

explicit midpoint method

$$y'(t) = -y^2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

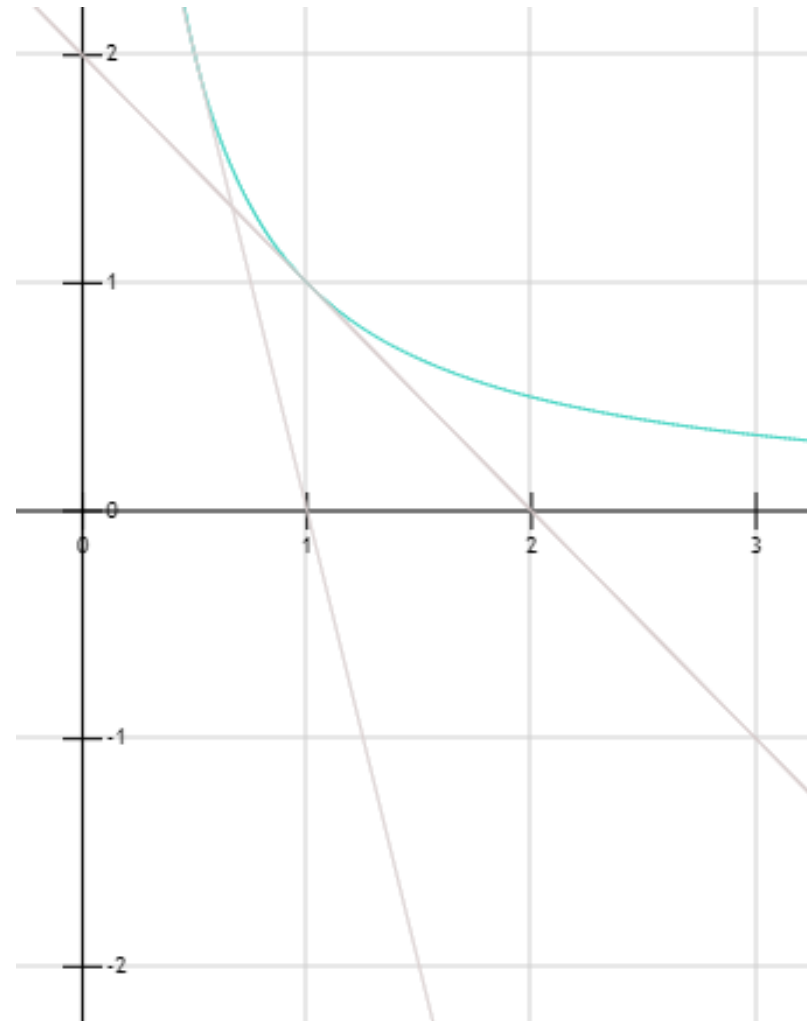
$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$



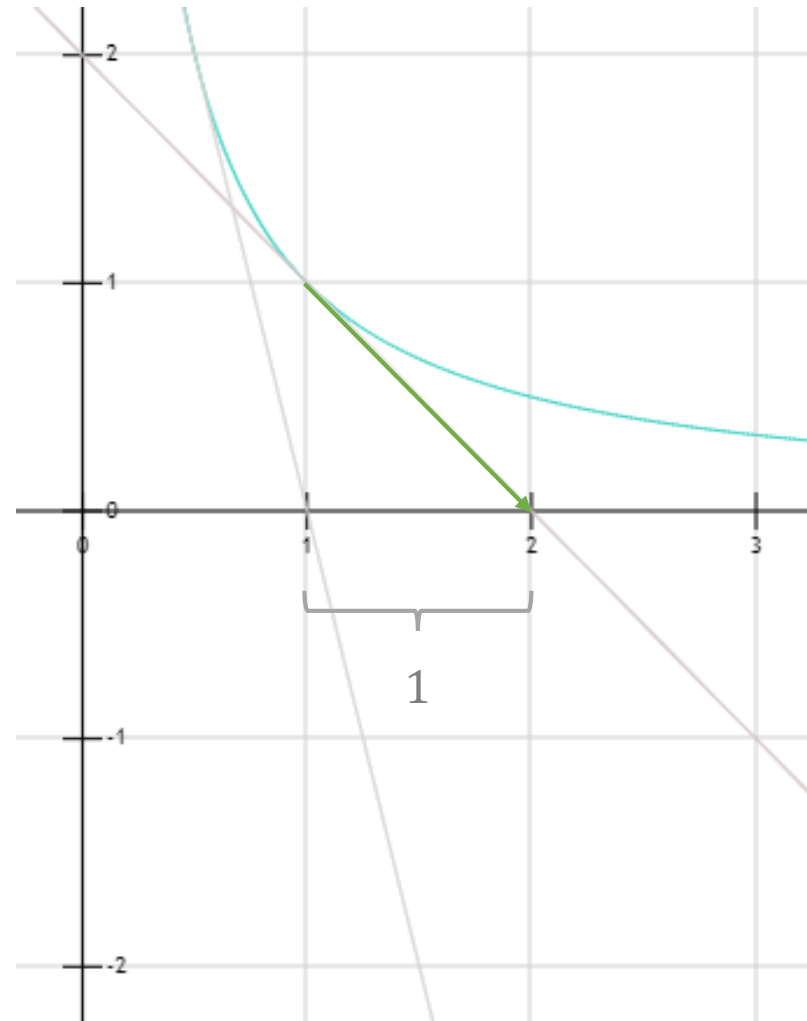
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

$$f(t, 1) = m_1 \in \mathbb{R}$$



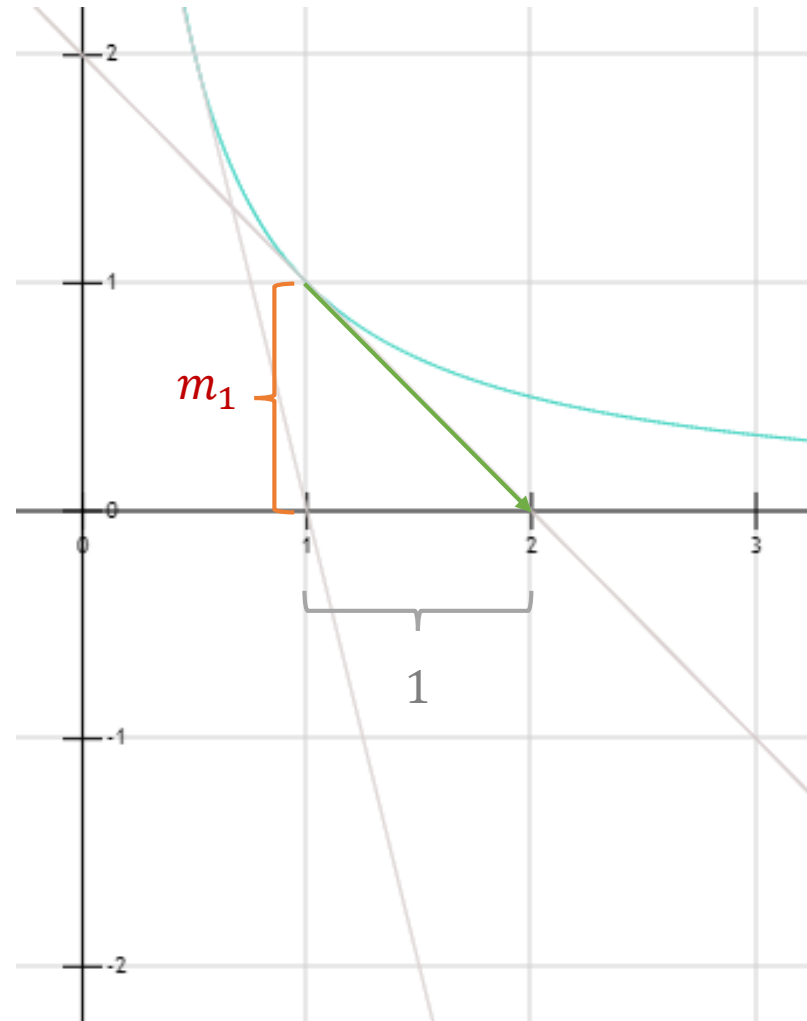
explicit midpoint method

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explicit midpoint method

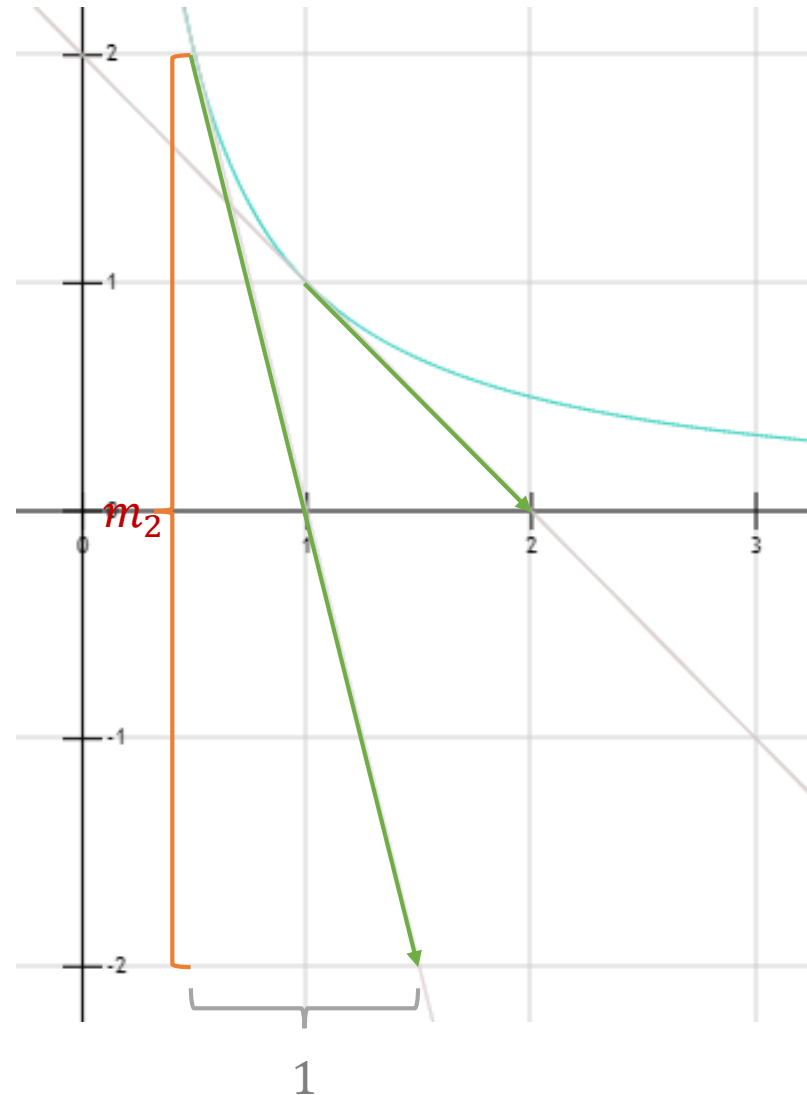
$$y'(t) = f(t, y) = -y^2$$

slope field

$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

$$f(t, 1) = m_1 \in \mathbb{R}$$

$$f(t, 2) = m_2 \in \mathbb{R}$$



explicit midpoint method

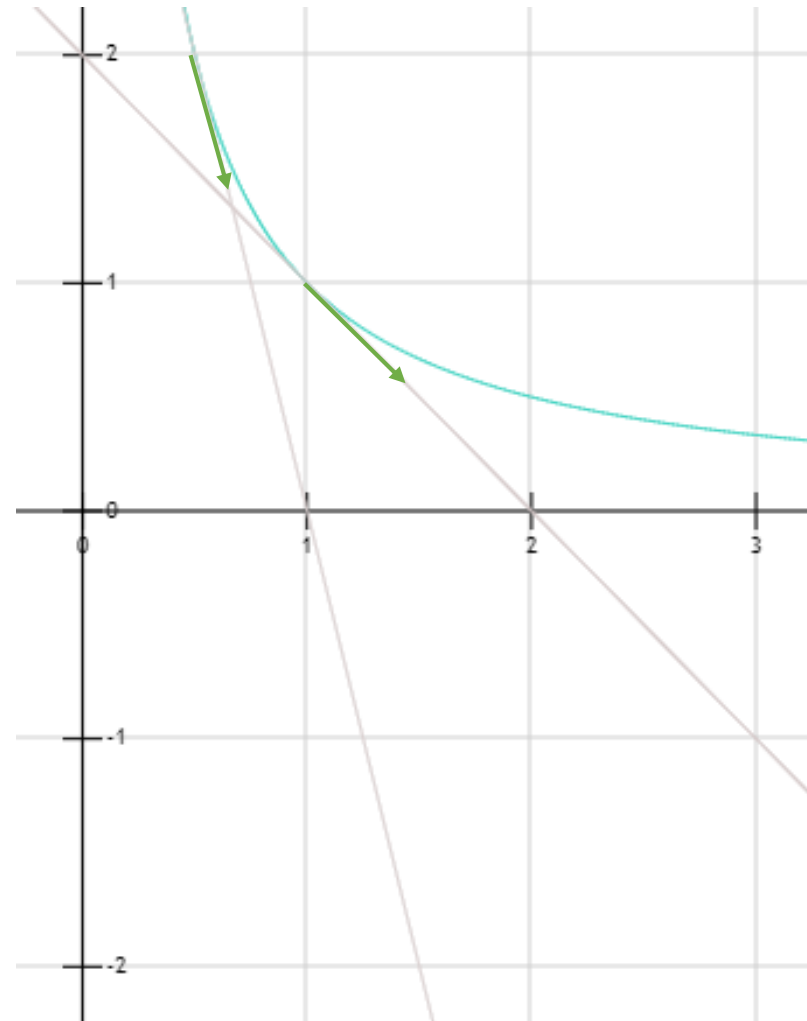
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slope field

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$$f(t, 1) = m_1 \in \mathbb{R}$$

$$f(t, 2) = m_2 \in \mathbb{R}$$

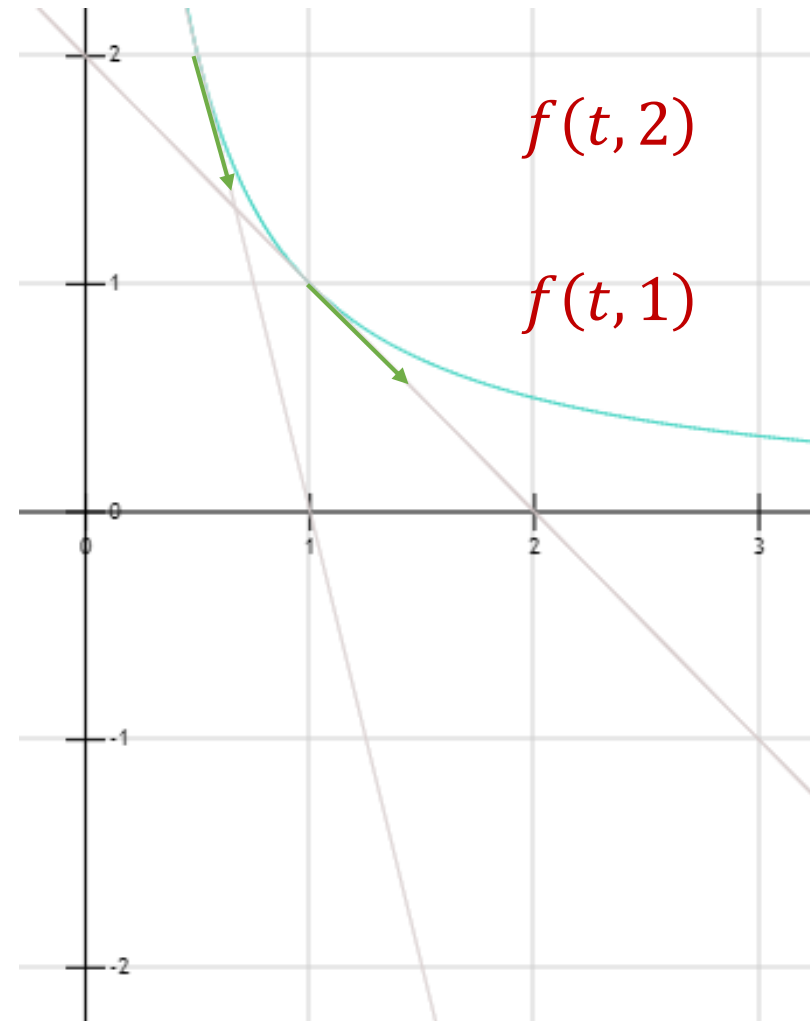


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

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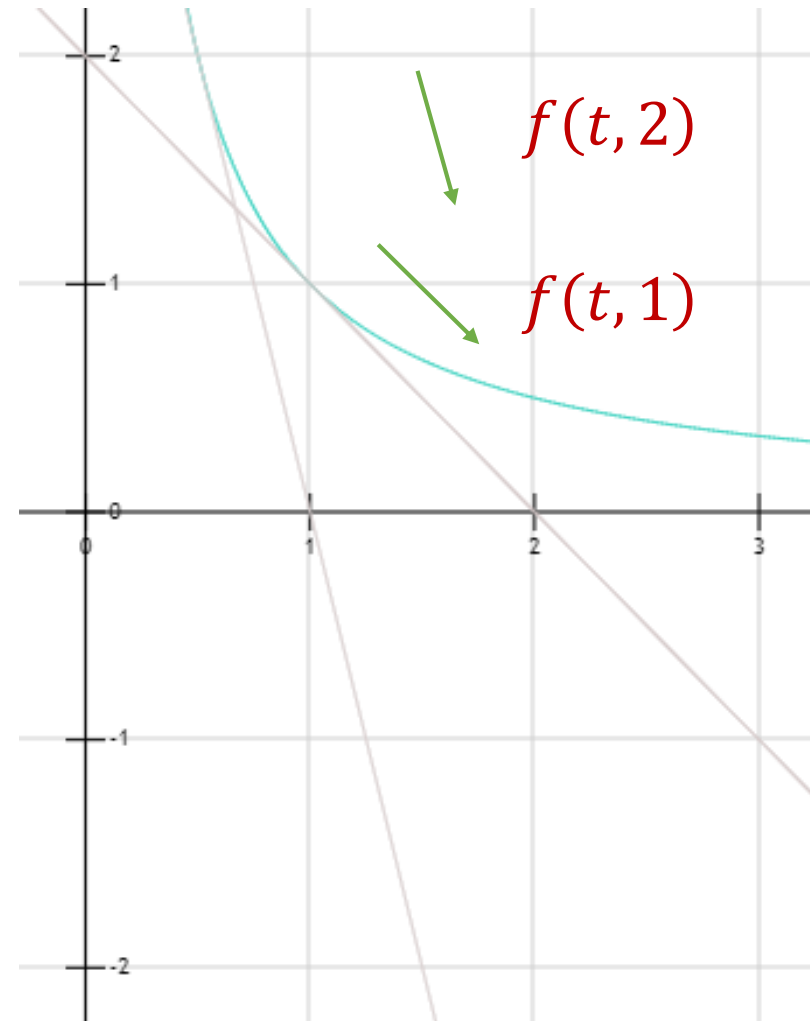


explicit midpoint method

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slope field

$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

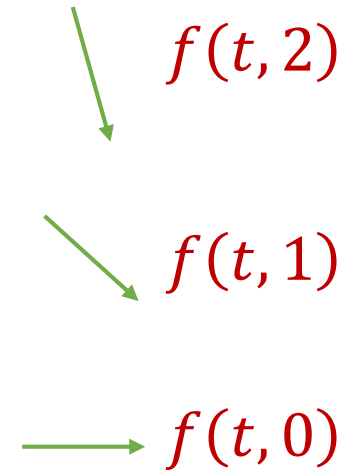


explicit midpoint method

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slope field

$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

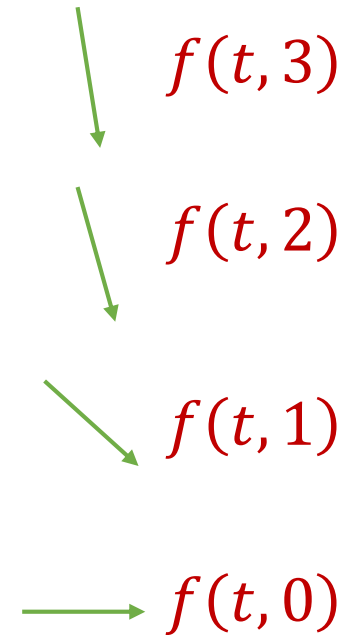


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

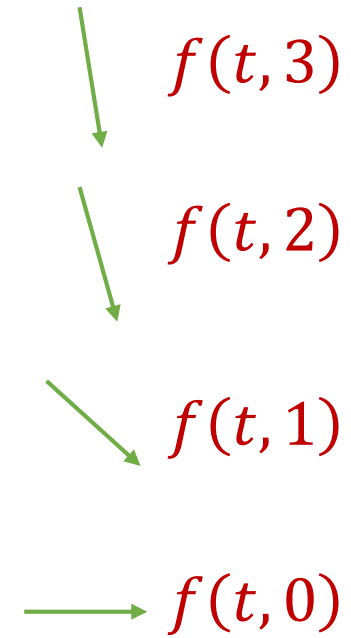
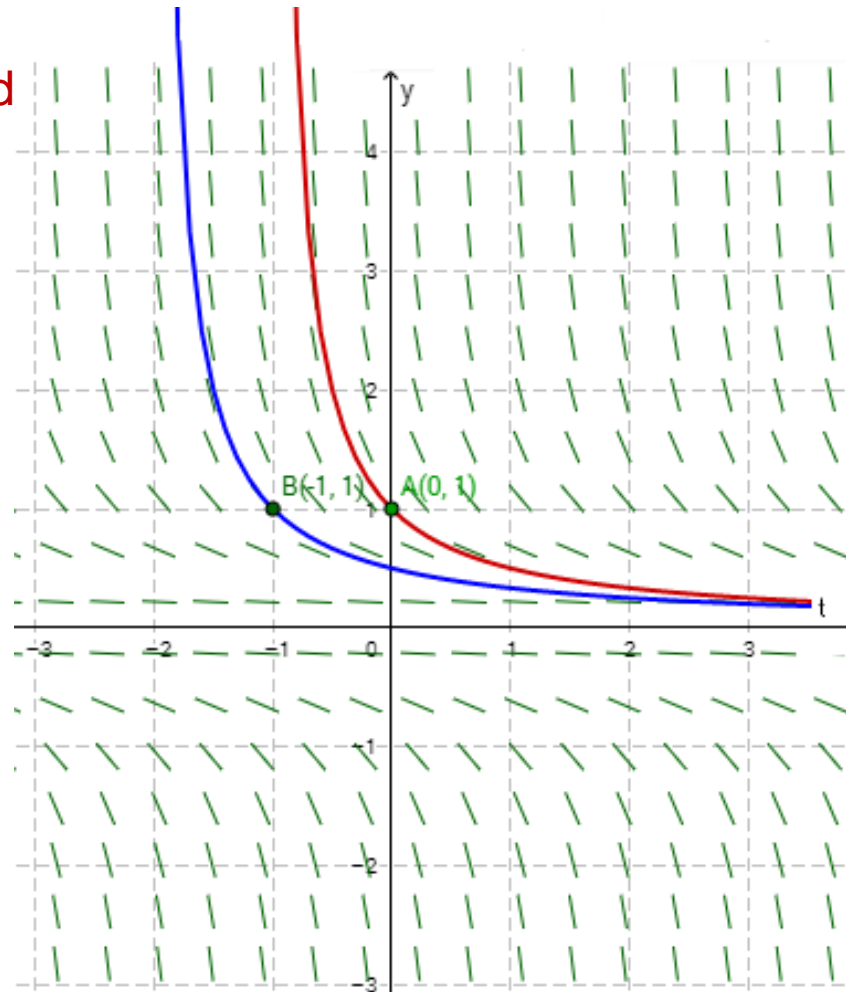
$$f: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

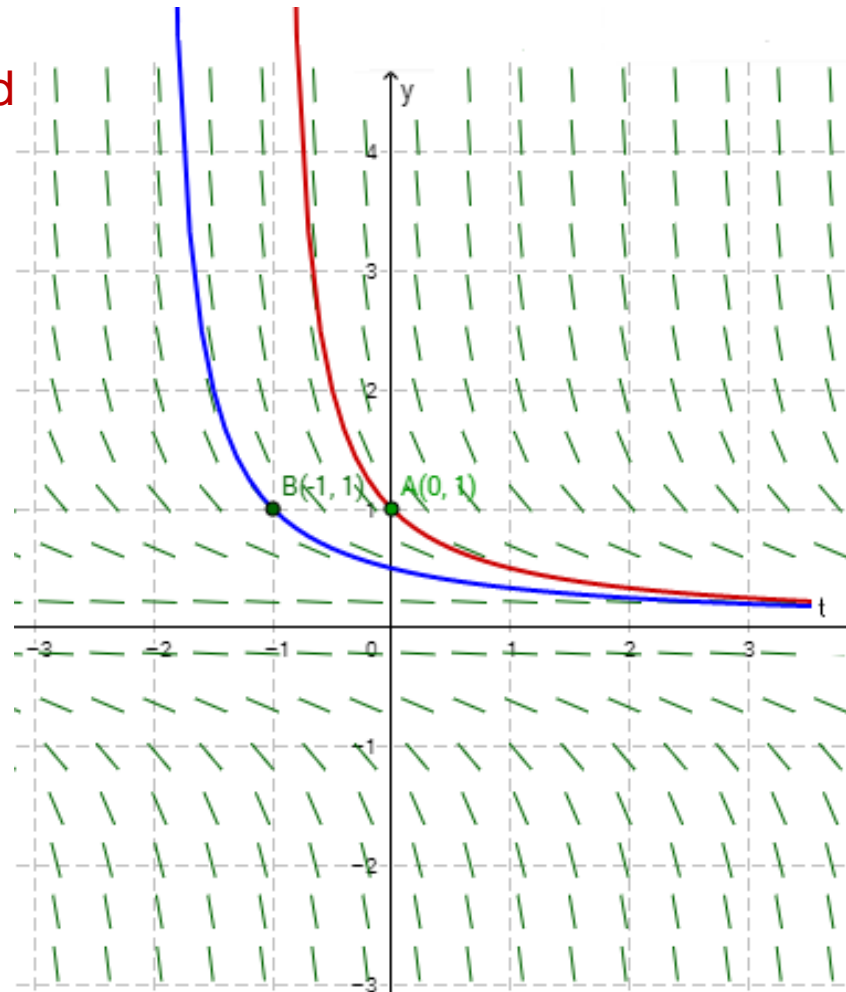
slope field



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field



- ↓ $f(t, 3)$
- ↓ $f(t, 2)$
- ↓ $f(t, 1)$
- $f(t, 0)$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

explicit midpoint method

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$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2 dt$$

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$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2 dt$$

$$\frac{dy}{y^2} = -dt$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$\int y^{-2} dy = - \int dt$$

$$dy = -y^2 dt$$

$$\frac{dy}{y^2} = - dt$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2 dt$$

$$\frac{dy}{y^2} = -dt$$

$$\int y^{-2} dy = - \int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2 dt$$

$$\frac{dy}{y^2} = -dt$$

$$\int y^{-2} dy = - \int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

$$c_1 - c_2 = c \Rightarrow \frac{1}{y} = t + c$$

$$1 = y(t + c)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$\int y^{-2} dy = - \int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

$$c_1 - c_2 = c \Rightarrow \frac{1}{y} = t + c$$

$$1 = y(t + c)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$y(0) = 1 \Rightarrow c = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$y(0) = 1 \Rightarrow c = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$

$$y(-1) = 1 \Rightarrow c = 2 \Rightarrow y(t) = \frac{1}{t + 2}$$

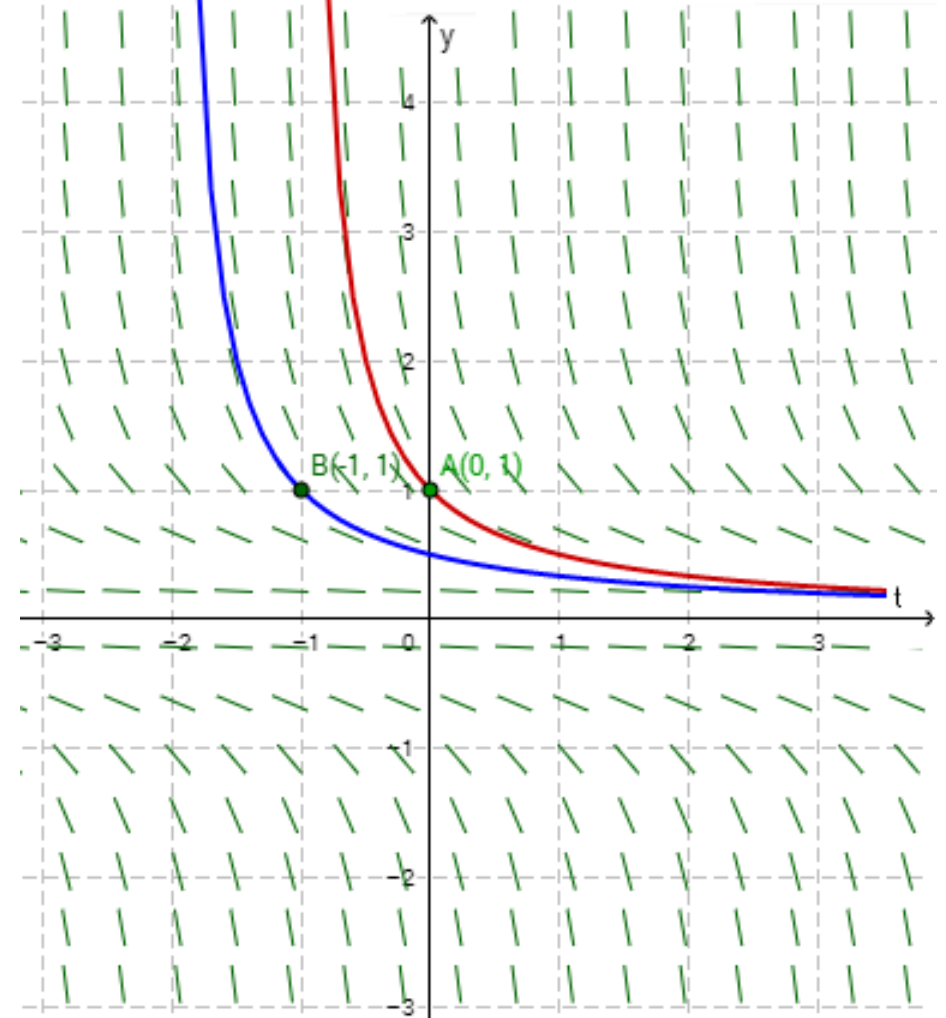
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(t) = \frac{1}{t + c}$$

$$y(-1) = 1 \Rightarrow y(t) = \frac{1}{t + 2}$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0 = 1$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \quad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$
$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$
$$y_0 = 1$$

$$y_1 = y_0 + 0.5 f \left(t_0 + \frac{0.5}{2}, y_0 + \frac{0.5}{2} f(t_0, y_0) \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \qquad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$
$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0 = 1; t_0 = 0 \Rightarrow$$

$$y_1 = y_0 + 0.5 f \left(t_0 + \frac{0.5}{2}, y_0 + \frac{0.5}{2} f(t_0, y_0) \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \qquad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$
$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0 = 1; t_0 = 0 \Rightarrow$$

$$y_1 = 1 + 0.5 f \left(0 + \frac{0.5}{2}, 1 + \frac{0.5}{2} f(0, 1) \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$f(0, 1)$$

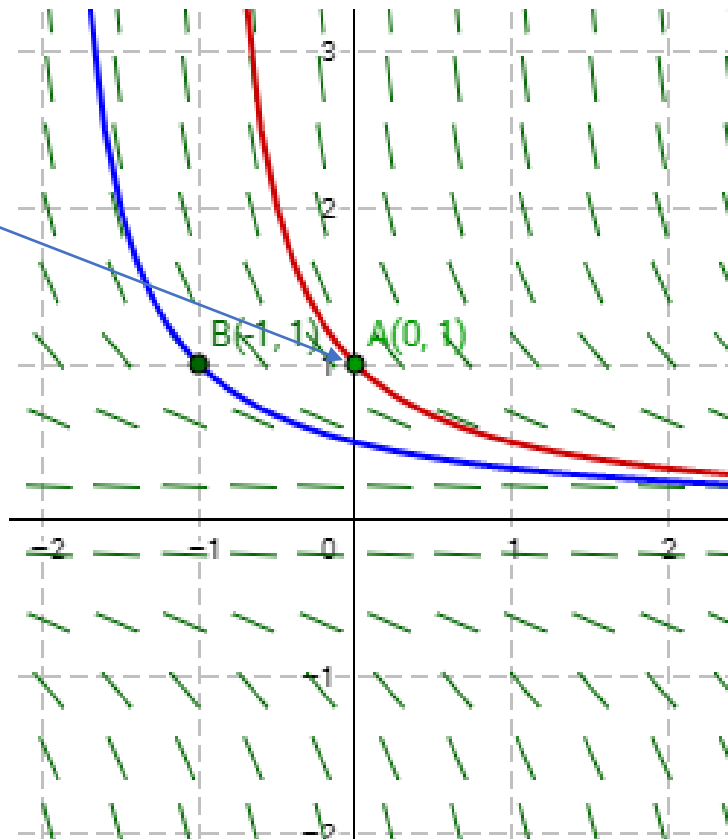
$$y_1 = 1 + 0.5f\left(0 + \frac{0.5}{2}, 1 + \frac{0.5}{2}f(0, 1)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$f(0, 1)$

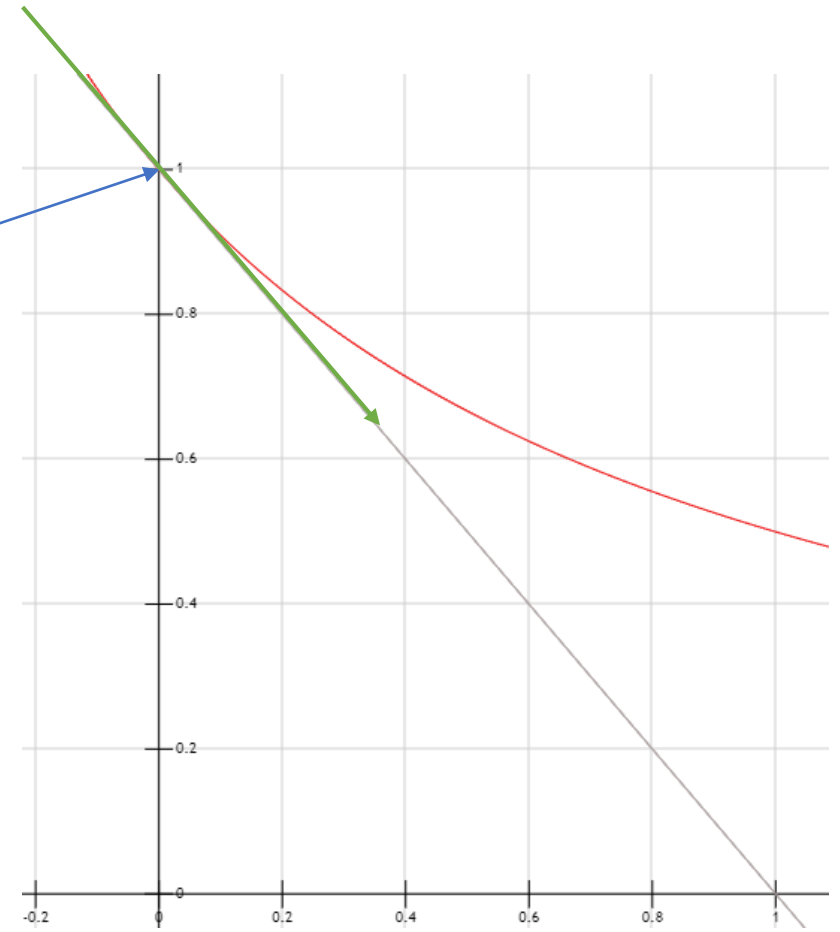
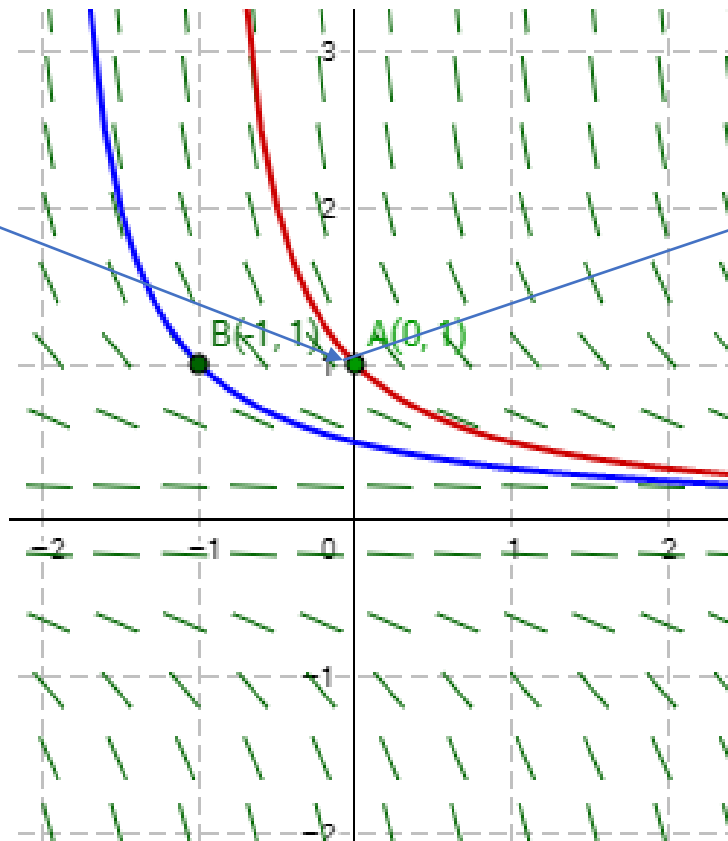


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$f(0, 1)$

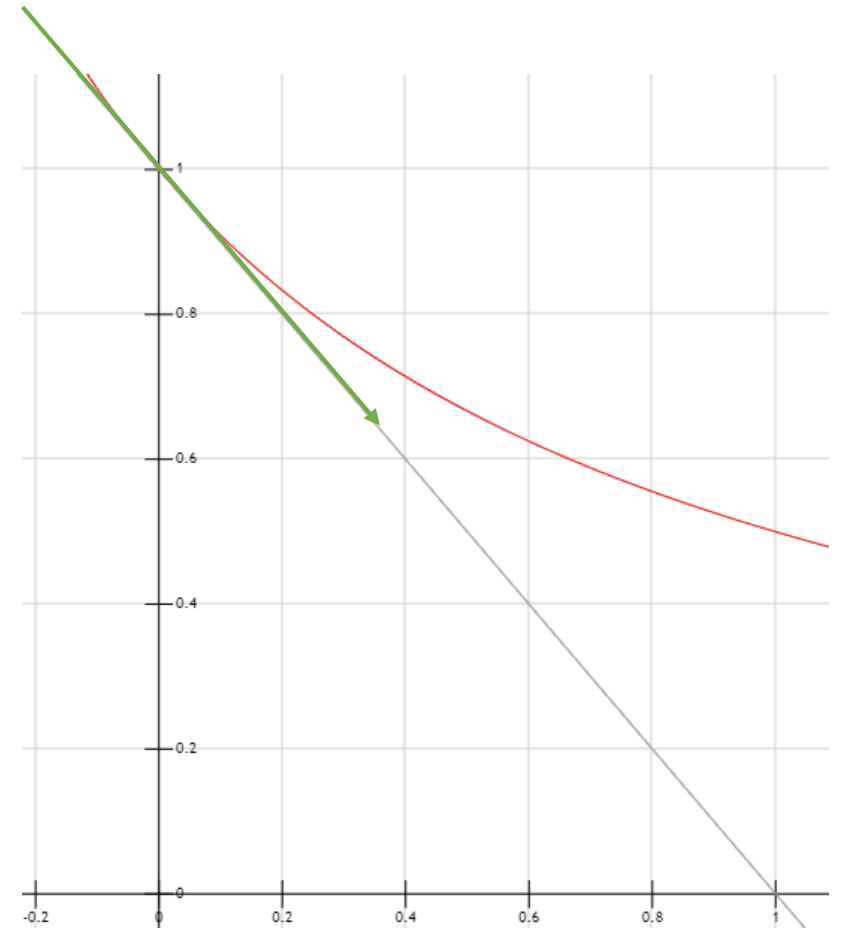


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0, 1) = -1^2 = -1$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0, 1) = -1^2 = -1$$

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explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0, 1) = -1^2 = -1$$

$$y_1 = 1 + 0.5f\left(0 + \frac{0.5}{2}, 1 - \frac{0.5}{2}\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \qquad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$
$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0 = 1; t_0 = 0 \Rightarrow$$

$$y_1 = 1 + 0.5 f \left(0 + \frac{0.5}{2}, 1 - \frac{0.5}{2} \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

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$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0 = 1; t_0 = 0 \Rightarrow$$

$$y_1 = 1 + 0.5 f(0.25, 0.75)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$f(0.25, 0.75)$$

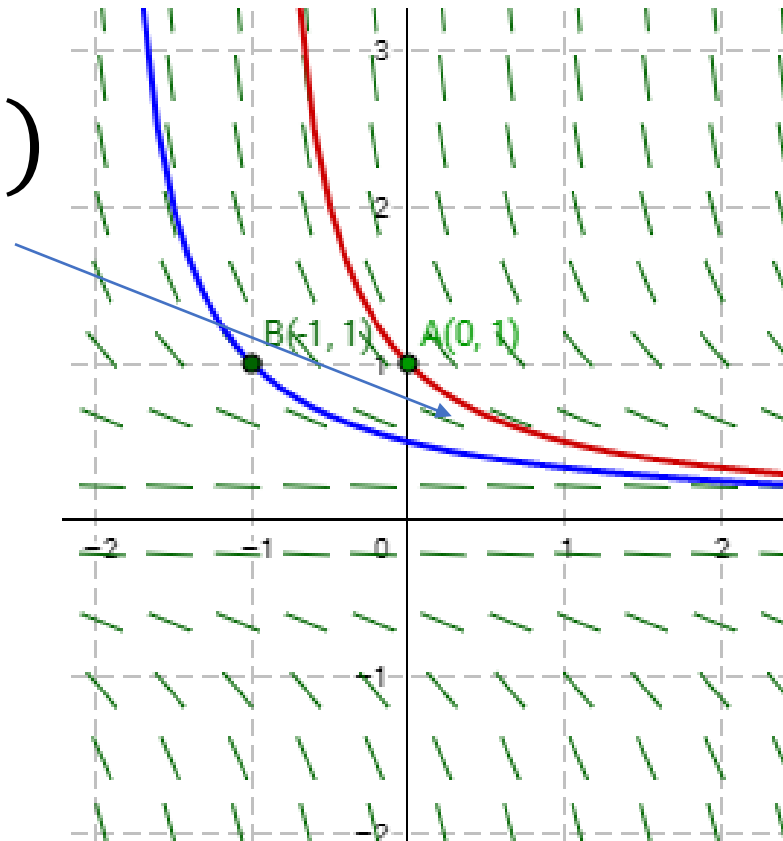
$$y_1 = 1 + 0.5f(0.25, 0.75)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$f(0.25, 0.75)$

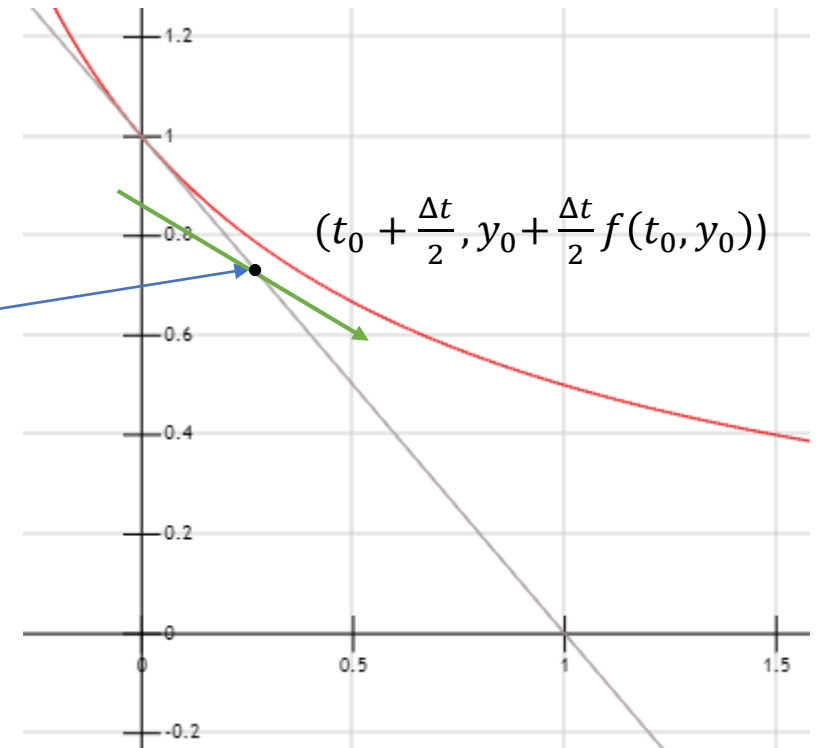
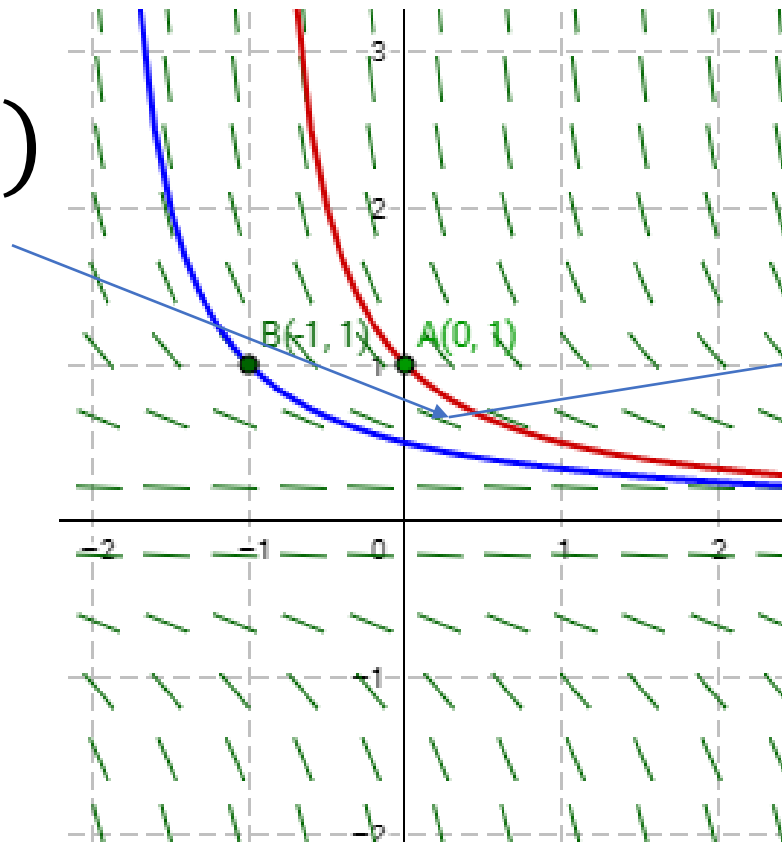


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

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$f(0.25, 0.75)$

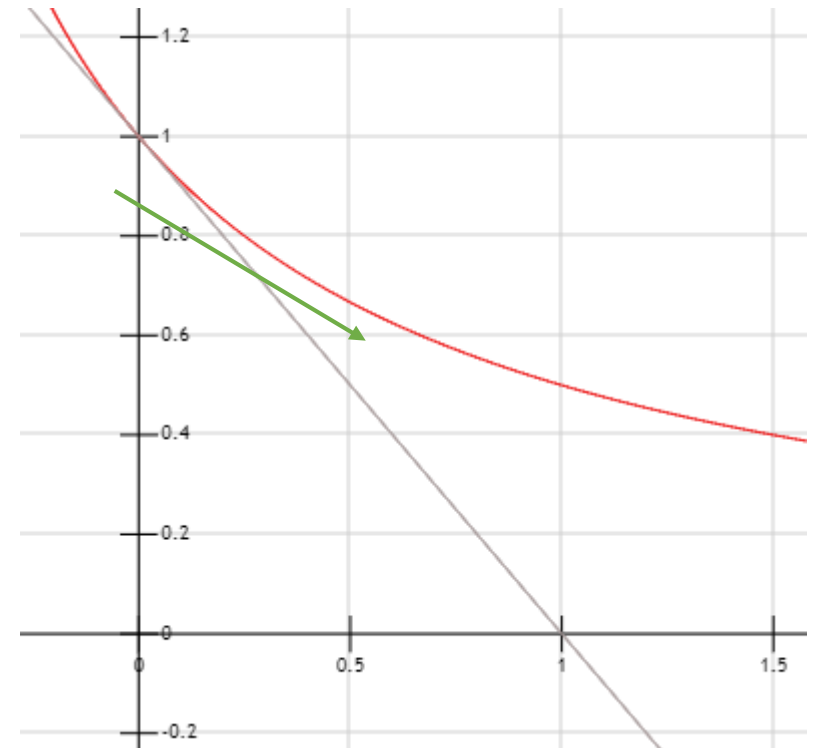


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.75^2 = -0.5625$$

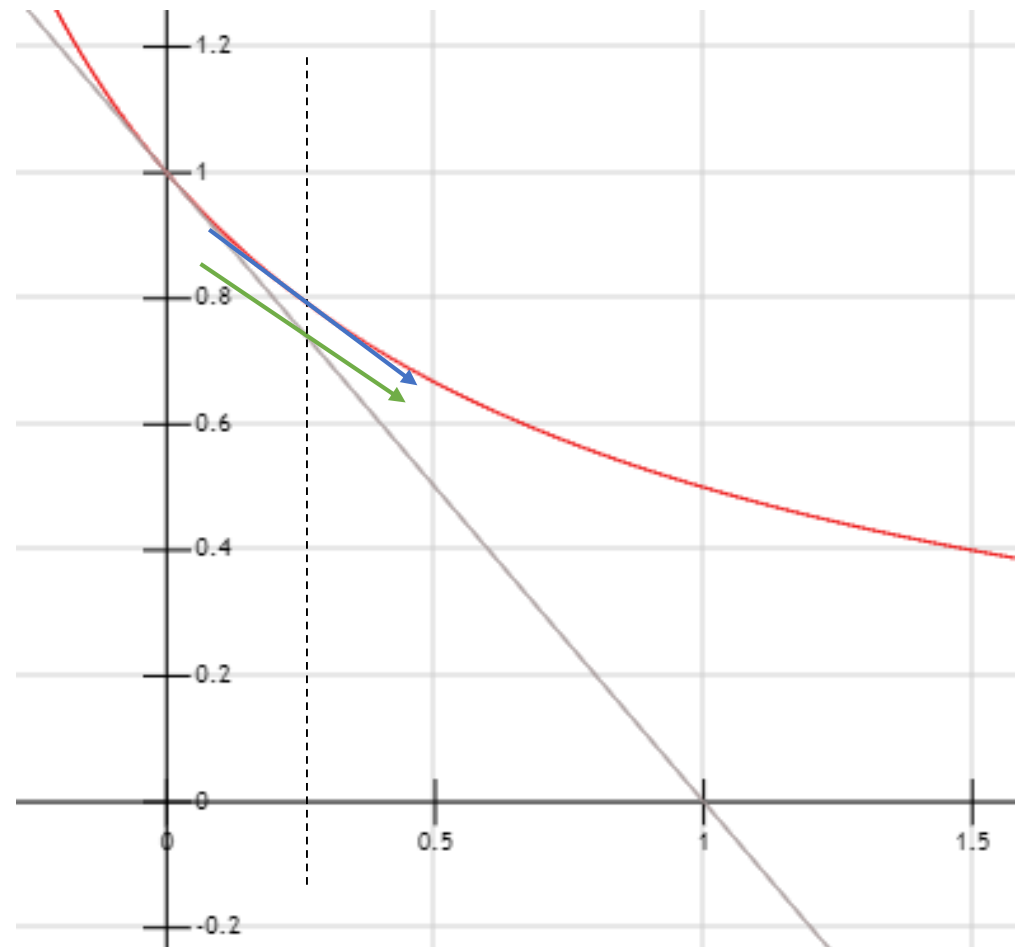


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explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 1 + 0.5f(0.25, 0.75)$$


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 1 - \frac{0.5625}{2}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 0.71875$$

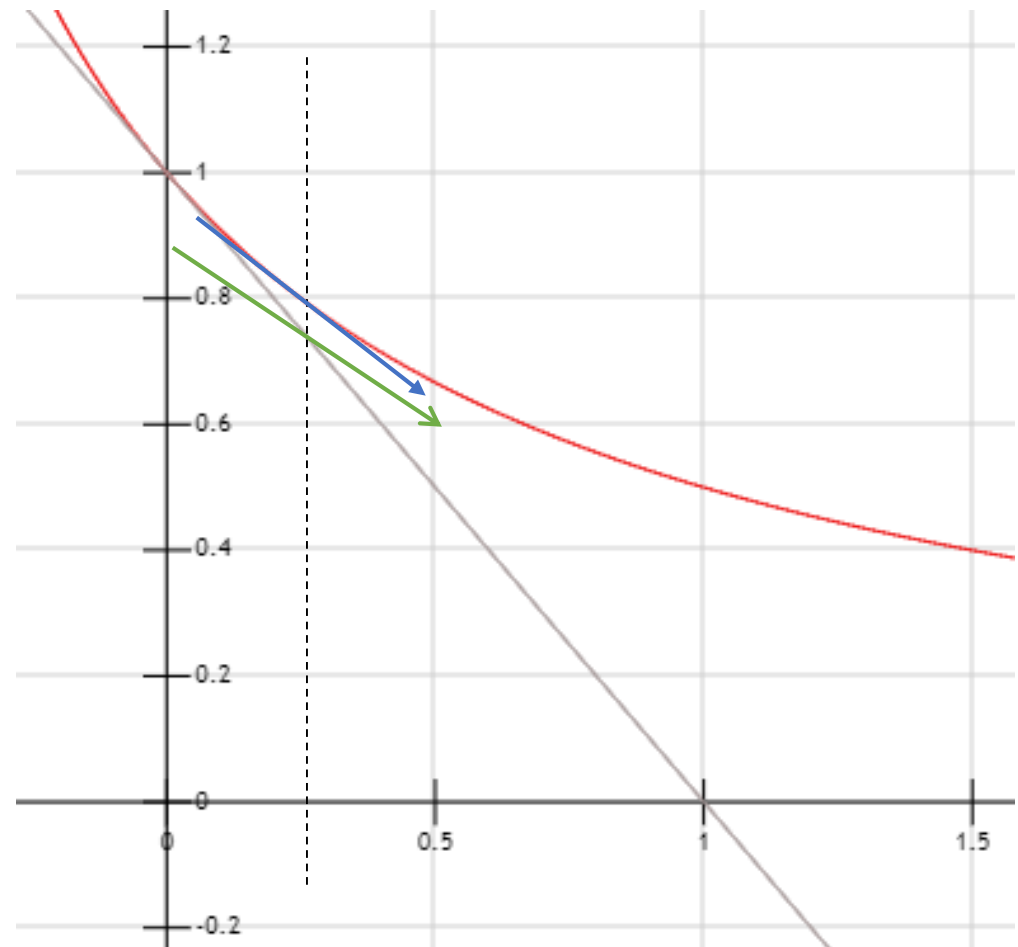
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



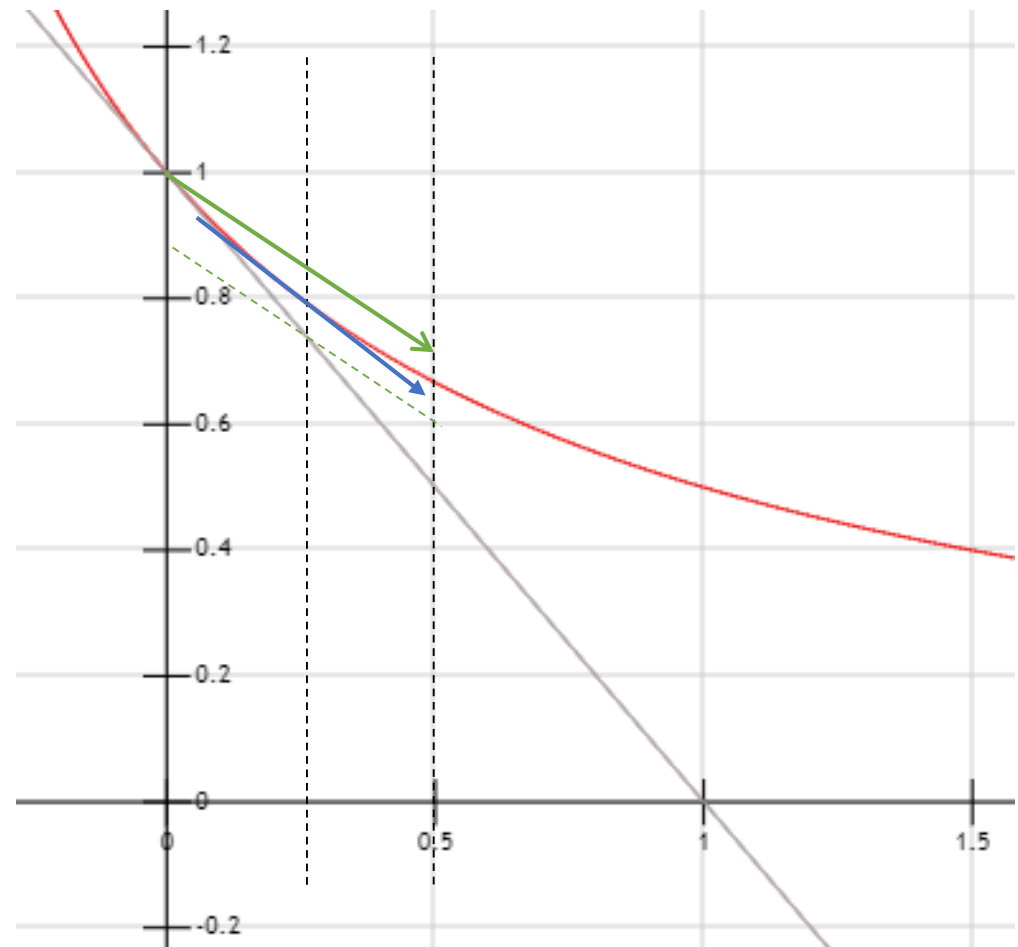
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



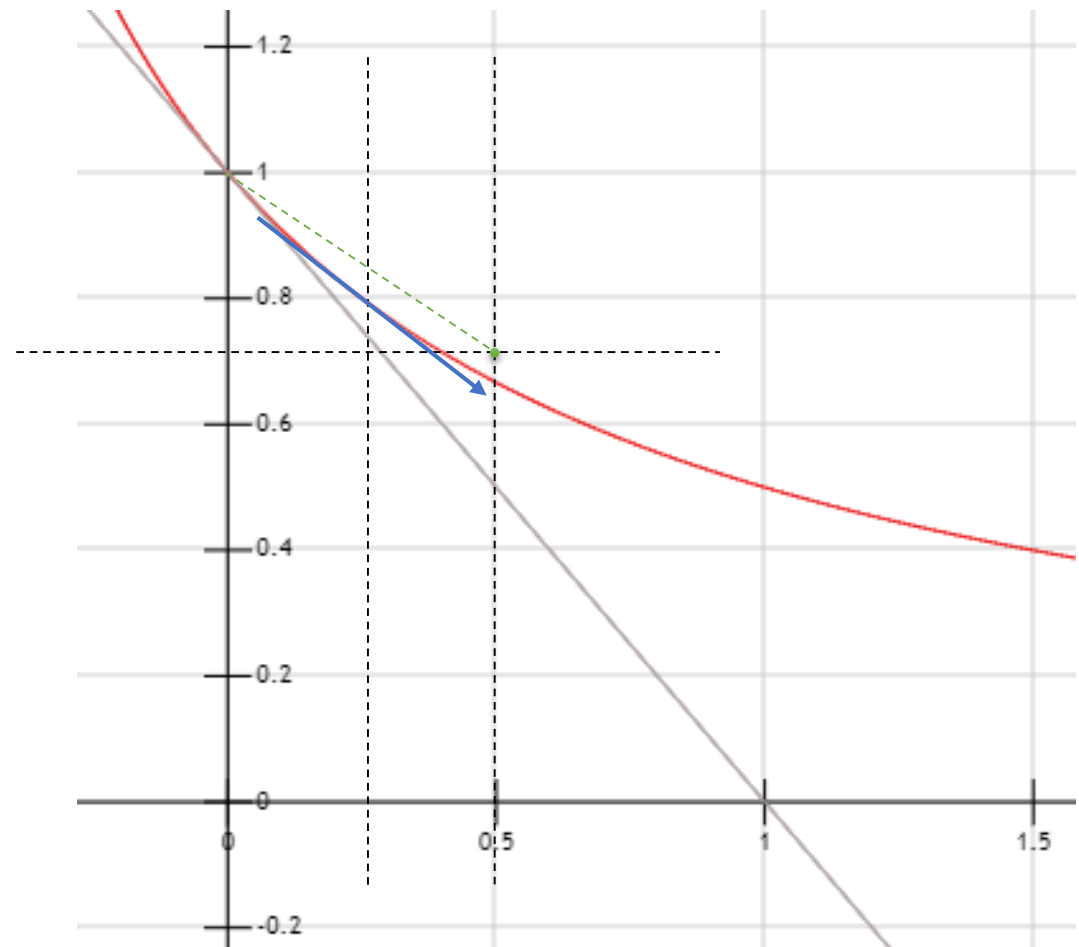
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0=1; t_0=0; y_1=0.71875; t_1=t_0 + \Delta t = \frac{1}{2}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \qquad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$
$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0=1; t_0=0; y_1=0.71875; t_1 = \frac{1}{2}$$

$$y_2 = y_1 + 0.5 f \left(t_1 + \frac{0.5}{2}, y_1 + \frac{0.5}{2} f(t_1, y_1) \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2 \qquad y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f \left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n) \right)$$

$$y_0=1; t_0=0; y_1=0.71875; t_1 = \frac{1}{2}$$

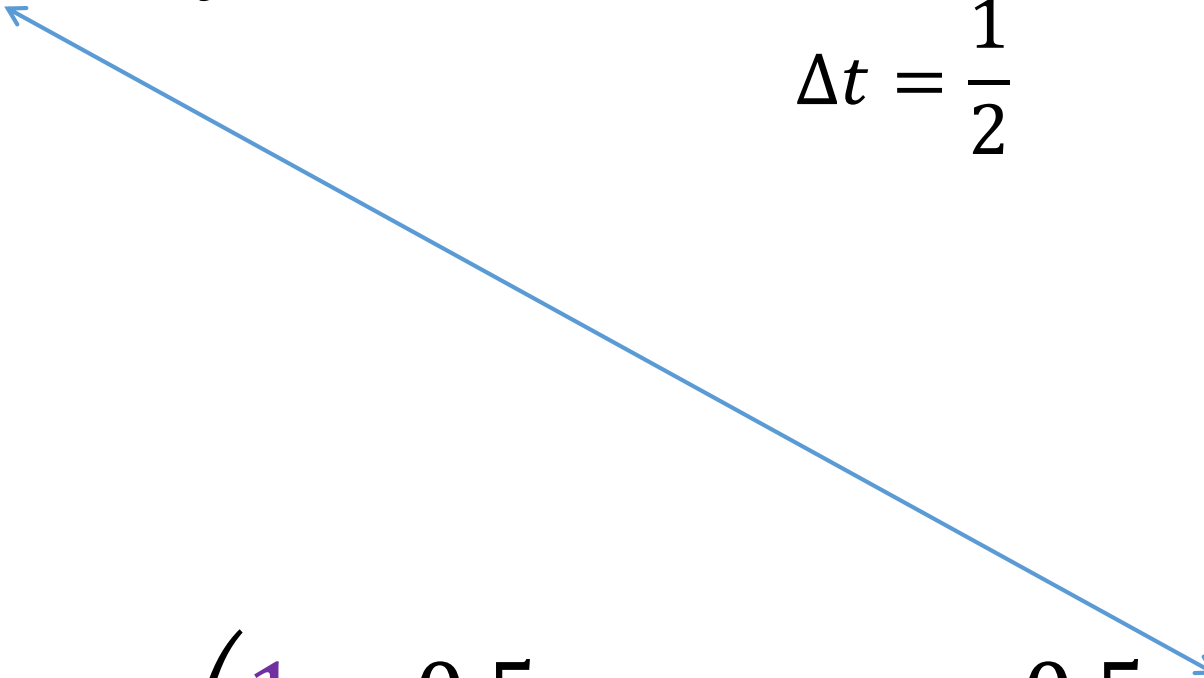
$$y_2 = 0.71875 + 0.5 f \left(\frac{1}{2} + \frac{0.5}{2}, 0.71875 + \frac{0.5}{2} f \left(\frac{1}{2}, 0.71875 \right) \right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1$$

$$\Delta t = \frac{1}{2}$$


$$y_2 = 0.71875 + 0.5f\left(\frac{1}{2} + \frac{0.5}{2}, 0.71875 + \frac{0.5}{2}f\left(\frac{1}{2}, 0.71875\right)\right)$$