Linear First Order Differential Equations

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 $a = dv / dt$ 

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g = 9.78 m / s<sup>2</sup>

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$$\frac{dv}{dt} = a$$

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## dv = a dt

Linear First Order Differential Equations

$$v = s / t$$
  
a = dv / dt  
g = 9.78 m / s<sup>2</sup>

$$\int dv = \int a \, dt$$

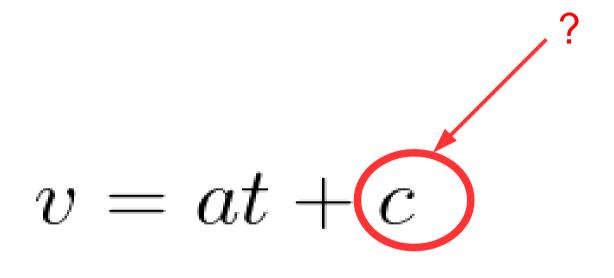
Linear First Order Differential Equations

$$v = s / t$$
  
a = dv / dt  
g = 9.78 m / s<sup>2</sup>

## v = at + c

Linear First Order Differential Equations

v = s / ta = dv / dt g = 9.78 m / s<sup>2</sup>



Linear First Order Differential Equations

v = s / ta = dv / dt g = 9.78 m / s<sup>2</sup>

 $t = 0 \rightarrow s = 0$  ? v = at + c

Linear First Order Differential Equations

$$v = s / t$$
  
a(t) = dv / dt  
a(t) = t - 3t<sup>2</sup> + t<sup>3</sup>

Linear First Order Differential Equations

$$v = s / t$$
  
a(t) = dv / dt  
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$$\frac{dv}{dt} = t - 3t^2 + t^3$$

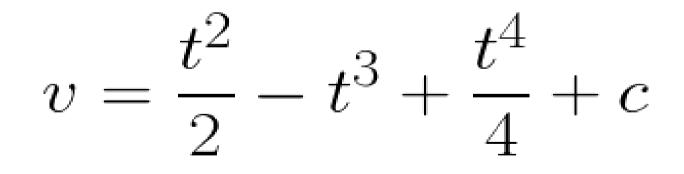
Linear First Order Differential Equations

$$v = s / t$$
  
a(t) = dv / dt  
a(t) = t - 3t<sup>2</sup> + t<sup>3</sup>

$$dv = tdt - 3t^2dt + t^3dt$$

Linear First Order Differential Equations

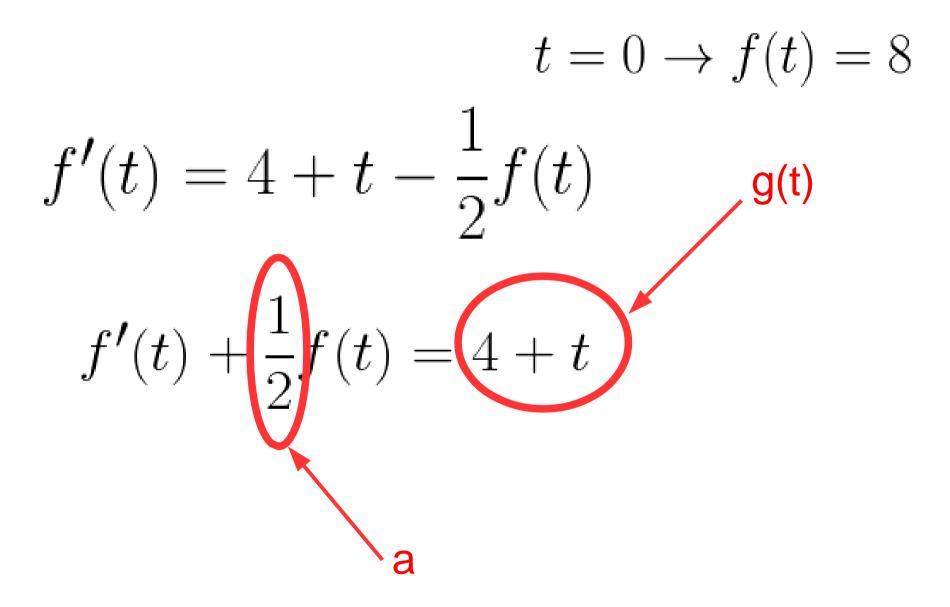
$$v = s / t$$
  
a(t) = dv / dt  
a(t) = t - 3t<sup>2</sup> + t<sup>3</sup>



Linear First Order Differential Equations

 $t = 0 \to f(t) = 8$  $f'(t) = 4 + t - \frac{1}{2}f(t)$ 

Linear First Order Differential Equations



Linear First Order Differential Equations

 $f'(t) + af(t) = g(t) /_{\mu(t)}$  $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$ 

Linear First Order Differential Equations

f'(t) + af(t) = g(t) $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$ 

Linear First Order Differential Equations

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Linear First Order Differential Equations

f'(t) + af(t) = g(t) $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$  $\mu'(t) = a\mu(t)$  $\int \frac{\mu'(t)}{\mu(t)} dt = \int a dt$ 

Linear First Order Differential Equations

f'(t) + af(t) = g(t) $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $\int \frac{\mu'(t)}{\mu(t)} dt = \int a dt$  $[ln(f(x))]' = \frac{f'(x)}{f(x)}$  $|ln|\mu(t)| = at + b$ 

Linear First Order Differential Equations

f'(t) + af(t) = g(t) $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $|ln|\mu(t)| = at + b$  $\mu(t) = ce^{at}$ 

Linear First Order Differential Equations

## $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$ $\mu(t) = ce^{at}$

Linear First Order Differential Equations

 $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $\mu(t) = ce^{at}$ 

 $ce^{at}f'(t) + ace^{at}f(t) = ce^{at}g(t)$ 

Linear First Order Differential Equations

 $\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$  $\mu(t) = ce^{at}$ 

 $ce^{at}f'(t) + ace^{at}f(t) = ce^{at}g(t)$ 

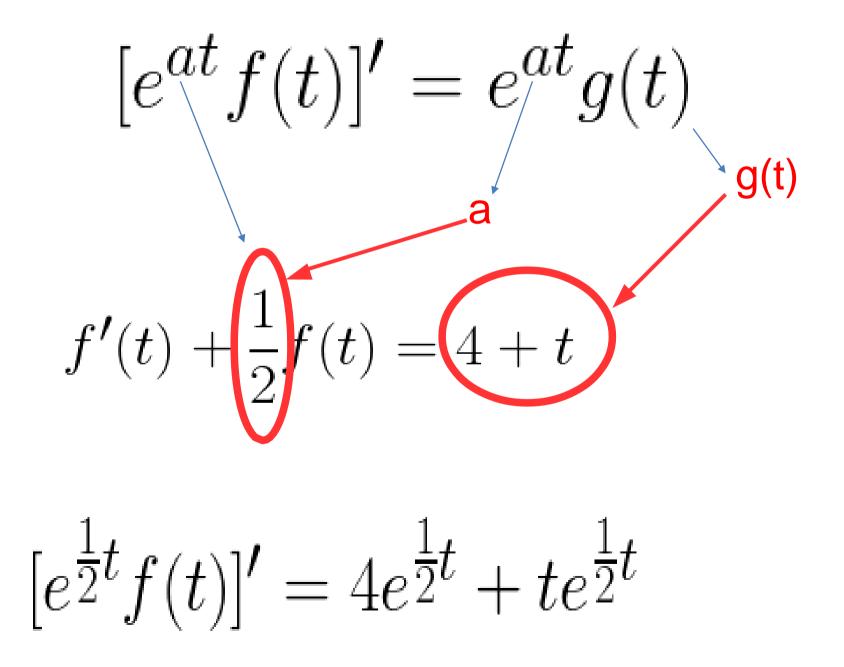
Linear First Order Differential Equations

$$e^{at}f'(t) + ae^{at}f(t) = e^{at}g(t)$$

$$[e^{at}f(t)]' = e^{at}g(t)$$

$$[e^{f(x)}]' = f(x)'e^{f(x)}$$

Linear First Order Differential Equations



Linear First Order Differential Equations

$$[e^{\frac{1}{2}t}f(t)]' = 4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$$
$$e^{\frac{1}{2}t}f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$
$$e^{\frac{1}{2}t}f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$
$$f^{(x)\nu(x)\,dx = u(x)\nu(x) - \int u(x)\nu'(x)\,dx}$$

Linear First Order Differential Equations

$$e^{\frac{1}{2}t}f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$
$$t = 0 \to f(t) = 8$$

Linear First Order Differential Equations

$$e^{\frac{1}{2}t}f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$
$$t = 0 \to f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

Linear First Order Differential Equations

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$
$$t = 0 \rightarrow f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

Getting Numerical with Ealer's Method

 $f(t + \Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$  $t < \xi < t + \Delta t$ 

Cetting Numerical with Ealer's Method

 $f(t + \Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$  $t < \xi < t + \Delta t$  $f'(t) = 4 + t - \frac{1}{2}f(t)$  $t = 0 \to f(t) = 8$  $\Delta t = 0.1$ 

Getting Numerical with Ealer's Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$
  $\Delta t = 0.1$   
 $t = 0 \to f(t) = 8$ 

$$f(0) = 8$$
  
$$f(0+0.1) = f(0) + 0.1f'(0) = 8 + 0.1(4+0-\frac{8}{2}) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$
$$f(0.1) = 4 + 2*0.1 + 4e^{-0.1/2} = 8.005$$

Getting Numerical with Euler's Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \qquad \Delta t = 0.1$$

$$t = 0 \to f(t) = 8$$

$$f(0 + 0.1) = f(0) + 0.1f'(0) = 8 + 0.1(4 + 0 - \frac{8}{2}) = 8$$

$$f(0.1 + 0.1) = f(0.1) + 0.1f'(0.1) = 8 + 0.1(4 + 0.1 - \frac{8}{2}) = 8.01$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} = 8.0193$$

Getting Numerical with Runge-Kutta Method

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$
  

$$k_1 = hf(x_n, y_n),$$
  

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}),$$
  

$$k_3 = hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}),$$
  

$$k_4 = hf(x_{n+1}, y_n + k_3).$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$
  $\Delta t = 0.1$   
 $t = 0 \to f(t) = 8$ 

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$
  $\Delta t = 0.1$   
 $t = 0 \to f(t) = 8$ 

$$f(0) = 8$$
  

$$k_1 = \Delta t f'(t) = 0.1(4 + 0 - \frac{8}{2}) = 0$$
  

$$k_2 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_1}{2})) = 0.1(4 + 0.05 - 4) = 0.005$$
  

$$k_3 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_2}{2})) = 0.1(4 + 0.05 - \frac{8.0025}{2}) = 0.004875$$
  

$$k_4 = 0.1(4 + (0 + 0.1) - \frac{1}{2}(f(0) + k_3)) = 0.1(4 + 0.1 - \frac{8.004875}{2}) = 0.00976$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

Getting Numerical with Range-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \qquad \Delta t = 0.1 \\ t = 0 \to f(t) = 8$$

$$f(0) = 8$$

 $\begin{array}{c} f(0.1)\approx f(0)+\frac{k_{1}+2(k_{2}+k_{3})+k_{4}}{6}=8+\frac{2(0.005+0.004875)+0.00976}{6}=8.00492\end{array}$ 

$$k_{1} = \Delta t f'(0.1) = 0.1(4 + 0.1 - \frac{8.00492}{2}) = 0.00975$$

$$k_{2} = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_{1}}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.004875}{2}) = 0.0145$$

$$k_{3} = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_{2}}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.00726}{2}) = 0.0144$$

$$k_{4} = 0.1(4 + (0.1 + 0.1) - \frac{1}{2}(f(0.1) + k_{3})) = 0.1(4 + 0.2 - \frac{8.00492 + 0.0144}{2}) = 0.019$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$
  $\Delta t = 0.1$   
 $t = 0 \to f(t) = 8$ 

$$f(0) = 8$$

$$\begin{split} f(0.1) &\approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = \\ 8.00492 \\ f(0.2) &\approx f(0.1) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8.00492 + \frac{0.00975 + 2(0.0145 + 0.0144) + 0.019}{6} = \\ 8.0194 \end{split}$$

$$f(0.1) = 4 + 2 * 0.1 + 4e^{-0.1/2} = 8.005$$
  
$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} = 8.0193$$