## Linear First Order Differential Equations

$$
\begin{aligned}
& \mathrm{v}=\mathrm{s} / \mathrm{t} \\
& \mathrm{a}=\mathrm{dv} / \mathrm{dt}
\end{aligned}
$$

## Linear First Order Differential Equations

$$
\begin{aligned}
& \mathrm{v}=\mathrm{s} / \mathrm{t} \\
& \mathrm{a}=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{~g}=9.78 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

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\end{aligned}
$$

$\frac{d v}{d t}=a$

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$$

$$
d v=a d t
$$

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\end{aligned}
$$

$$
v=a t+c
$$

Linear First Order Differential Equations

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& \mathrm{a}=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{~g}=9.78 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
t=0 \rightarrow s=0
$$

$$
v=a t+c
$$

## Linear First Order Differential Equations

$$
\begin{aligned}
& \mathrm{v}=\mathrm{s} / \mathrm{t} \\
& \mathrm{a}(\mathrm{t})=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{a}(\mathrm{t})=\mathrm{t}-3 \mathrm{t}^{2}+\mathrm{t}^{3}
\end{aligned}
$$

## Linear First Order Differential Equations

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\begin{aligned}
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& \mathrm{a}(\mathrm{t})=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{a}(\mathrm{t})=\mathrm{t}-3 \mathrm{t}^{2}+\mathrm{t}^{3}
\end{aligned}
$$

$$
\frac{d v}{d t}=t-3 t^{2}+t^{3}
$$

## Linear First Order Differential Equations

$$
\begin{aligned}
& \mathrm{v}=\mathrm{s} / \mathrm{t} \\
& \mathrm{a}(\mathrm{t})=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{a}(\mathrm{t})=\mathrm{t}-3 \mathrm{t}^{2}+\mathrm{t}^{3}
\end{aligned}
$$

$d v=t d t-3 t^{2} d t+t^{3} d t$

## Linear First Order Differential Equations

$$
\begin{aligned}
& \mathrm{v}=\mathrm{s} / \mathrm{t} \\
& \mathrm{a}(\mathrm{t})=\mathrm{dv} / \mathrm{dt} \\
& \mathrm{a}(\mathrm{t})=\mathrm{t}-3 \mathrm{t}^{2}+\mathrm{t}^{3}
\end{aligned}
$$

$$
v=\frac{t^{2}}{2}-t^{3}+\frac{t^{4}}{4}+c
$$

Linear First Order Differential Equations

$$
\begin{aligned}
& t=0 \rightarrow f(t)=8 \\
& f^{\prime}(t)=4+t-\frac{1}{2} f(t)
\end{aligned}
$$

Linear First Order Differential Equations

$$
\begin{array}{r}
t=0 \rightarrow f(t)=8 \\
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad g^{g(t)}
\end{array}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
f^{\prime}(t)+a f(t)=g(t) / \mu(t) \\
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t)
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
f^{\prime}(t)+a f(t)=g(t) \\
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\mu(t) f^{\prime}(t)+\mu^{\prime}(t) f(t)=[\mu(t) f(t)]^{\prime}
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
f^{\prime}(t)+a f(t)=g(t) \\
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\mu(t) f^{\prime}(t)+\underbrace{\mu^{\prime}(t) f(t)=[\mu(t) f(t)]^{\prime}} \\
\mu^{\prime}(t)=a \mu(t)
\end{gathered}
$$

## Linear First Order Differential Equations

$$
f^{\prime}(t)+a f(t)=g(t)
$$

$$
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t)
$$

$$
\mu(t) f^{\prime}(t)+\mu^{\prime}(t) f(t)=[\mu(t) f(t)]^{\prime}
$$

$$
\int \frac{\mu^{\prime}(t)}{\mu(t)} d t=\int a d t
$$

## Linear First Order Differential Equations

$$
f^{\prime}(t)+a f(t)=g(t)
$$

$$
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t)
$$

$$
[\ln (f(x))]^{\prime}=\frac{f^{\prime}(x)}{f(x)}
$$

$$
\begin{aligned}
& \int \frac{\mu^{\prime}(t)}{\mu(t)} d t=\int a d t \\
& \ln |\mu(t)|=a t+b
\end{aligned}
$$

## Linear First Order Differential Egations

$$
\begin{gathered}
f^{\prime}(t)+a f(t)=g(t) \\
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\ln |\mu(t)|=a t+b \\
\mu(t)=c e^{a t}
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\mu(t)=c e^{a t}
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\mu(t)=c e^{a t} \\
c e^{a t} f^{\prime}(t)+a c e^{a t} f(t)=c e^{a t} g(t)
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
\mu(t) f^{\prime}(t)+a \mu(t) f(t)=\mu(t) g(t) \\
\mu(t)=c e^{a t}
\end{gathered}
$$

$$
<e^{a t} f^{\prime}(t)+a<e^{a t} f(t)=\ltimes e^{a t} g(t)
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
e^{a t} f^{\prime}(t)+a e^{a t} f(t)=e^{a t} g(t) \\
{\left[e^{a t} f(t)\right]^{\prime}=e^{a t} g(t)}
\end{gathered}
$$

$$
\left[e^{f(x)}\right]^{\prime}=f(x)^{\prime} e^{f(x)}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
{\left[e^{a t} f(t)\right]^{\prime}=e^{a t} g(t)} \\
f^{\prime}(t)+\frac{1}{2} f(t)=4(4+t \\
{\left[e^{\frac{1}{2} t} f(t)\right]^{\prime}=4 e^{\frac{1}{2} t}+t e^{\frac{1}{2} t}}
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\left[e^{\frac{1}{2} t} f(t)\right]^{\prime}=4 e^{\frac{1}{2} t}+t e^{\frac{1}{2} t}
$$

$e^{\frac{1}{2} t} f(t)=8 e^{\frac{1}{2} t}+2 t e^{\frac{1}{2} t}-4 e^{\frac{1}{2} t}+c$
per partes

$$
\int u^{\prime}(x) v(x) d x=u(x) v(x)-\int u(x) v^{\prime}(x) d x
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
e^{\frac{1}{2} t} f(t)=8 e^{\frac{1}{2} t}+2 t e^{\frac{1}{2} t}-4 e^{\frac{1}{2} t}+c \\
t=0 \rightarrow f(t)=8
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
e^{\frac{1}{2} t} f(t)=8 e^{\frac{1}{2} t}+2 t e^{\frac{1}{2} t}-4 e^{\frac{1}{2} t}+c \\
t=0 \rightarrow f(t)=8 \\
f(t)=4+2 t+4 e^{-t / 2}
\end{gathered}
$$

## Linear First Order Differential Equations

$$
\begin{gathered}
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \\
t=0 \rightarrow f(t)=8 \\
f(t)=4+2 t+4 e^{-t / 2}
\end{gathered}
$$

Getting Nowersial witt Euler s' Method

$$
\begin{aligned}
& \quad f(t+\Delta t)=f(t)+\Delta t f^{\prime}(t)+\frac{h^{2}}{2} f^{\prime \prime}(\xi) \\
& t<\xi<t+\Delta t
\end{aligned}
$$

## Getting Nowersial witt Euler's Method

$$
\begin{gathered}
f(t+\Delta t)=f(t)+\Delta t f^{\prime}(t)+\frac{h^{2}}{2} f^{\prime \prime}(\xi) \\
t<\xi<t+\Delta t \\
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \\
t=0 \rightarrow f(t)=8 \\
\Delta t=0.1
\end{gathered}
$$

## Getting Nanerrical witt Euler s' Method

$$
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \begin{aligned}
\Delta t & =0.1 \\
t=0 & \rightarrow f(t)=8
\end{aligned}
$$

$$
f(0)=8
$$

$$
f(0+0.1)=f(0)+0.1 f^{\prime}(0)=8+0.1\left(4+0-\frac{8}{2}\right)=8
$$

$$
f(t)=4+2 t+4 e^{-t / 2}
$$

$$
f(0.1)=4+2 * 0.1+4 e^{-0.1 / 2}=8.005
$$

## Getting Numerical witt Euler 's Method

$$
\begin{gathered}
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \Delta t=0.1 \\
f(0)=8 \quad t=0 \rightarrow f(t)=8 \\
f(0+0.1)=f(0)+0.1 f^{\prime}(0)=8+0.1\left(4+0-\frac{8}{2}\right)=8 \\
f(0.1+0.1)=f(0.1)+0.1 f^{\prime}(0.1)=8+0.1\left(4+0.1-\frac{8}{2}\right)=8.01 \\
f(t)=4+2 t+4 e^{-t / 2} \\
f(0.2)=4+2 * 0.2+4 e^{-0.2 / 2}= \\
8.0193
\end{gathered}
$$

Getting Nanervical with Range-Kutta Method

$$
\begin{aligned}
y_{n+1} & =y_{n}+\frac{1}{6}\left(k_{1}+2 k_{2}+2 k_{3}+k_{4}\right) \\
k_{1} & =h f\left(x_{n}, y_{n}\right) \\
k_{2} & =h f\left(x_{n}+\frac{h}{2}, y_{n}+\frac{k_{1}}{2}\right) \\
k_{3} & =h f\left(x_{n}+\frac{h}{2}, y_{n}+\frac{k_{2}}{2}\right) \\
k_{4} & =h f\left(x_{n+1}, y_{n}+k_{3}\right)
\end{aligned}
$$

Getting Namersical with Range-Nutta Metrod

$$
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \begin{aligned}
\Delta t & =0.1 \\
t=0 & \rightarrow f(t)=8
\end{aligned}
$$

$$
f(0)=8
$$

## Getting Nanervical with Runge-Katta Metiod

$$
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \begin{aligned}
\Delta t & =0.1 \\
t=0 & \rightarrow f(t)=8
\end{aligned}
$$

$$
f(0)=8
$$

$$
k_{1}=\Delta t f^{\prime}(t)=0.1\left(4+0-\frac{8}{2}\right)=0
$$

$$
k_{2}=0.1\left(4+\left(0+\frac{0.1}{2}\right)-\frac{1}{2}\left(f(0)+\frac{k_{1}}{2}\right)\right)=0.1(4+0.05-4)=0.005
$$

$$
k_{3}=0.1\left(4+\left(0+\frac{0.1}{2}\right)-\frac{1}{2}\left(f(0)+\frac{k_{2}}{2}\right)\right)=0.1\left(4+0.05-\frac{8.0025}{2}\right)=
$$

$$
0.004875
$$

$$
k_{4}=0.1\left(4+(0+0.1)-\frac{1}{2}\left(f(0)+k_{3}\right)\right)=0.1\left(4+0.1-\frac{8.004875}{2}\right)=
$$ 0.00976

$$
f(0.1) \approx f(0)+\frac{k_{1}+2\left(k_{2}+k_{3}\right)+k_{4}}{6}=8+\frac{2(0.005+0.004875)+0.00976}{6}=
$$

## Getting Nanervical with Runge-Katta Metiod

$$
\begin{gathered}
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \Delta t=0.1 \\
t=0 \rightarrow f(t)=8 \\
f(0)=8
\end{gathered}
$$

$$
f(0.1) \approx f(0)+\frac{k_{1}+2\left(k_{2}+k_{3}\right)+k_{4}}{6}=8+\frac{2(0.005+0.004875)+0.00976}{6}=
$$

$$
8.00492
$$

$$
k_{1}=\Delta t f^{\prime}(0.1)=0.1\left(4+0.1-\frac{8.00492}{2}\right)=0.00975
$$

$k_{2}=0.1\left(4+\left(0.1+\frac{0.1}{2}\right)-\frac{1}{2}\left(f(0.1)+\frac{k_{1}}{2}\right)\right)=0.1\left(4+0.15-\frac{8.00492+0.004875}{2}\right)=$ 0.0145
$k_{3}=0.1\left(4+\left(0.1+\frac{0.1}{2}\right)-\frac{1}{2}\left(f(0.1)+\frac{k_{2}}{2}\right)\right)=0.1\left(4+0.15-\frac{8.00492+0.00726}{2}\right)=$ 0.0144
$k_{4}=0.1\left(4+(0.1+0.1)-\frac{1}{2}\left(f(0.1)+k_{3}\right)\right)=0.1\left(4+0.2-\frac{8.00492+0.0144}{2}\right)=$ 0.019

## Getting Namerical with Range-Kutta Metiod

$$
\begin{gathered}
f^{\prime}(t)=4+t-\frac{1}{2} f(t) \quad \begin{array}{c}
\Delta t=0.1 \\
t=0 \rightarrow f(t)=8 \\
f(0)=8
\end{array}
\end{gathered}
$$

$$
f(0.1) \approx f(0)+\frac{k_{1}+2\left(k_{2}+k_{3}\right)+k_{4}}{6}=8+\frac{2(0.005+0.004875)+0.00976}{6}=
$$

$$
8.00492
$$

$$
f(0.2) \approx f(0.1)+\frac{k_{1}+2\left(k_{2}+k_{3}\right)+k_{4}}{6}=8.00492+\frac{0.00975+2(0.0145+0.0144)+0.019}{6}=
$$ 8.0194

$$
\begin{aligned}
& f(0.1)=4+2 * 0.1+4 e^{-0.1 / 2}=8.005 \\
& f(0.2)=4+2 * 0.2+4 e^{-0.2 / 2}= \\
& 8.0193
\end{aligned}
$$

