Computational Logic

Description Logic $\mathcal{ALC}$

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Ontology

- Ontology – formal conceptualization of a domain
  - knowledge base
  - represented in formal language
  - dealing with terminology in some area of discourse

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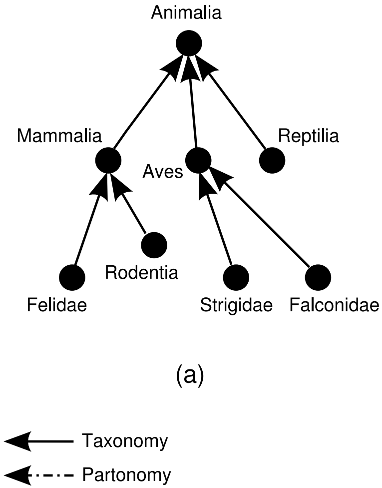
- Logic – semantics and reasoning

Example Ontologies



Vocabulary:

1. Individual symbols:
   $a, b, \ldots \in N_{\mathsf{I}}$
2. Concept symbols:
   $A, B, C, D, \ldots \in N_{\mathsf{C}}$
3. Role symbols:
   $R, S, \ldots \in N_{\mathsf{R}}$

Concepts

1. Atomic concepts:
   $A, B, \ldots \in N_\mathsf{C}$
2. Complex concepts are smallest set expressions of the forms:
   $A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \exists R.C \mid \forall R.C$
   where $A \in N_\mathsf{C}$, $R \in N_\mathsf{R}$, and C, D are any complex concepts

Axioms

1. Subsumption (General Concept Inclusion, GCI):
   $C \sqsubseteq D$
   for any concepts C and D
2. Individual assertions:
   $a : C$ (also written $C(a)$)
   for any $a \in N_\mathrm{I}$ and any concept C
3. Role assertions:
   $a, b : R$ (also written $R(a, b)$)
   for any $a, b \in N_\mathrm{I}$ and any $R \in N_\mathrm{R}$

Knowledge Base

DL KB is composed of 2 parts:

1. TBox $\mathcal{T}$:
   finite set of GCI
   (terminological knowledge)
2. ABox $\mathcal{A}$:
   finite set of concept and role assertions
   (data, assertional knowledge)

$\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$ is interpretation of $\langle \mathcal{T}, \mathcal{A} \rangle$ if:

- domain Δ is a non-empty set
- $\cdot^{\mathcal{I}}$ is a function s.t.:
  $a^{\mathcal{I}} \in \Delta$ for all $a \in N_{\mathrm{I}}$
  $C^{\mathcal{I}} \subseteq \Delta$ for all concepts C
  $R^{\mathcal{I}} \subseteq \Delta \times \Delta$ for all $R \in N_{\mathrm{R}}$
- and for any C, D and R, the following restrictions hold:
  $\neg C^{\mathcal{I}} = \Delta \setminus C^{\mathcal{I}}$
  $C \sqcap D^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$
  $C \sqcup D^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
  $\exists R.C^{\mathcal{I}} = \{x \in \Delta \mid \exists y \in \Delta : \langle x, y \rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
  $\forall R.C^{\mathcal{I}} = \{x \in \Delta \mid \forall y \in \Delta : \langle x, y \rangle \in R^{\mathcal{I}} \implies y \in C^{\mathcal{I}}\}$

Semantics: Satisfiability & Satisfaction

- concept C is satisfiable if $C^{\mathcal{I}} \neq \emptyset$ for some interpretation $\mathcal{I}$
- $\mathcal{I}$ satisfies ($\models$) axioms as follows:
  $\mathcal{I} \models C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
  $\mathcal{I} \models a : C$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
  $\mathcal{I} \models a, b : C$ if $\langle a^{\mathcal{I}}, b^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$

Semantics: Model

Interpretation $\mathcal{I} = \langle \Delta, \cdot^{\mathcal{I}} \rangle$ is a model of $\langle \mathcal{T}, \mathcal{A} \rangle$ if it satisfies every axiom in $\mathcal{T}$ and $\mathcal{A}$

KB $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable if it has a model

Reasoning Tasks

Satisfiability of concepts:
C is satisfiable w.r.t. $\langle \mathcal{T}, \mathcal{A} \rangle$ if there is a model $\mathcal{I}$ of $\langle \mathcal{T}, \mathcal{A} \rangle$ such that $C^{\mathcal{I}} \neq \emptyset$

Entailment:
$\langle \mathcal{T}, \mathcal{A} \rangle$ entails $C \sqsubseteq D$ ($\langle \mathcal{T}, \mathcal{A} \rangle \models C \sqsubseteq D$) if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in every model $\mathcal{I}$ of $\langle \mathcal{T}, \mathcal{A} \rangle$