow{\sqsubseteq} 2' : D_2 \in \mathfrak{B}$, $2' : D_2 \xrightarrow{\sqsubseteq} 3 : D_3 \in \mathfrak{B}$,
3. $\mathfrak{T} \models_\epsilon 1 : C_1 \sqsubseteq D_1$.

The subsumption relation between C_1 and D_1 that holds in $\mathcal{T}_1$ does not propagate to $\mathcal{T}_3$, since in each distributed model $\mathfrak{I}$ of $\mathfrak{T}$ the interpretations of the two concepts "on the way" – ${C_2}^{\mathcal{I}_2}$ and ${D_2}^{\mathcal{I}_{2'}}$ – are totally unrelated.

By composition of the bridge rules that are available here we derive the inclusions: ${C_3}^{\mathcal{I}_3} \subseteq r_{23}({C_2}^{\mathcal{I}_2}) \subseteq r_{23}(r_{12}({C_1}^{\mathcal{I}_1}))$ and $r_{2'3}(r_{12'}({D_1}^{\mathcal{I}_1})) \subseteq r_{2'3}({D_2}^{\mathcal{I}_{2'}}) \subseteq {D_3}^{\mathcal{I}_3}$. However $r_{23}(r_{12}({C_1}^{\mathcal{I}_1}))$ and $r_{2'3}(r_{12'}({D_1}^{\mathcal{I}_1}))$ are not related in $\Delta^{\mathcal{I}_3}$, it does not help that ${C_1}^{\mathcal{I}_1} \subseteq {D_1}^{\mathcal{I}_1}$ in $\Delta^{\mathcal{I}_1}$.

Another pattern in which subsumption does not propagate as we would expect, has been already outlined in Example 2. We now revisit this example more formally.

Example 4. Let $\mathfrak{T}$ be a distributed TBox with set of bridge rules $\mathfrak{B}$, local TBoxes $\mathcal{T}_0$, $\mathcal{T}_1$, and $\mathcal{T}_2$ and local concepts $\mathsf{Beverage}_0 \in \mathcal{T}_0$, $\mathsf{Wine}_1 \in \mathcal{T}_1$, $\mathsf{Drink}_2 \in \mathcal{T}_2$, and $\mathsf{Beaujolais}_2 \in \mathcal{T}_2$. There are three bridge rules: $0 : \mathsf{Beverage}_0 \xrightarrow{\sqsupseteq} 1 : \mathsf{Wine}_1 \in \mathfrak{B}$, $1 : \mathsf{Wine}_1 \xrightarrow{\sqsupseteq} 2 : \mathsf{Beaujolais}_2 \in \mathfrak{B}$ and $0 : \mathsf{Beverage}_0 \xrightarrow{\sqsubseteq} 2 : \mathsf{Drink}_2 \in \mathfrak{B}$. Given a model of $\mathfrak{T}$, again we get some semantic constraints within $\Delta^{\mathcal{I}_2}$, particularly

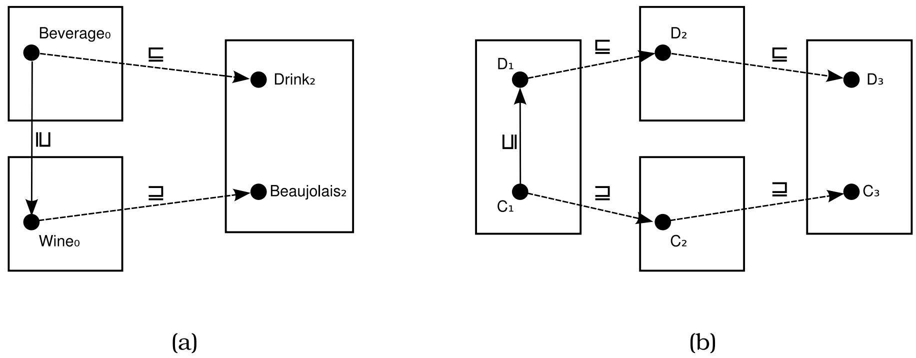

Fig. 2. Depiction of two distributed ontologies of Examples 2, 3 (a), and Example 4 (b). Local subsumption is indicated by solid arrows and bridge rules are indicated by dashed arrows.

$\mathsf{Beaujolais_2}^{\mathcal{I}_2} \subseteq r_{12}\Big(\mathsf{Wine_1}^{\mathcal{I}_1}\Big)$ and $r_{02}\Big(\mathsf{Beverage_0}^{\mathcal{I}_0}\Big) \subseteq \mathsf{Drink_2}^{\mathcal{I}_2}$ thanks to two of the bridge rules. However, the third bridge rule does not help anyhow to establish relation between $r_{12}\Big(\mathsf{Wine_1}^{\mathcal{I}_1}\Big)$ and $r_{02}\Big(\mathsf{Beverage_0}^{\mathcal{I}_0}\Big)$.

As we have seen above in Examples 3 and 4, in DDL subsumption does not always propagate to remote ontologies in more complex cases as we would intuitively expect. One possible explanation that offers here is the observation, that we have stressed in the examples: all the semantic constraints generated by remote bridge rules do not propagate to remote parts of the system, as they do in the special case characterized by Theorem 2. In the next section, we propose an adjustment to the semantics of DDL that enables subsumption propagation even in such cases.

4 DDL under Compositional Consistency

In Package-based Description Logics (P-DL) [6] so called *compositional consistency* condition is imposed on the importing relation in a distributed ontology environment. This condition is applied on the DDL framework as follows.

Definition 2. *Given a distributed interpretation $\mathfrak{I}$ with domain relation r, we say that r (and also $\mathfrak{I}$) satisfies compositional consistency if for each $i, j, k \in I$ and for each $x \in \Delta^{\mathcal{I}_i}$ with $r_{ij}(x) = d$ we have $r_{jk}(d) = r_{ik}(x)$.*

We say that DDL is under compositional consistency if only domain relations that satisfy the compositional consistency condition are allowed in distributed interpretations. The adjusted semantics actually extends the original one, in the sense that if some subsumption formula Φ is entailed by a distributed TBox

$\mathfrak{T}$ in the original semantics, then it is also entailed by $\mathfrak{T}$ under compositional consistency. The only difference is that in the adjusted semantics possibly some more subsumption formulae are entailed in addition.

Theorem 3. *Given a distributed TBox $\mathfrak{T}$ and a subsumption formula Φ, if $\mathfrak{T} \models_\epsilon \Phi$ according to the original semantics, then $\mathfrak{T} \models_\epsilon \Phi$ also holds in DDL under compositional consistency.*

Proof. This follows from the fact that each model that satisfies compositional consistency is also a model in the original DDL semantics. If a formula Φ is satisfied in each model from the set of models of $\mathfrak{T}$ according to the original semantics, it is also satisfied by each model from its subset – the set of models of $\mathfrak{T}$ that we obtain under compositional consistency. □

In the next section we study subsumption propagation in DDL under compositional consistency.

5 Subsumption Propagation in DDL under Compositional Consistency

In Examples 3 and 4 we have argued that if certain patterns occur in the system of distributed ontologies, then, under the original semantics of DDL, the subsumption does not propagate to remote ontologies as we would intuitively expect. Let us now reconsider Example 3 and verify what happens if the compositional consistency condition is enforced. By composition of the bridge rules that are available in the example we derive the inclusions: ${C_3}^{\mathcal{I}_3} \subseteq r_{23}\big({C_2}^{\mathcal{I}_2}\big) \subseteq r_{23}\big(r_{12}\big({C_1}^{\mathcal{I}_1}\big)\big)$ and $r_{2'3}\big(r_{12'}\big({D_1}^{\mathcal{I}_1}\big)\big) \subseteq r_{2'3}\big({D_2}^{\mathcal{I}_{2'}}\big) \subseteq {D_3}^{\mathcal{I}_3}$. Compositional consistency implies $r_{23}\big(r_{12}\big({C_1}^{\mathcal{I}_1}\big)\big) = r_{13}\big({C_1}^{\mathcal{I}_1}\big)$ and $r_{13}\big({D_1}^{\mathcal{I}_1}\big) = r_{2'3}\big(r_{12'}\big({D_1}^{\mathcal{I}_1}\big)\big)$. Finally, since $\mathfrak{T} \models_\epsilon 1 : C_1 \sqsubseteq D_1$ we are now able to derive $r_{13}\big({C_1}^{\mathcal{I}_1}\big) \subseteq r_{13}\big({D_1}^{\mathcal{I}_1}\big)$ and hence ${C_3}^{\mathcal{I}_3} \subseteq {D_3}^{\mathcal{I}_3}$.

Theorem 4 below provides a more general characterization of cases when subsumption propagates to remote ontologies. This characterization generalizes both Examples 3 and 4.

Theorem 4. *Given a distributed TBox, as illustrated in Fig. 3, with index set I and set of bridge rules $\mathfrak{B}$, that features $n+1$ local TBoxes $\mathcal{T}_0, \mathcal{T}_1, \dots, \mathcal{T}_n$ with concepts $E, F \in \mathcal{T}_0$, and $C_i, D_i \in \mathcal{T}_i$, for $1 \leq i \leq n$, and k with $1 \leq k \leq n$ such that*

1. $\mathfrak{T} \models_\epsilon i : C_i \sqsubseteq D_i$, *for* $1 \leq i \leq n$,
2. $i+1 : C_{i+1} \overset{\sqsupseteq}{\rightarrow} i : D_i \in \mathfrak{B}$, *for* $1 \leq i < k$,
3. $i : D_i \overset{\sqsubseteq}{\rightarrow} i+1 : C_{i+1} \in \mathfrak{B}$, *for* $k \leq i < n$,
4. $1 : C_1 \overset{\sqsupseteq}{\rightarrow} 0 : E \in \mathfrak{B}$ *and* $n : D_n \overset{\sqsubseteq}{\rightarrow} 0 : F \in \mathfrak{B}$.

In DDL under compositional consistency it follows that $\mathfrak{T} \models_\epsilon 0 : E \sqsubseteq F$.

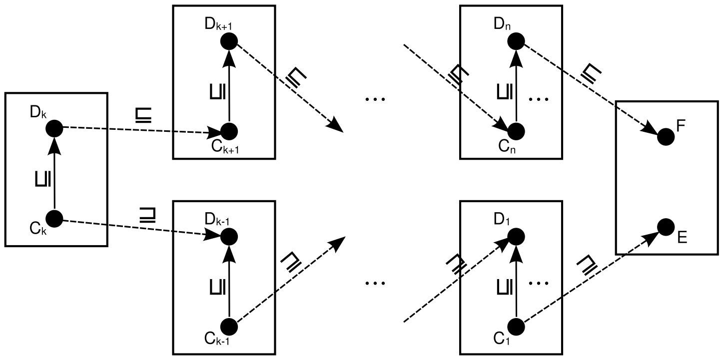

Fig. 3. Depiction of the distributed TBox from Theorem 4. Local subsumption is indicated by solid arrows and bridge rules are indicated by dashed arrows.

Proof. (Sketch.) The key observation is to realize the implications of compositional consistency on chaining domain relations. The chain of bridge rules $i+1 : C_{i+1} \stackrel{\sqsupseteq}{\rightarrow} i : D_i \in \mathfrak{B}$, for $1 \leq i < k$ together with the local subsumptions in the assumptions allows us to relate $r_{10}\big({C_1}^{\mathcal{I}_1}\big) \subseteq r_{10}\big(r_{21}\big(\cdots r_{kk-1}\big({C_k}^{\mathcal{I}_k}\big)\cdots\big)\big)$. From compositional consistency we derive $r_{10}\big(r_{21}\big(\cdots r_{kk-1}\big({C_k}^{\mathcal{I}_k}\big)\cdots\big)\big) = r_{k0}\big({C_k}^{\mathcal{I}_k}\big)$. For the other chain of bridge rules alike. Now it is easy to see that $E^{\mathcal{I}_0} \subseteq r_{10}\big({C_1}^{\mathcal{I}_1}\big) \subseteq r_{k0}\big({C_k}^{\mathcal{I}_k}\big) \subseteq r_{k0}\big({D_k}^{\mathcal{I}_k}\big) \subseteq r_{10}\big({D_1}^{\mathcal{I}_1}\big) \subseteq F^{\mathcal{I}_0}$. $\square$

In [2,3] and [5] subsumption propagation over complex concepts and interaction of concept union and intersection operators have been further studied. We leave out evaluation of DDL under compositional consistency in these complex cases for future work.

6 Other Properties

Besides subsumption propagation, throughout [1,2,3] other desiderata have been postulated for DDL (Properties 1–3). We show in this section that the monotonicity, directionality and local inconsistency properties are retained in DDL under compositional consistency.

Theorem 5. *The monotonicity property is satisfied in DDL under compositional consistency.*

Proof. Given $\mathcal{T}_i \in \mathfrak{T}$ and $C, D \in \mathcal{T}_i$ such that $\mathcal{T}_i \models_\epsilon C \sqsubseteq D$, only models of $\mathcal{T}_i$ and $\mathcal{I}^\epsilon$ are allowed in the $\mathcal{I}_i$ slot of any distributed model $\mathfrak{I}$ of $\mathfrak{T}$. Since in each of these $C \sqsubseteq D$ holds, we get $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$ directly from the definition. $\square$

Theorem 6. *The directionality property is satisfied in DDL under compositional consistency.*

Proof. (Sketch.) Given $\mathfrak{T}$ with two local TBoxes $\mathcal{T}_i$ and $\mathcal{T}_j$ such that there is no directed path of bridge rules from $\mathcal{T}_i$ to $\mathcal{T}_j$ we shall prove that removing all bridge rules outgoing from $\mathcal{T}_i$ (and removing $\mathcal{T}_i$) from $\mathfrak{T}$ has no effect on $\{\phi \mid \mathfrak{T} \models_\epsilon j : \phi\}$. Indeed, removal of such bridge rules has no effect on $\mathcal{T}_j$ as bridge-rules outgoing from $\mathcal{T}_i$ only generate semantic constraints in $\mathcal{T}_j$ if they are connected to $\mathcal{T}_j$ by a directed path of bridge rules. This holds even under compositional consistency, since the compositional consistency condition only contributes to subsumption propagation along directed paths of bridge-rules – it only acts as reinforcement on top of the semantic constraints generated by bridge rules.

□

Theorem 7. *The local inconsistency property is satisfied in DDL under compositional consistency.*

Proof. Let $\mathfrak{T}$, $i \in I$, C and D be as in the assumptions of the theorem. If there is no distributed model $\mathfrak{I}$ of $\mathfrak{T}$ with $\mathcal{I}_i \neq \mathcal{I}^\epsilon$ then trivially $\mathfrak{T} \models_\epsilon i : C \sqsubseteq D$ and also $\mathfrak{T}(\epsilon_J) \models_{\mathrm{d}} i : C \sqsubseteq D$ for each $J \subseteq I$ such that $i \notin J$, since there is no d-model of $\mathfrak{T}(\epsilon_J)$.

In the other case we establish a 1-to-1 correspondence between ϵ-models of $\mathfrak{T}$ and the union of all d-models of $\mathfrak{T}(\epsilon_J)$ for each $J \subseteq I$ such that $i \notin J$. Let $\mathfrak{I}$ be an ϵ-model of $\mathfrak{T}$ and let $J = \{i \in I \mid \mathcal{I}_i = \mathcal{I}^\epsilon\}$ be the set of slots that are occupied with holes. Then $\mathfrak{I}$ is a d-model of $\mathfrak{T}(\epsilon_J)$ disregarding the slots of J. Also the other way around, given a d-model $\mathfrak{I}'$ of $\mathfrak{T}(\epsilon_J)$, we obtain an ϵ-model of $\mathfrak{T}$ by adding $\mathcal{I}^\epsilon$ for each slot of J. This proves the theorem. □

7 Related Work

In P-DL (Bao et. al [6]) the importing relation that provides semantics for importing concepts between independent ontologies (called packages) is much similar to the domain relation used in DDL. Compositional consistency is a requirement in P-DL and the importing relation is strictly one-to-one. Tanks to these requirements the local models in P-DL are viewed as partially overlapping, the semantics captures the intuitions of concept-importing (in contrast with concept correspondence in DDL) and transitivity of inter-module subsumption is ensured. This work, inspired with the results of P-DL, applies the compositional consistency requirement in DDL. The resulting adjusted semantics for DDL features improved subsumption propagation between remote ontologies without restricting domain relations to be strictly one-to-one. Another feature that distinguishes this work from P-DL is the employment of holes in order to fight inconsistency propagation.

DDL has been introduced by Borgida and Serafini in [1]. The core body of work on DDL includes [2,3]. Propagation of subsumption in DDL has also been studied by Homola in [5]. In [9,10], Ghidini and Serafini enrich DDL with heterogeneous mappings, that are mappings between concepts and roles. An alternative approach to provide semantics for a system of ontologies connected by semantic mapping has been proposed by Zimmermann in [11]. An extension of DDL called C-OWL has been introduced by Bouquet et al. in [12]. Herein several improvements were suggested, including a richer family of bridge rules allowing bridging between roles, etc. Another approach that deals with concept importing is that of Pan et al. [13].

Another approach to distributed and modular ontologies besides DDL is that of Cuenca Grau et al. [14,15] where ontologies are combined using $\mathcal{E}$-connections [16]. In this framework, inter-ontology roles are employed instead of bridge rules. While $\mathcal{E}$-connections and DDL are related [16,17], each maintains its own primary intuitions – in DDL inter-ontology subsumption is modeled directly with bridge rules, while the preference of links in the latter framework has lead to such results as automated ontology decomposition [18]. For further comparison of P-DL, $\mathcal{E}$-connections, and DDL see [7].

8 Conclusion and Future Work

In this paper we have studied subsumption propagation in DDL. Even if this issue has been addressed in literature [1,2,3,5], it has only been studied for special cases when only two ontologies are involved. We have focused on the general case with arbitrary number of ontologies involved, and we have studied whether subsumption propagates between remote ontologies that are not connected directly by bridge rules, but only indirectly by path of bridge rules of length two or greater. We have showed that there are cases when subsumption propagates in such complex setting. We have also described cases when the propagation does not occur even if we would expect it.

Inspired by P-DL [6], we have studied the case when the so called compositional consistency condition is required in domain relations in DDL. In thus adjusted semantics, subsumption propagates to remote ontologies to a far greater extent than under the original one. Specifically, in cases that we have discussed, when subsumption does not propagate under the original semantics as we would expect, under the compositional consistency requirement it does. However the resulting semantics keeps important DDL-style features that distinguish it from P-DL: inconsistency propagation is restrained with holes and domain relations are not restricted to one-to-one in accordance with some of the basic intuitions behind DDL. Properties described as desiderata for DDL: monotonicity, directionality, and local inconsistency (non-propagation) are retained.

Practical applicability of the resulting semantics in terms of development of a reasoning algorithm is left for future work.

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