explicit midpoint method

$$y'(t) = -y^2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

$$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

explicit midpoint method

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slope field

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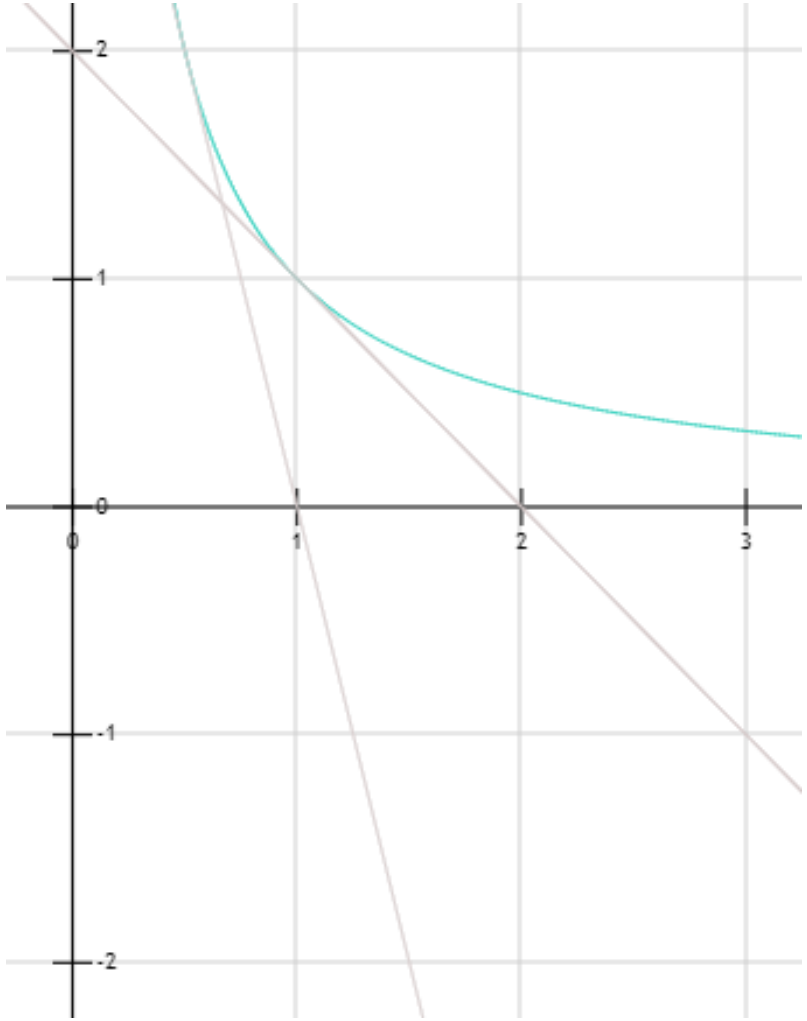


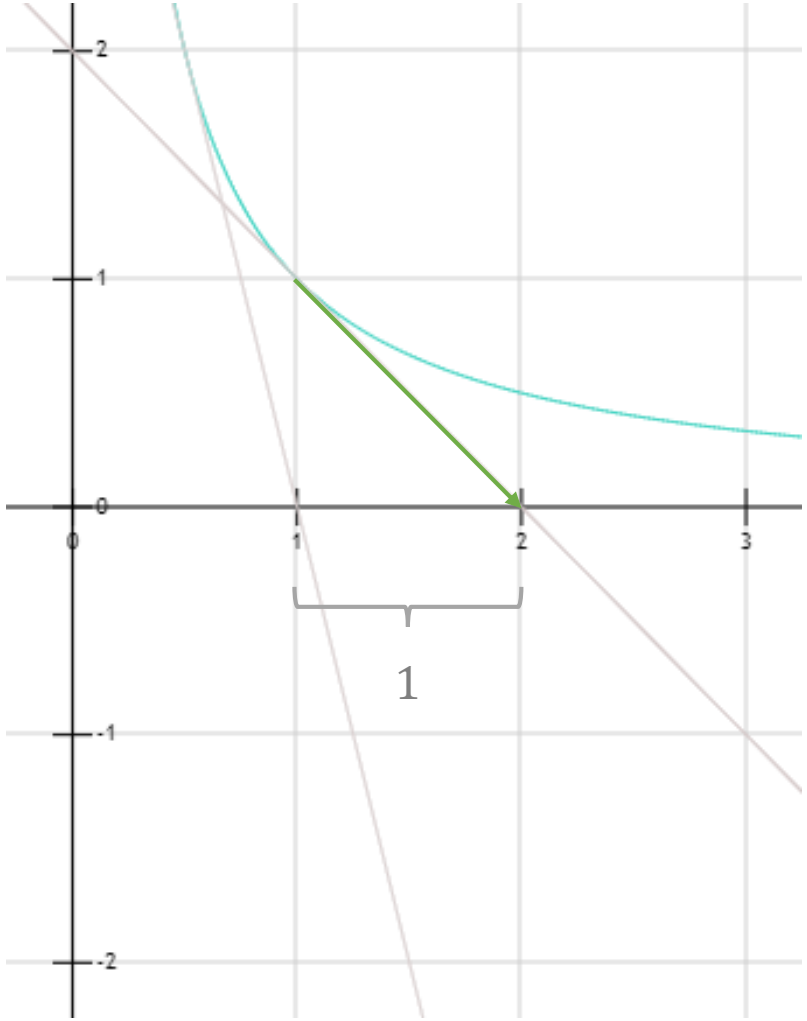
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field

$$f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$$

$$f(t, 1) = m_1 \in \mathbb{R}$$



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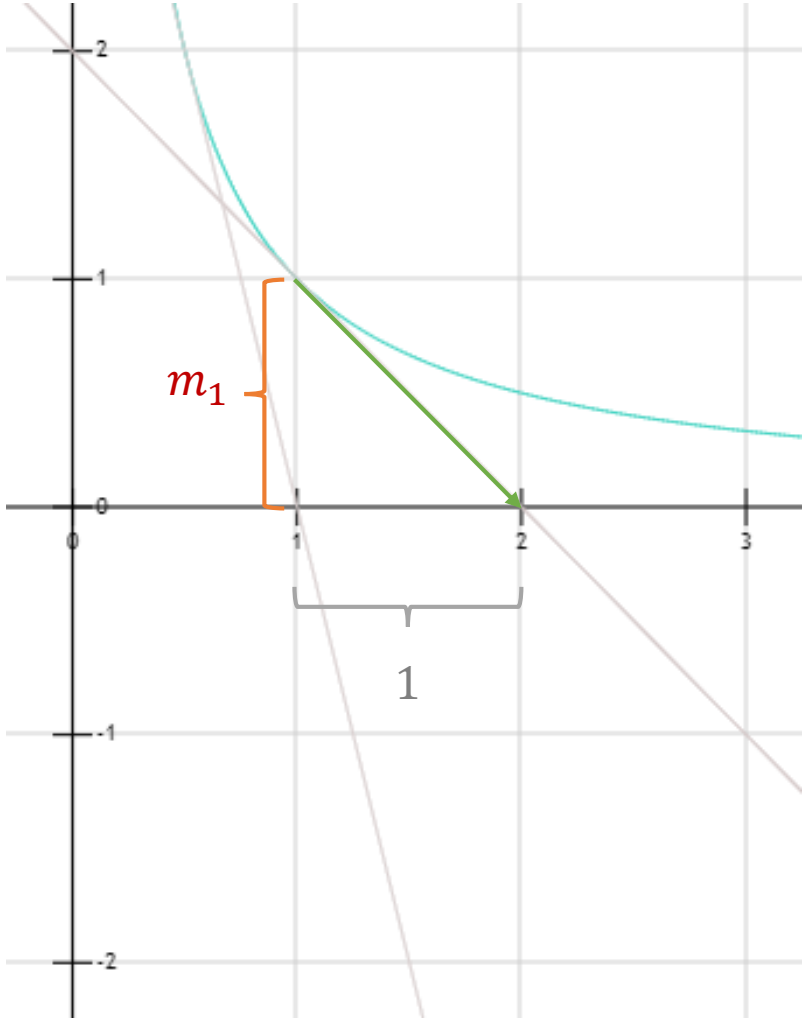

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slope field

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$$f(t, 1) = m_1 \in \mathbb{R}$$

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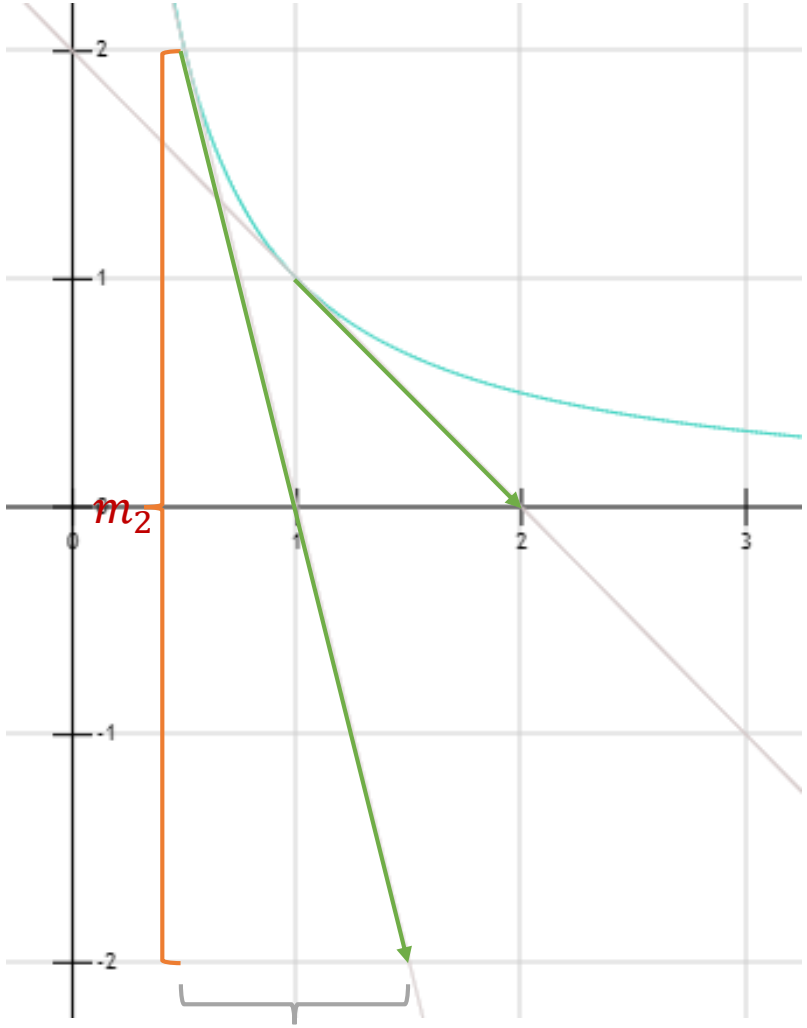

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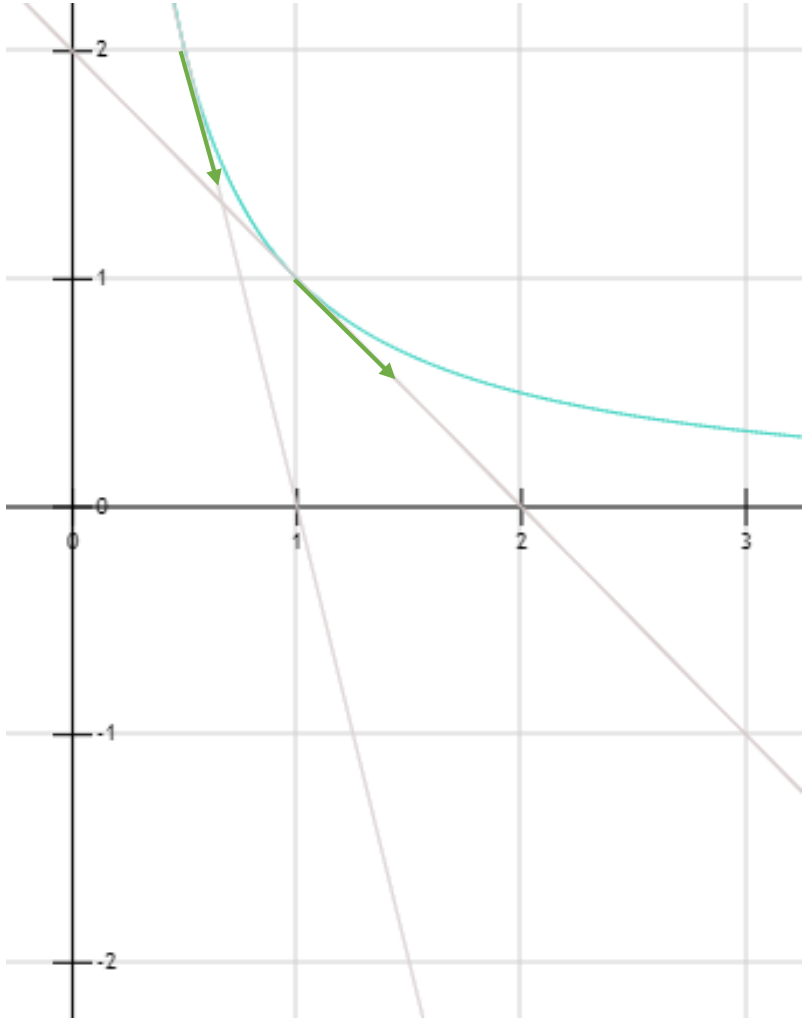

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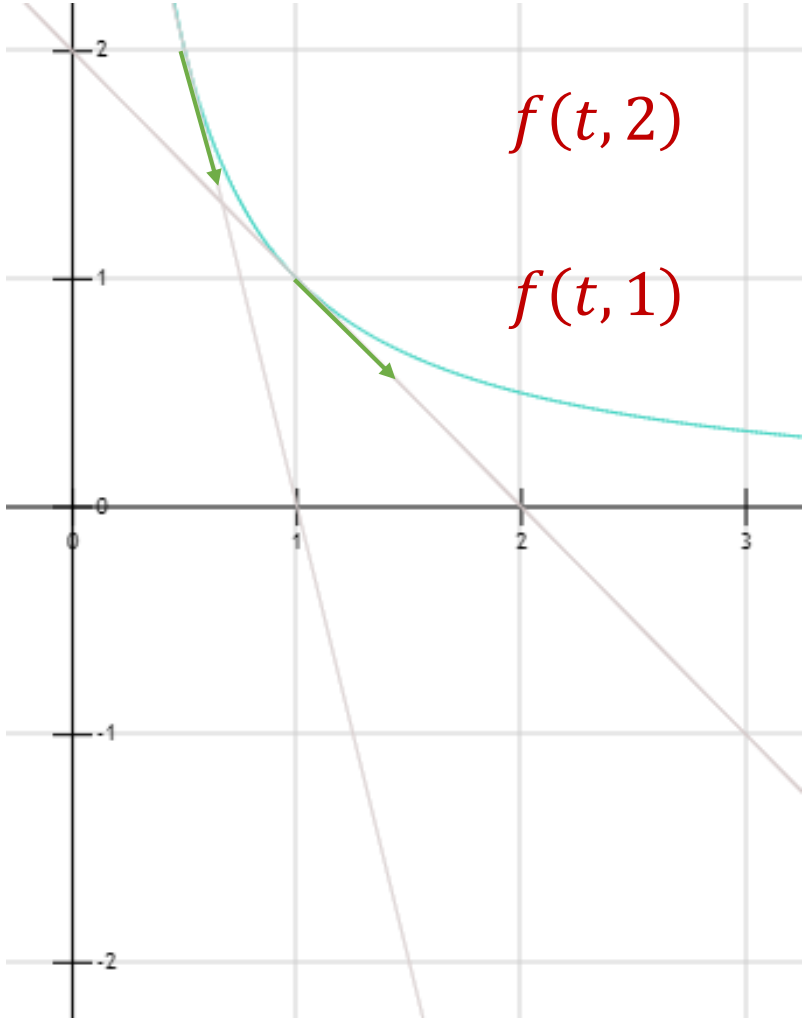

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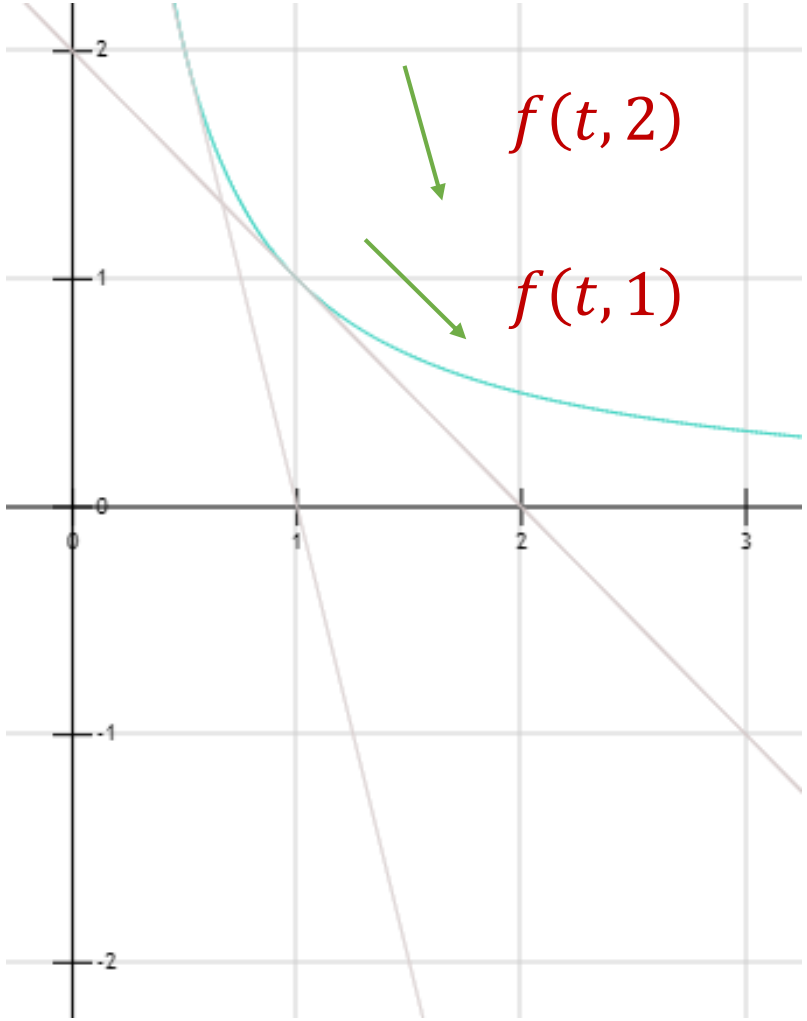

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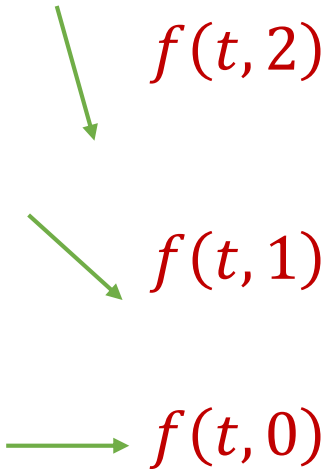

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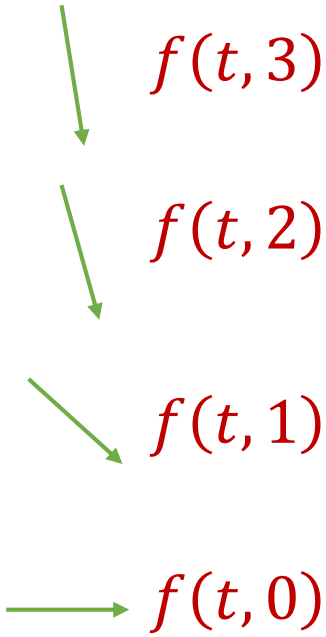

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$$y'(t) = f(t, y) = -y^2$$

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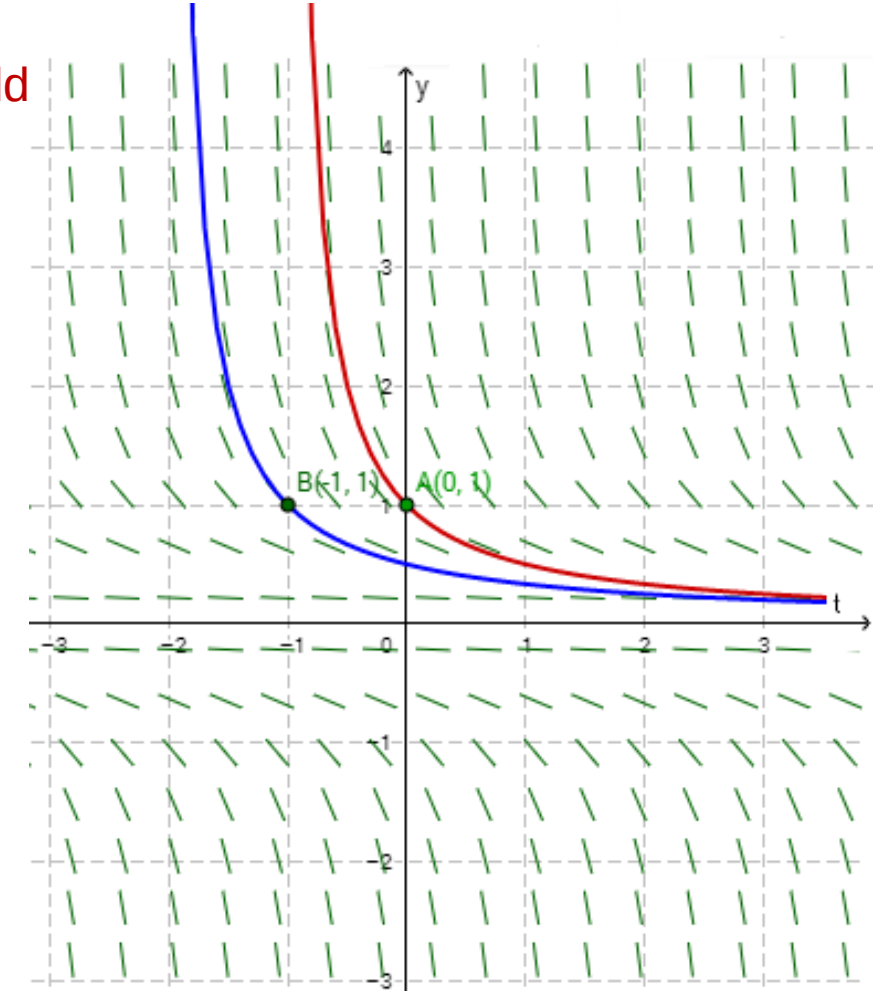


$f(t, 3)$

$f(t, 2)$

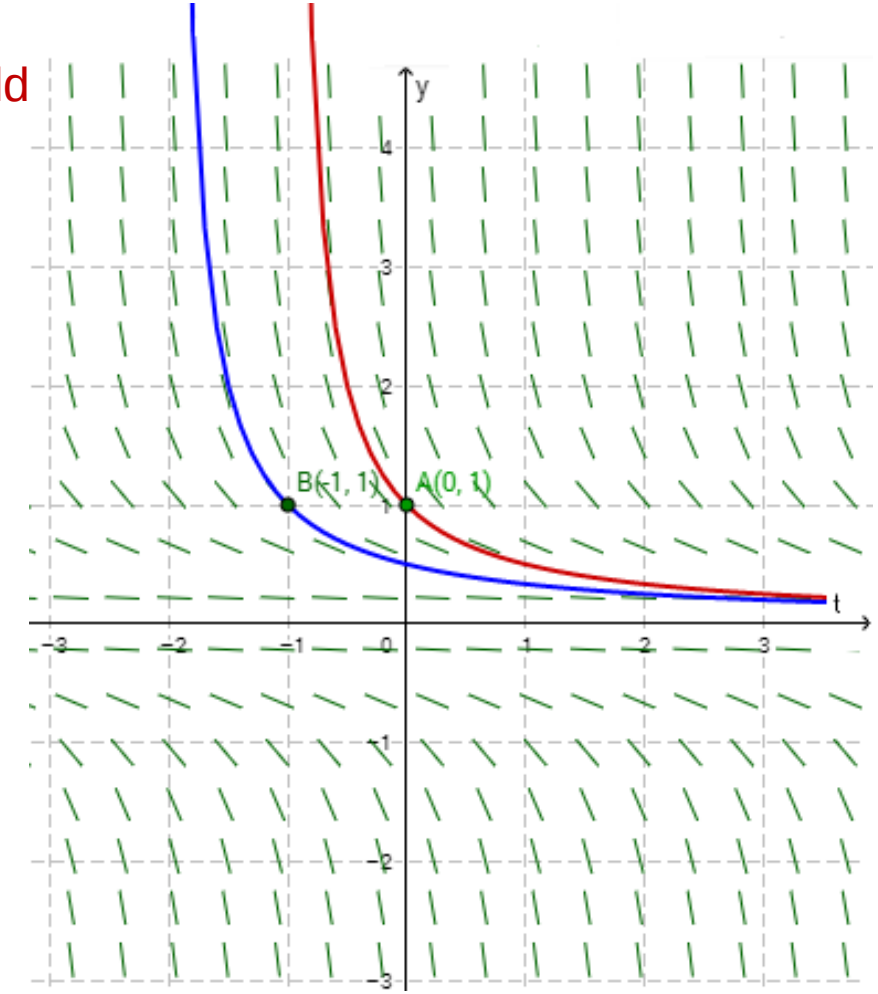
$f(t, 1)$

$f(t, 0)$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

slope field



$f(t, 3)$

$f(t, 2)$

$f(t, 1)$

$f(t, 0)$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2\, dt$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2\, dt$$

$$\frac{dy}{y^2} = -\, dt$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2\, dt$$

$$\frac{dy}{y^2} = -\, dt$$

$$\int y^{-2}\, dy = -\int dt$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2\, dt$$

$$\frac{dy}{y^2} = -\, dt$$

$$\int y^{-2}\, dy = -\int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

$$y'(t) = f(t, y) = -y^2$$

$$\frac{dy}{dt} = -y^2$$

$$dy = -y^2\, dt$$

$$\frac{dy}{y^2} = -\, dt$$

$$\int y^{-2}\, dy = -\int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

$$c_1 - c_2 = c \Rightarrow \frac{1}{y} = t + c$$

$$1 = y(t + c)$$

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$\int y^{-2}\, dy = -\int dt$$

$$-y^{-1} + c_1 = -t + c_2$$

$$c_1 - c_2 = c \Rightarrow \frac{1}{y} = t + c$$

$$1 = y(t + c)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$y(0) = 1 \Rightarrow c = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y = \frac{1}{t + c}$$

$$y(0) = 1 \Rightarrow c = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$

$$y(1) = -1 \Rightarrow c = 2 \Rightarrow y(t) = \frac{1}{t + 2}$$

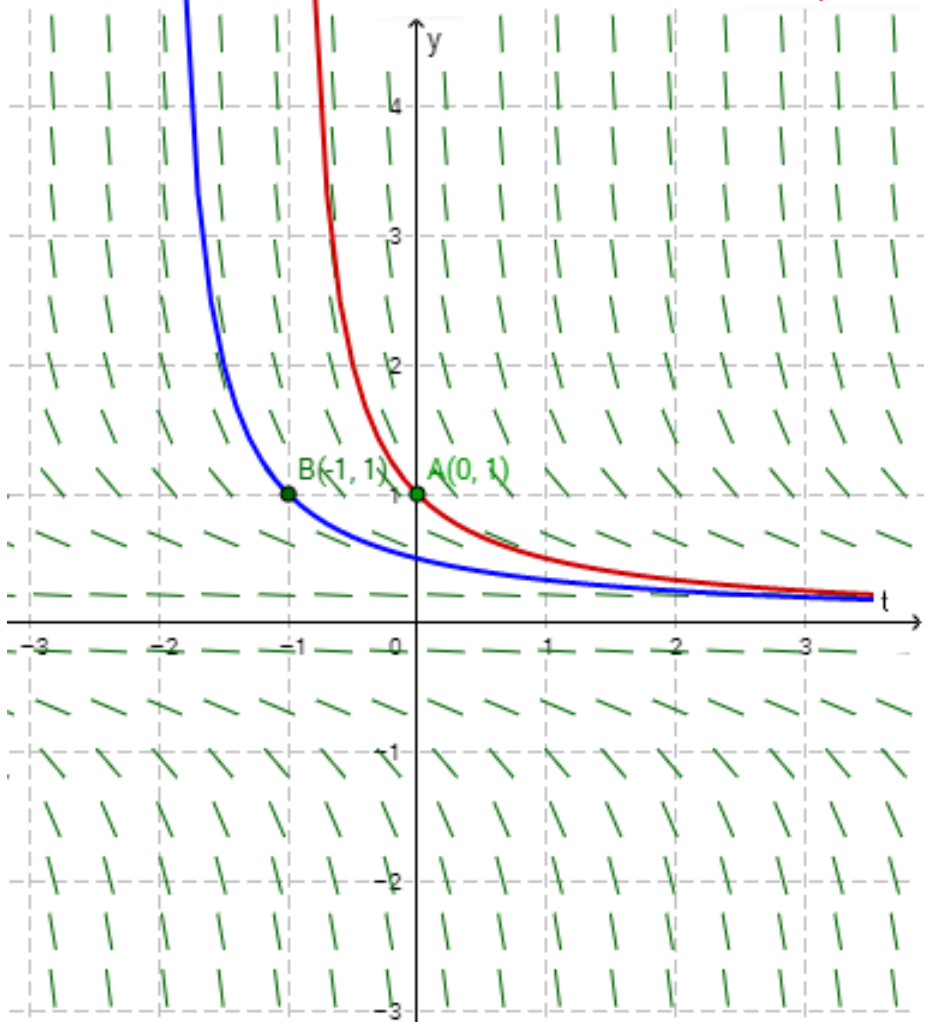
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(t) = \frac{1}{t + c}$$

$$y(1) = -1 \Rightarrow y(t) = \frac{1}{t + 2}$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t + 1}$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1

$$y_1 = y_0 + 0.5 f\left(t_0 + \frac{0.5}{2}, y_0 + \frac{0.5}{2} f(t_0, y_0)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

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$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1; t_0=0 $\Rightarrow$

$$y_1 = y_0 + 0.5 f\left(t_0 + \frac{0.5}{2}, y_0 + \frac{0.5}{2} f(t_0, y_0)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

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y_0=1; t_0=0 $\Rightarrow$

$$y_1 = 1 + 0.5 f\left(0 + \frac{0.5}{2}, 1 + \frac{0.5}{2} f(0,1)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$f(0,1)$$

$$y_1 = 1 + 0.5 f\left(0 + \frac{0.5}{2}, 1 + \frac{0.5}{2} f(0,1)\right)$$

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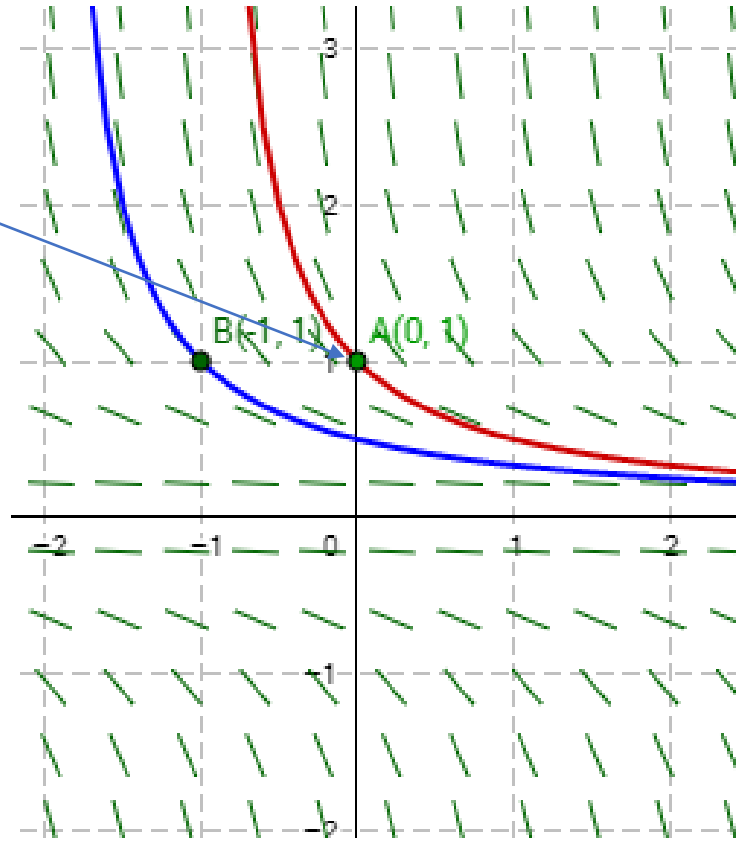
$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0,1)$$

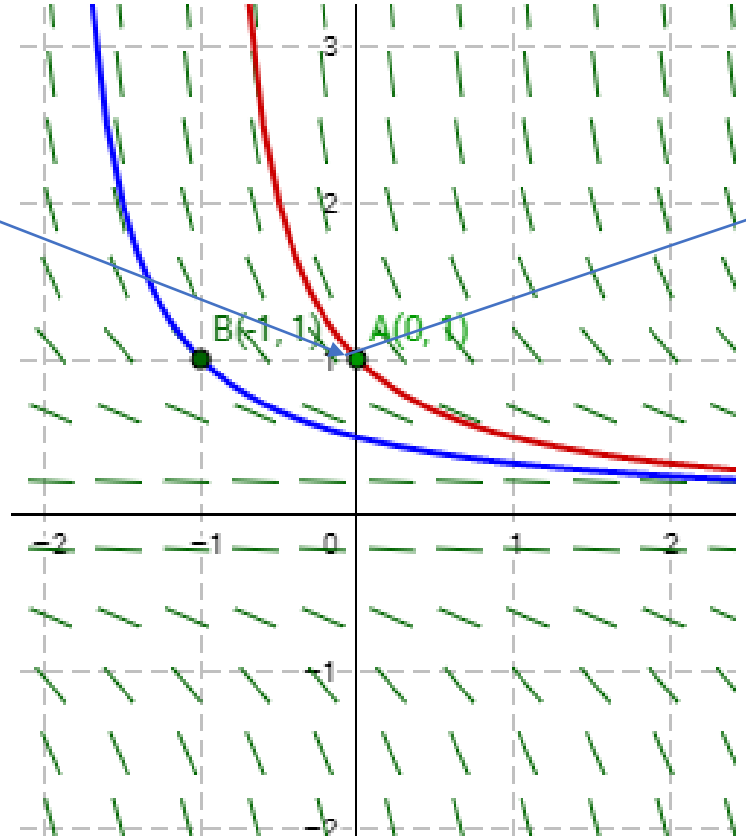


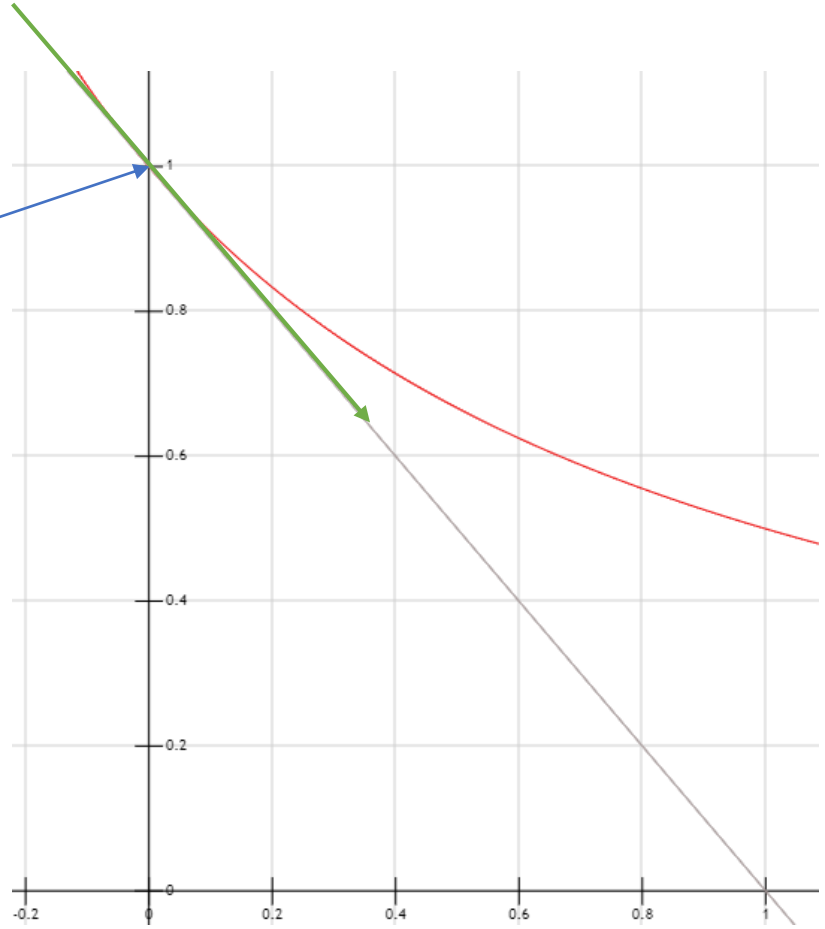
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0,1)$$



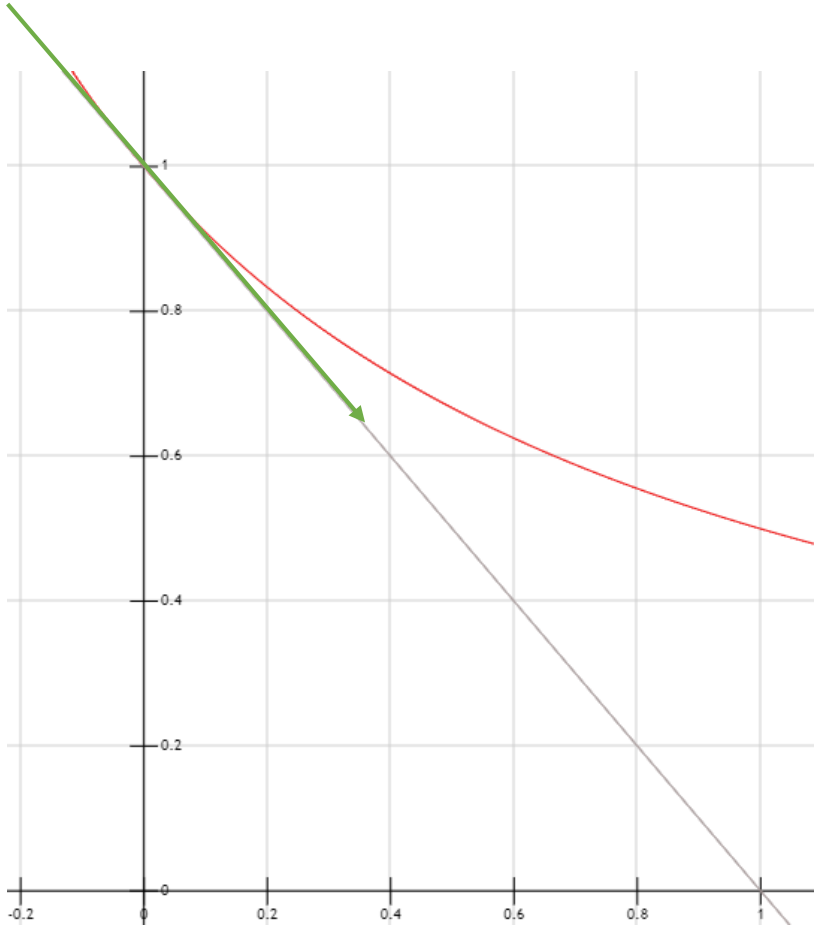


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0,1) = -1^2 = -1$$



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$$y'(t) = f(t, y) = -y^2$$

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explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

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$$y'(t) = f(t, y) = -y^2$$

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y_0=1; t_0=0 $\Rightarrow$

$$y_1 = 1 + 0.5 f\left(0 + \frac{0.5}{2}, 1 - \frac{0.5}{2}\right)$$

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y_0=1; t_0=0 $\Rightarrow$

$$y_1 = 1 + 0.5 f(0.25, 0.75)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$f(0.25, 0.75)$$

$$y_1 = 1 + 0.5 f(0.25, 0.75)$$

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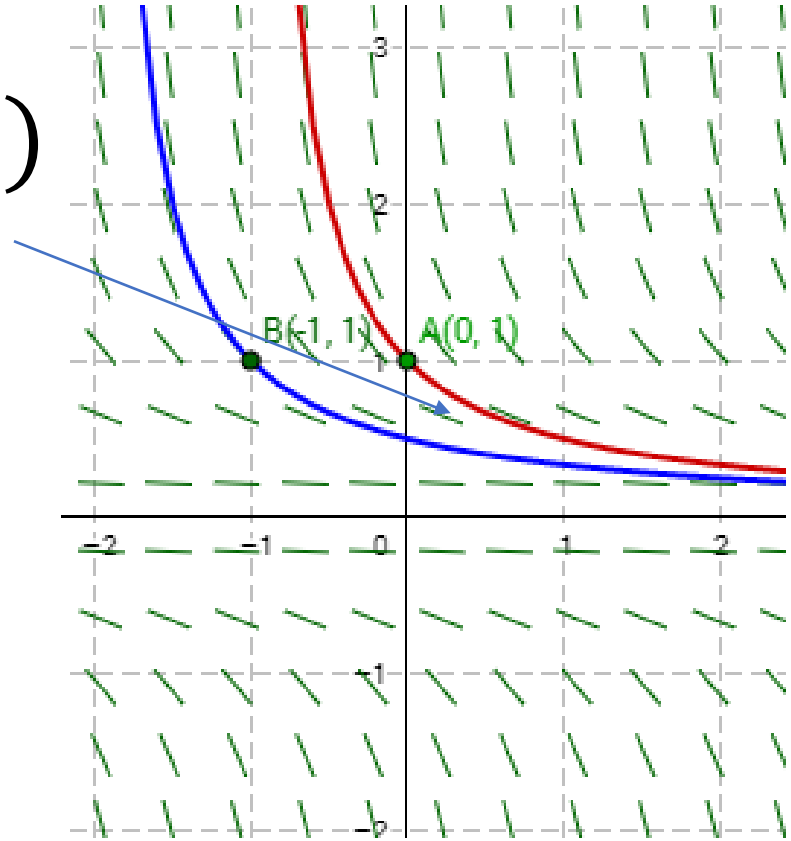
$$f(0.25, 0.75)$$



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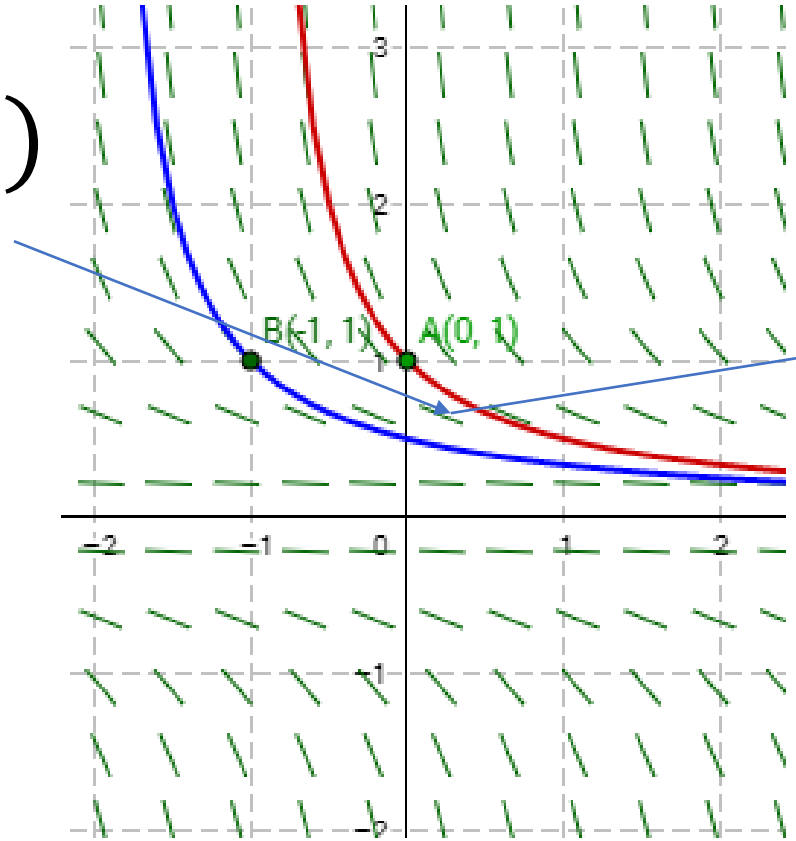
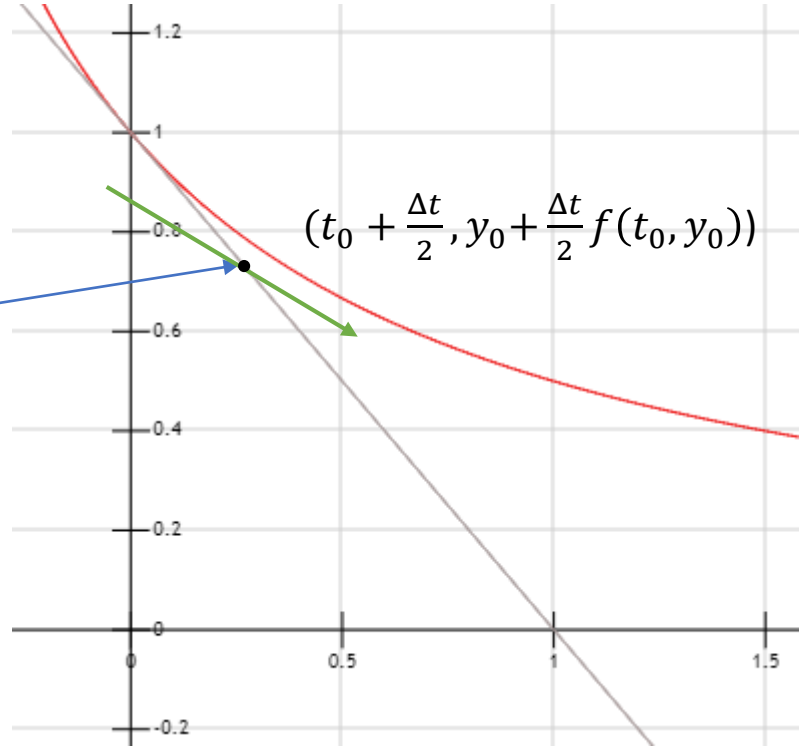
$$f(0.25, 0.75)$$



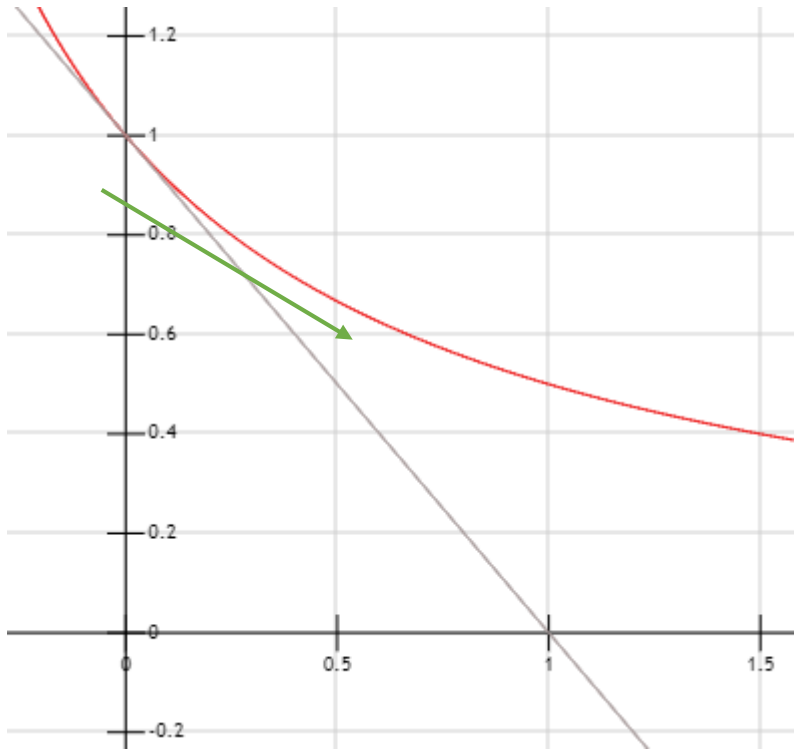


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.75^2 = -0.5625$$

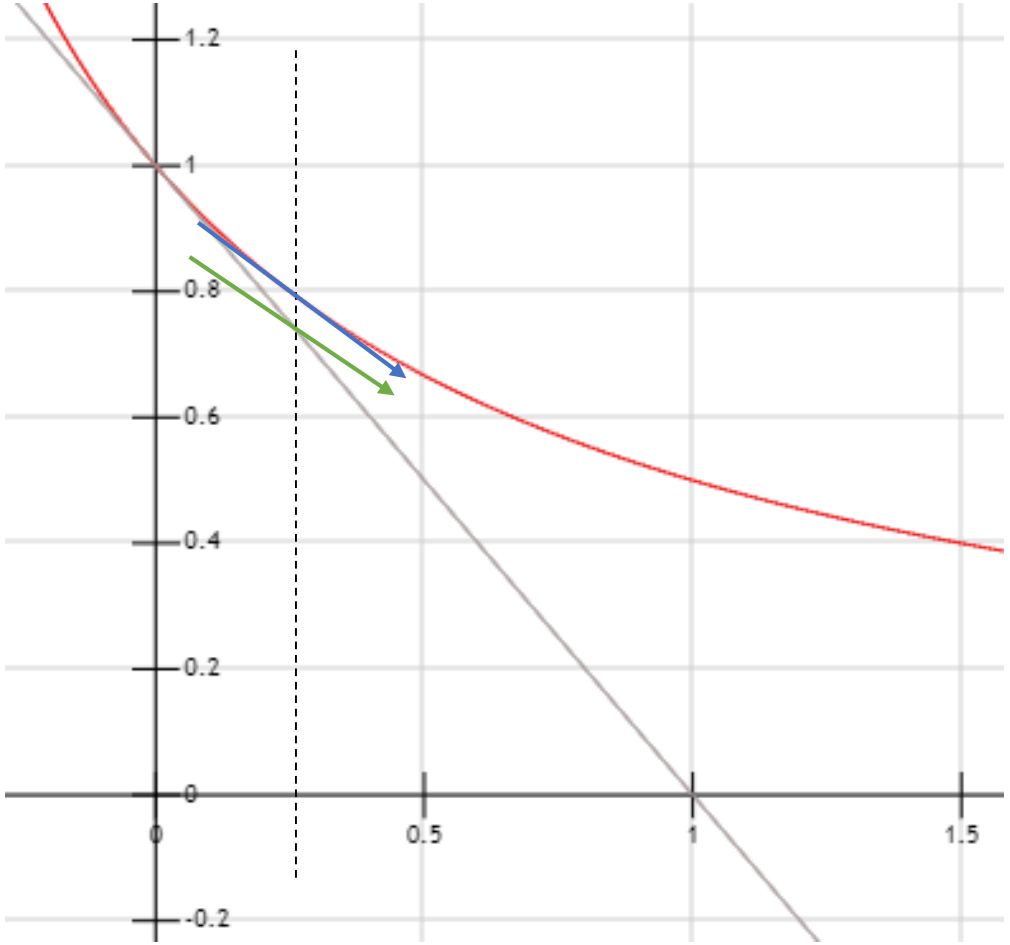


explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 1 + 0.5 f(0.25, 0.75)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 1 - \frac{0.5625}{2}$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75) = -0.5625$$

$$y_1 = 0.71875$$

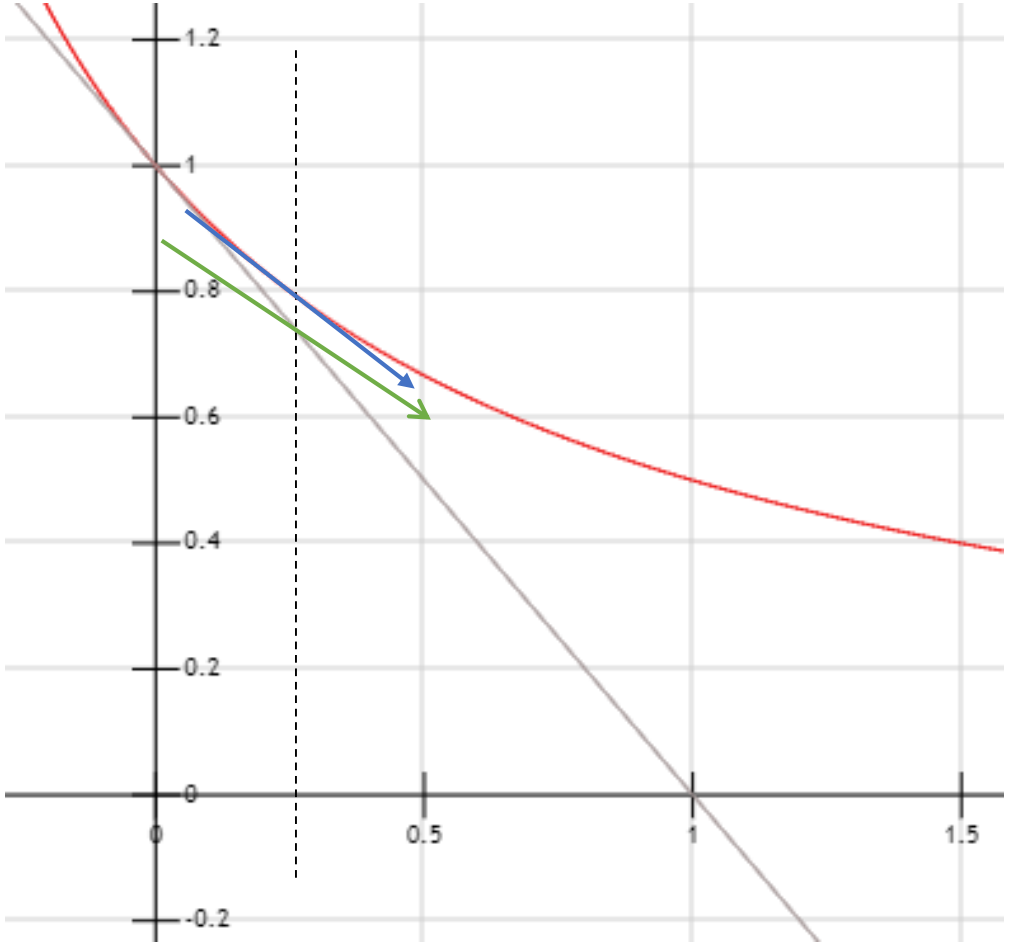
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



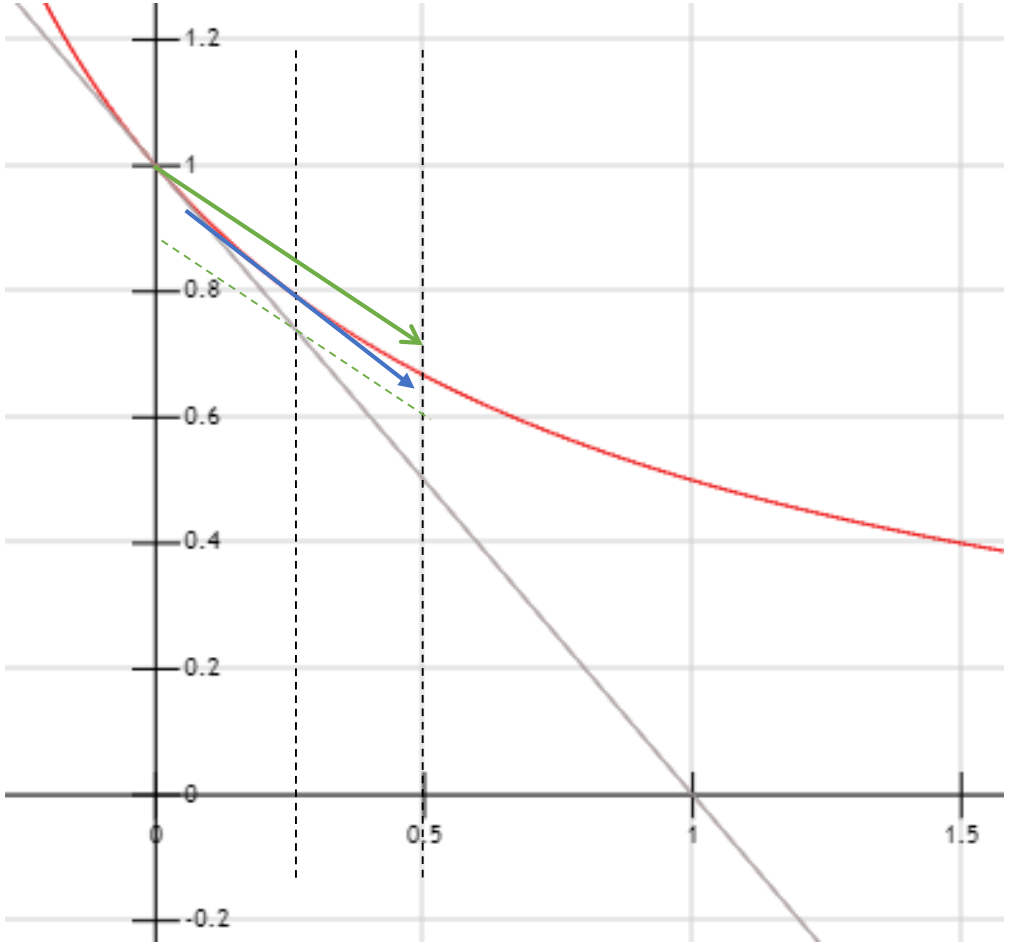
explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



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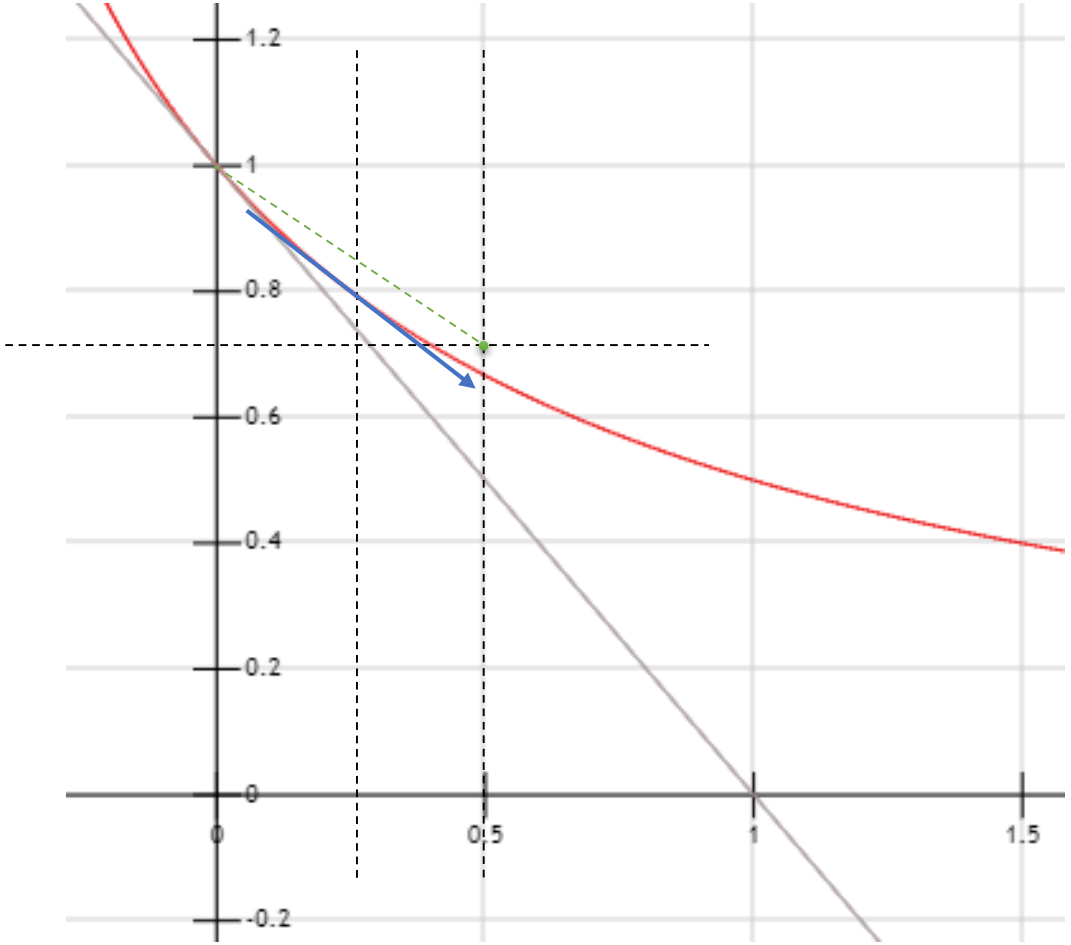
$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$f(0.25, 0.75)$$

$$y_1 = 0.71875$$



explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1; t_0=0; y_1= 0.71875; t_1= t_0 + $\Delta t = \frac{1}{2}$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1; t_0=0; y_1= 0.71875; $t_1 = \frac{1}{2}$

$$y_2 = y_1 + 0.5 f\left(t_1 + \frac{0.5}{2}, y_1 + \frac{0.5}{2} f(t_1, y_1)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1 \Rightarrow y(t) = \frac{1}{t+1}$$

$$\Delta t = \frac{1}{2}$$

$$y_{n+1} = y_n + \Delta t f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} f(t_n, y_n)\right)$$

y_0=1; t_0=0; y_1= 0.71875; $t_1 = \frac{1}{2}$

$$y_2 = 0.71875 + 0.5 f\left(\frac{1}{2} + \frac{0.5}{2}, 0.71875 + \frac{0.5}{2} f\left(\frac{1}{2}, 0.71875\right)\right)$$

explicit midpoint method

$$y'(t) = f(t, y) = -y^2$$

$$y(0) = 1$$

$$\Delta t = \frac{1}{2}$$

$$y_2 = 0.71875 + 0.5 f\left(\frac{1}{2} + \frac{0.5}{2}, 0.71875 + \frac{0.5}{2} f\left(\frac{1}{2}, 0.71875\right)\right)$$