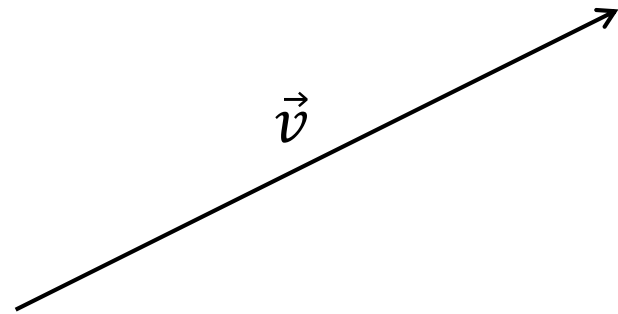


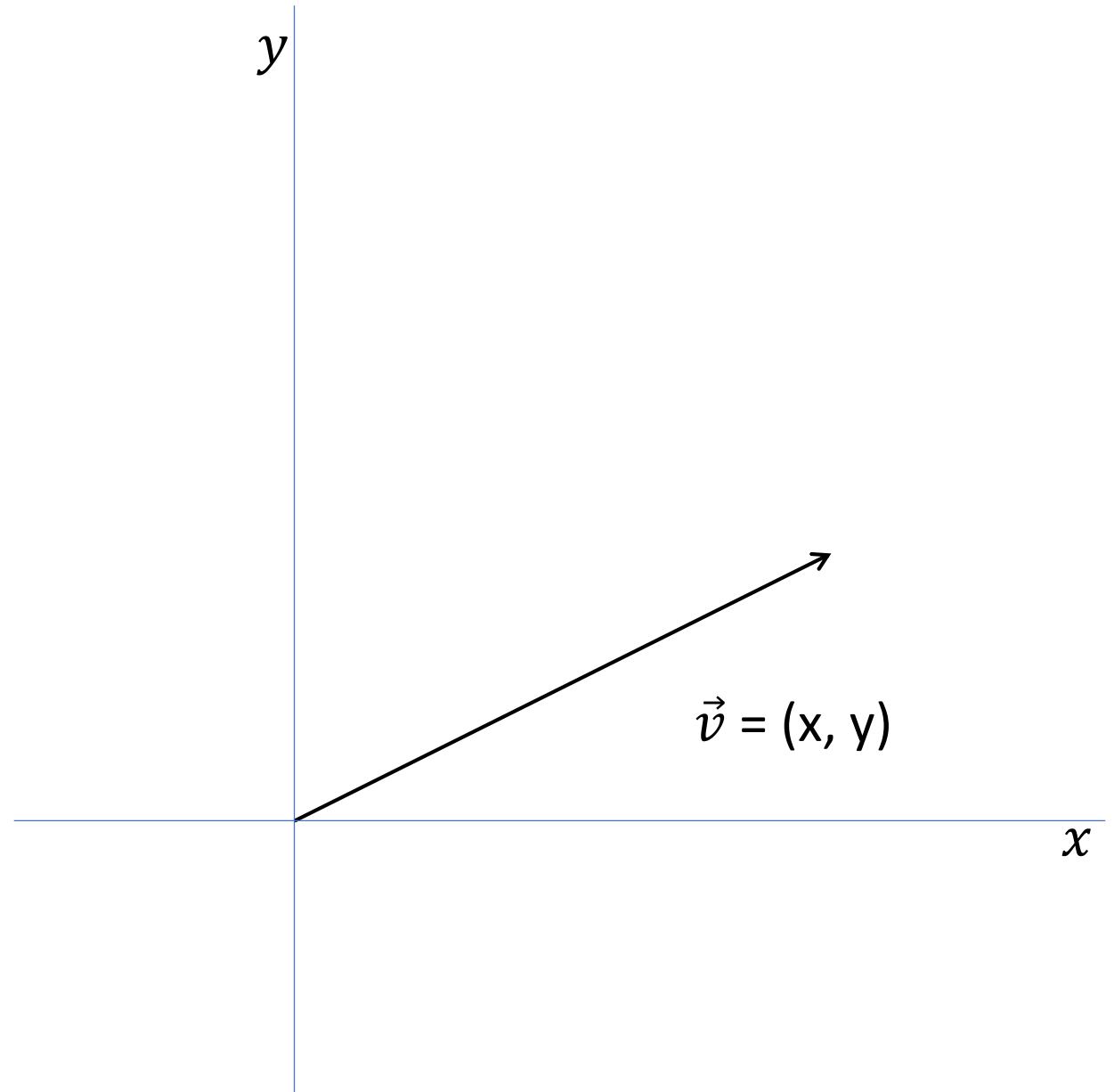
Euclidean vector

- direction
- magnitude / length



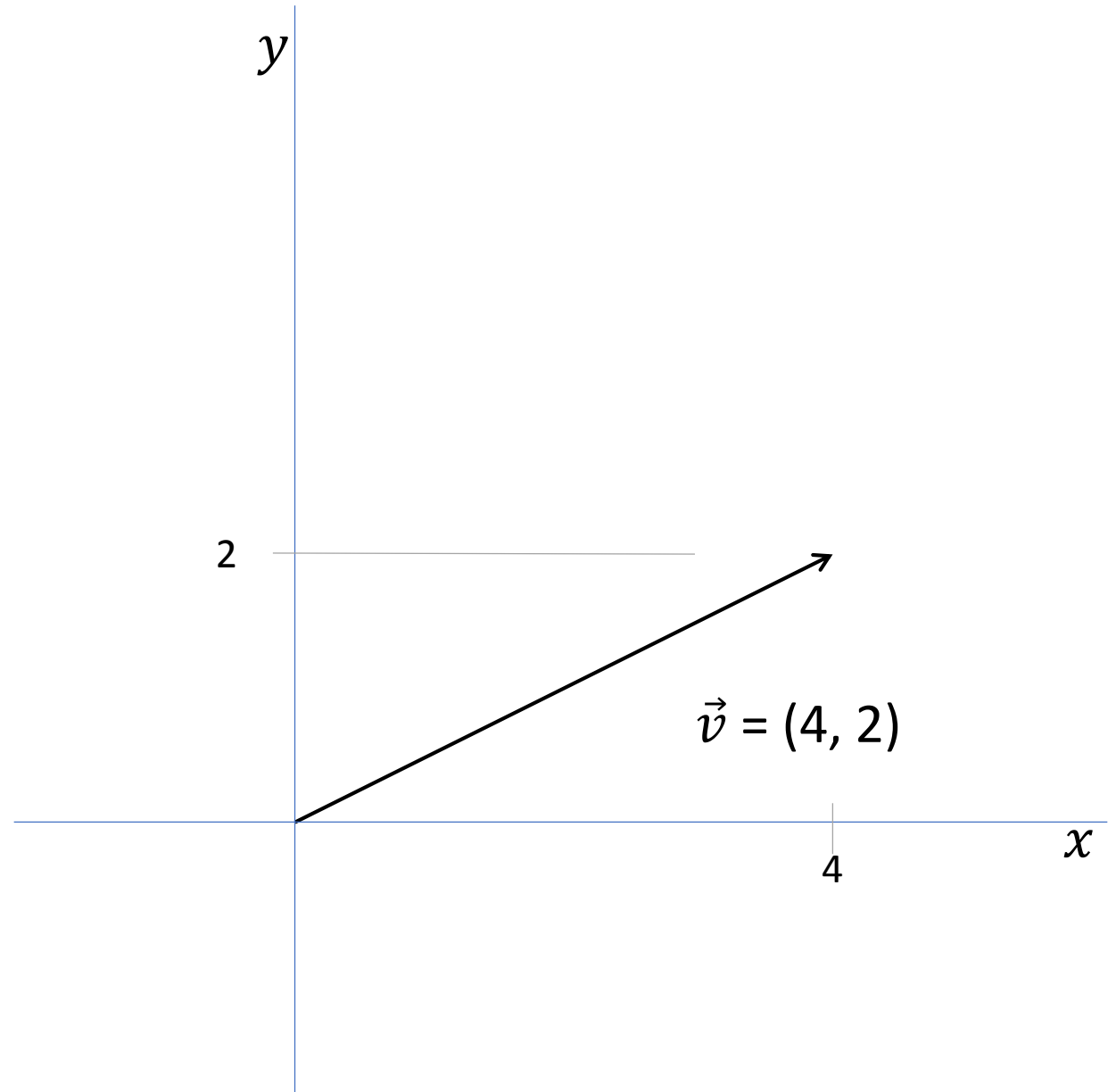
Euclidean vector

- direction
- magnitude / length



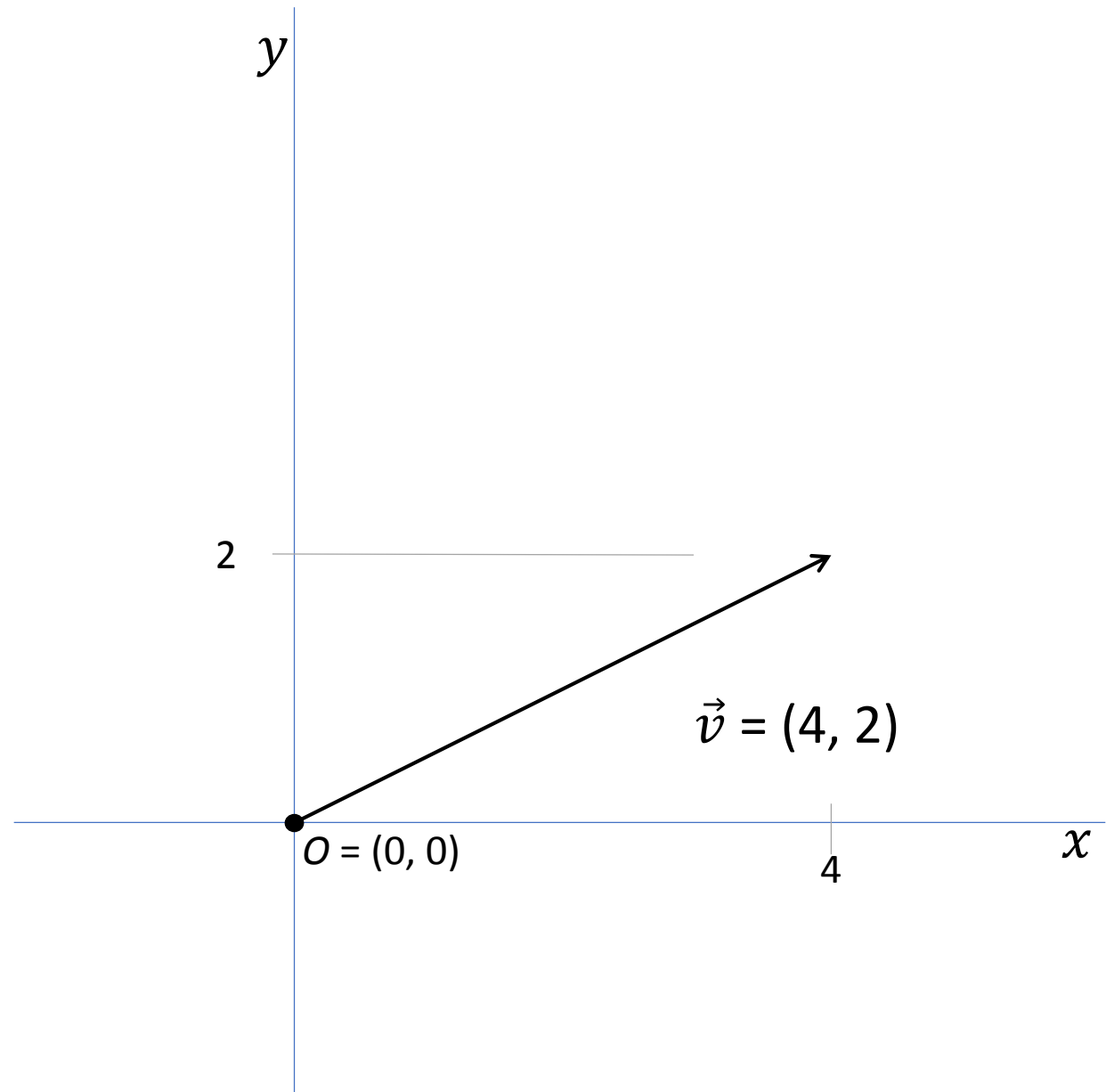
Euclidean vector

- direction
- magnitude / length



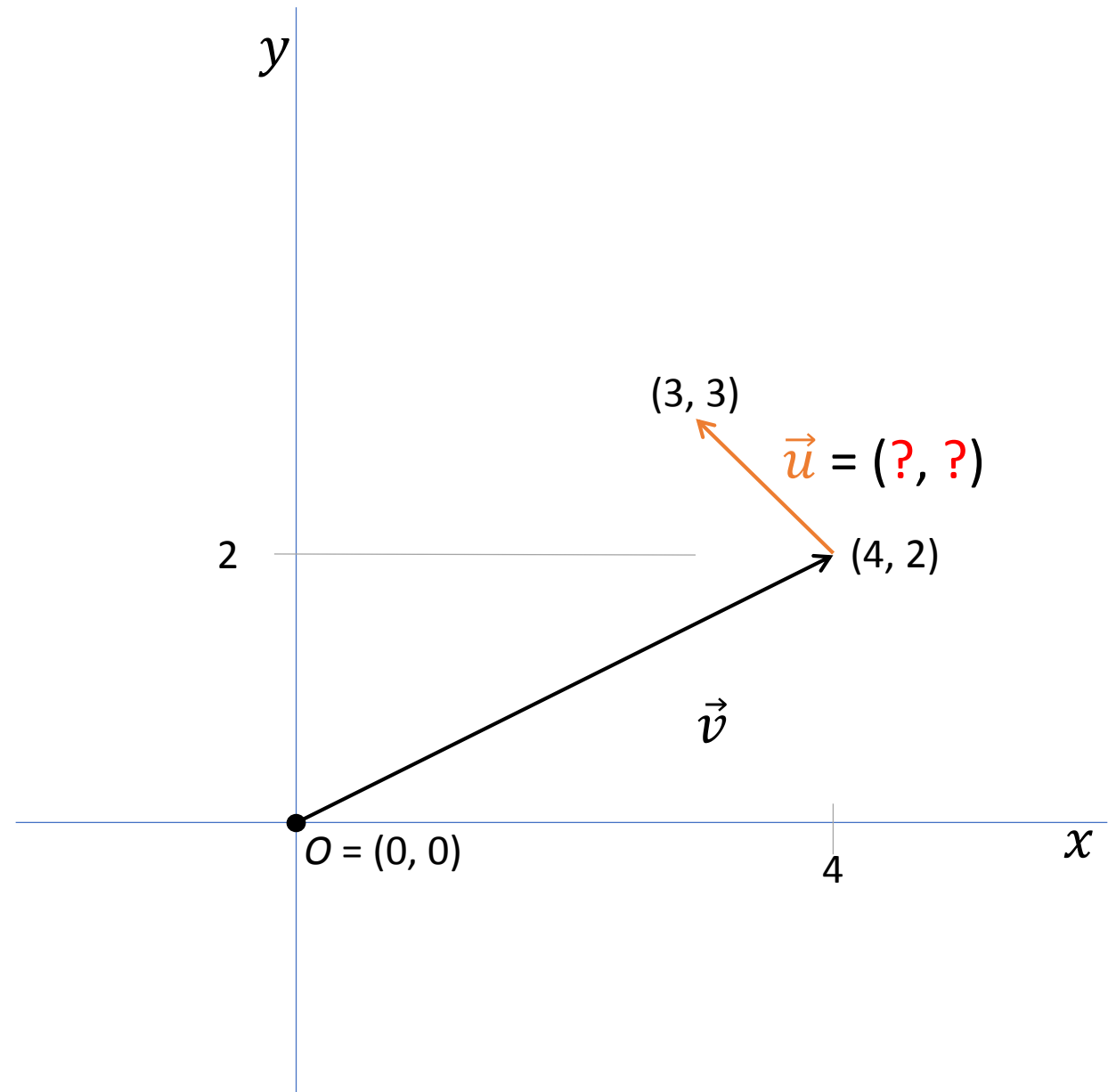
Euclidean vector

- direction
- magnitude / length



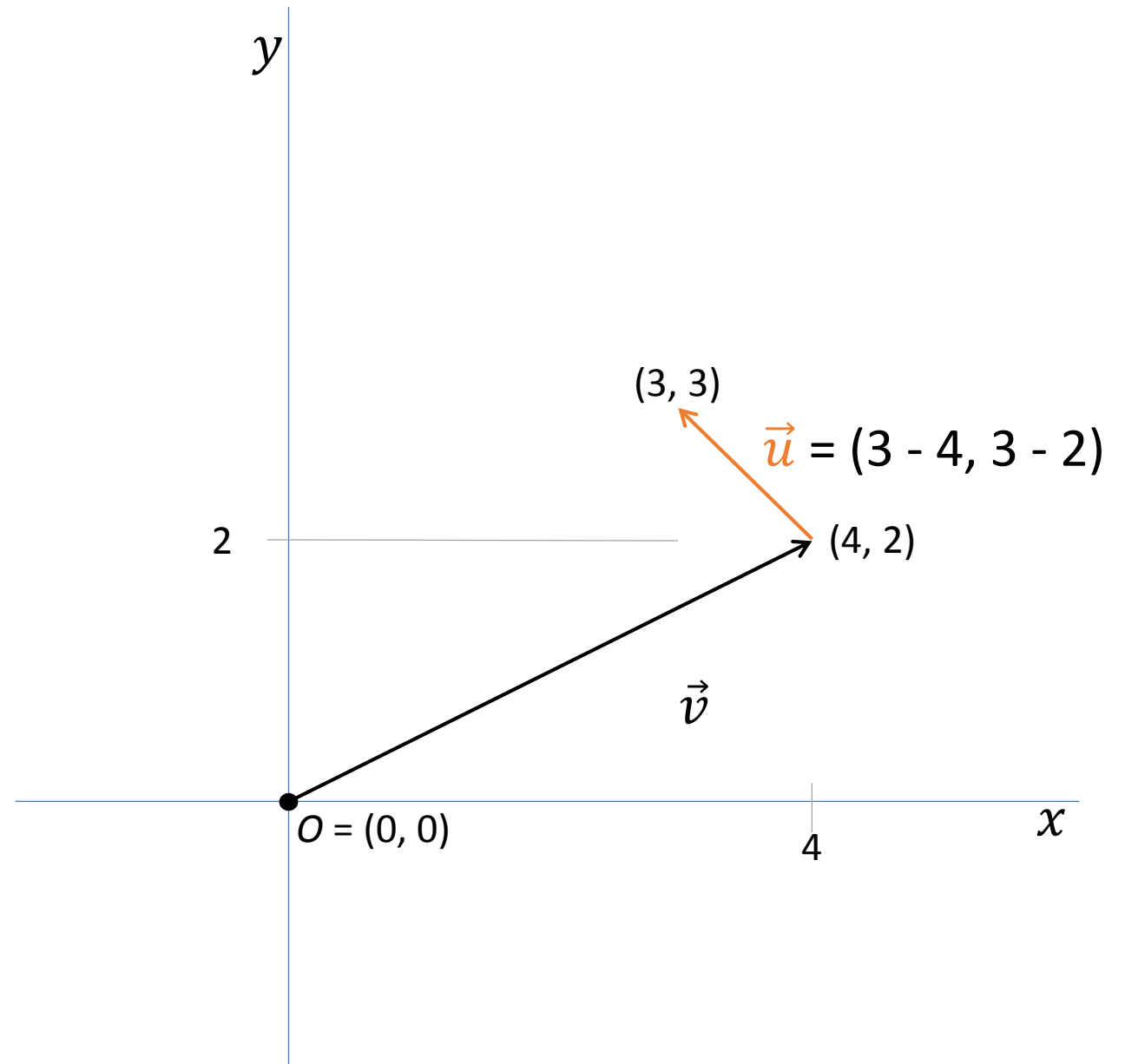
Euclidean vector

- direction
- magnitude / length



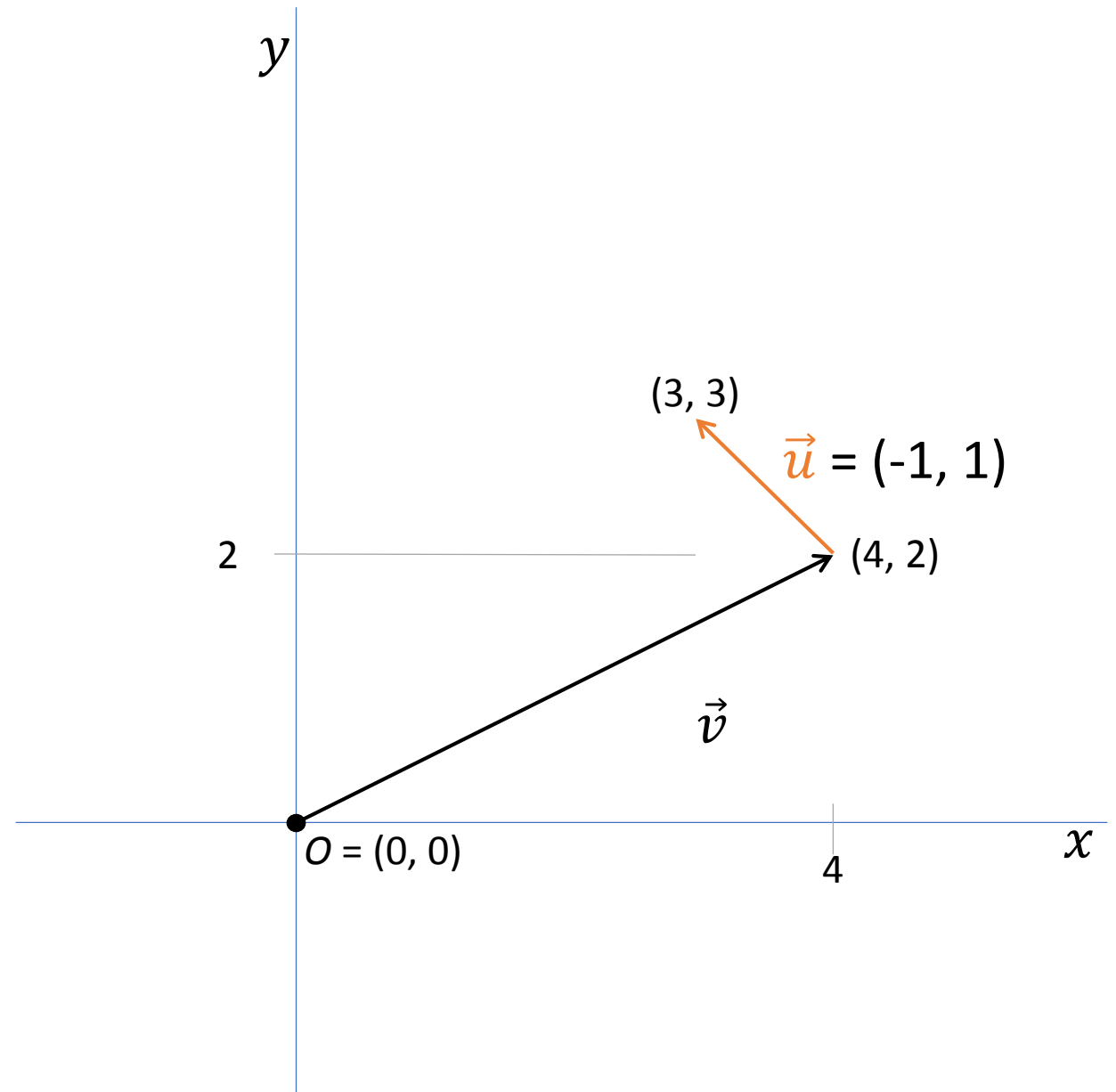
Euclidean vector

- direction
- magnitude / length



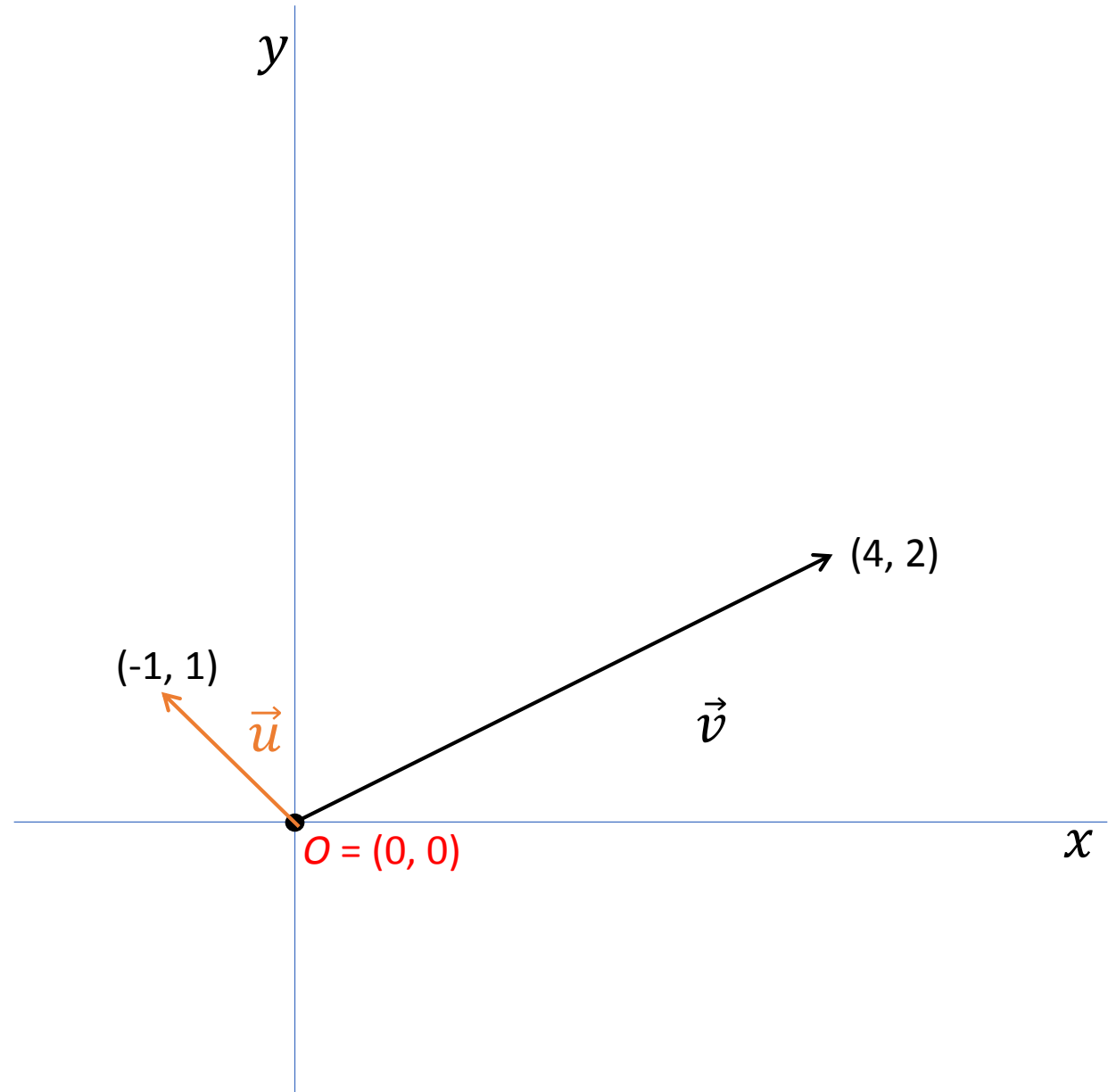
Euclidean vector

- direction
- magnitude / length



Euclidean vector

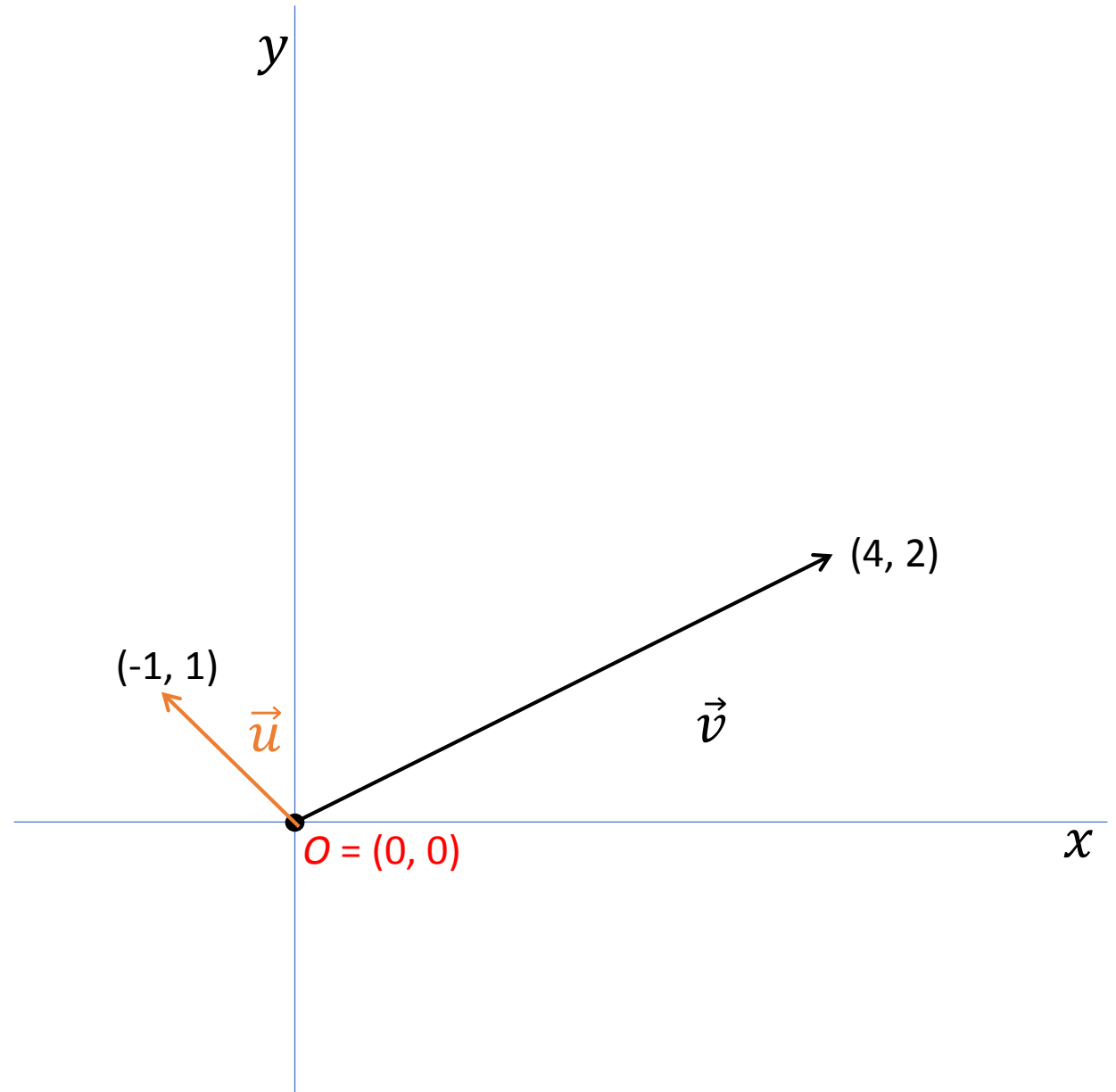
- direction
- magnitude / length



Euclidean vector

- direction
- magnitude / length

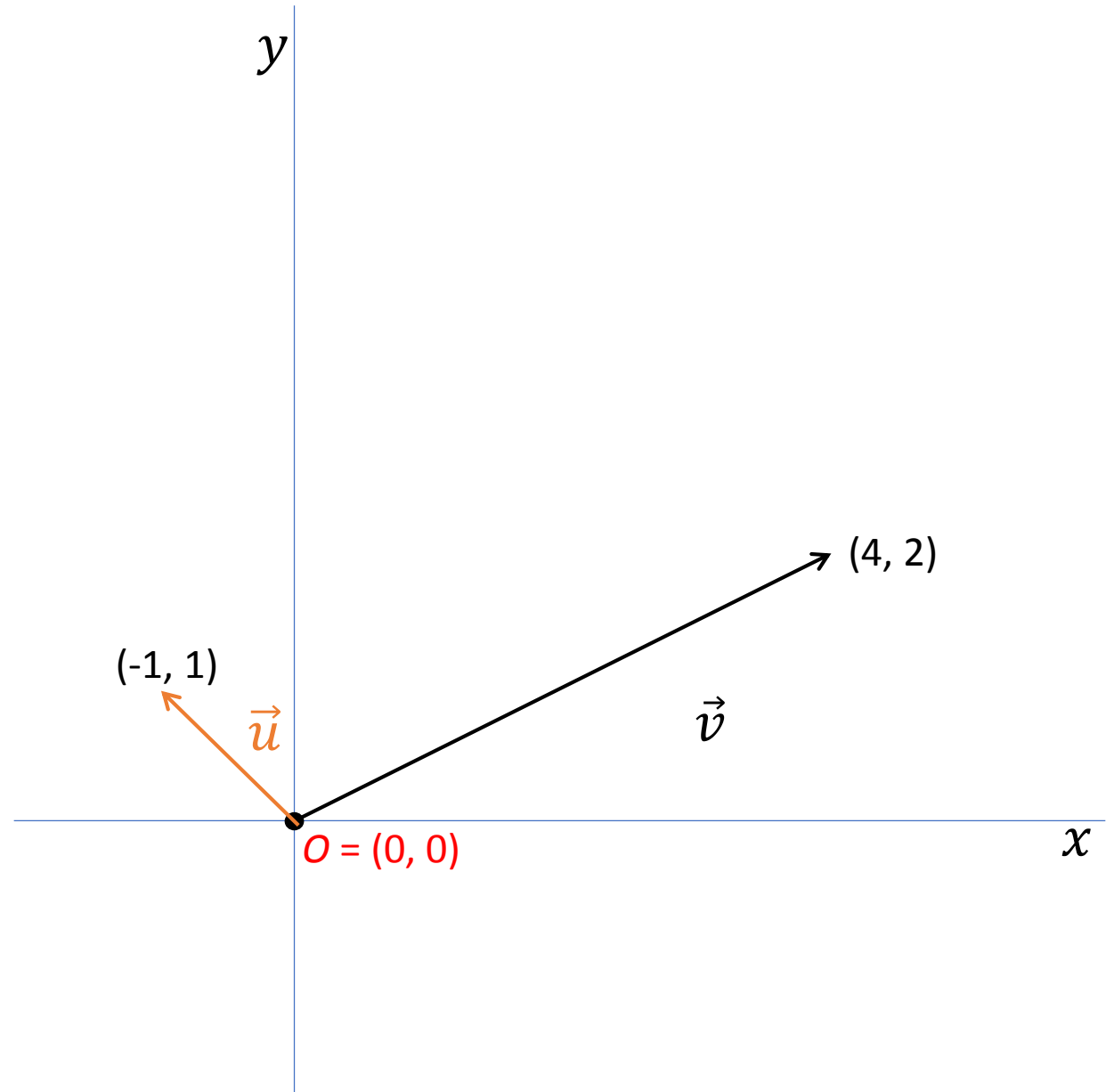
$$\vec{w} = \vec{v} + \vec{u}$$



Euclidean vector

- direction
- magnitude / length

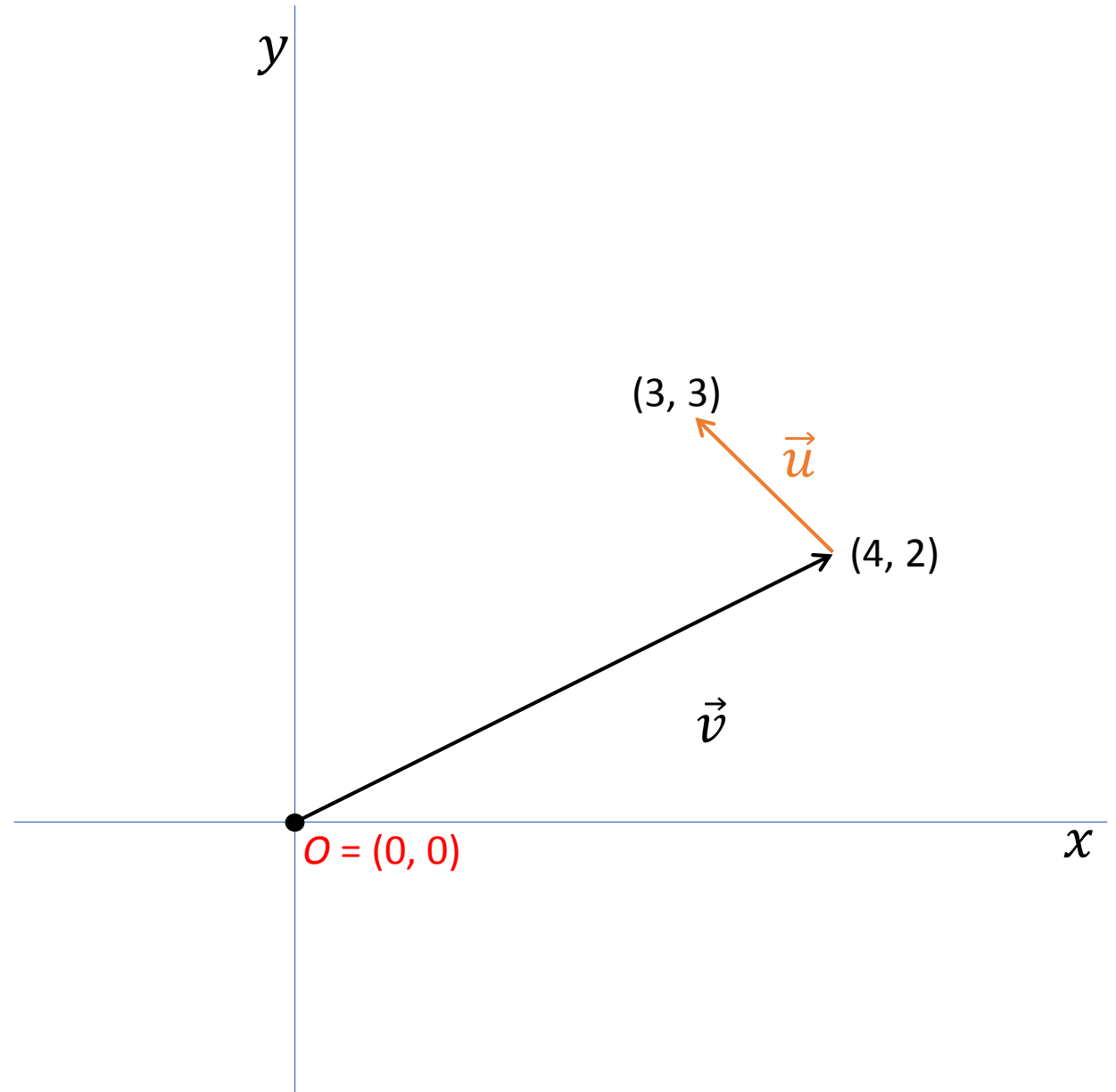
$$\vec{w} = (4, 2) + (-1, 1)$$



Euclidean vector

- direction
- magnitude / length

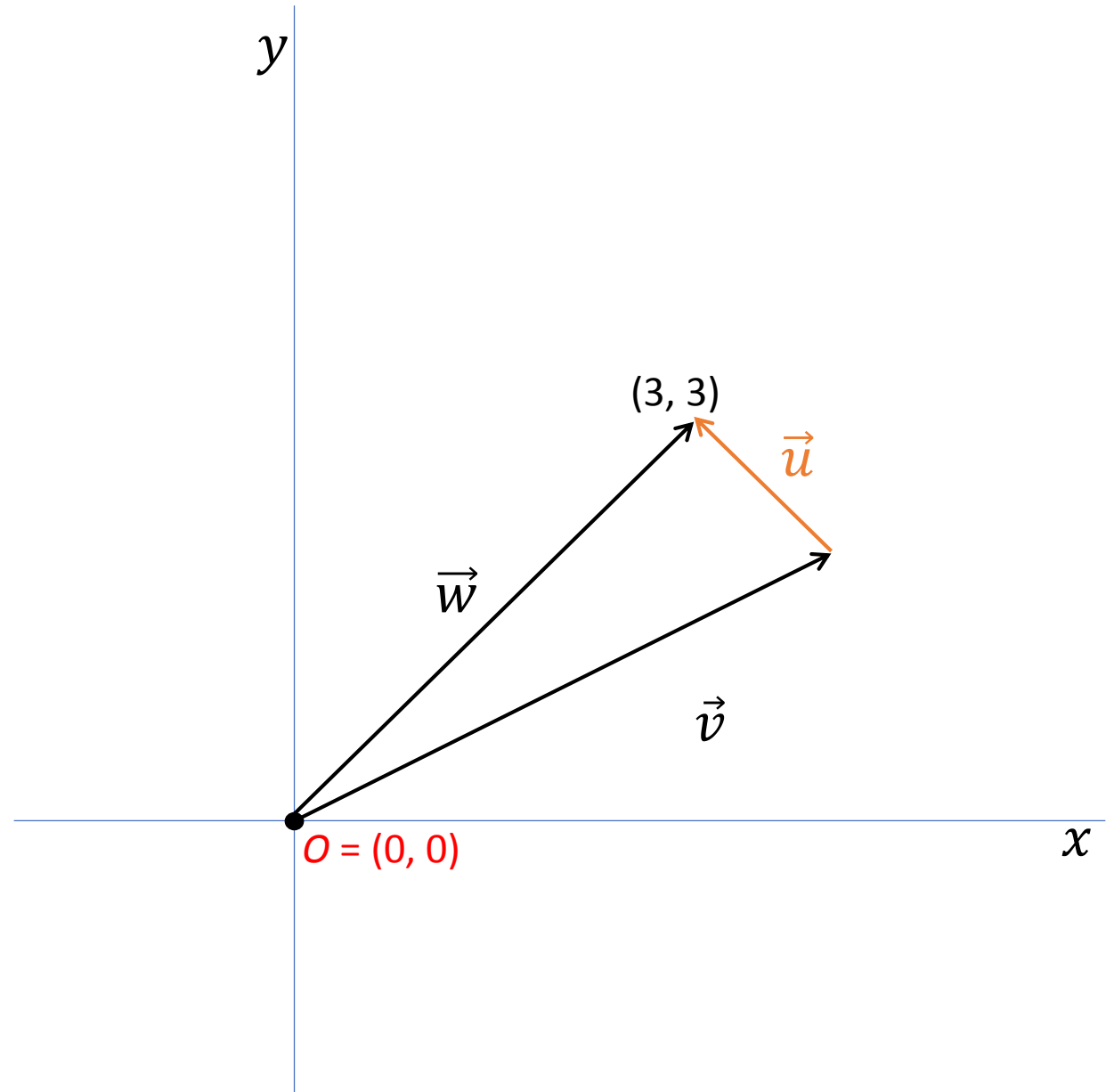
$$\vec{w} = (4 - 1, 2 + 1)$$



Euclidean vector

- direction
- magnitude / length

$$\vec{w} = (3, 3)$$

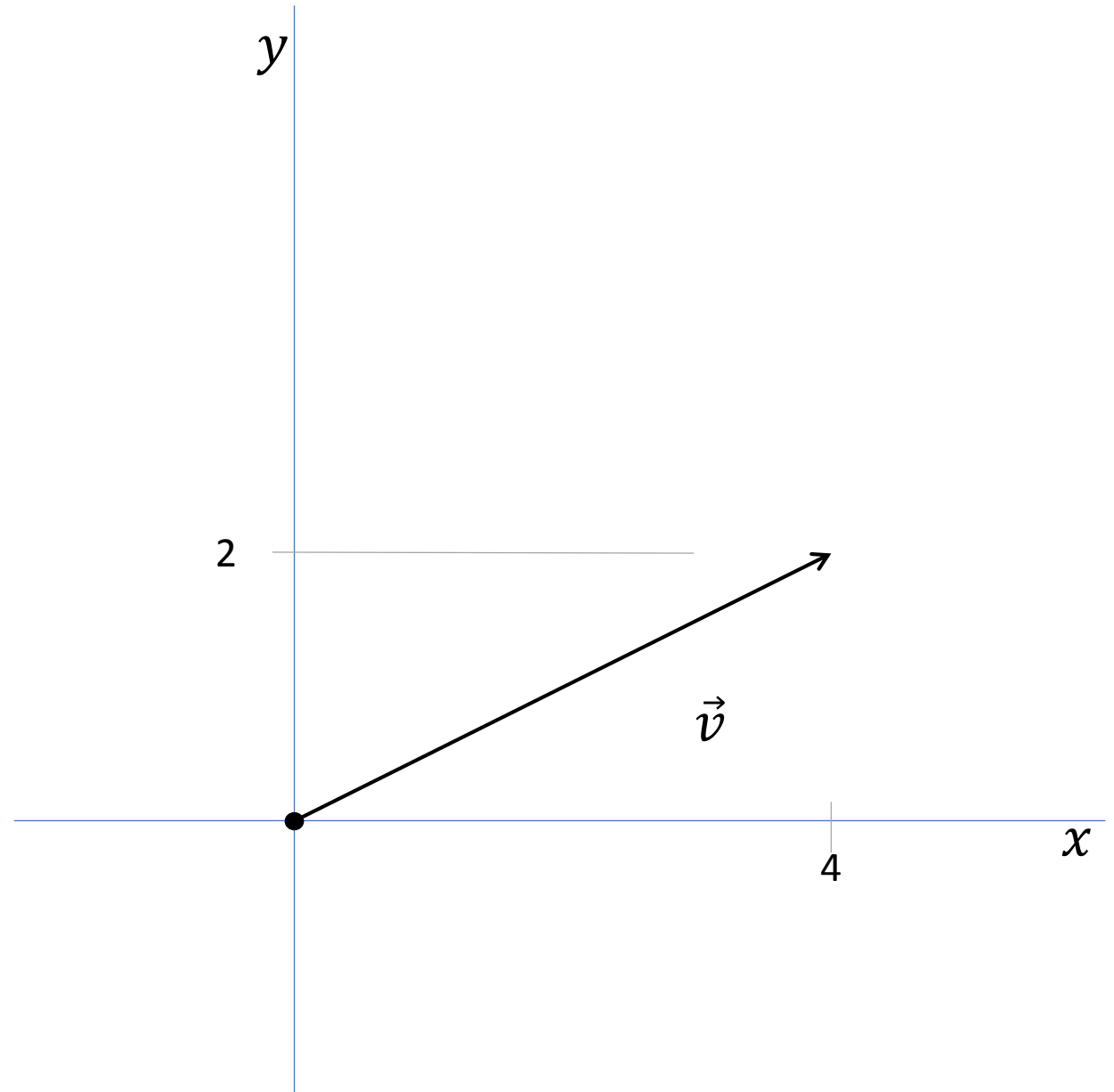


Euclidean vector

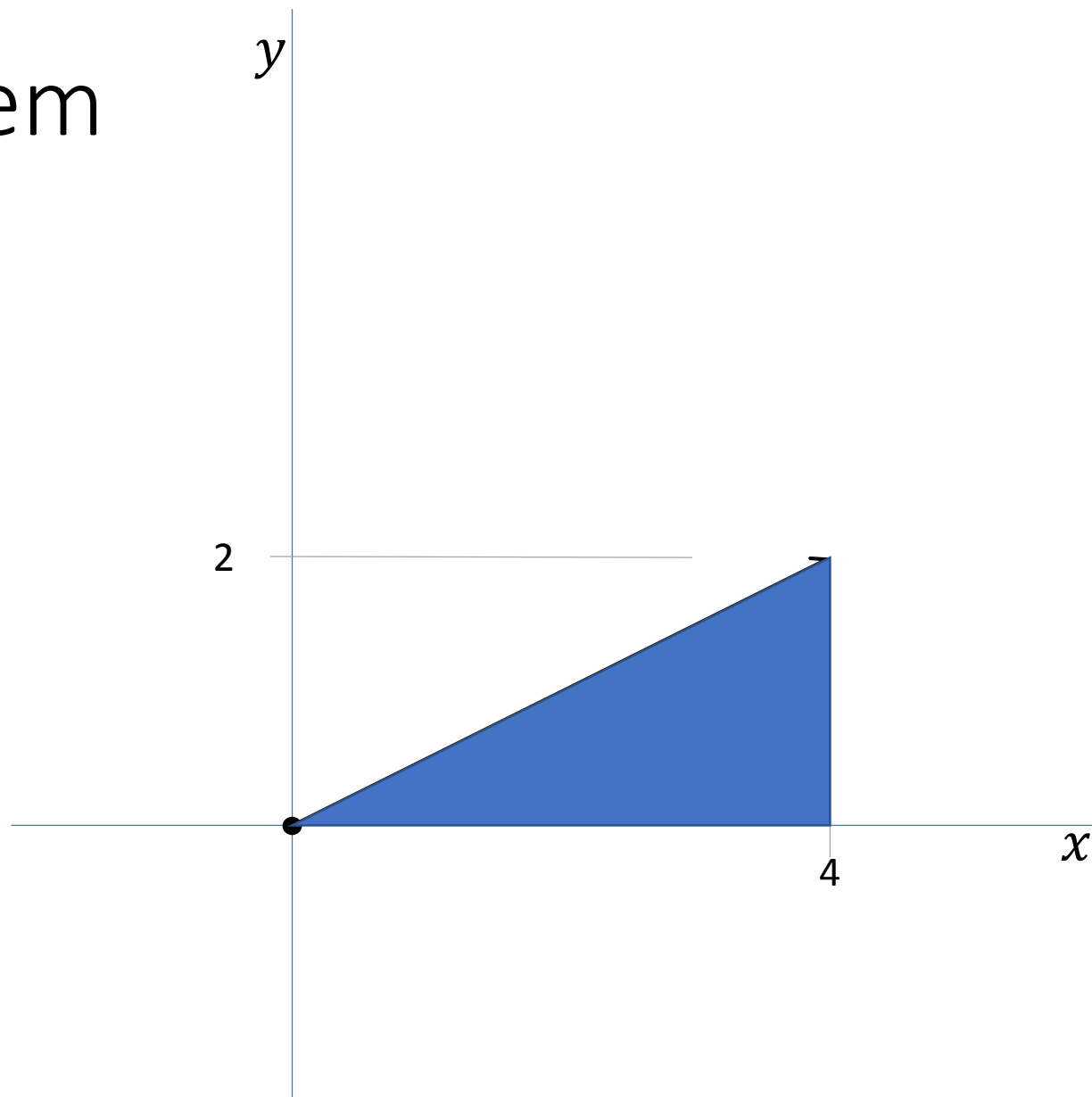
- direction

- magnitude / length

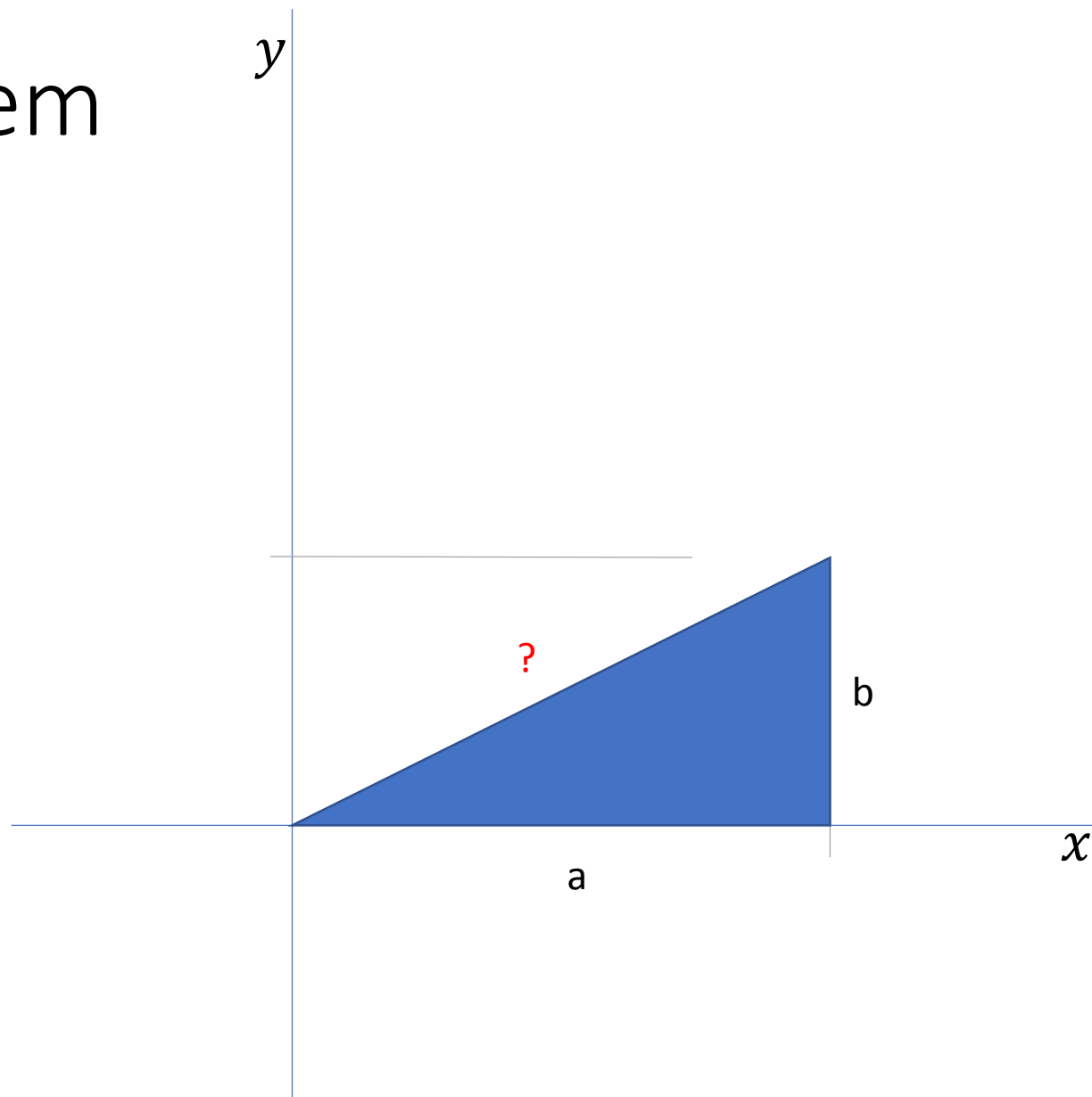
$$|\vec{v}| = ?$$



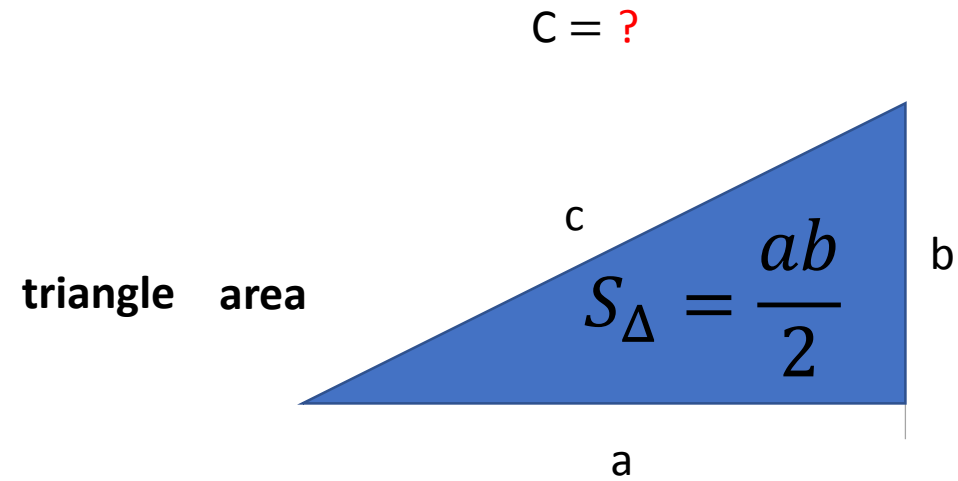
Pythagorean theorem



Pythagorean theorem



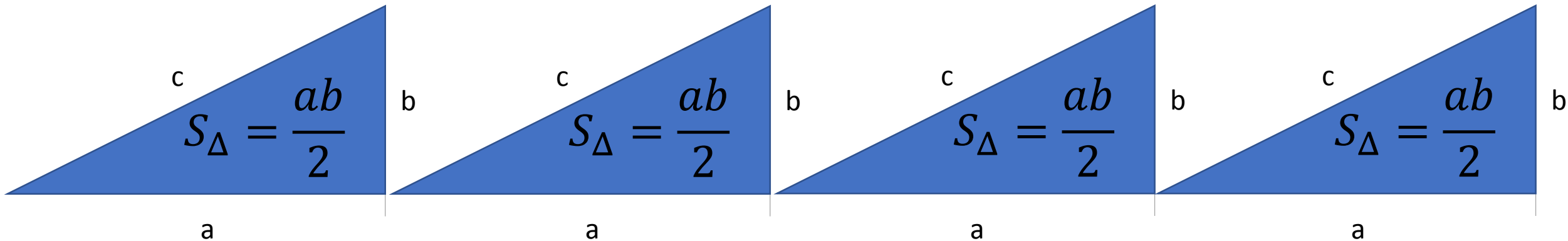
Pythagorean theorem



Pythagorean theorem

$c = ?$

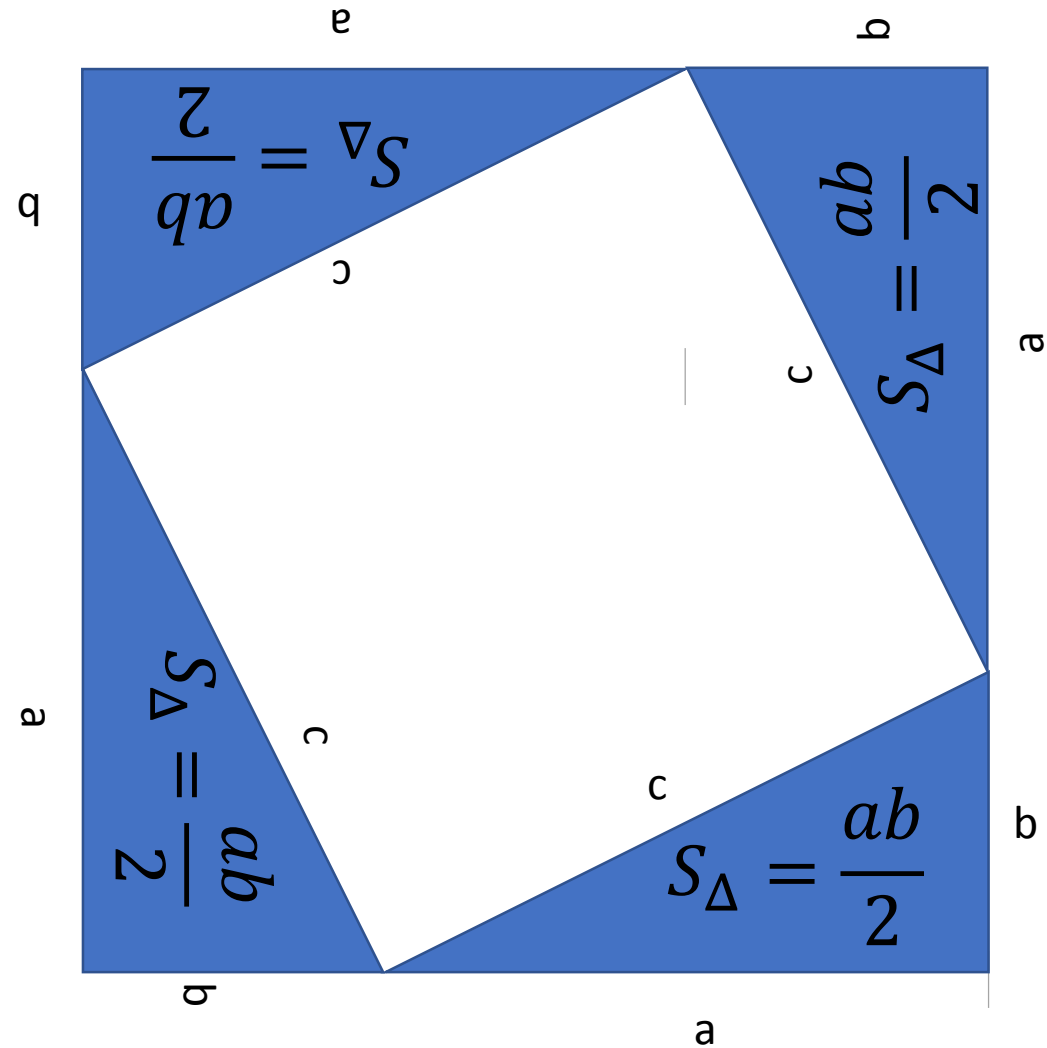
$$S = 4S_{\Delta} = 2ab$$



Pythagorean theorem

$c = ?$

$$S = 4S_{\Delta} = 2ab$$

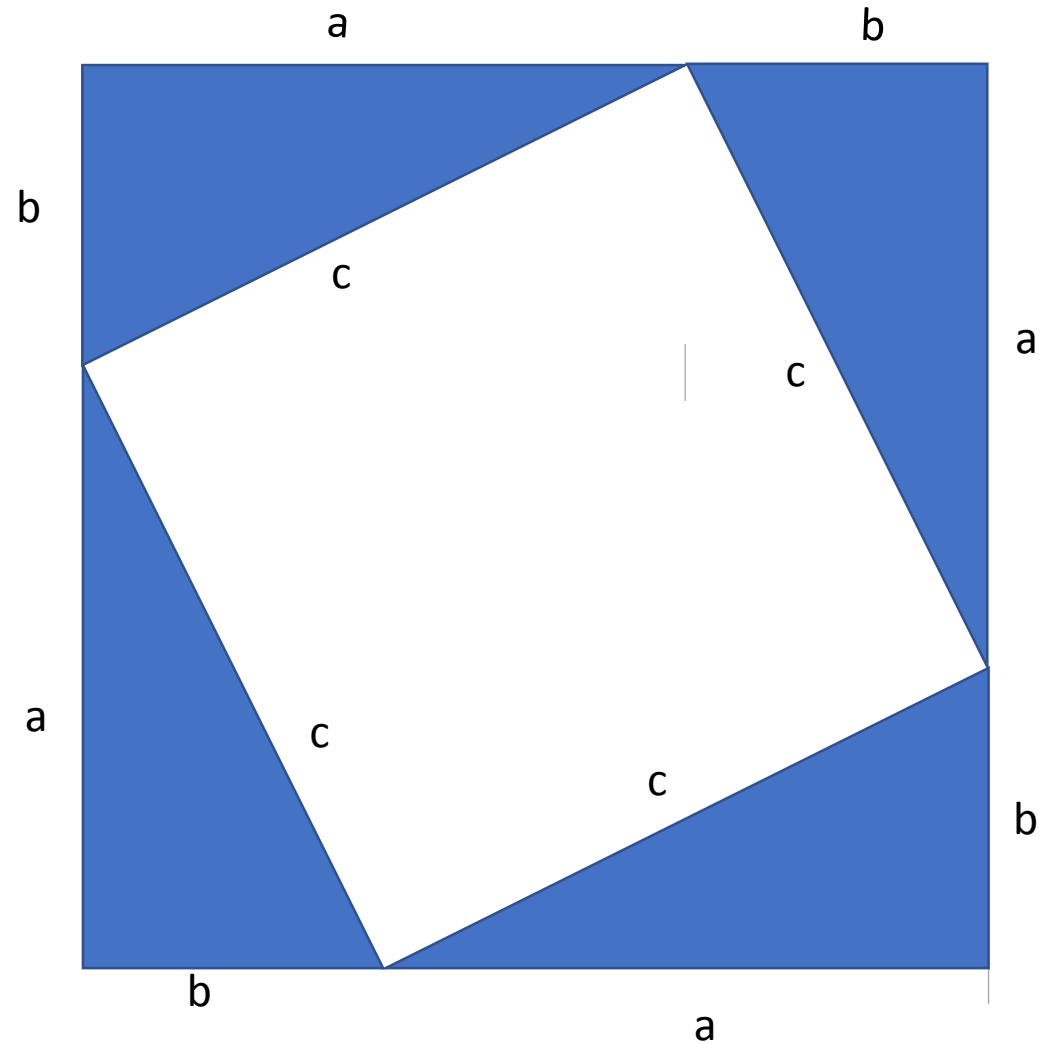


Pythagorean theorem

$c = ?$

$$S = 4S_{\Delta} = 2ab$$

$$S_{\blacksquare} = (a + b)^2$$

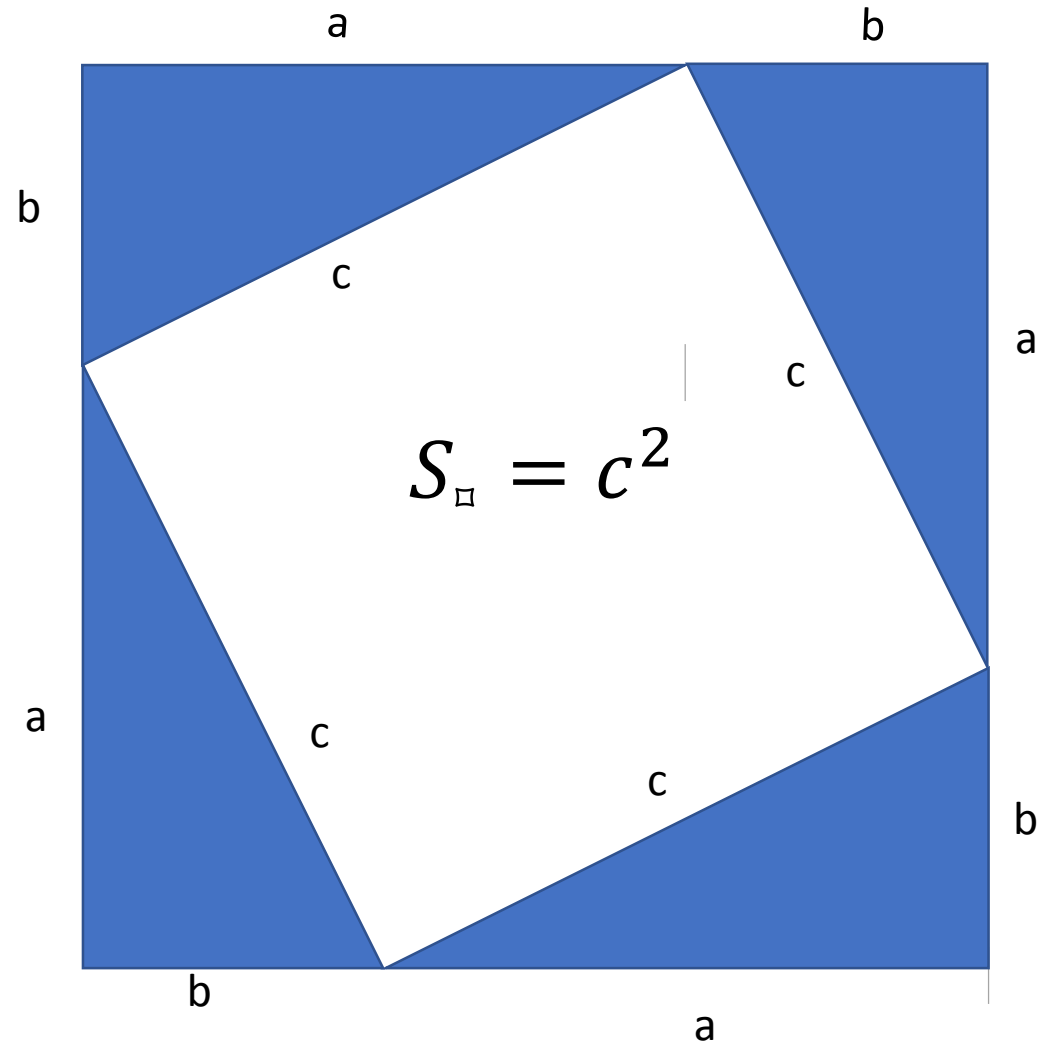


Pythagorean theorem

$c = ?$

$$S = 4S_{\Delta} = 2ab$$

$$S_{\blacksquare} = (a + b)^2$$



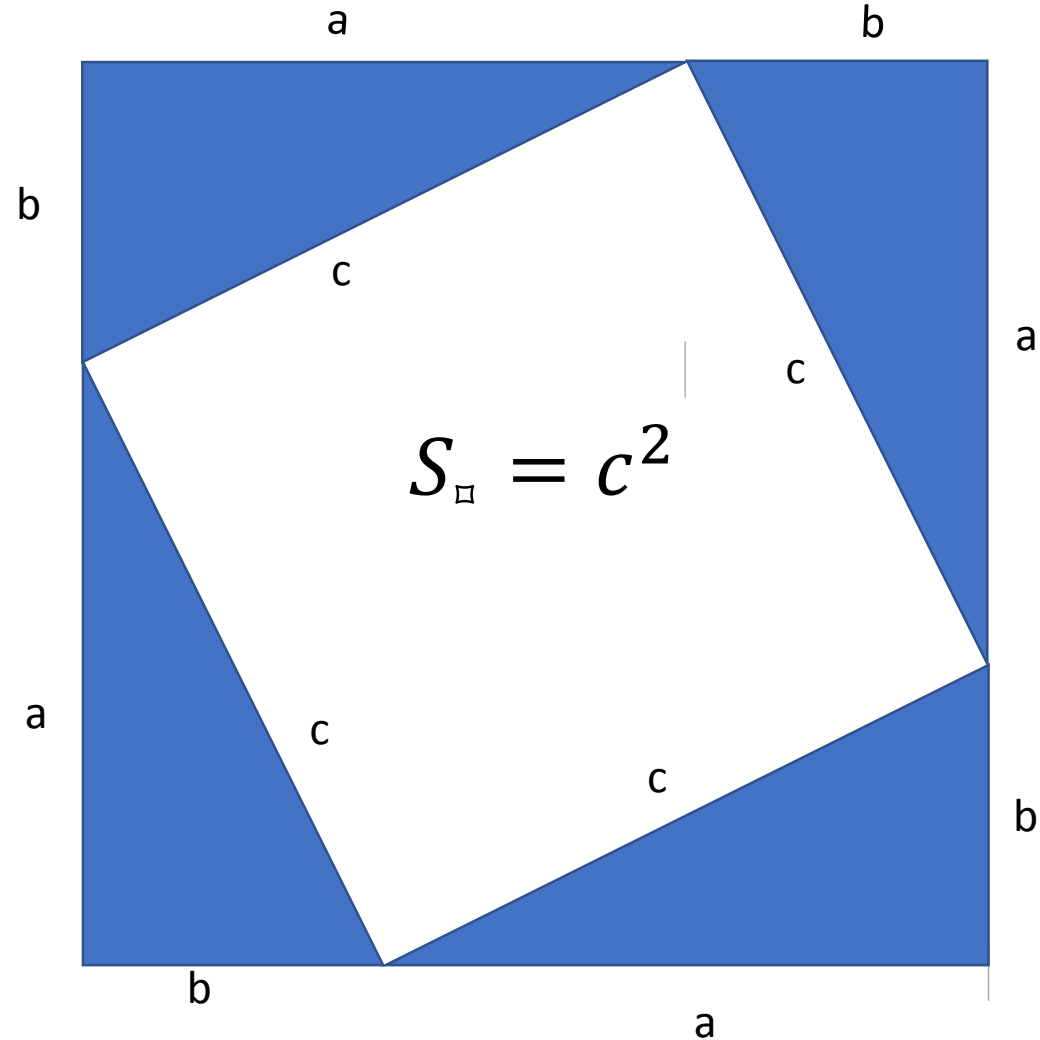
Pythagorean theorem

$c = ?$

$$S = 4S_{\Delta} = 2ab$$

$$S = S_{\blacksquare} - S_{\square}$$

$$S_{\blacksquare} = (a + b)^2$$



Pythagorean theorem

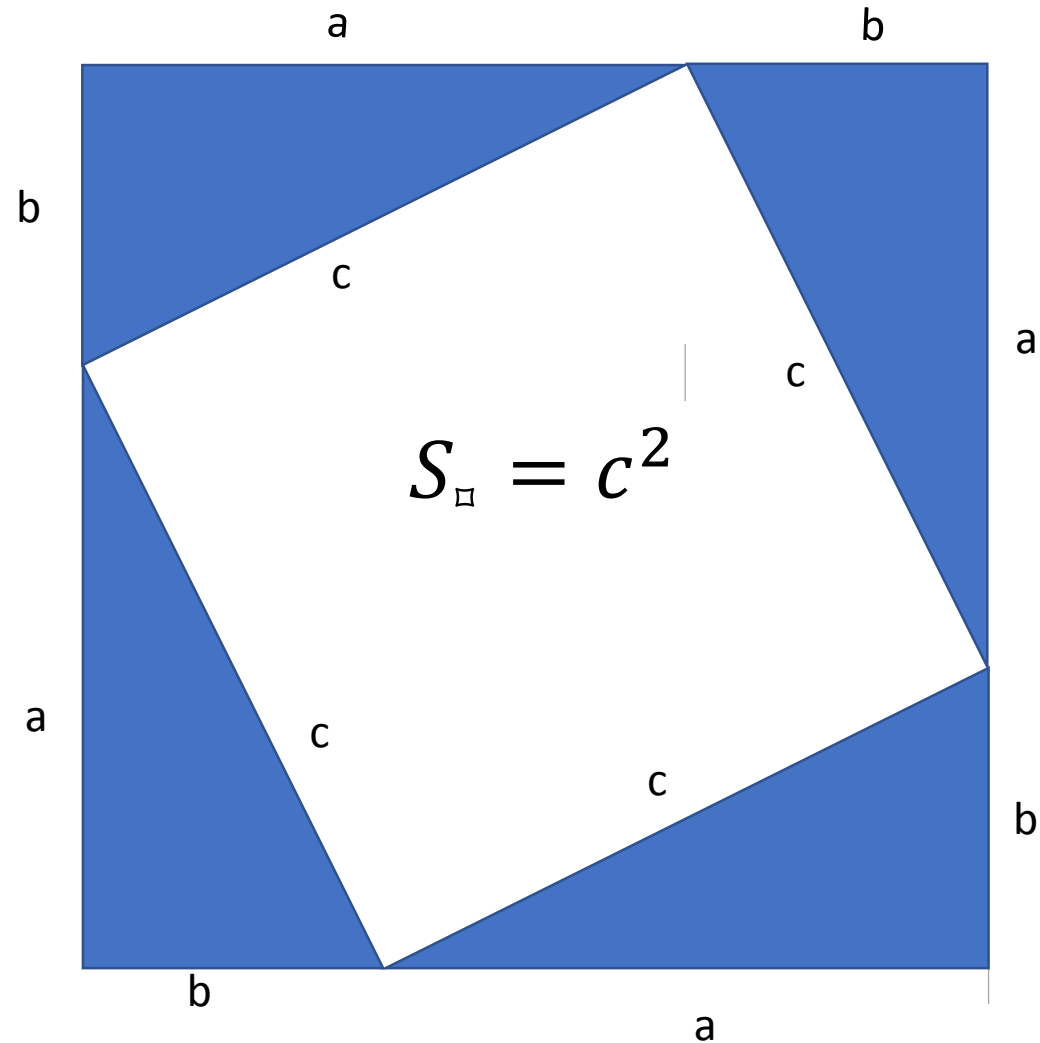
$$S = 4S_{\Delta} = 2ab$$

$$S = S_{\blacksquare} - S_{\square}$$

$$c = ?$$

$$2ab = (a + b)^2 - c^2$$

$$S_{\blacksquare} = (a + b)^2$$



Pythagorean theorem

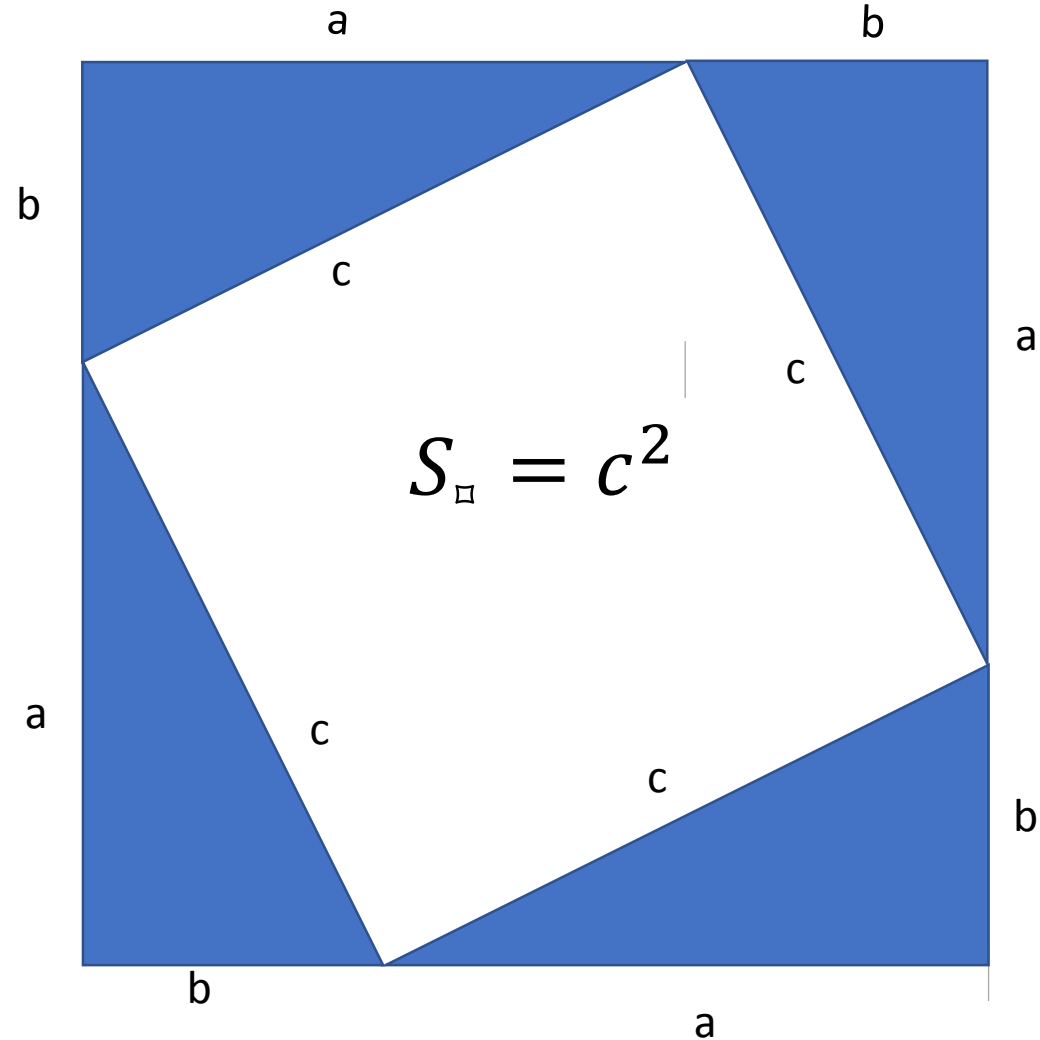
$$S = 4S_{\Delta} = 2ab$$

$$S = S_{\blacksquare} - S_{\square}$$

$c = ?$

$$2ab = a^2 + 2ab + b^2 - c^2$$

$$S_{\blacksquare} = (a + b)^2$$



Pythagorean theorem

$$S = 4S_{\Delta} = 2ab$$

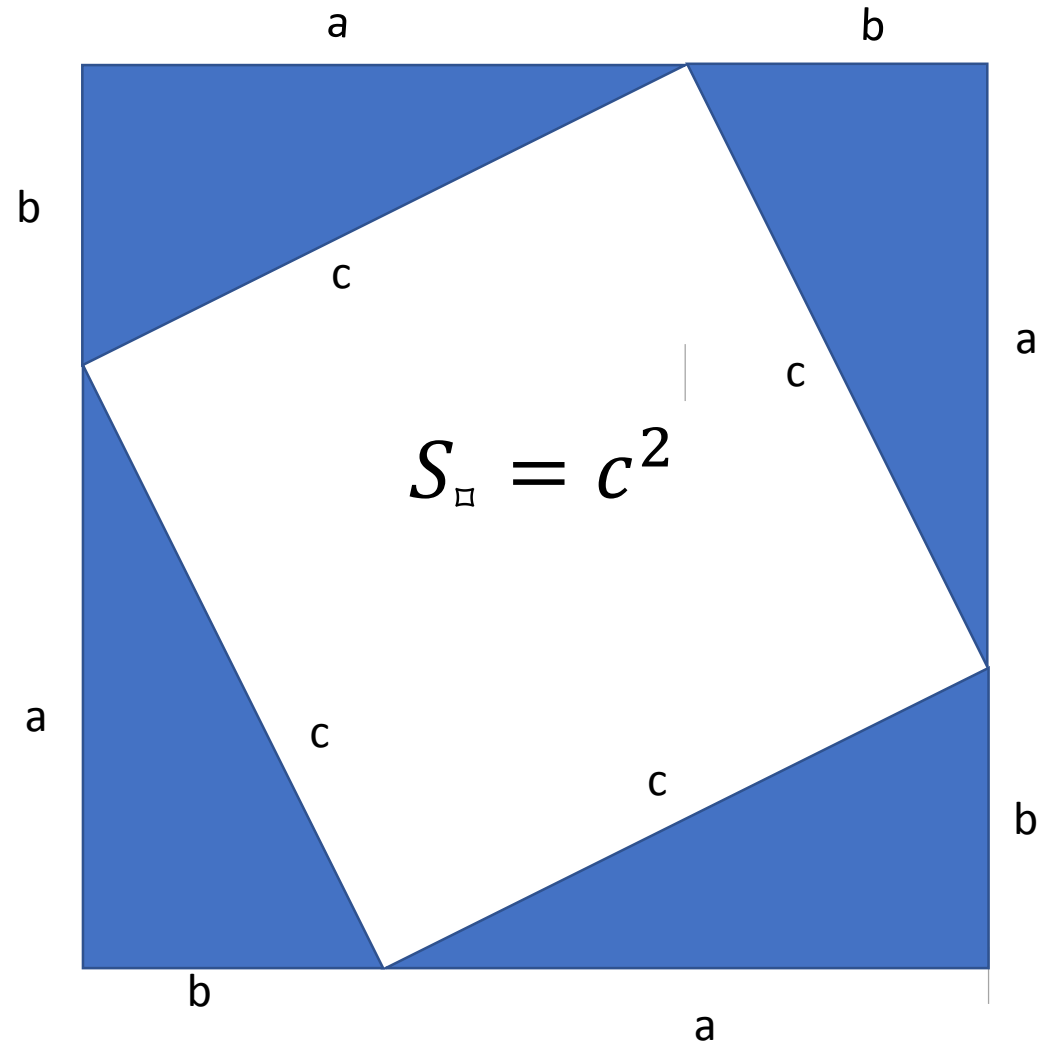
$$S = S_{\blacksquare} - S_{\square}$$

$c = ?$

$$2ab = a^2 + 2ab + b^2 - c^2$$

$$c^2 = a^2 + b^2$$

$$S_{\blacksquare} = (a + b)^2$$

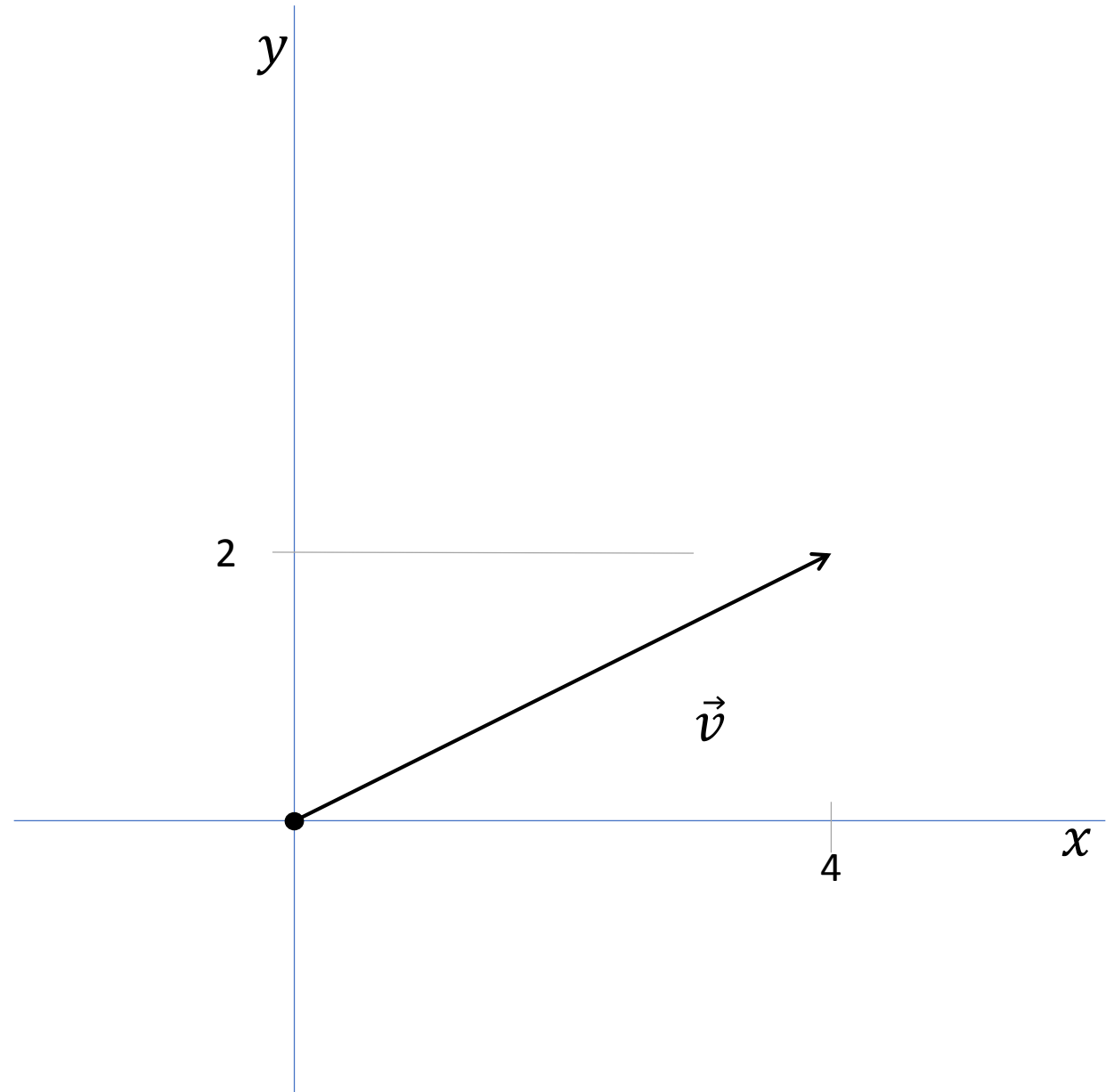


Euclidean vector

- direction

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2}$$

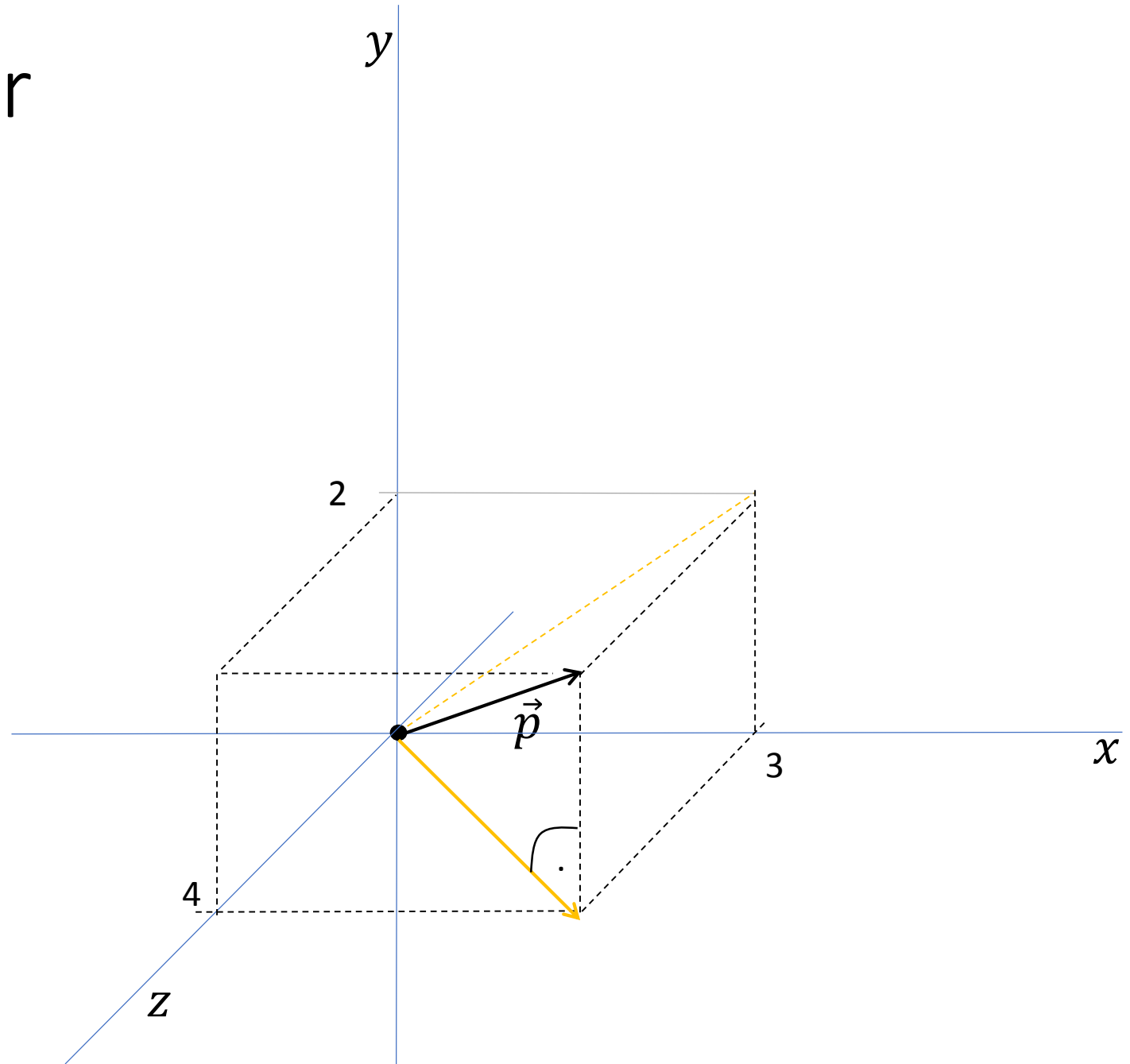


Euclidean vector

- direction

- magnitude / length

$$|\vec{p}| = ?$$



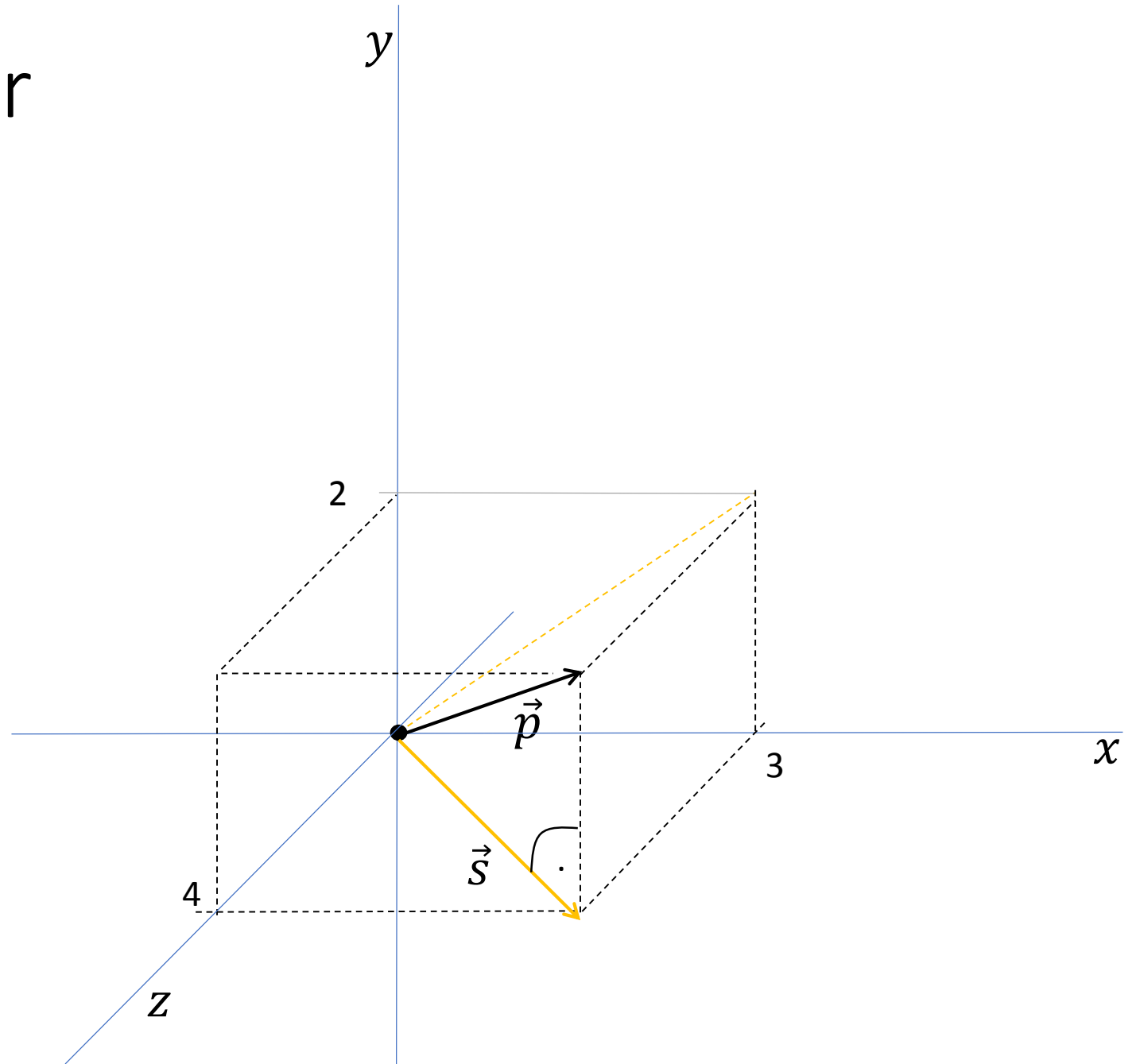
Euclidean vector

- direction

- magnitude / length

$$|\vec{p}| = ?$$

$$|\vec{s}| = ?$$



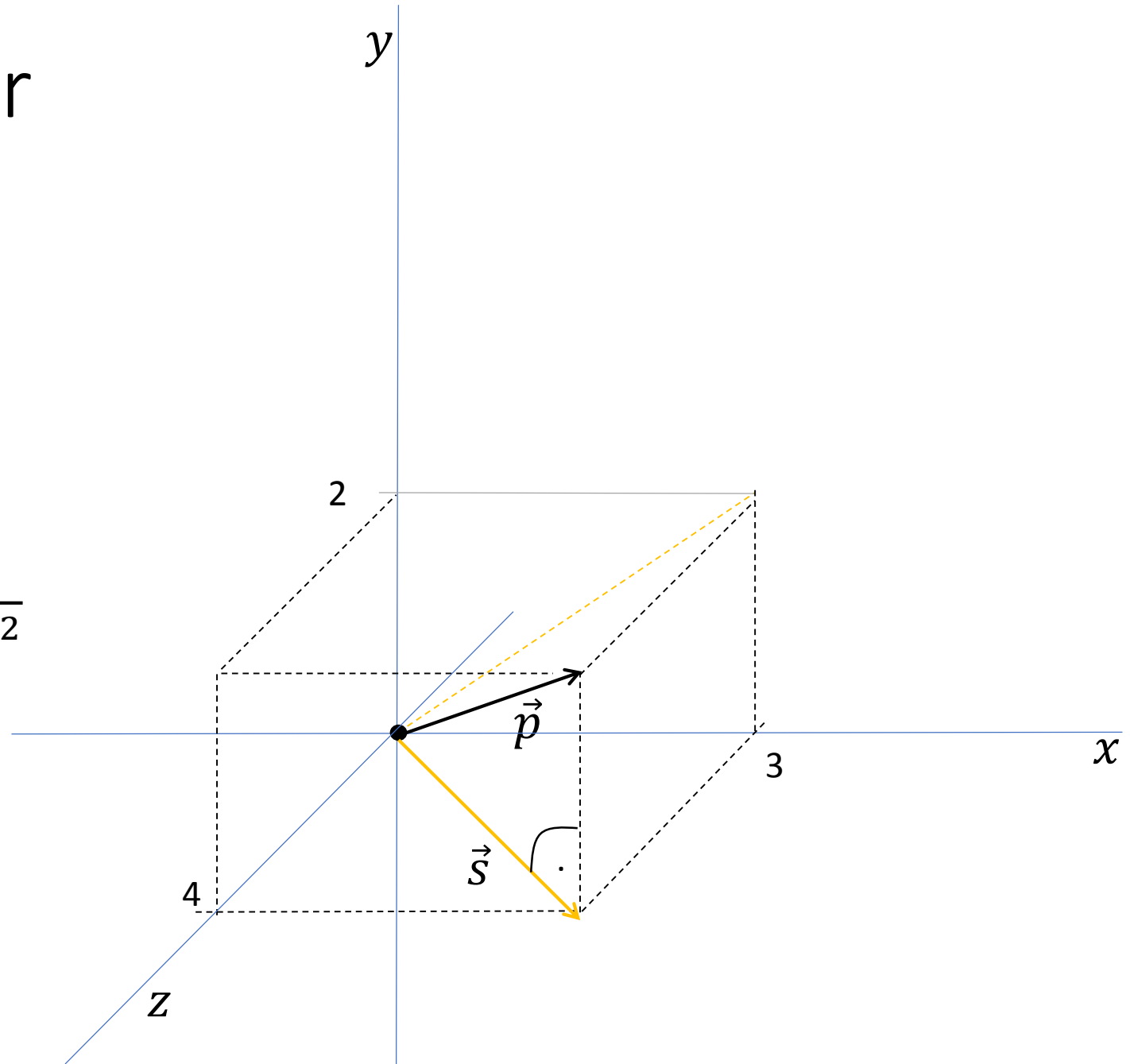
Euclidean vector

- direction

- magnitude / length

$$|\vec{p}| = ?$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$



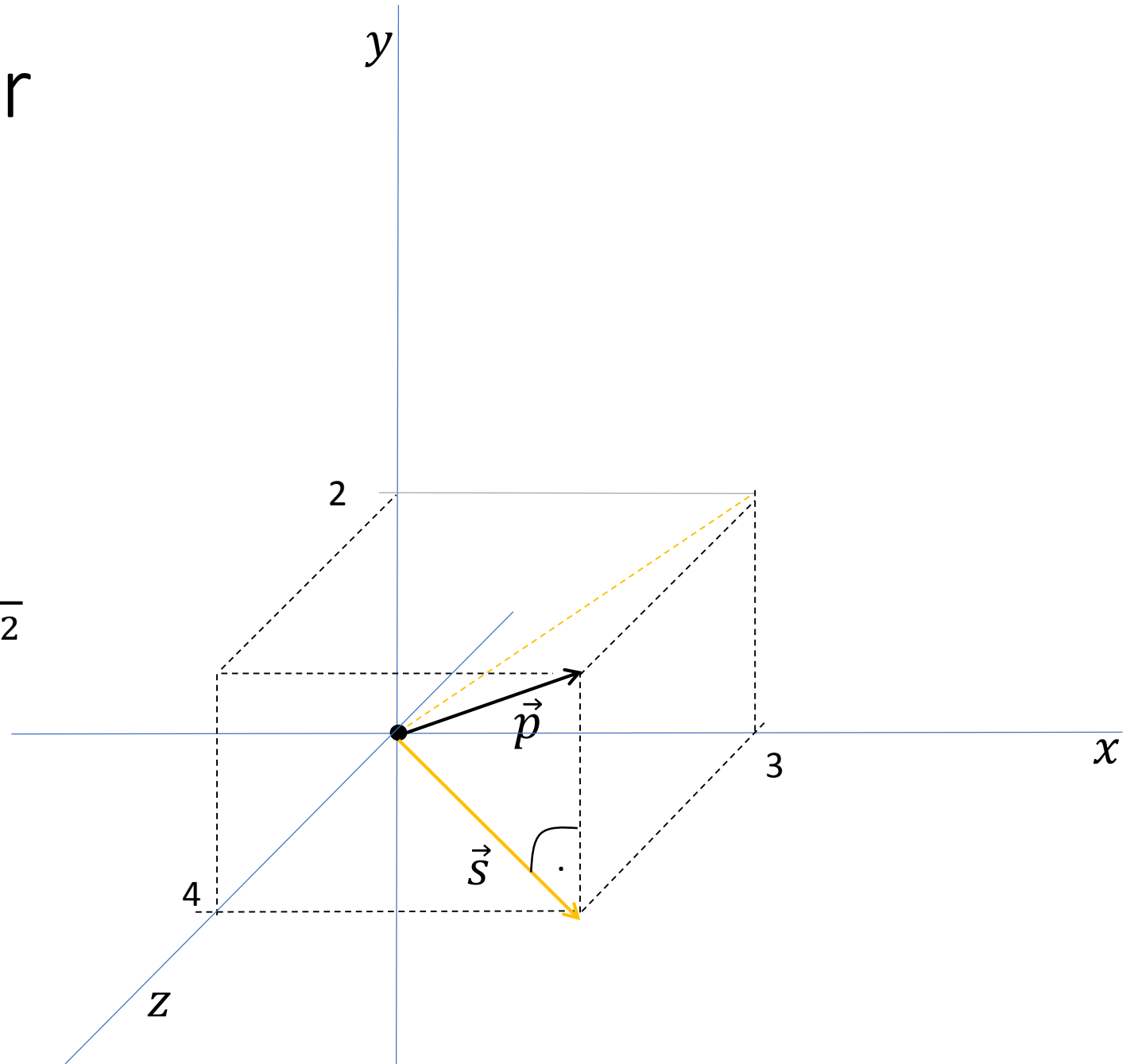
Euclidean vector

- direction

- magnitude / length

$$|\vec{p}| = \sqrt{|\vec{s}|^2 + y^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$



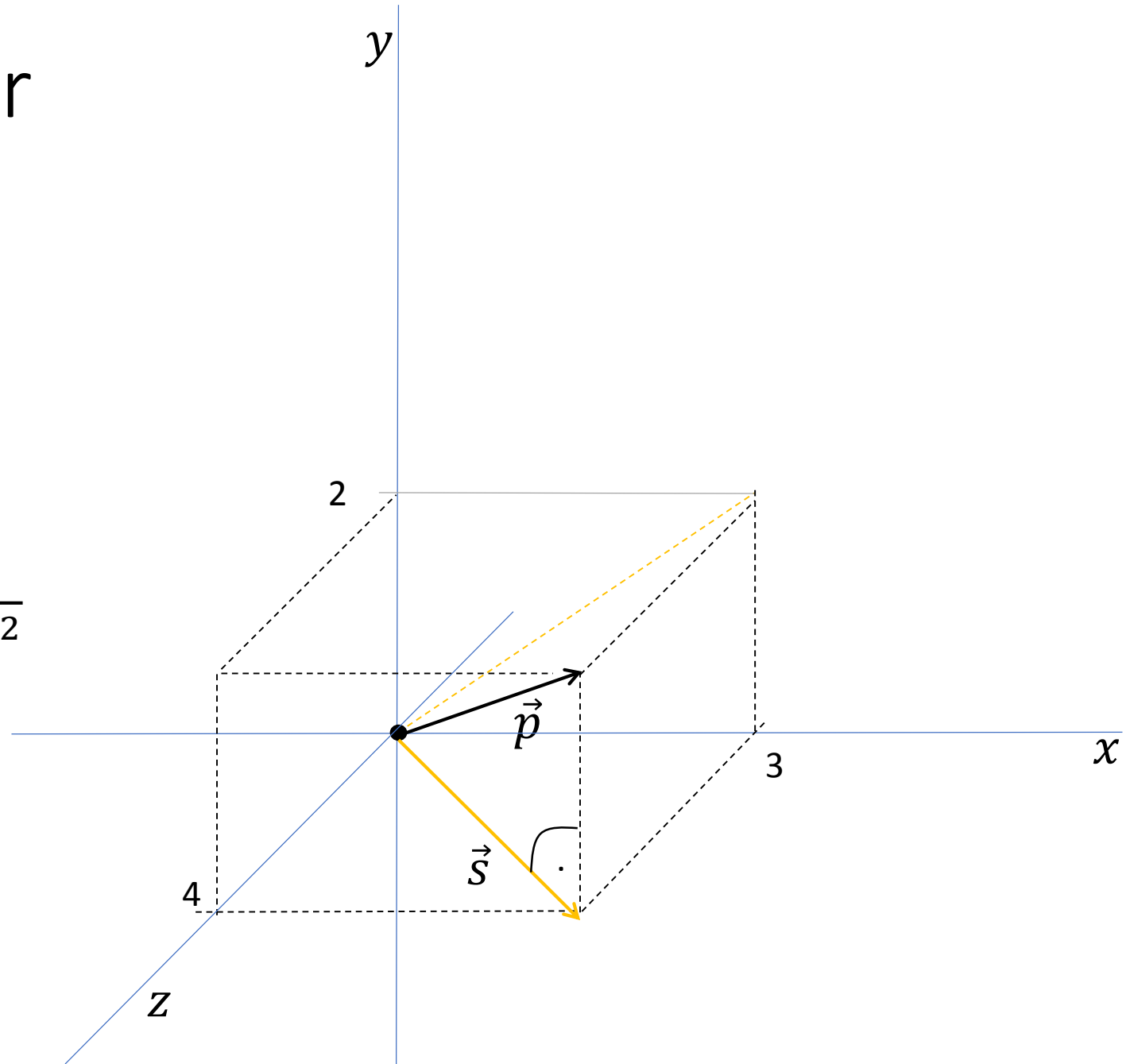
Euclidean vector

- direction

- magnitude / length

$$|\vec{p}| = \sqrt{\sqrt{x^2 + z^2}^2 + y^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$



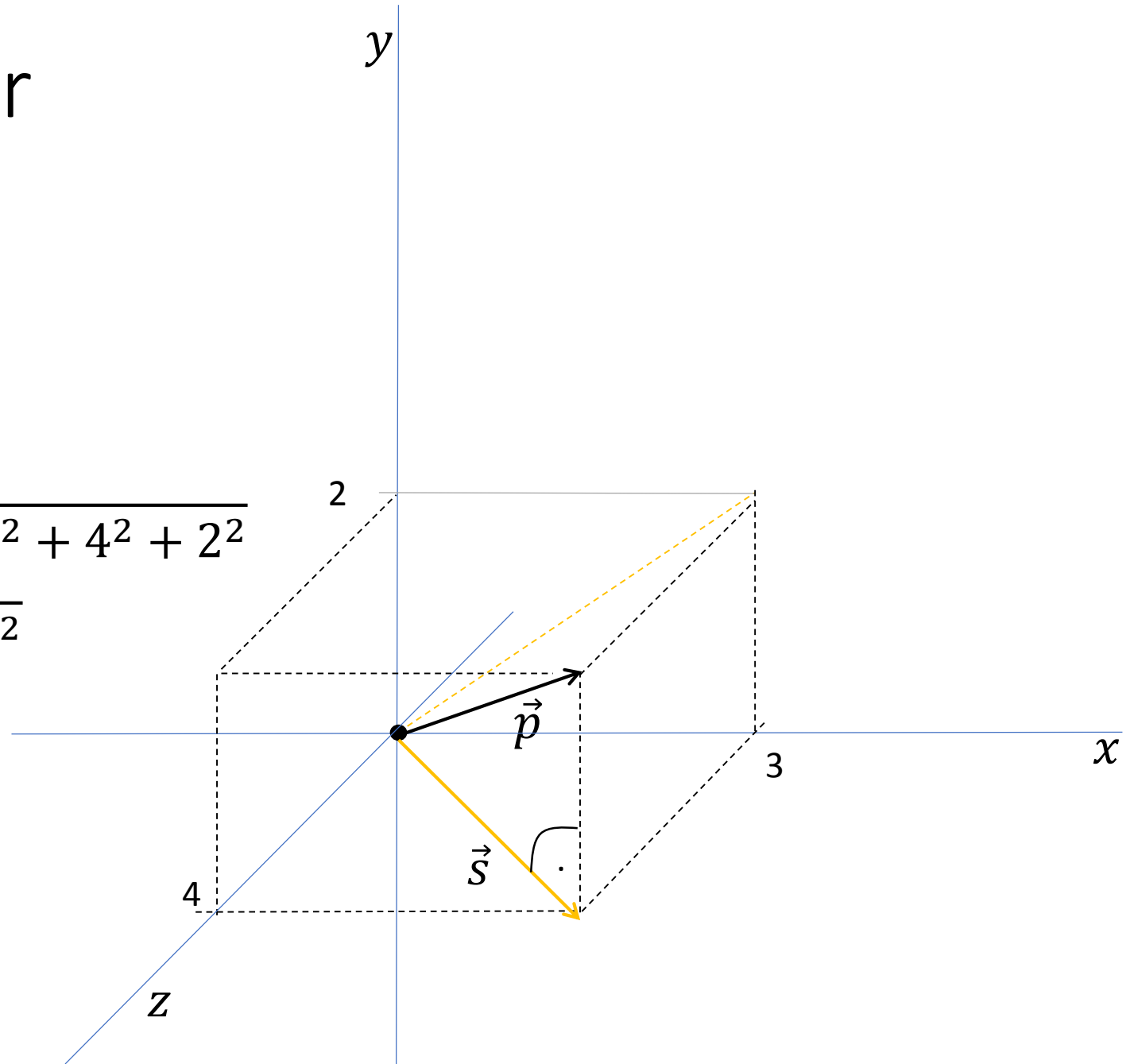
Euclidean vector

- direction

- magnitude / length

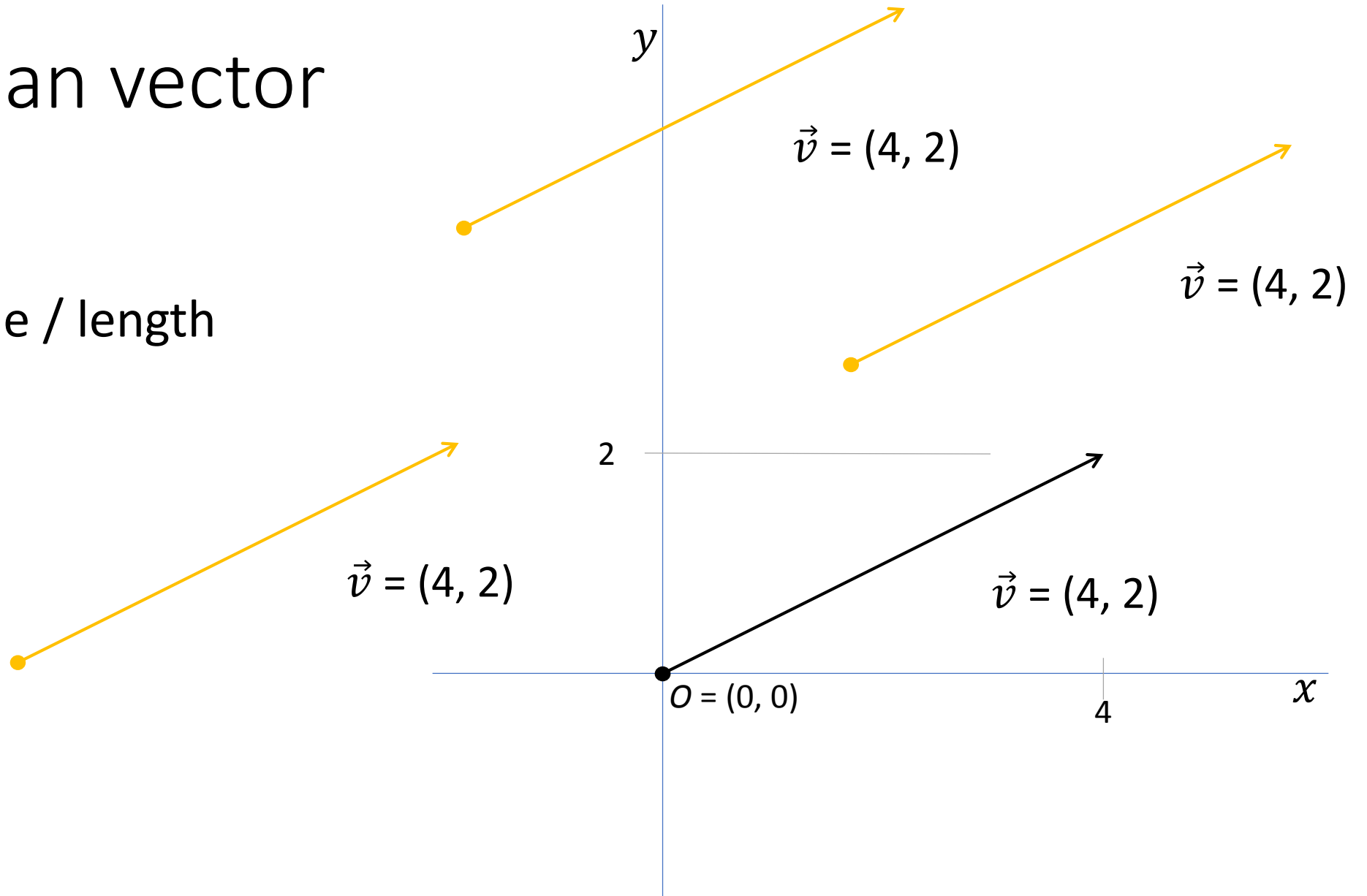
$$|\vec{p}| = \sqrt{x^2 + z^2 + y^2} = \sqrt{3^2 + 4^2 + 2^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$



Euclidean vector

- direction
- magnitude / length




```

#include <stdio.h>
#include <math.h>

class Triple {
public:
    float x, y, z;

    Triple() : x(0), y(0), z(0) {}

    Triple(const float x, const float y, const float z)
        : x(x), y(y), z(z) {}
};

class Vector3D: public Triple {
public:
    Vector3D(const float x, const float y, const float z)
        : Triple(x, y, z) {}

    float magnitude() const {
        return sqrt(x*x + y*y + z*z);
    }
};

class Point3D: public Triple {
public:
    Point3D(const float x, const float y, const float z)
        : Triple(x, y, z) {}

    Point3D translate(const Vector3D &vector) {
        x += vector.x;
        y += vector.y;
        z += vector.z;
    }

    Vector3D operator-(const Point3D& p) const {
        return Vector3D(x - p.x, y - p.y, z - p.z);
    }
};

int main(void) {

    Point3D p0( 0.0f, 0.0f, 1.0f);
    Point3D p1( 1.0f, 2.0f, 3.0f);

    Vector3D v = p1 - p0;
    printf ("v=(%f, %f, %f)\n", v.x, v.y, v.z); //v=(1.000000, 2.000000, 2.000000)

    p0.translate(v);
    printf ("p0=(%f, %f, %f)\n", p0.x, p0.y, p0.z); //p0=(1.000000, 2.000000, 3.000000)

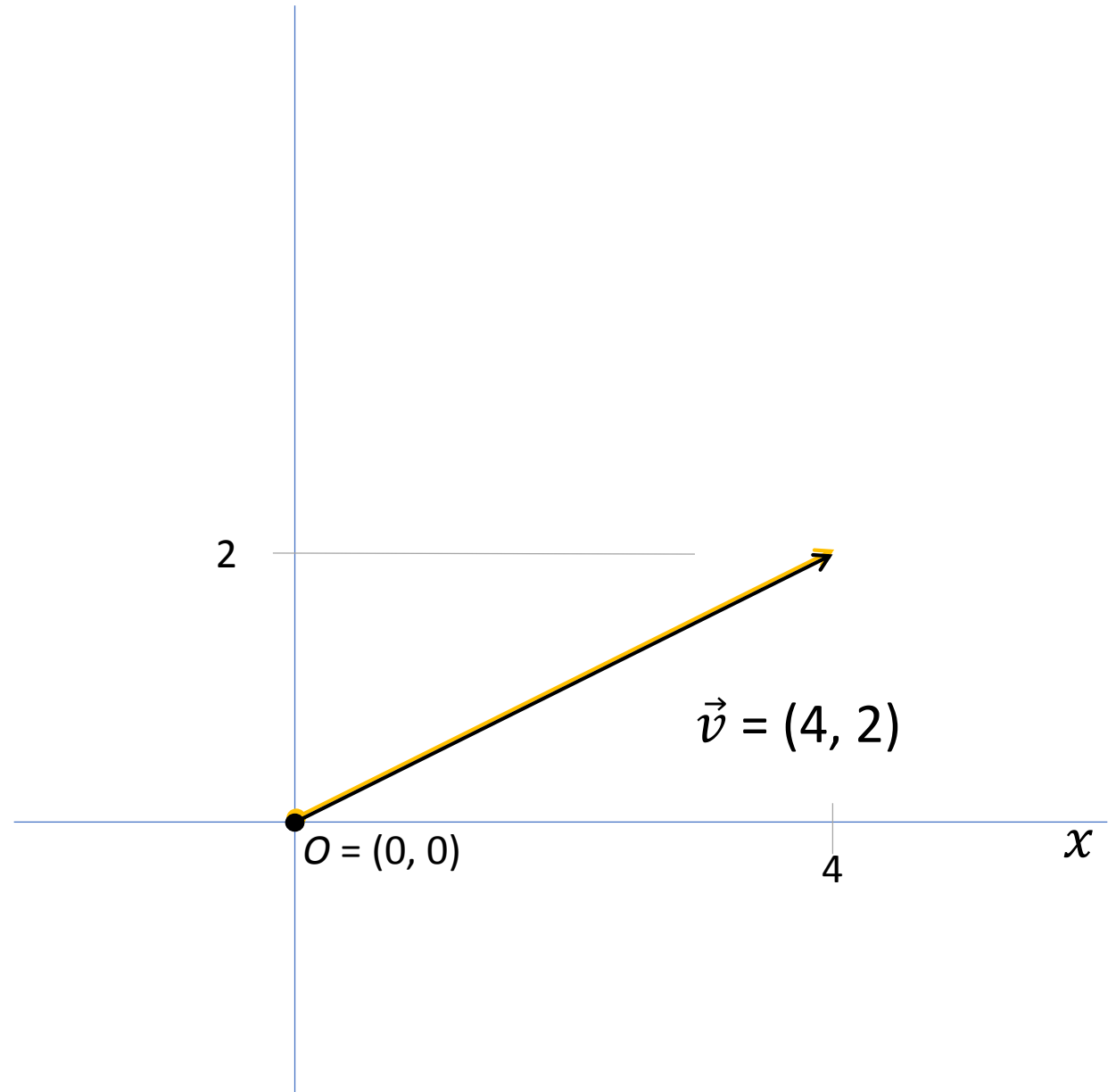
    v.translate(Vector3D( 1.0f, 2.0f, 3.0f)); //error
    float len = p0.magnitude(); //error

    return 0;
}

```

Normalized vector

- direction
- magnitude / length

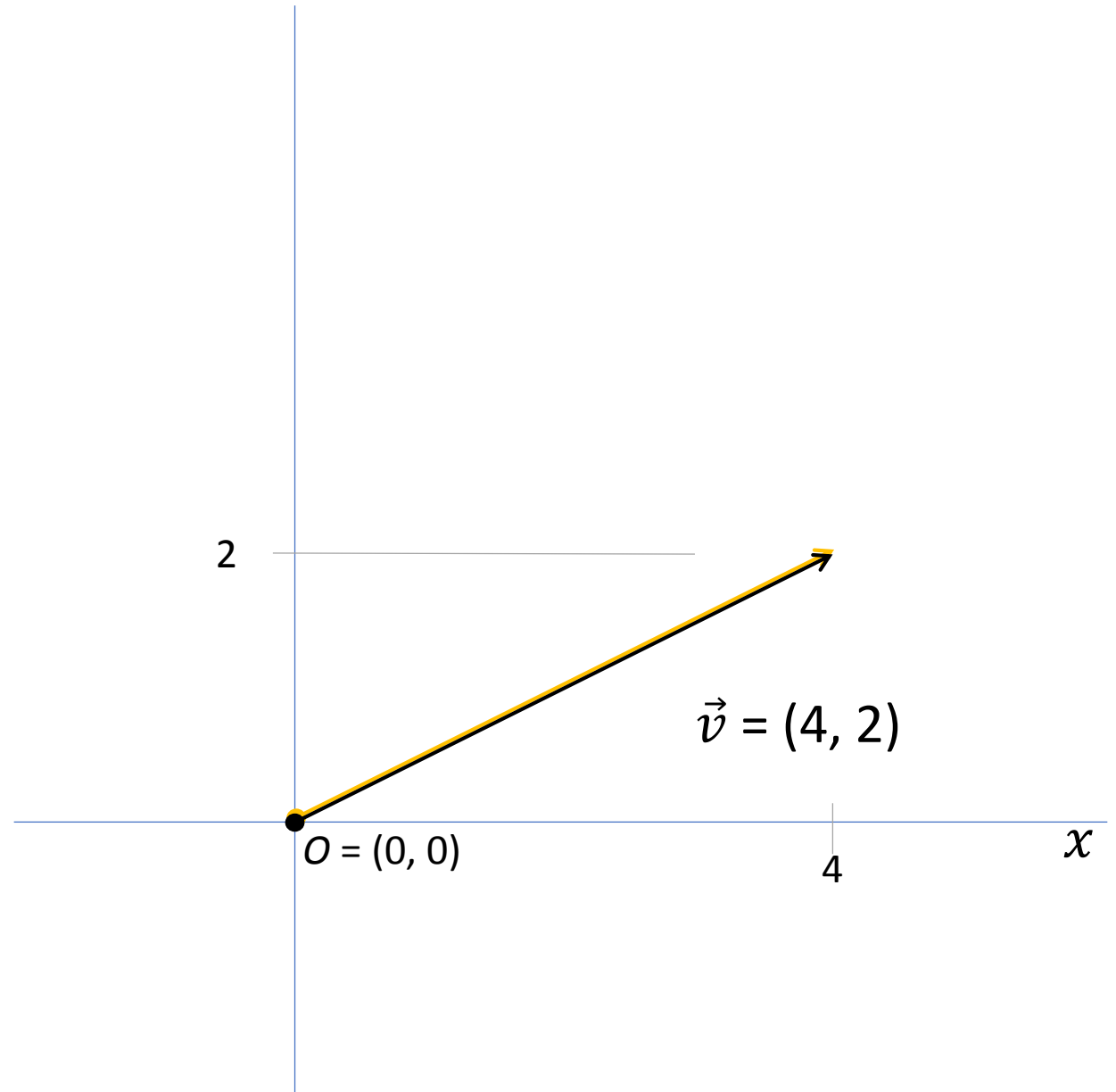


Normalized vector

- direction

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = \sqrt{20}$$



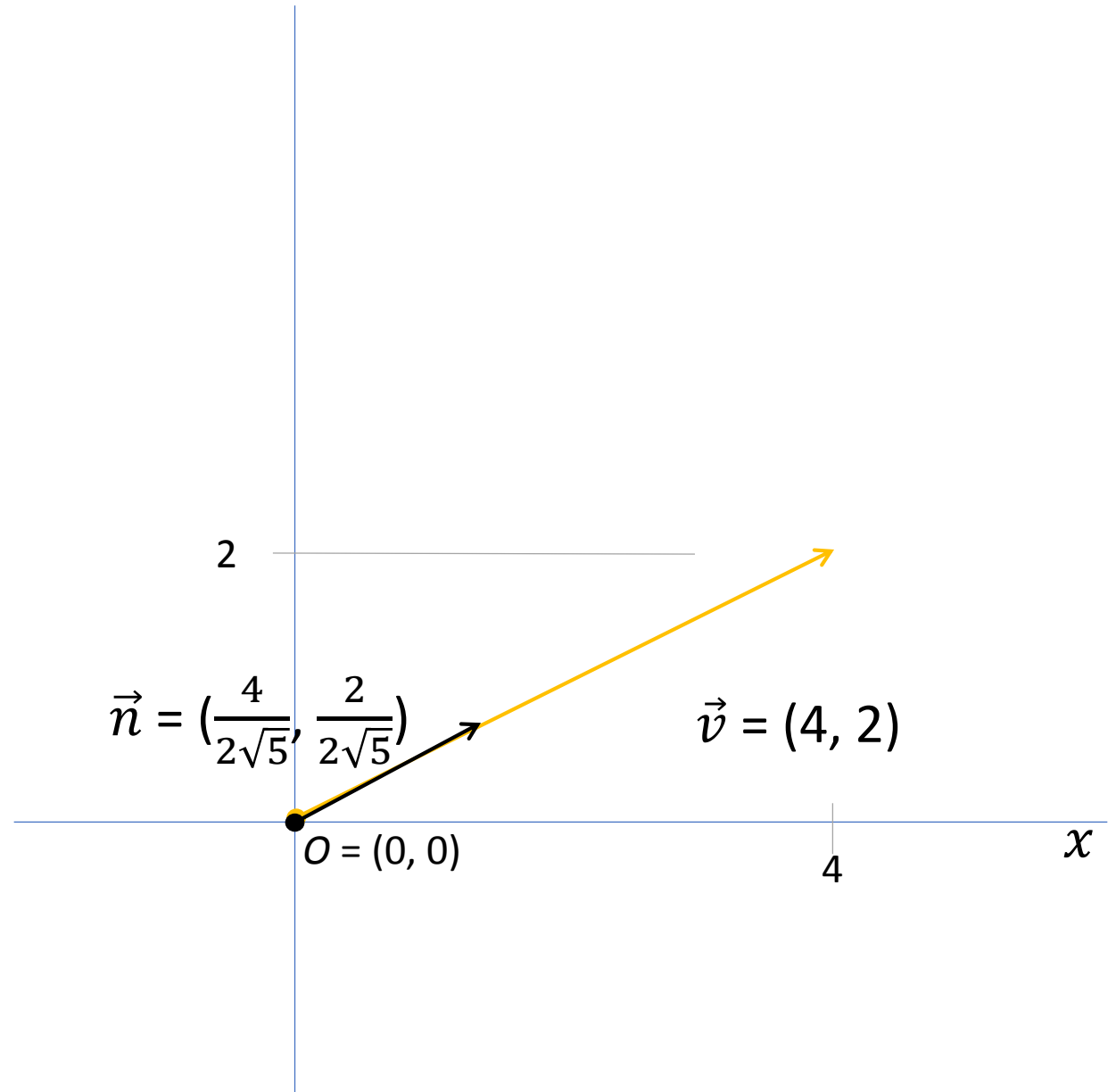
Normalized vector

- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$



Normalized vector

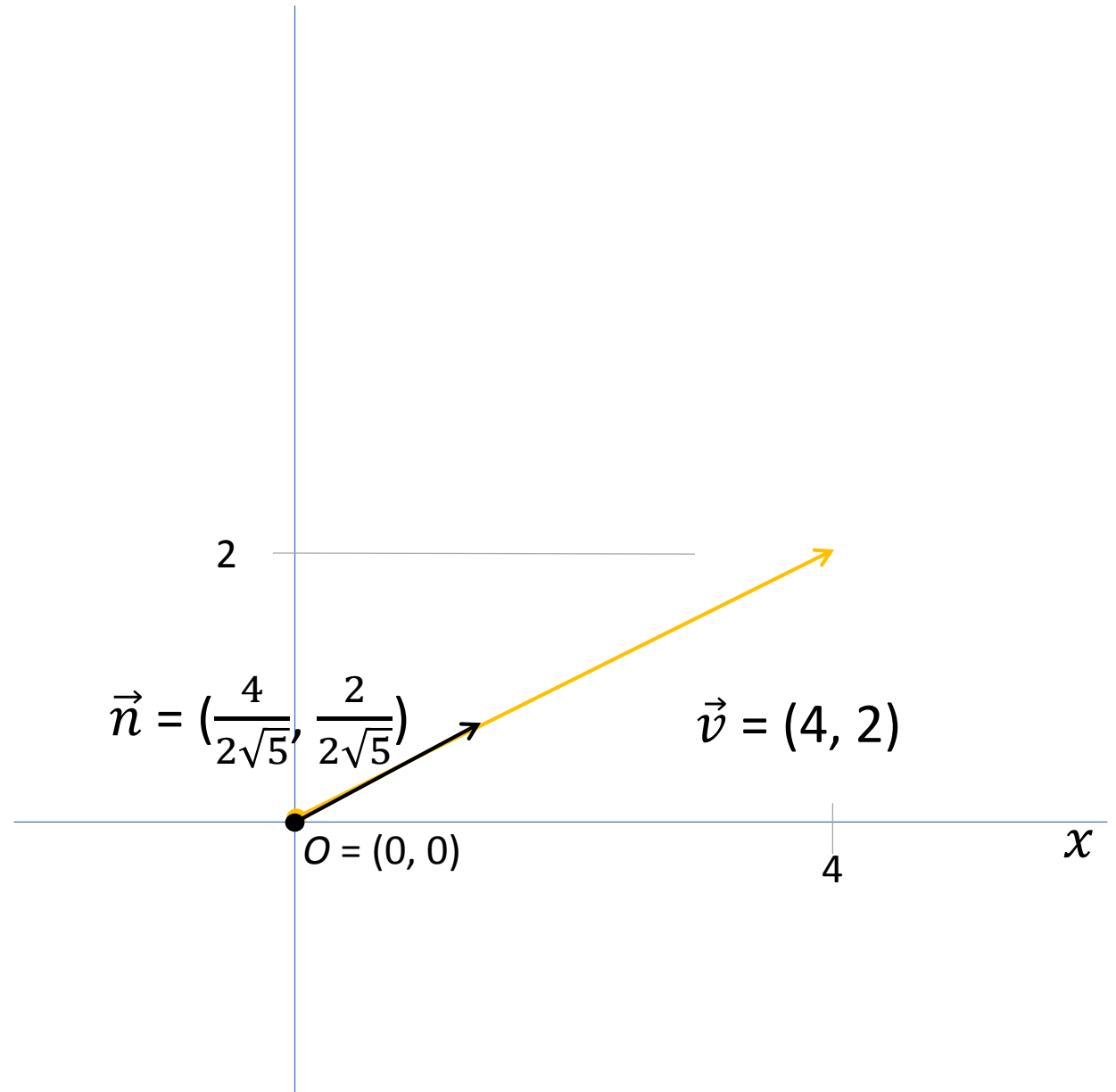
- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

$$|\vec{n}| = ?$$



Normalized vector

- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

$$|\vec{n}| = \sqrt{\left(\frac{4}{2\sqrt{5}}\right)^2 + \left(\frac{2}{2\sqrt{5}}\right)^2} = \sqrt{\frac{16+4}{2 \times 2 \times 5}}$$



Normalized vector

- direction

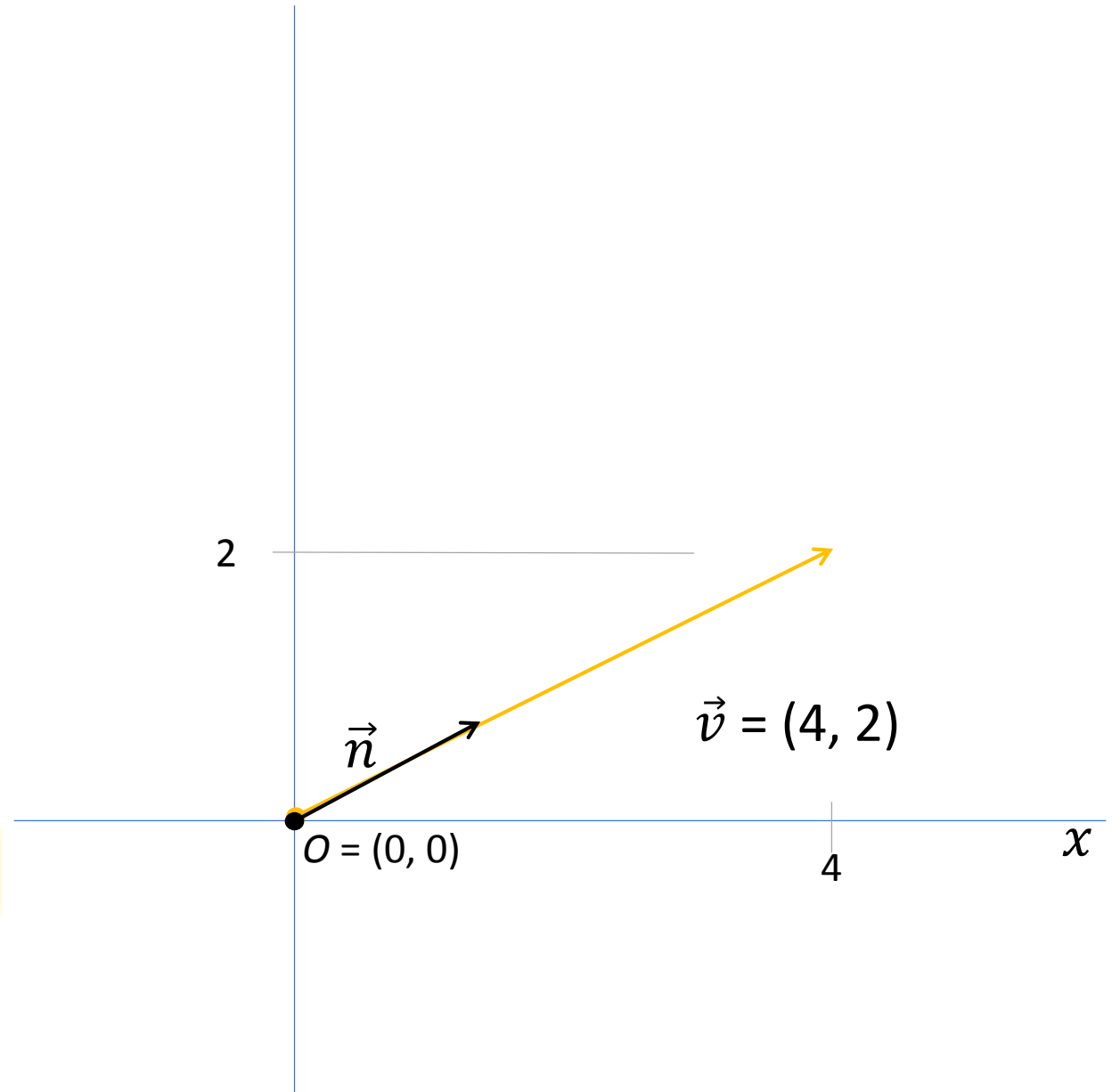
$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

- scalar vector multiplication

$$c \vec{n} = ?$$



Normalized vector

- direction

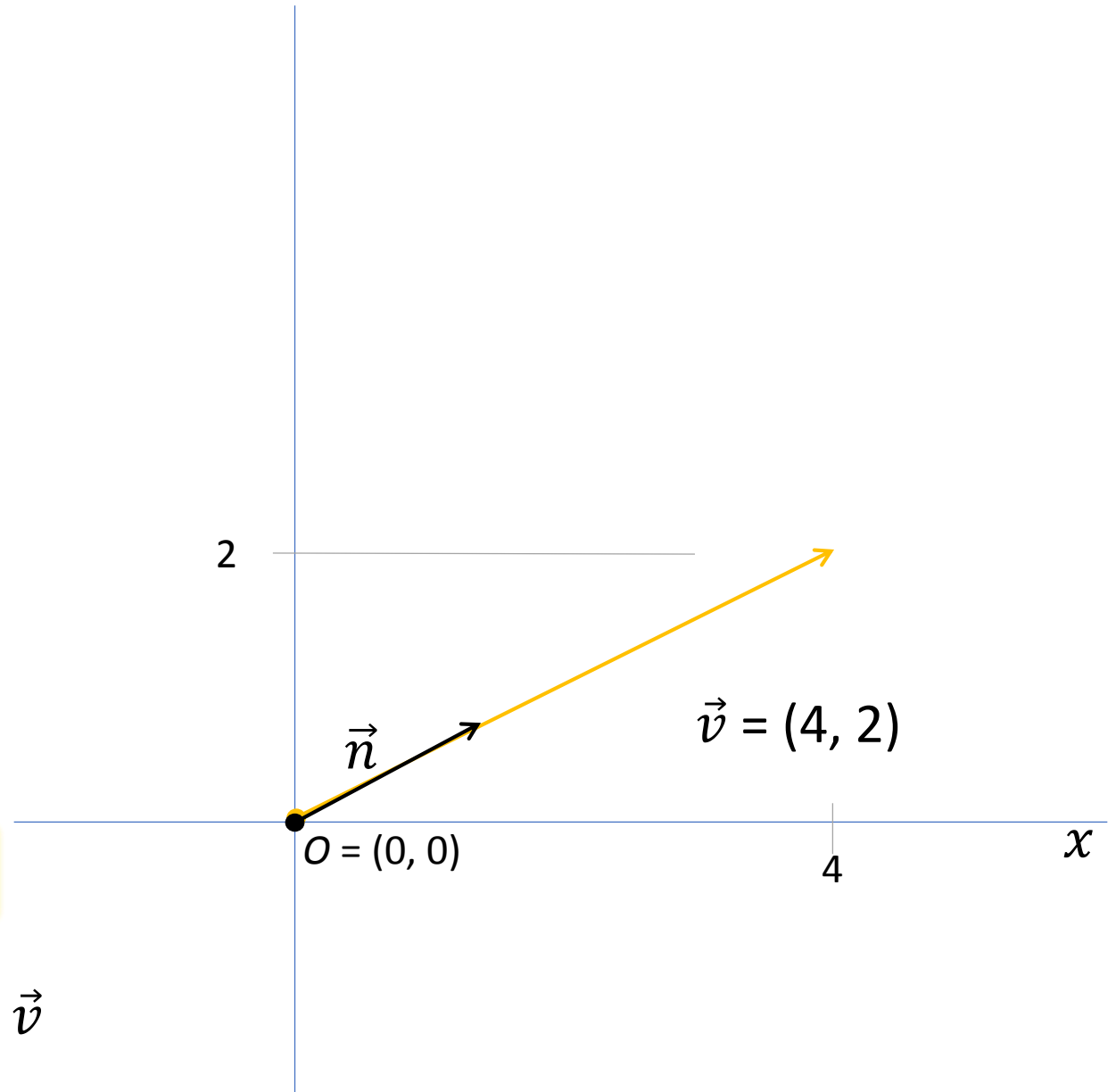
$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

- scalar vector multiplication

$$c \vec{n} = 2\sqrt{5} \left(\frac{4}{2\sqrt{5}}, \frac{2}{2\sqrt{5}} \right) = (4, 2) = \vec{v}$$



Normalized vector

- direction

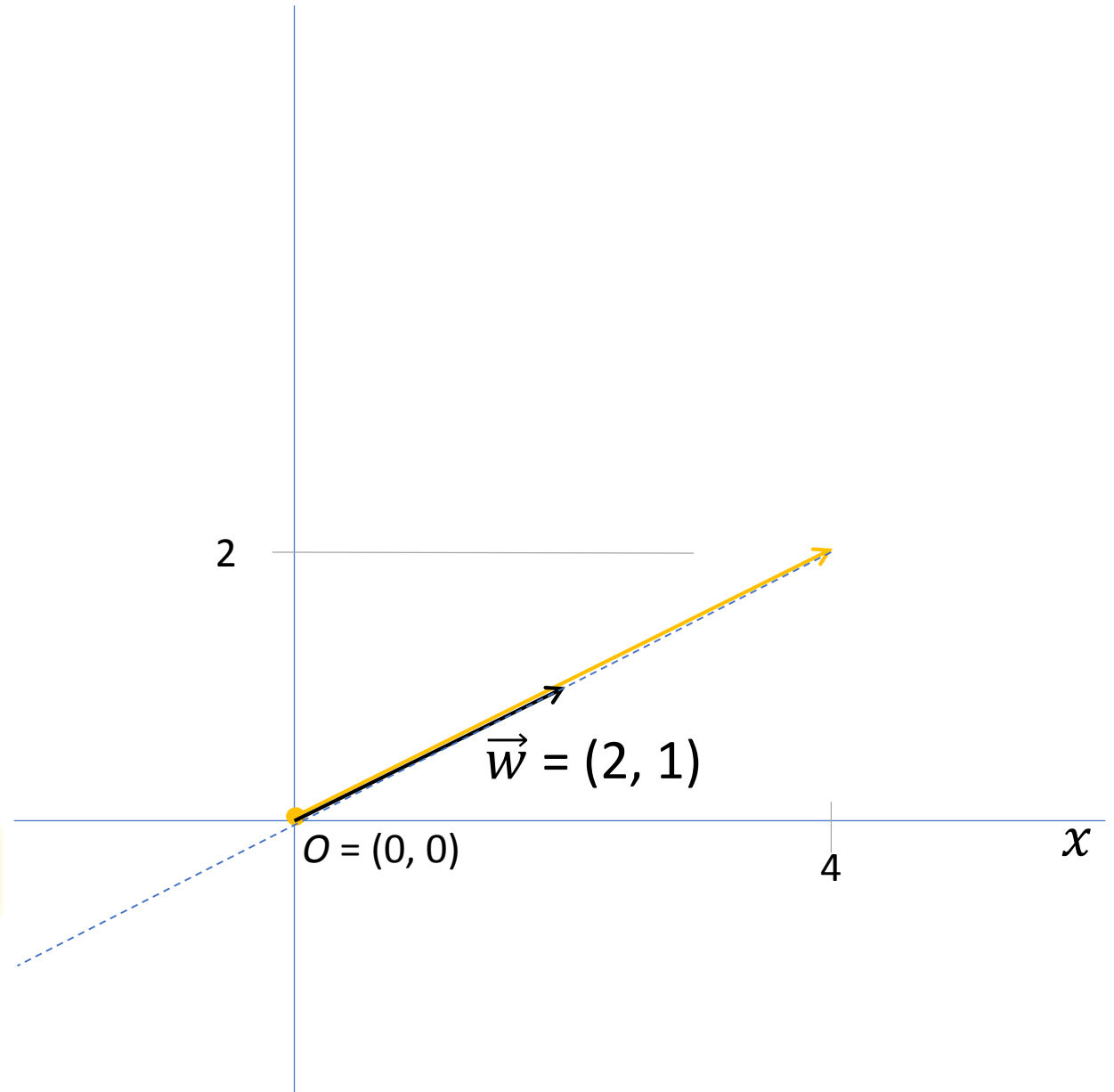
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2, 1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$2 \vec{w} = 2 (2, 1) = (4, 2)$$



Normalized vector

- direction

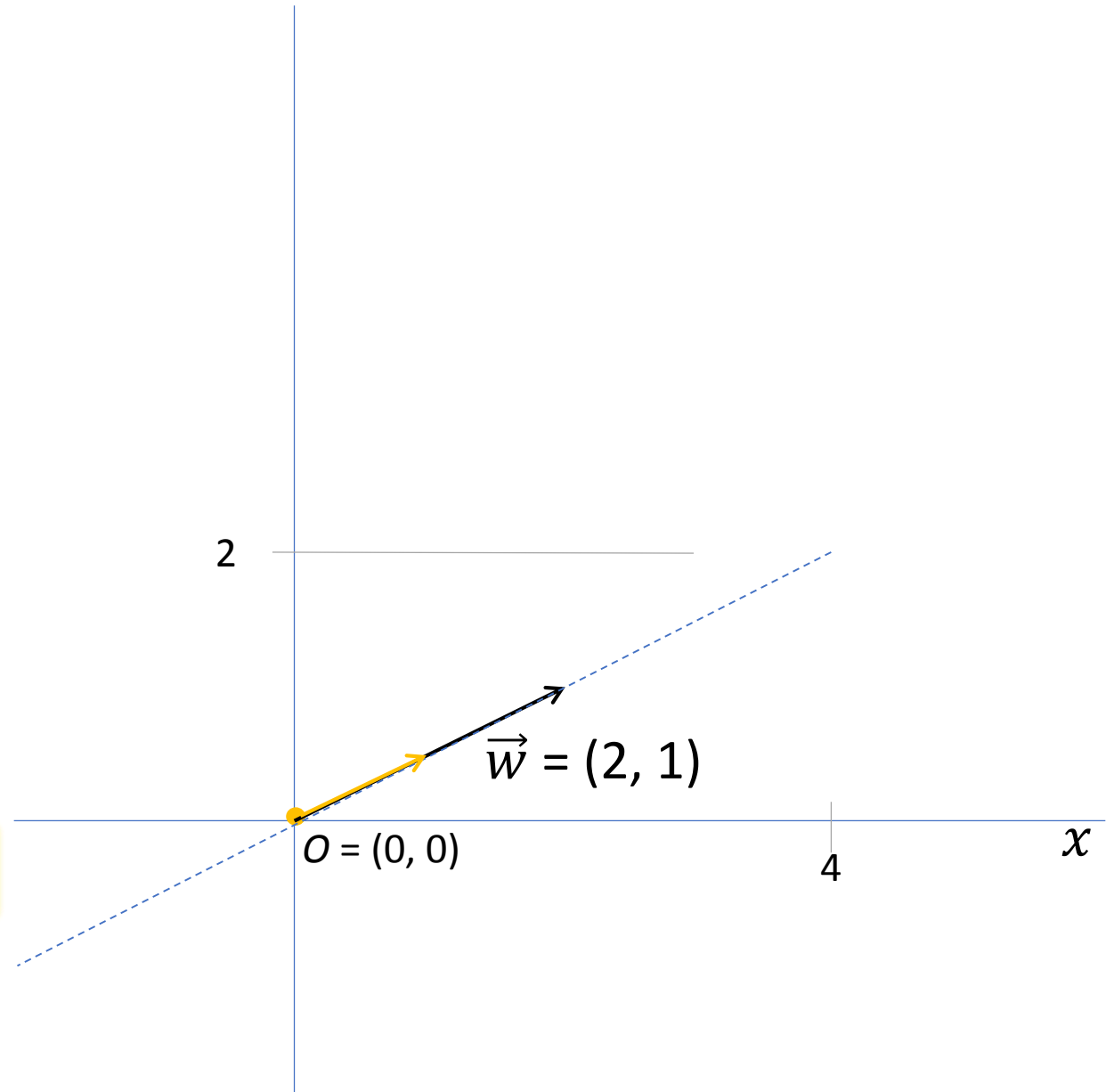
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2, 1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$\frac{1}{2} \vec{w} = \frac{1}{2} (2, 1) = \left(1, \frac{1}{2}\right)$$



Normalized vector

- direction

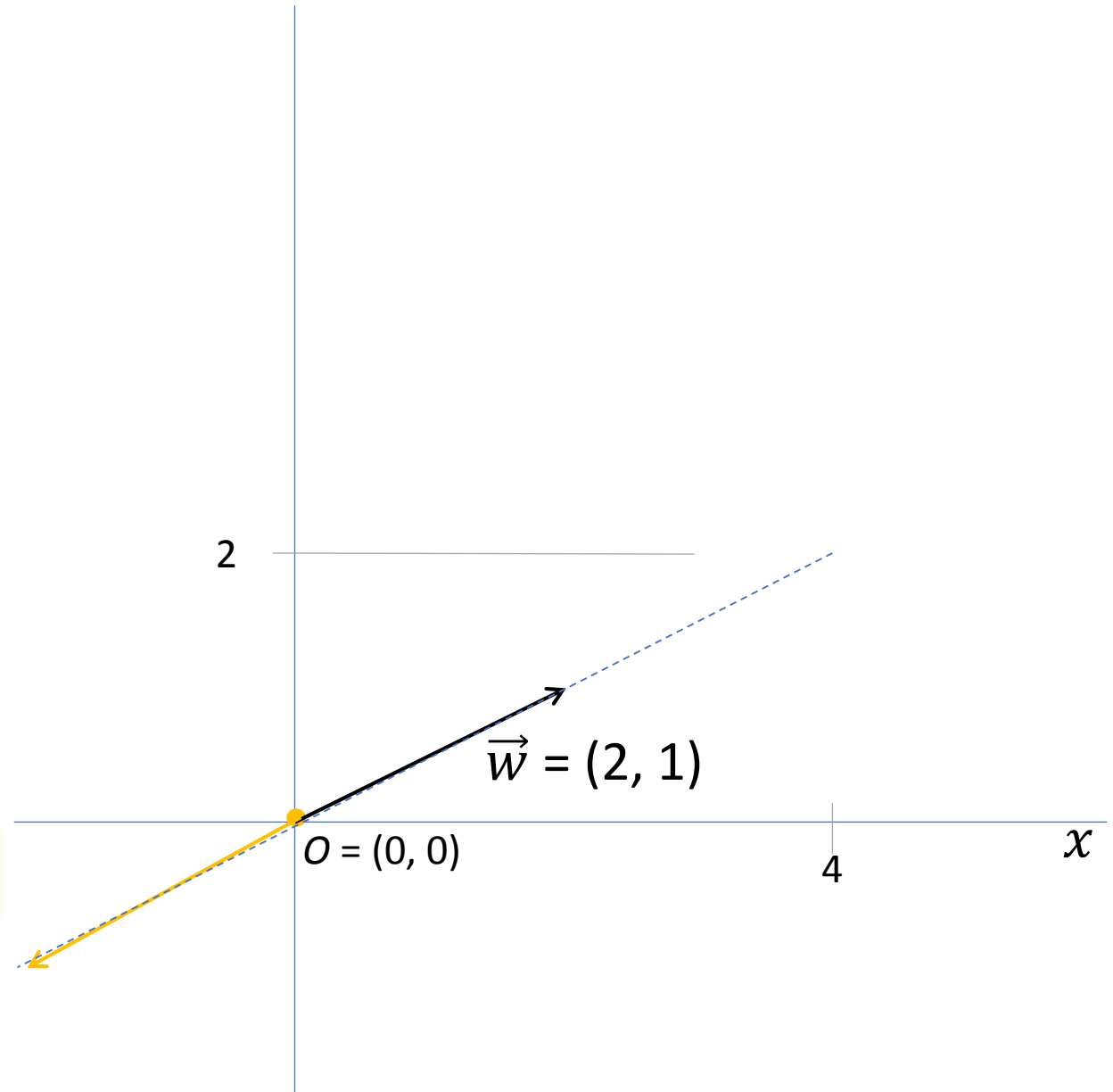
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2, 1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$-1 \vec{w} = -1 (2, 1) = (-2, -1) = -\vec{w}$$



vector (1,0,0) (0,1,0) (0,0,1) (1,1,1)

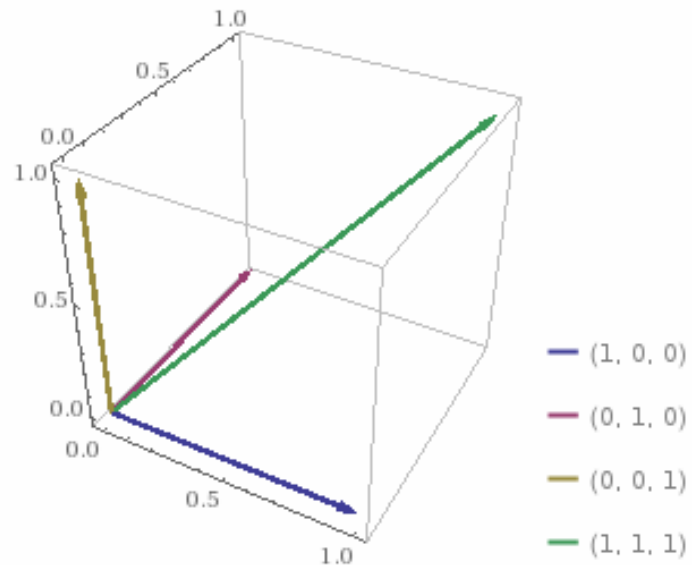


<http://m.wolframalpha.com/>

Input interpretation:

{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)}

Vector plot:



Vector lengths:

Approximate forms

(1, 0, 0)	1
(0, 1, 0)	1
(0, 0, 1)	1
(1, 1, 1)	$\sqrt{3}$

- Linear combination

$$\vec{e}_1 = (1, 0, 0)$$

$$\vec{e}_2 = (0, 1, 0)$$

$$\vec{e}_3 = (0, 0, 1)$$

- $x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3 = (x, y, z)$
- $\vec{e}_1 + \vec{e}_2 + \vec{e}_3 = (1, 1, 1)$

Matrix

- rectangular array arranged in rows and columns

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

- # rows = 2
- # columns = 3

Matrix

- rectangular array arranged in rows and columns

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

- # rows = 2
- # columns = 3

$$2 \times 3$$

Matrix

- Transpose

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$3 \times 2$$

Matrix

- Transpose

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

3×2

- vector as a matrix

$$(1 \ 2 \ 3)^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

3×1

Matrix

- Matrix = 2D array

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

- Vector = a row or column

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Matrix

$$2x - y = 0$$
$$2x + y = 2$$

plot {2x-y = 0} {2x+y = 2}



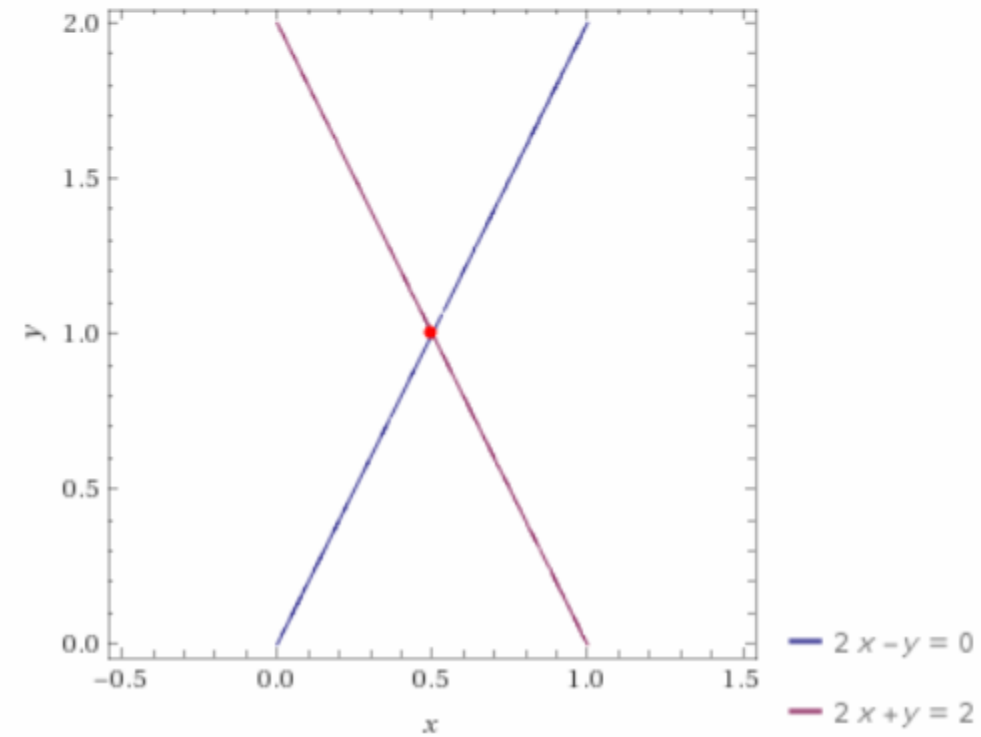
Input interpretation:

plot

$$2x - y = 0$$

$$2x + y = 2$$

Implicit plot:



Matrix

$$2x - y = 0$$

$$2x + y = 2$$

multiplication

$$\begin{pmatrix} 2x - y \\ 2x + y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

plot {2x-y = 0} {2x+y = 2}



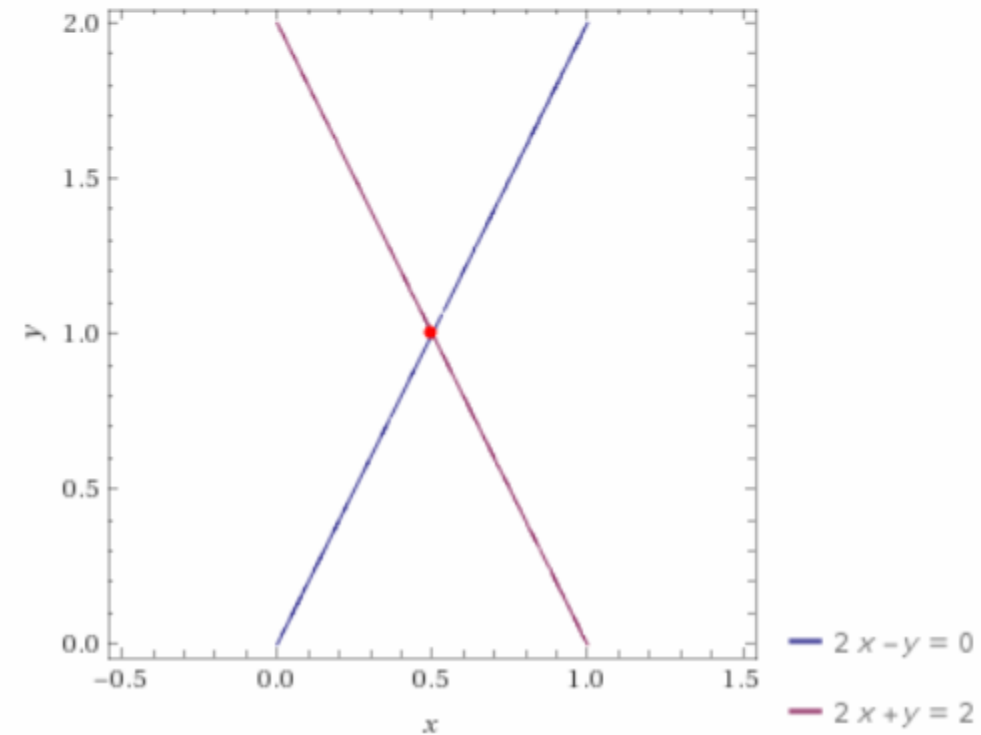
Input interpretation:

plot

$$2x - y = 0$$

$$2x + y = 2$$

Implicit plot:



Matrix

$$2x - y = 0$$

$$2x + y = 2$$

multiplication

$$\begin{pmatrix} 2x - y \\ 2x + y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

plot {2x-y = 0} {2x+y = 2}



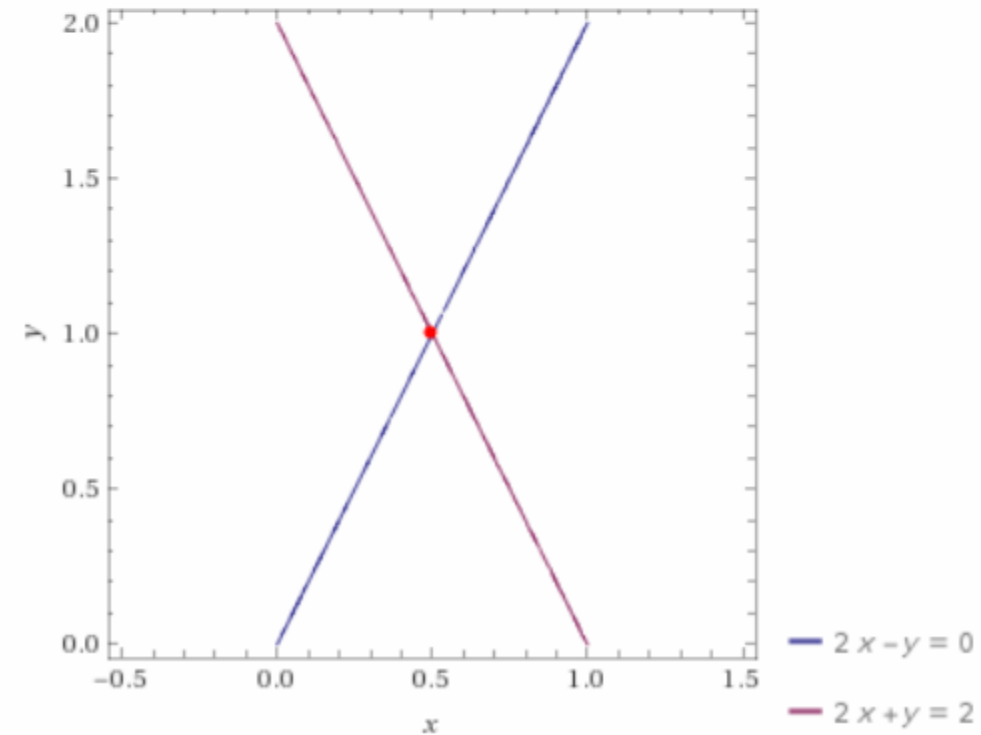
Input interpretation:

plot

$$2x - y = 0$$

$$2x + y = 2$$

Implicit plot:



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{aligned} \begin{pmatrix} 2 & -1 & | & 0 \\ 2 & 1 & | & 2 \end{pmatrix} &\sim \begin{pmatrix} 1 & -\frac{1}{2} & | & 0 \\ 2 & 1 & | & 2 \end{pmatrix} \sim \\ \sim \begin{pmatrix} 1 & -\frac{1}{2} & | & 0 \\ 0 & 2 & | & 2 \end{pmatrix} &\sim \begin{pmatrix} 1 & -\frac{1}{2} & | & 0 \\ 0 & 1 & | & 1 \end{pmatrix} \sim \\ &\sim \begin{pmatrix} 1 & 0 & | & \frac{1}{2} \\ 0 & 1 & | & 1 \end{pmatrix} \end{aligned}$$

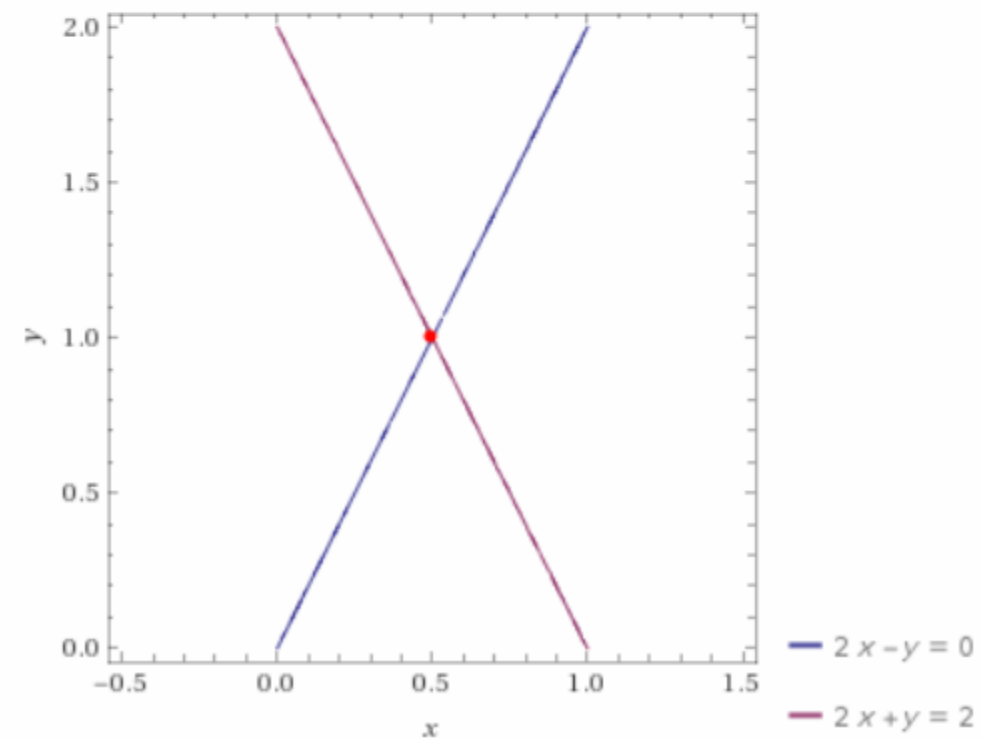
plot {2x-y = 0} {2x+y = 2}



Input interpretation:

plot $2x - y = 0$
 $2x + y = 2$

Implicit plot:



Matrix

$$\begin{aligned} \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & 1 & 2 \end{array} \right) &\sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 2 & 1 & 2 \end{array} \right) \sim \\ &\sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 2 & 2 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 \end{array} \right) \sim \\ &\sim \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 1 \end{array} \right) \end{aligned}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

plot {2x-y = 0} {2x+y = 2}



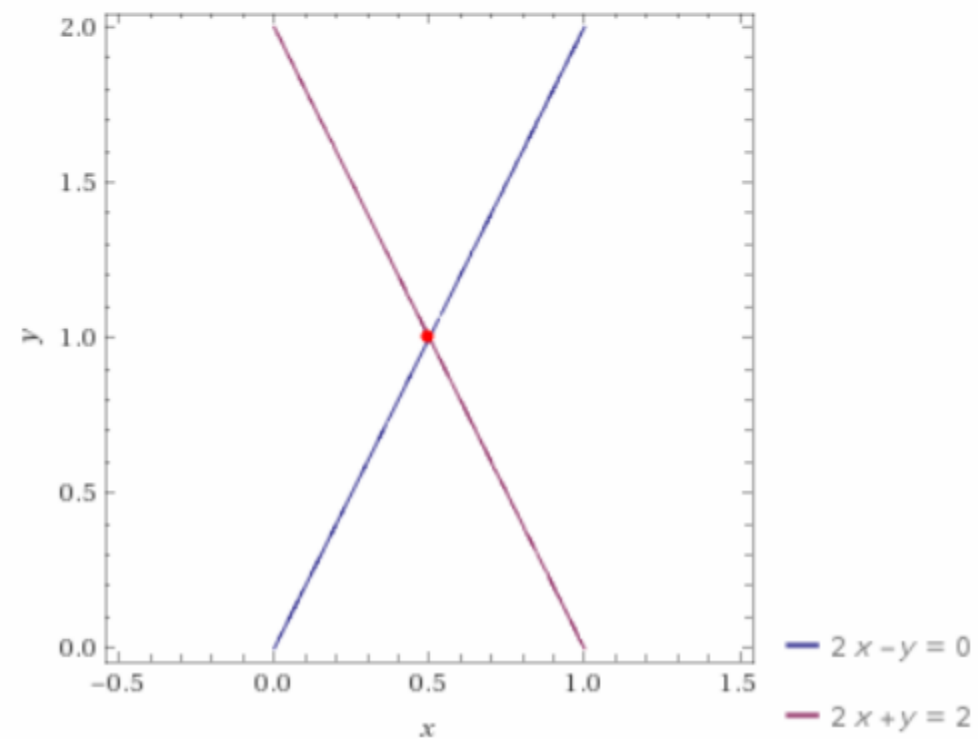
Input interpretation:

plot

2x - y = 0

2x + y = 2

Implicit plot:



Matrix

$$\begin{pmatrix} 2 & -1 & | & 0 \\ 2 & 1 & | & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & | & \frac{1}{2} \\ 0 & 1 & | & 1 \end{pmatrix}$$
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

Diagram illustrating the row reduction of the augmented matrix $\begin{pmatrix} 2 & -1 & | & 0 \\ 2 & 1 & | & 2 \end{pmatrix}$ to the identity matrix $\begin{pmatrix} 1 & 0 & | & \frac{1}{2} \\ 0 & 1 & | & 1 \end{pmatrix}$. The resulting system of equations is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$. Green arrows show the transformation of the first row from $(2, -1, 0)$ to $(1, 0, \frac{1}{2})$. Blue arrows show the transformation of the second row from $(2, 1, 2)$ to $(0, 1, 1)$.

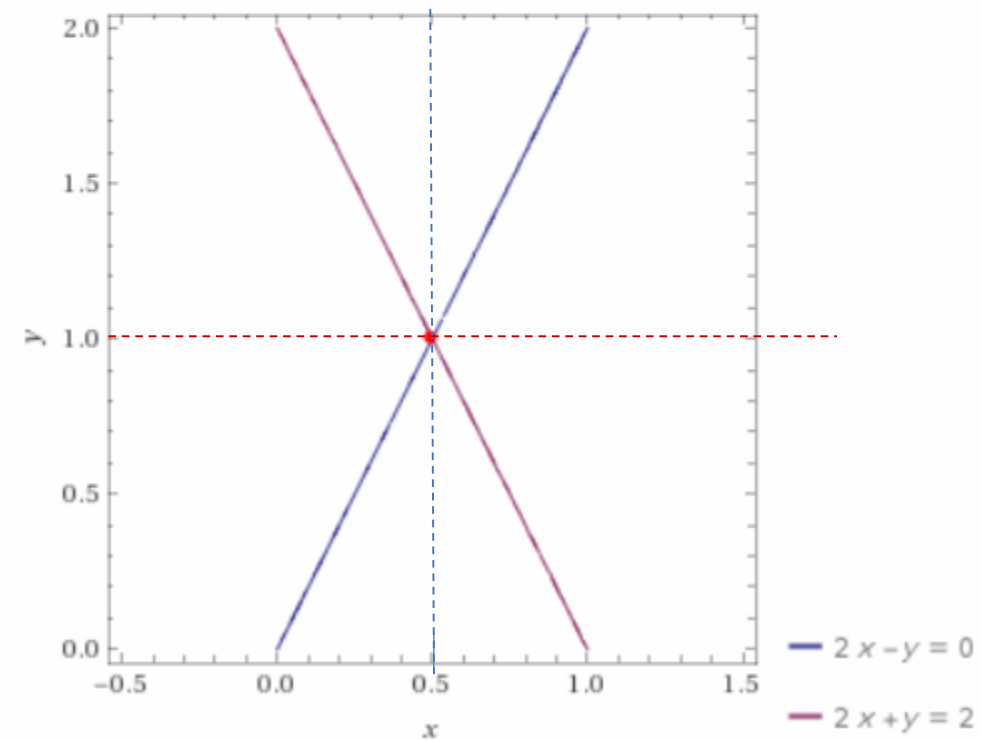
plot {2x-y = 0} {2x+y = 2}



Input interpretation:

plot $2x - y = 0$
 $2x + y = 2$

Implicit plot:



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

plot {2x-y = 0} {2x+y = 2}



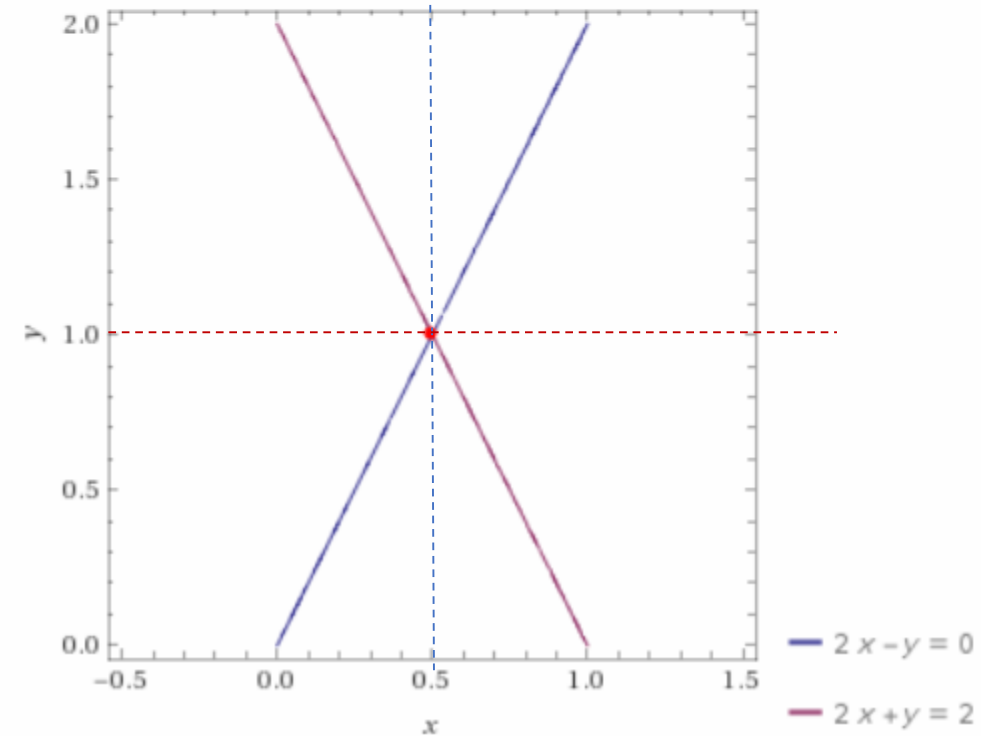
Input interpretation:

plot

$$2x - y = 0$$

$$2x + y = 2$$

Implicit plot:



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

plot {2x-y = 0} {2x+y = 2}



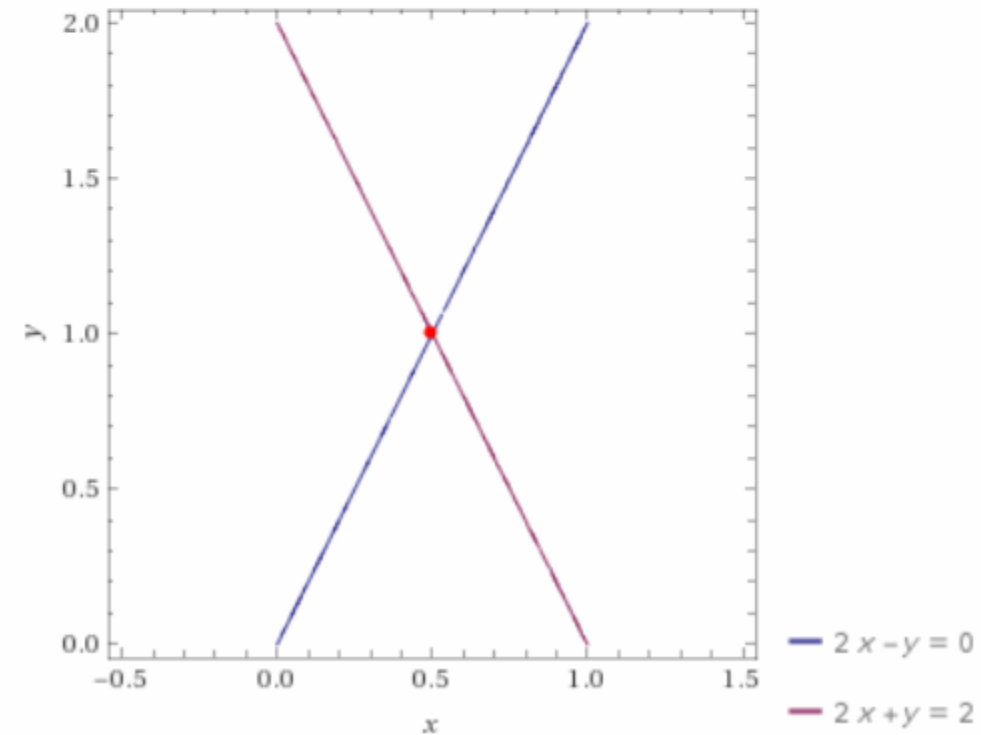
Input interpretation:

plot

$$2x - y = 0$$

$$2x + y = 2$$

Implicit plot:



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

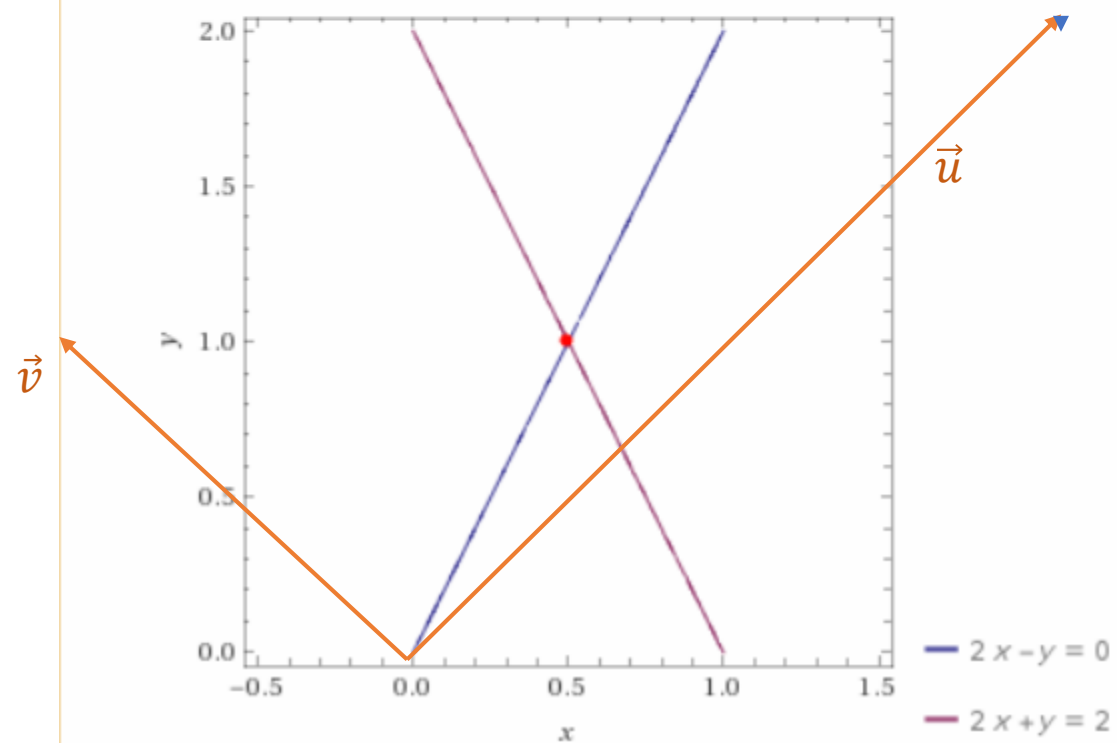
plot {2x-y = 0} {2x+y = 2}



Input interpretation:

plot $2x - y = 0$
 $2x + y = 2$

Implicit plot:



Matrix

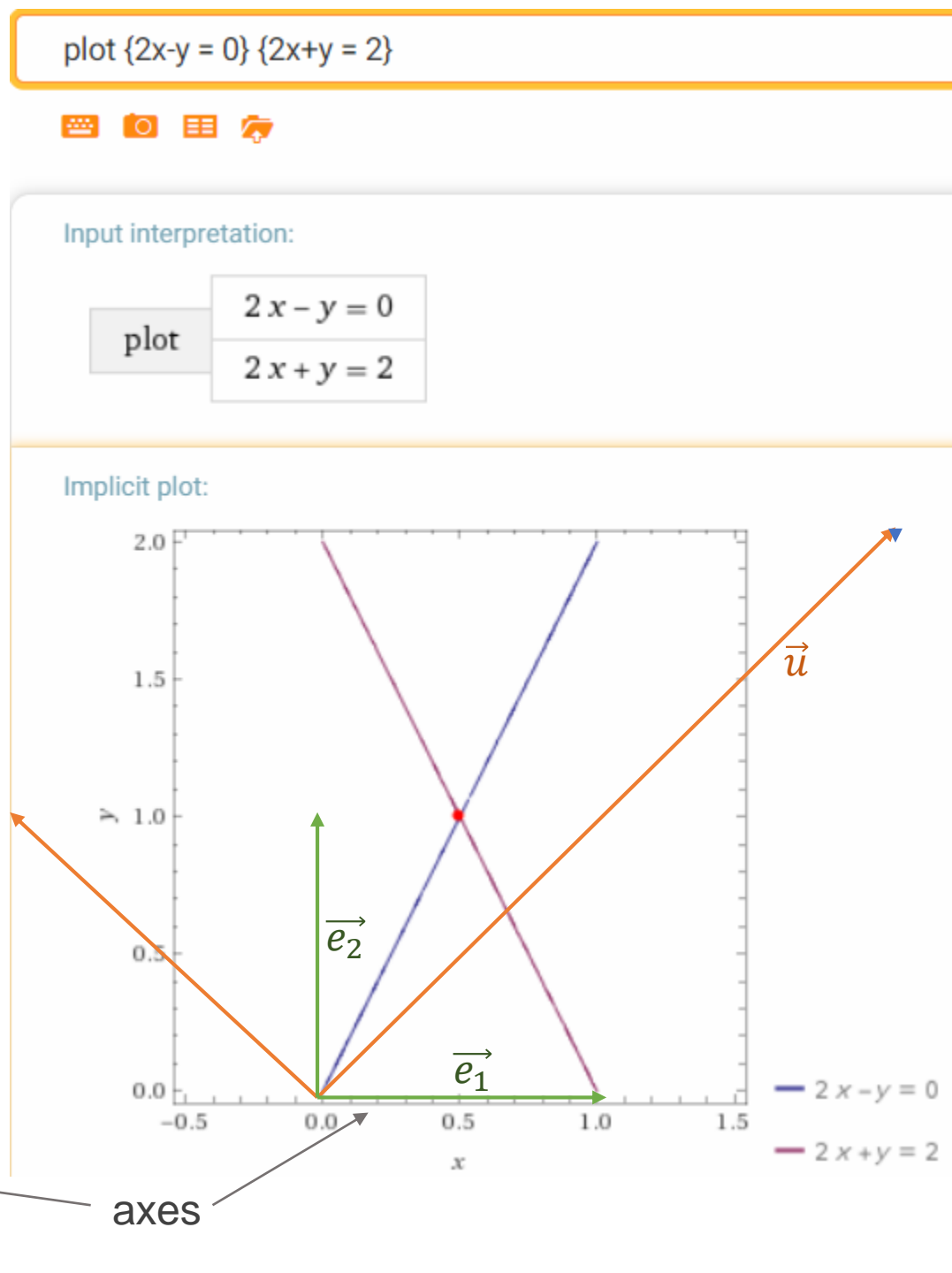
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

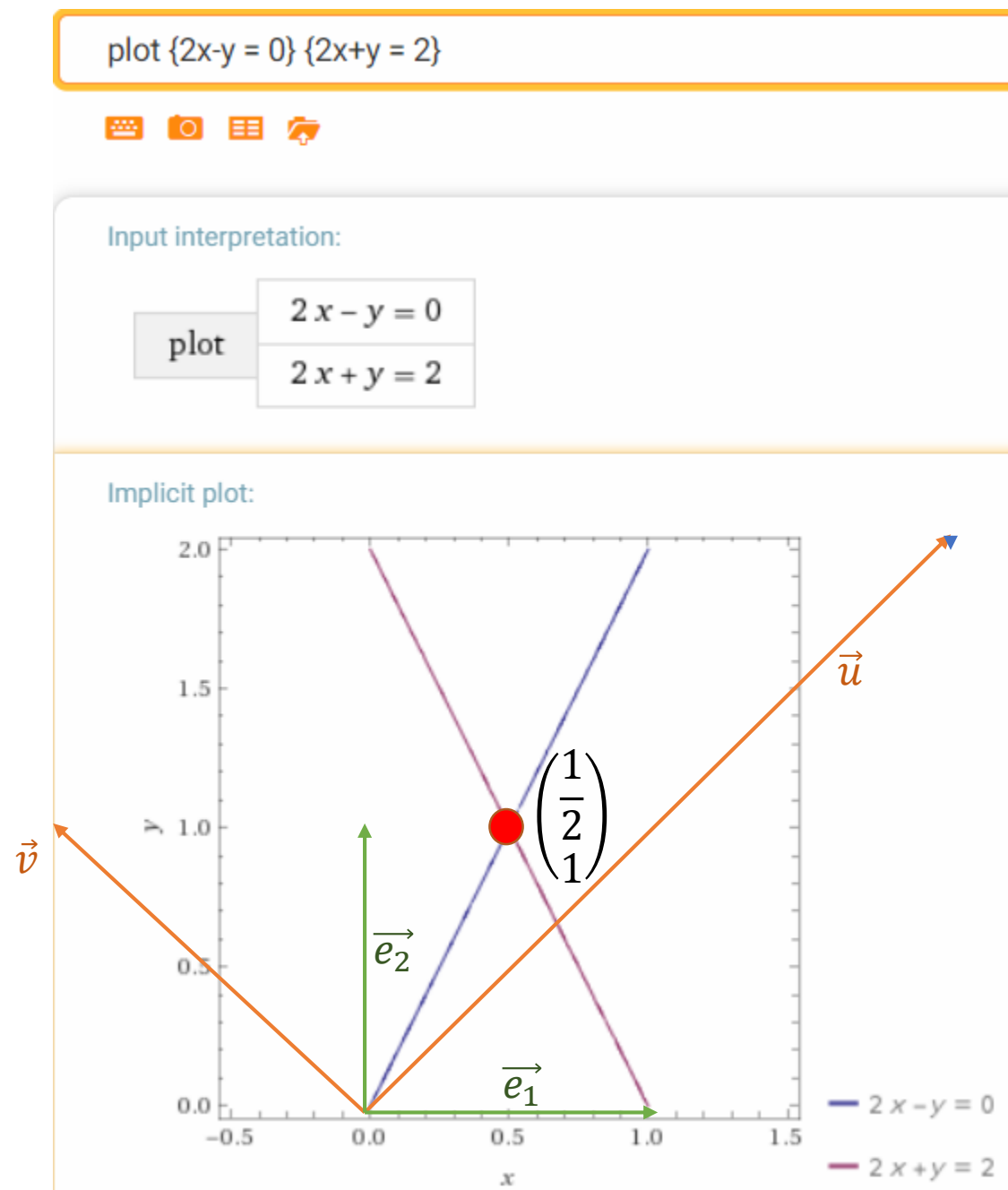
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

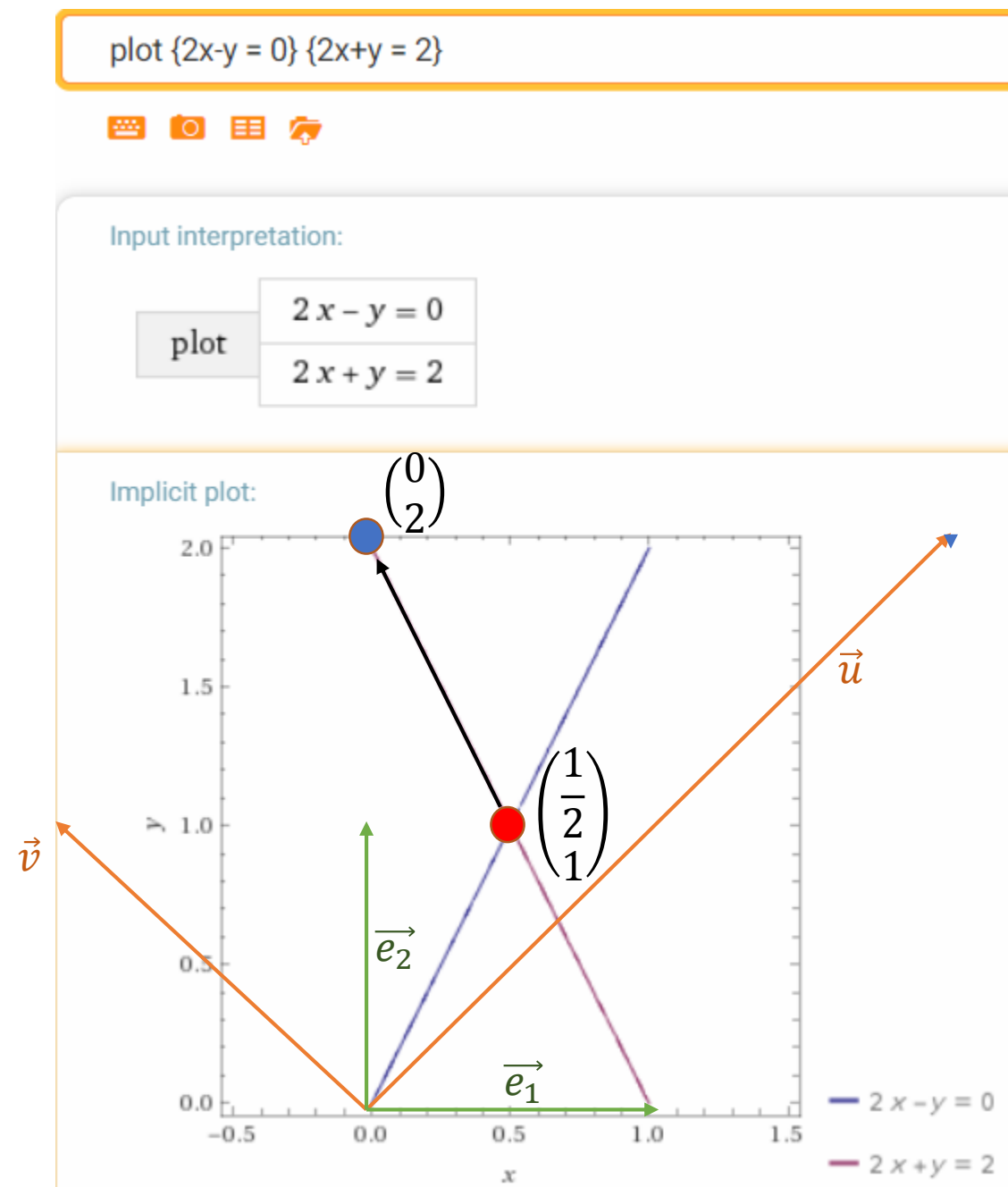
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

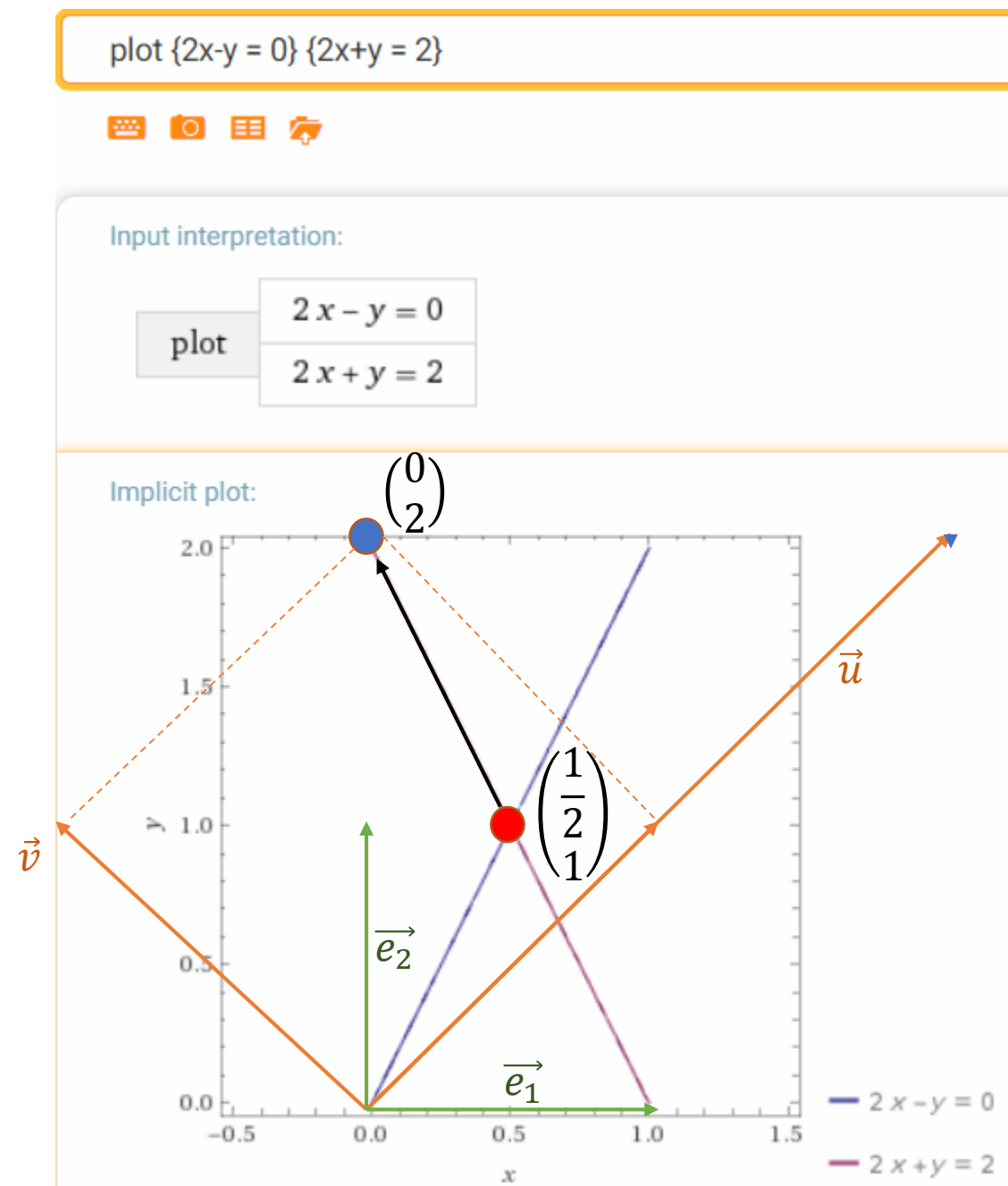
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$x\vec{u} + y\vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

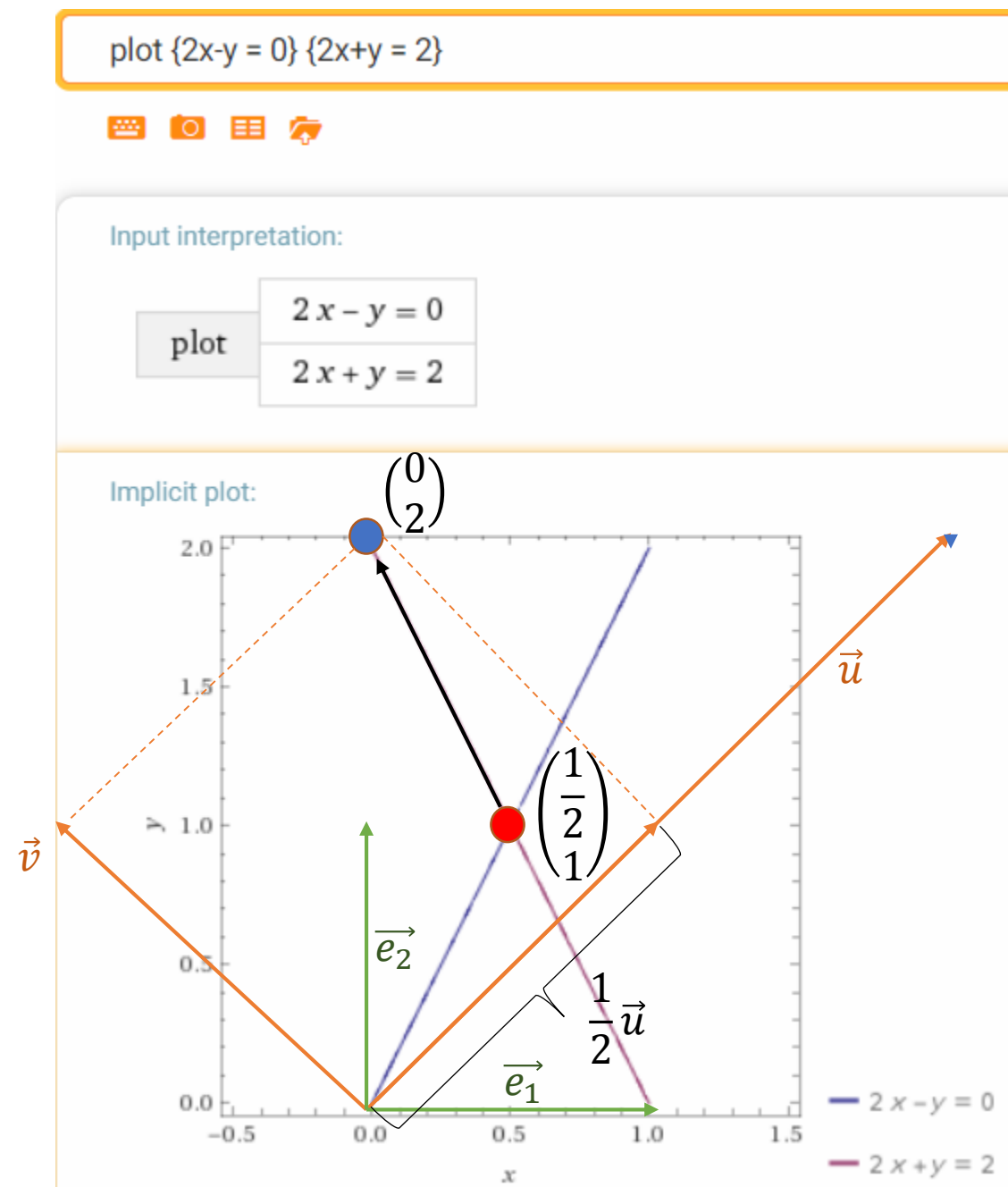
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\frac{1}{2}\vec{u} + y\vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

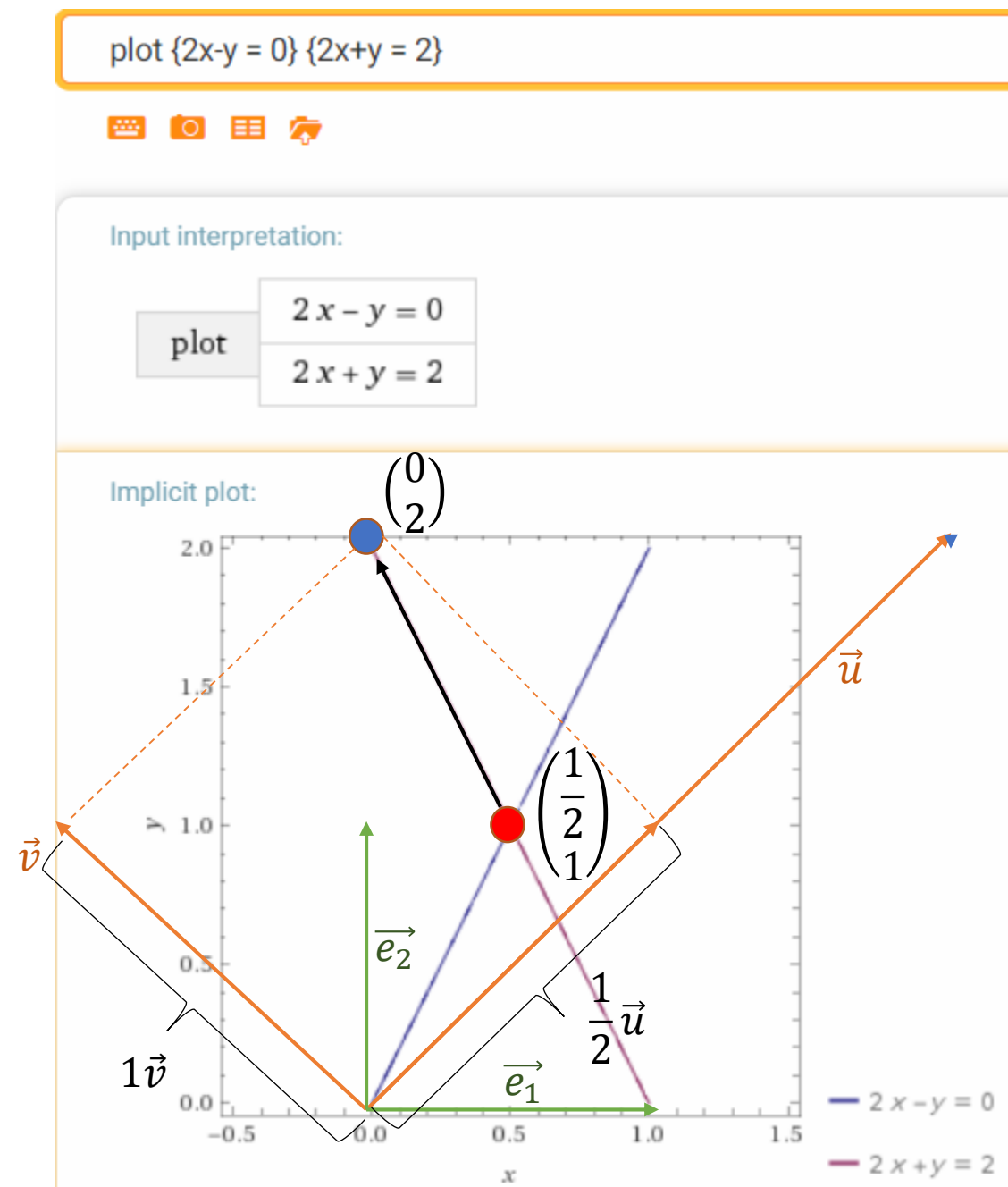
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\frac{1}{2} \vec{u} + 1 \vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

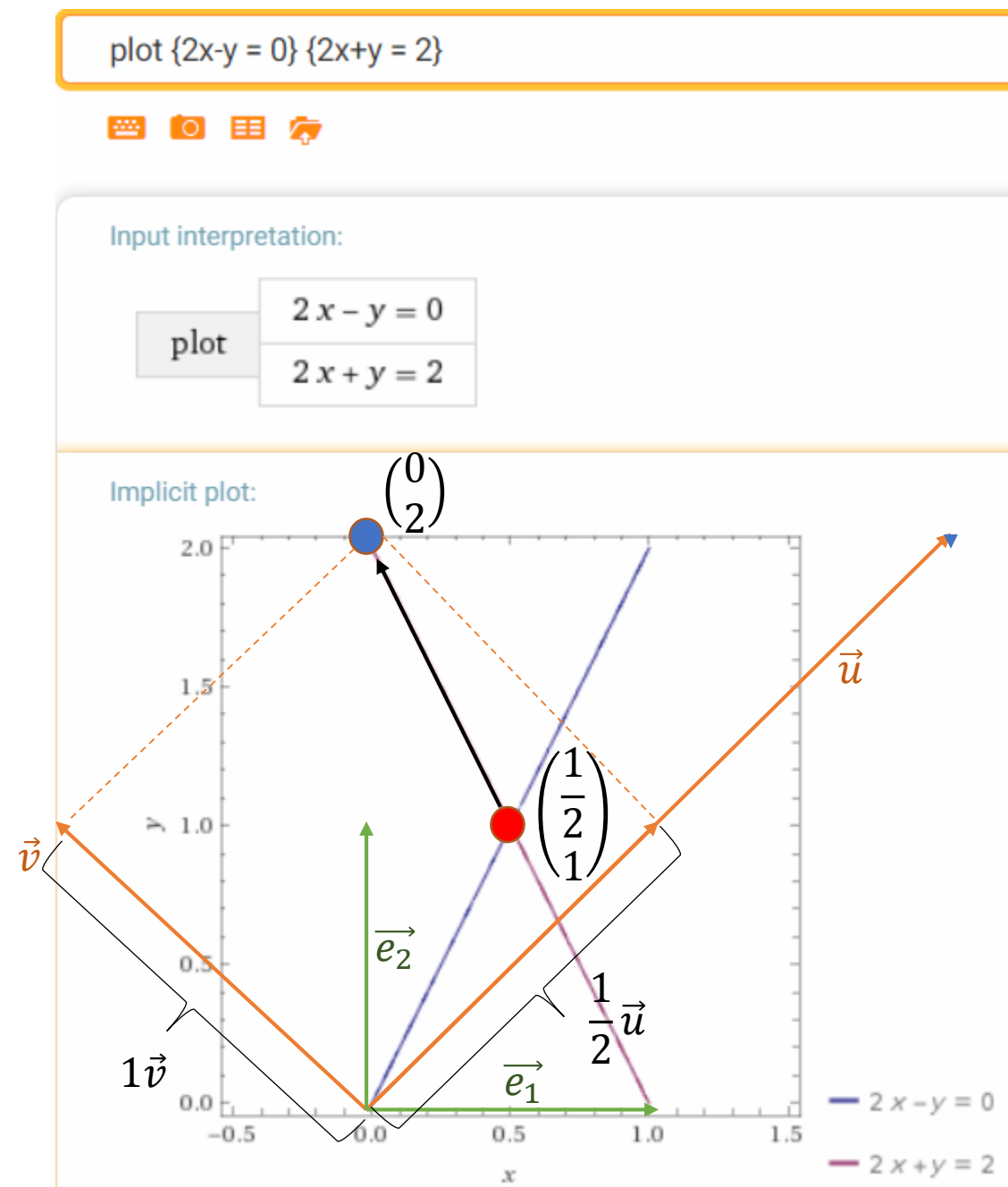


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

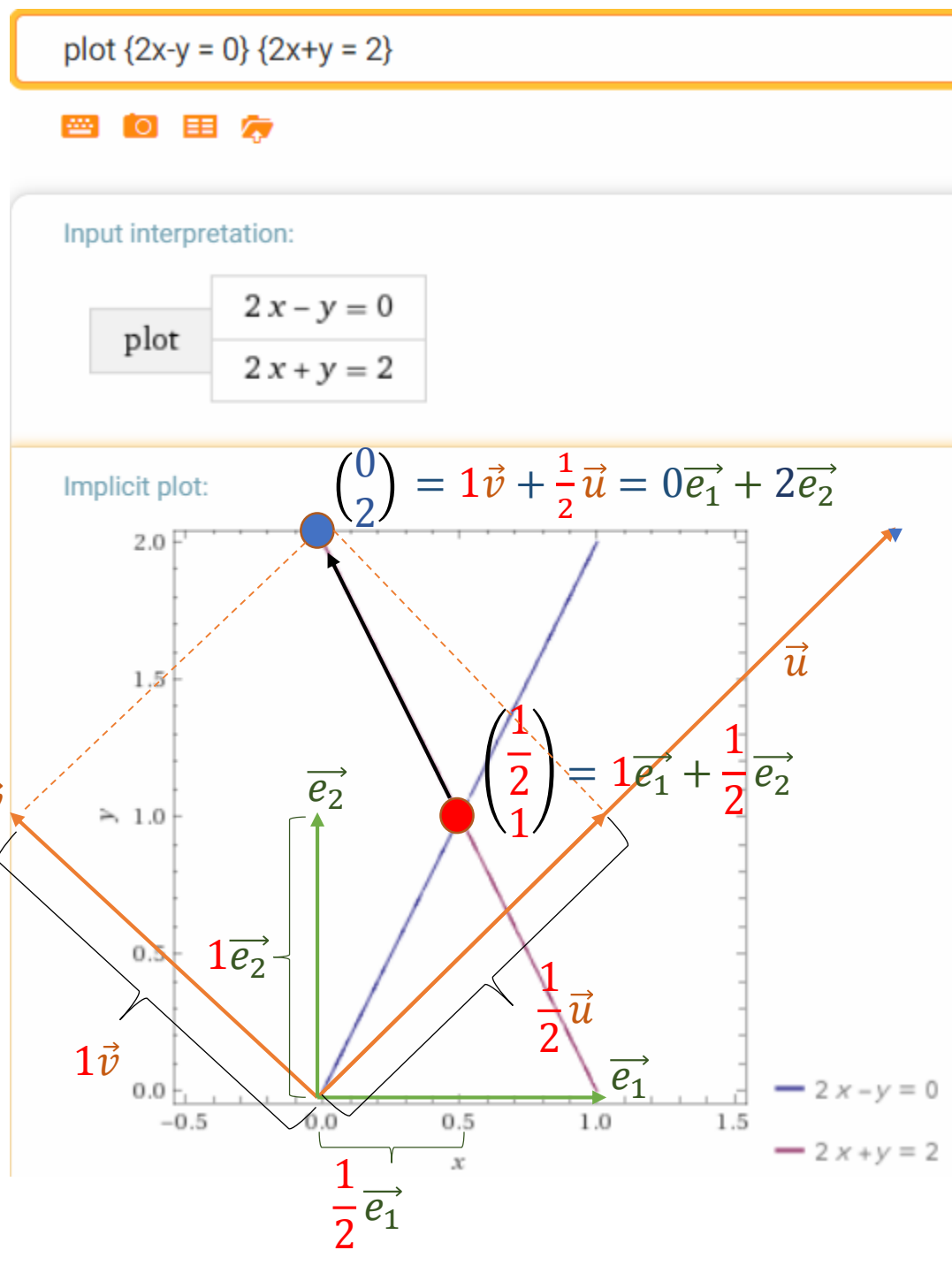


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

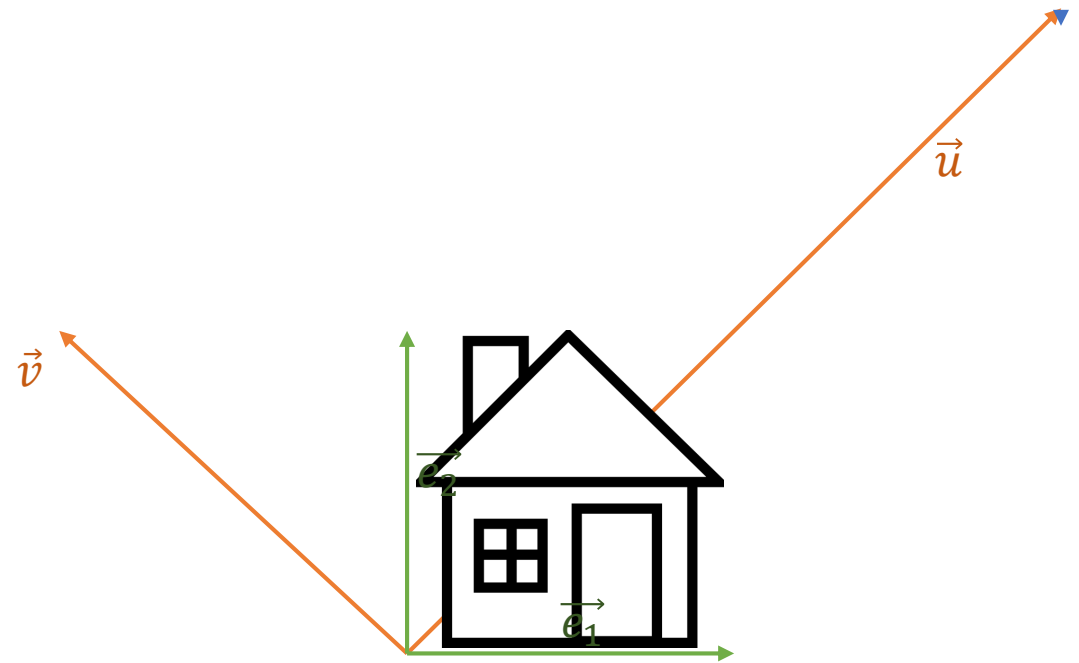


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

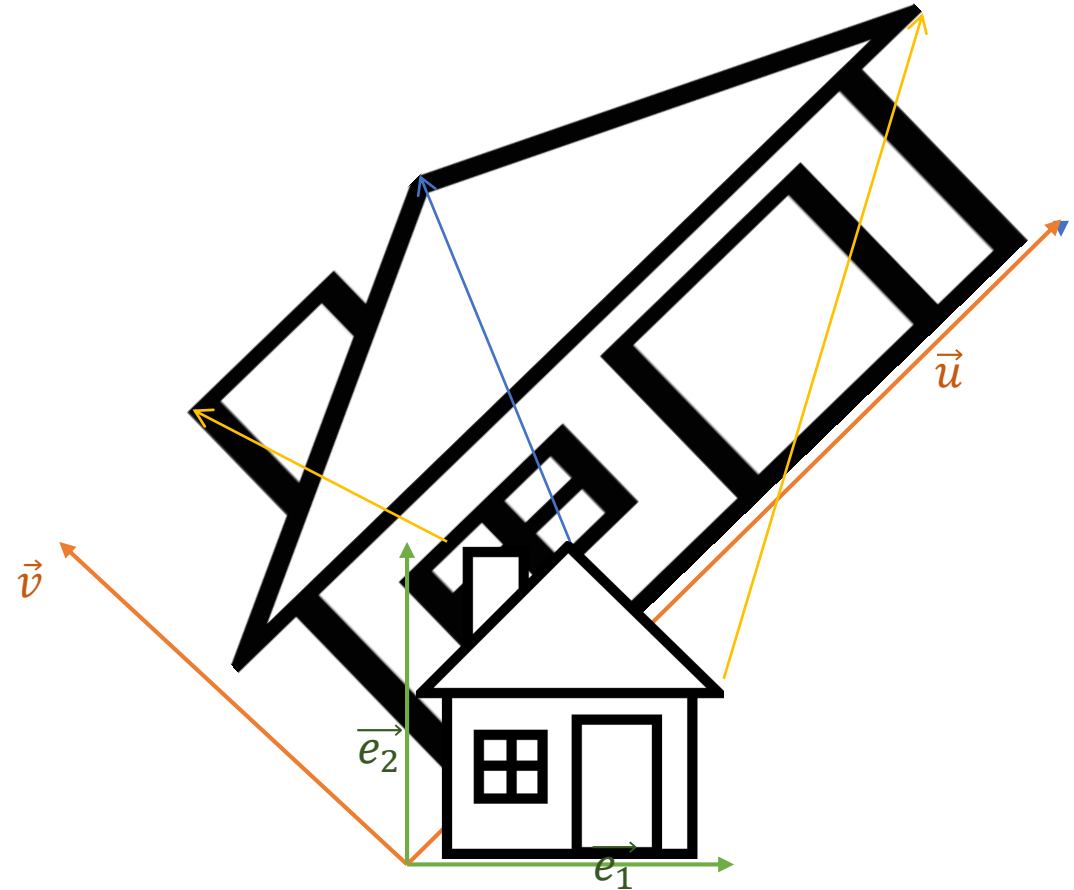


Matrix as a transformation

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

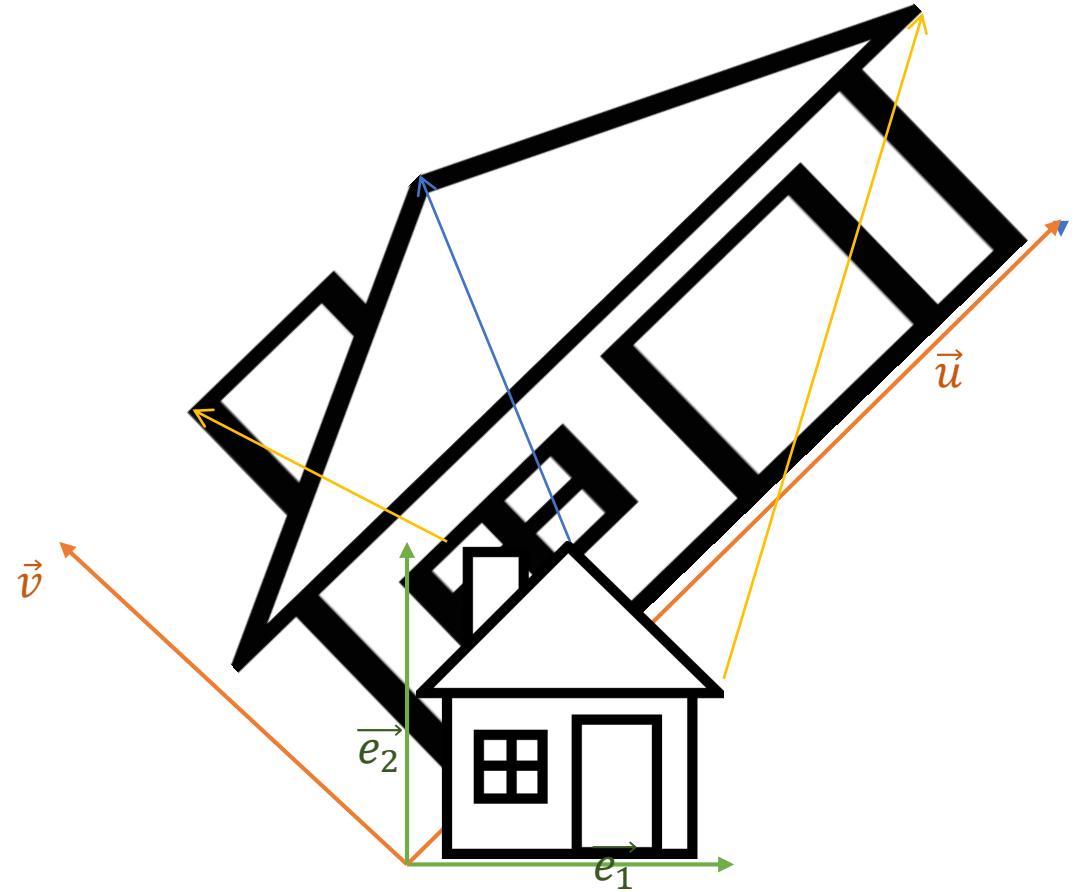


Matrix as a transformation

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



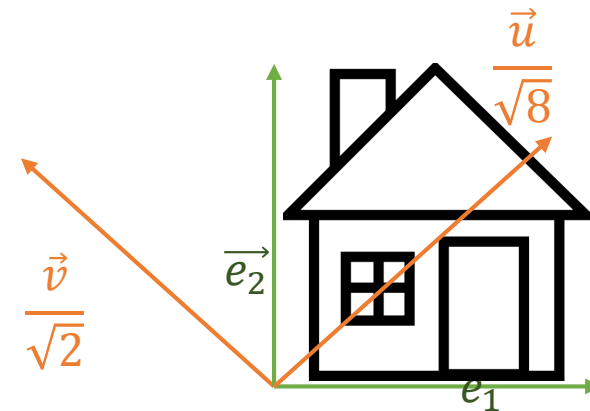
Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



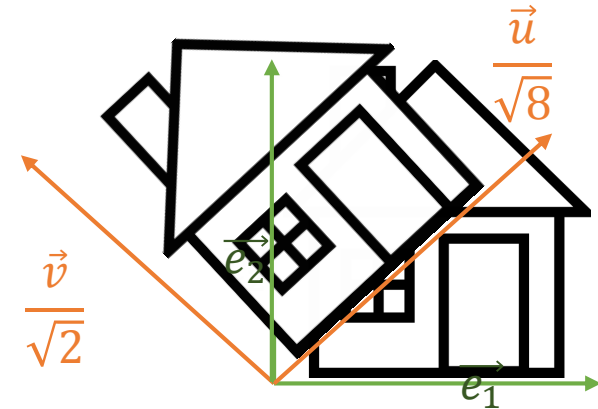
Matrix as a transformation

$$\begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



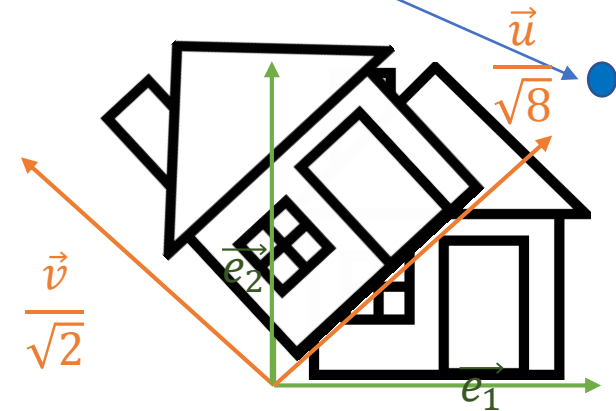
Matrix as a transformation

$$\begin{pmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



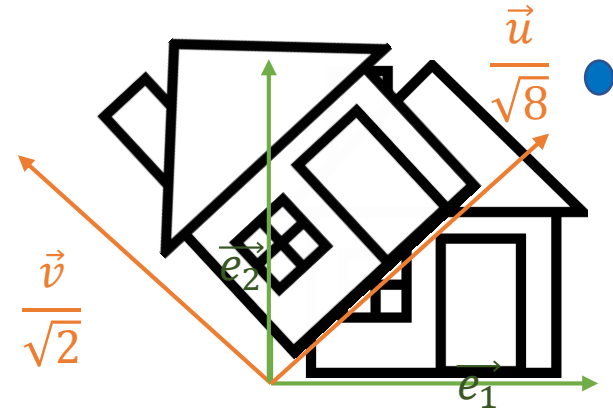
Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = ?$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



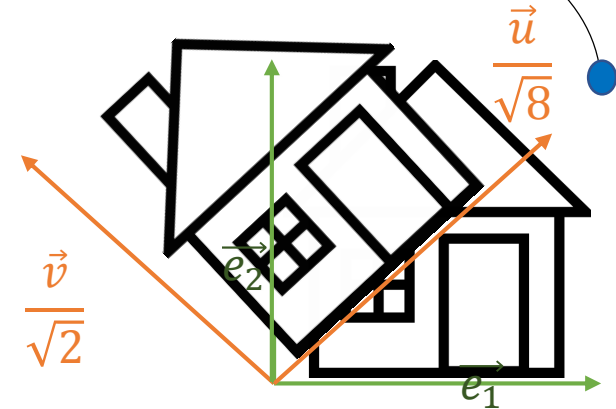
Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

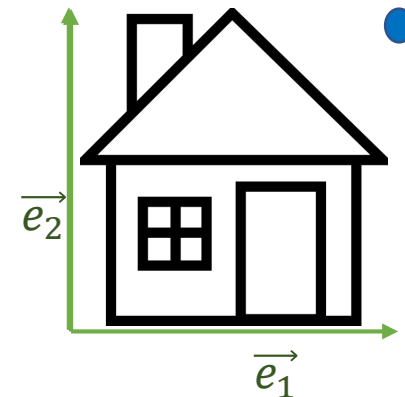
$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix as a transformation

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = ?$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

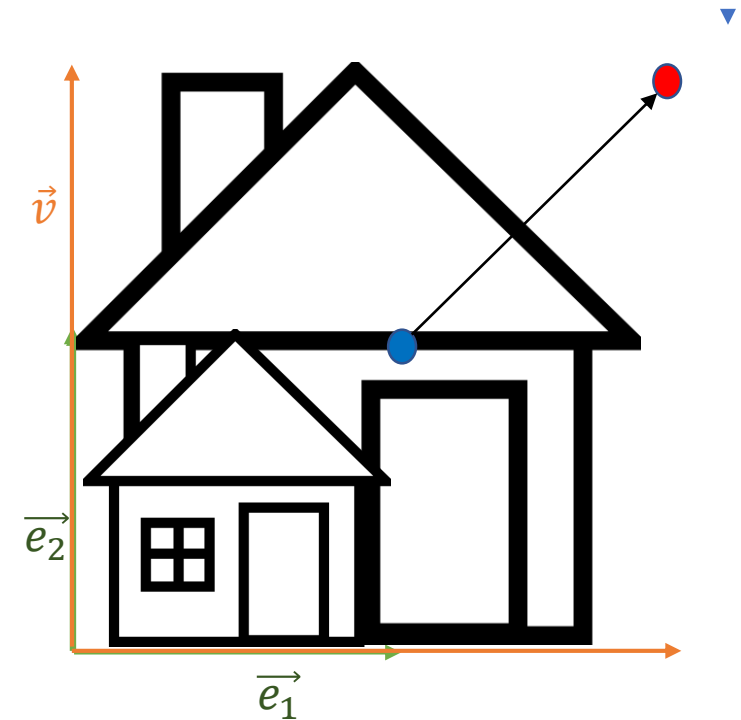


Matrix as a transformation

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}; \vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Dot product

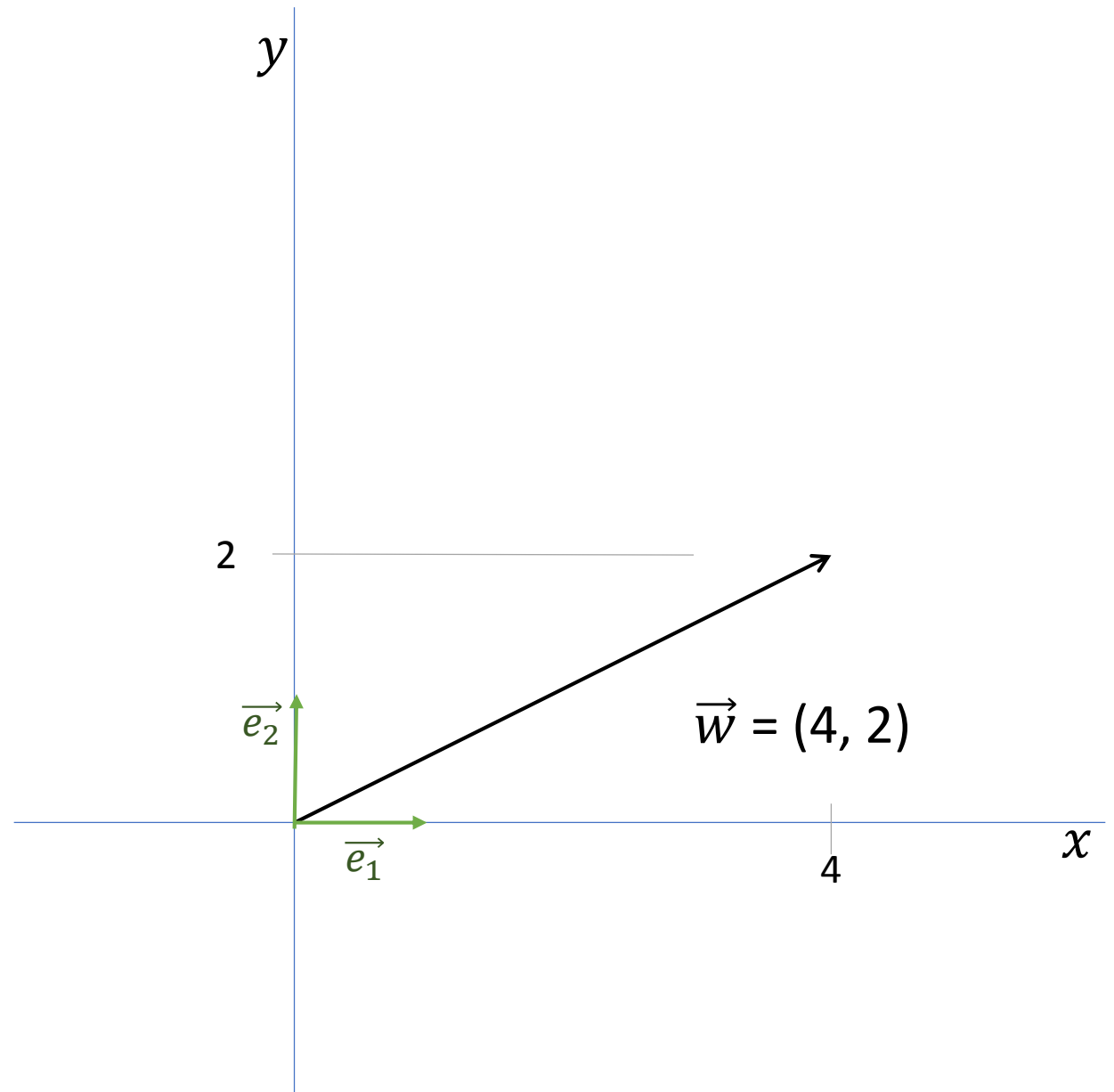
$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}; \vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = (a_x \ a_y \ a_z) \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z$$

Dot product

$$\vec{e}_1 \cdot \vec{w} = ?$$

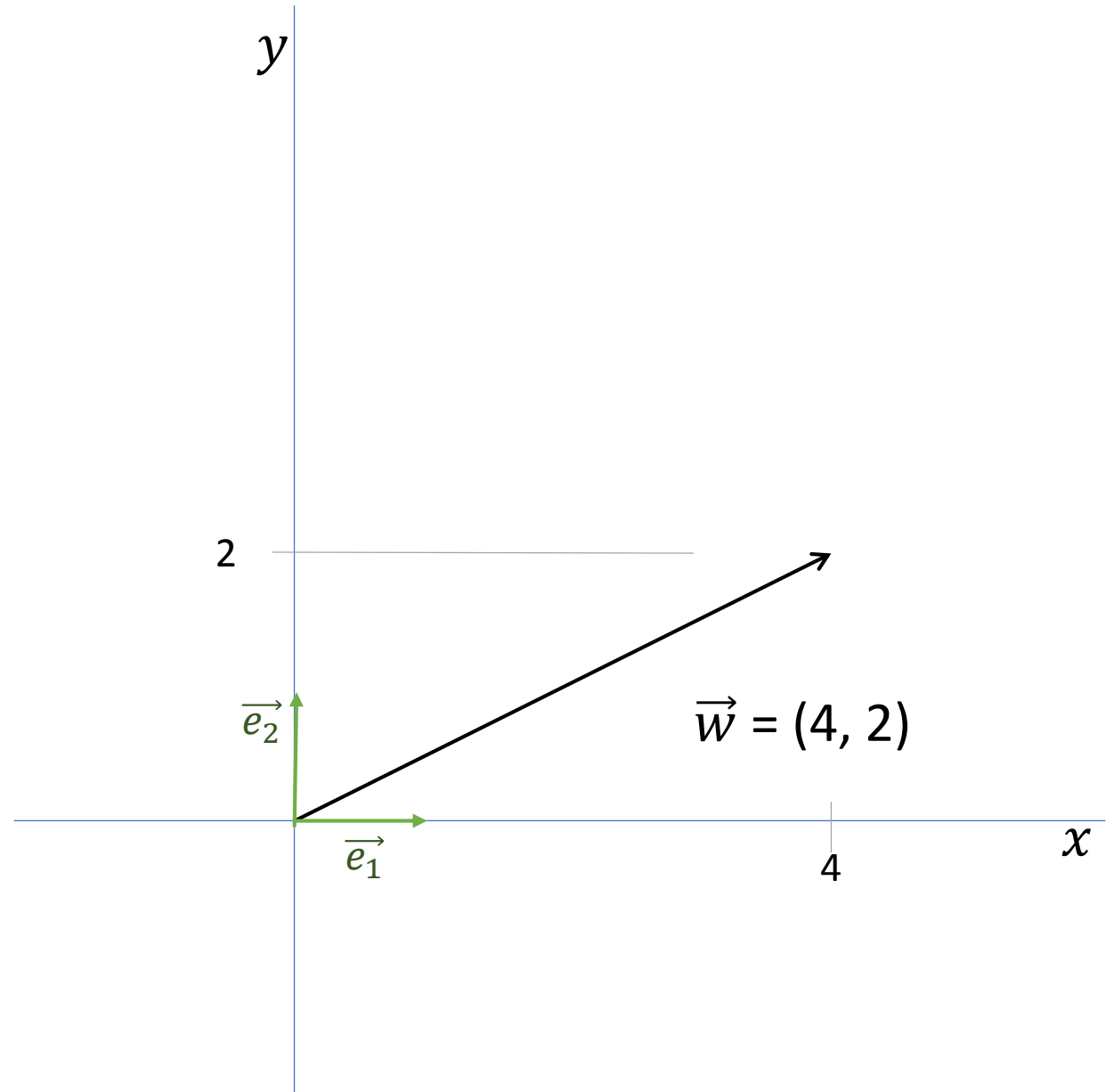
$$\vec{e}_2 \cdot \vec{w} = ?$$



Dot product

$$\vec{e}_1 \cdot \vec{w} = (1 \ 0) \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 4$$

$$\vec{e}_2 \cdot \vec{w} = (0 \ 1) \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2$$

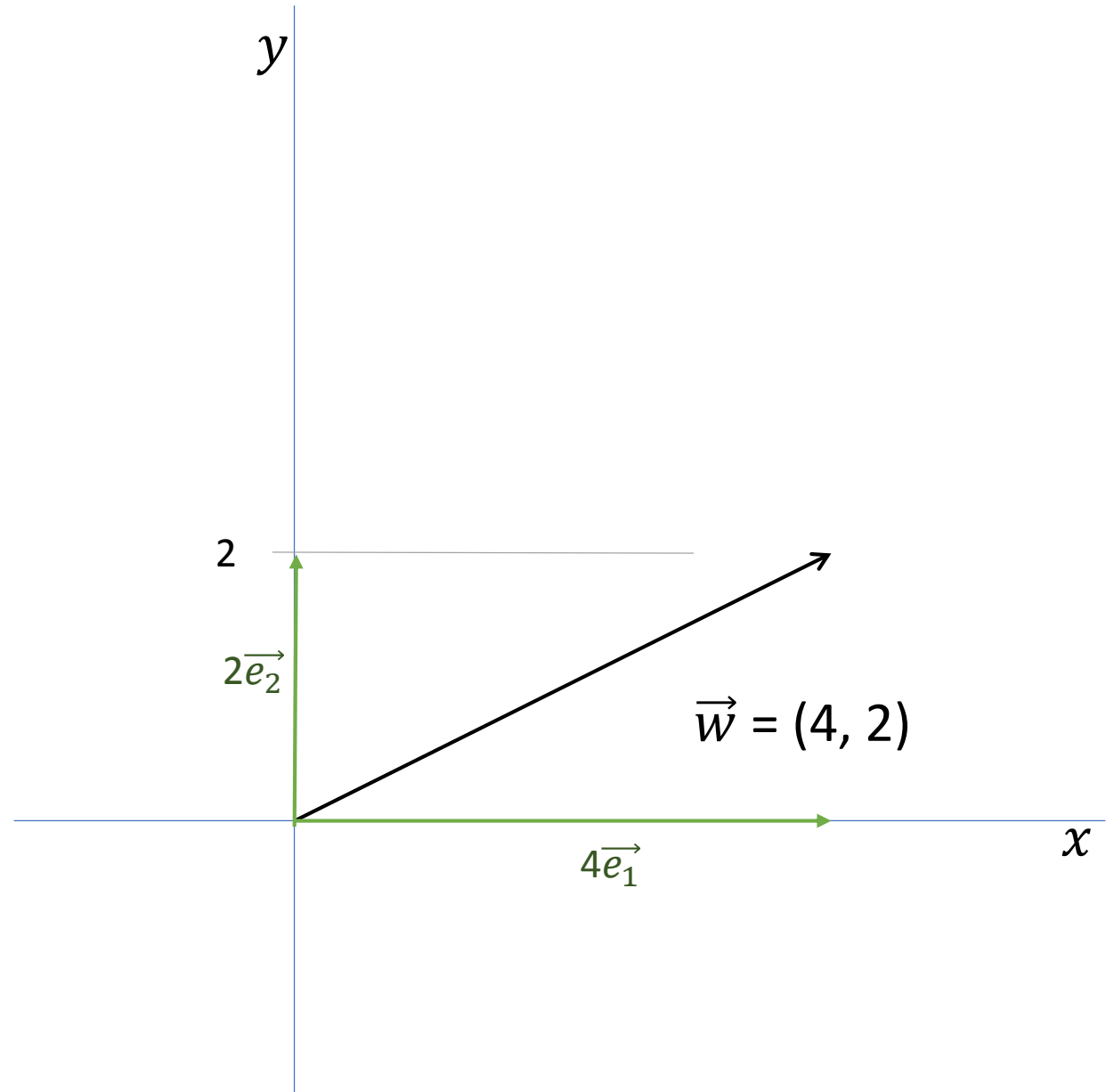


Dot product

$$\vec{e}_1 \cdot \vec{w} = (1 \ 0) \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 4$$

$$\vec{e}_2 \cdot \vec{w} = (0 \ 1) \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2$$

$$\vec{w} = 4\vec{e}_1 + 2\vec{e}_2$$



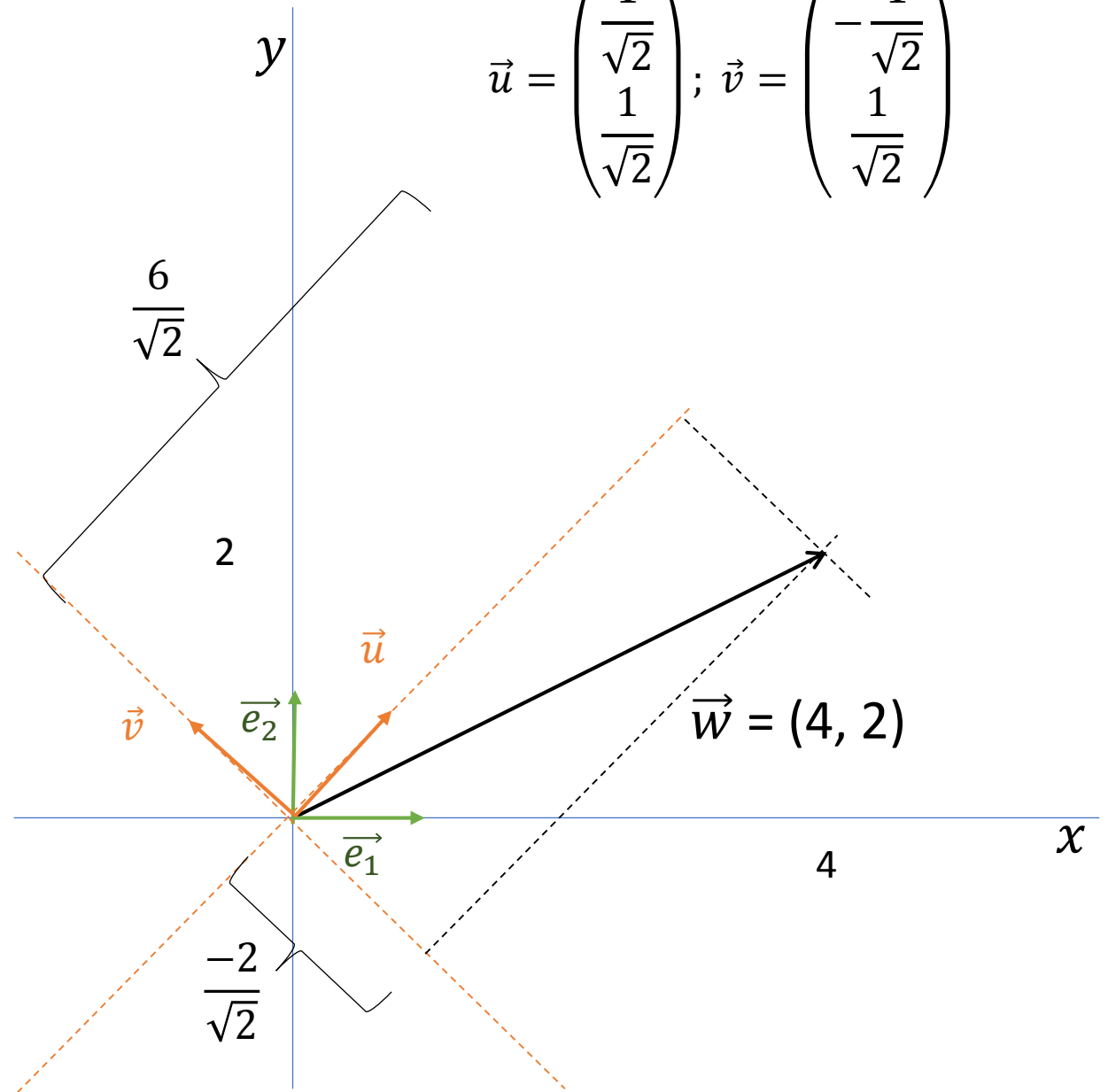
Dot product

$$\vec{u} = \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \vec{v} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} \cdot \vec{w} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \frac{6}{\sqrt{2}}$$

$$\vec{v} \cdot \vec{w} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \frac{-2}{\sqrt{2}}$$

$$\vec{w} = \frac{6}{\sqrt{2}} \vec{u} + \frac{-2}{\sqrt{2}} \vec{v}$$



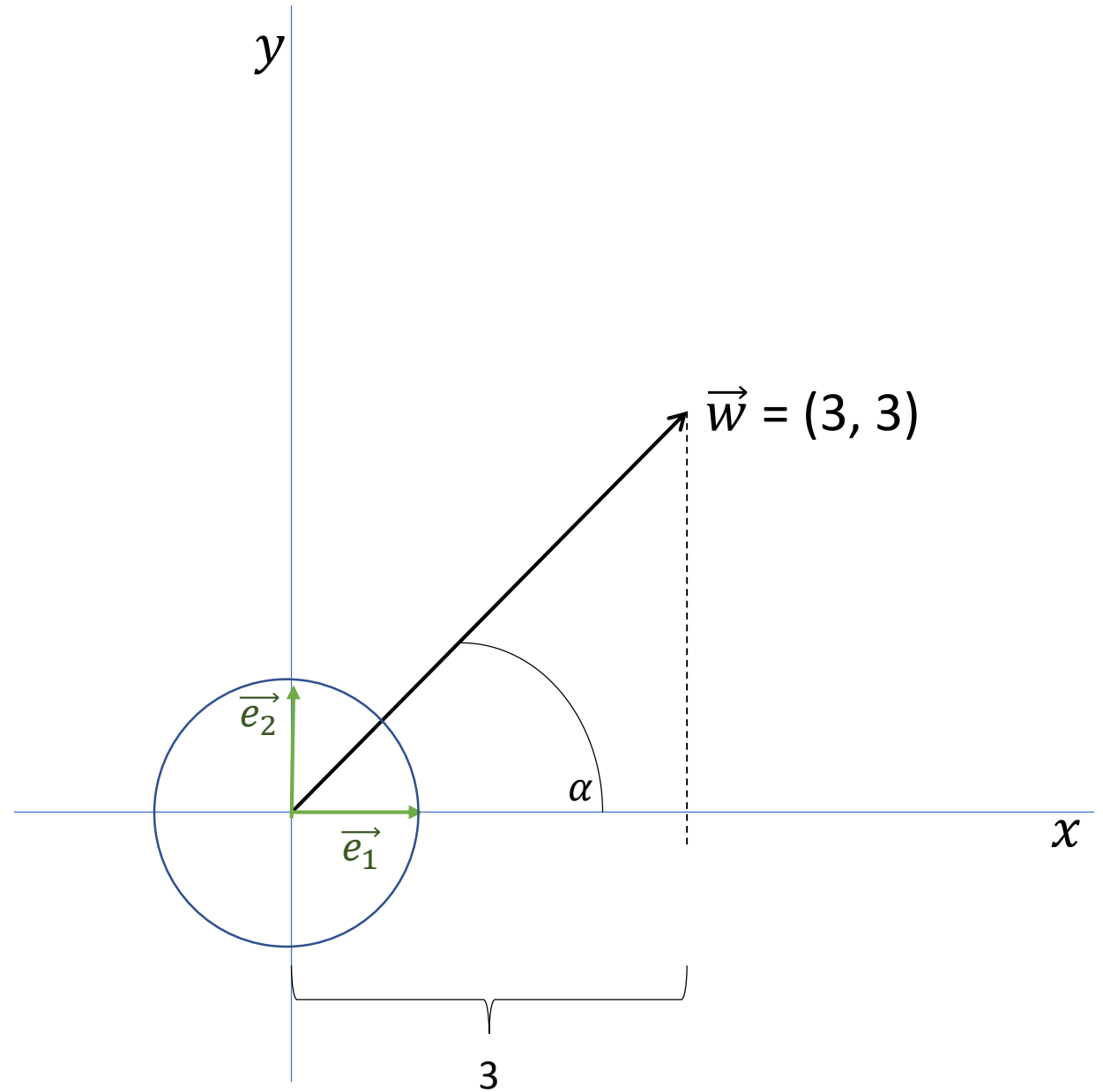
Dot product

$$\vec{e}_1 \cdot \vec{w} = (1 \ 0) \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3$$

$$|\vec{w}| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$|\vec{w}| \cos \alpha = \vec{e}_1 \cdot \vec{w}$$

$$\frac{\vec{e}_1 \cdot \vec{w}}{|\vec{w}|} = \frac{3}{3\sqrt{2}} = \cos 45^\circ$$



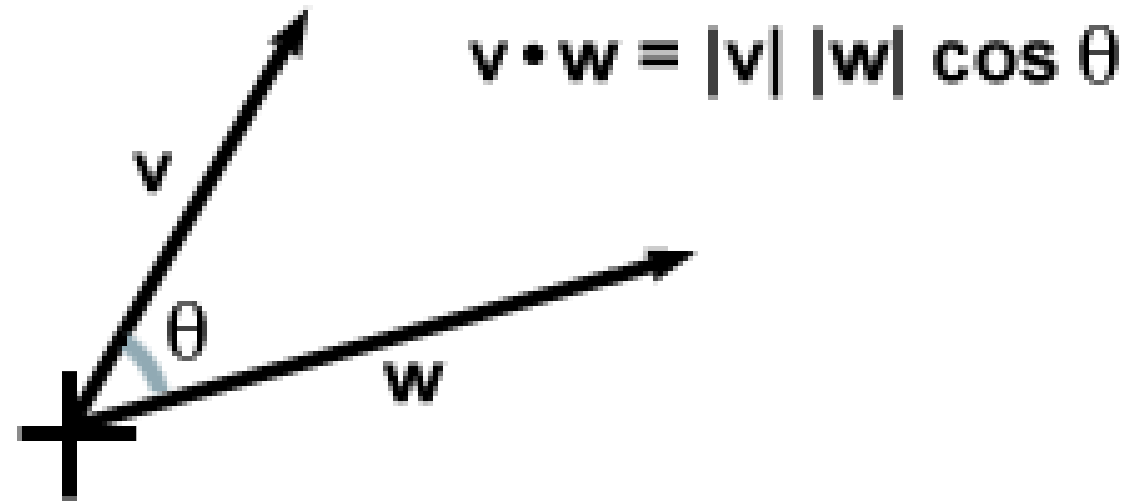
Dot product

$$\vec{v} \cdot \vec{w} = (1 \ 0) \begin{pmatrix} 3 \\ 3 \end{pmatrix} = 3$$

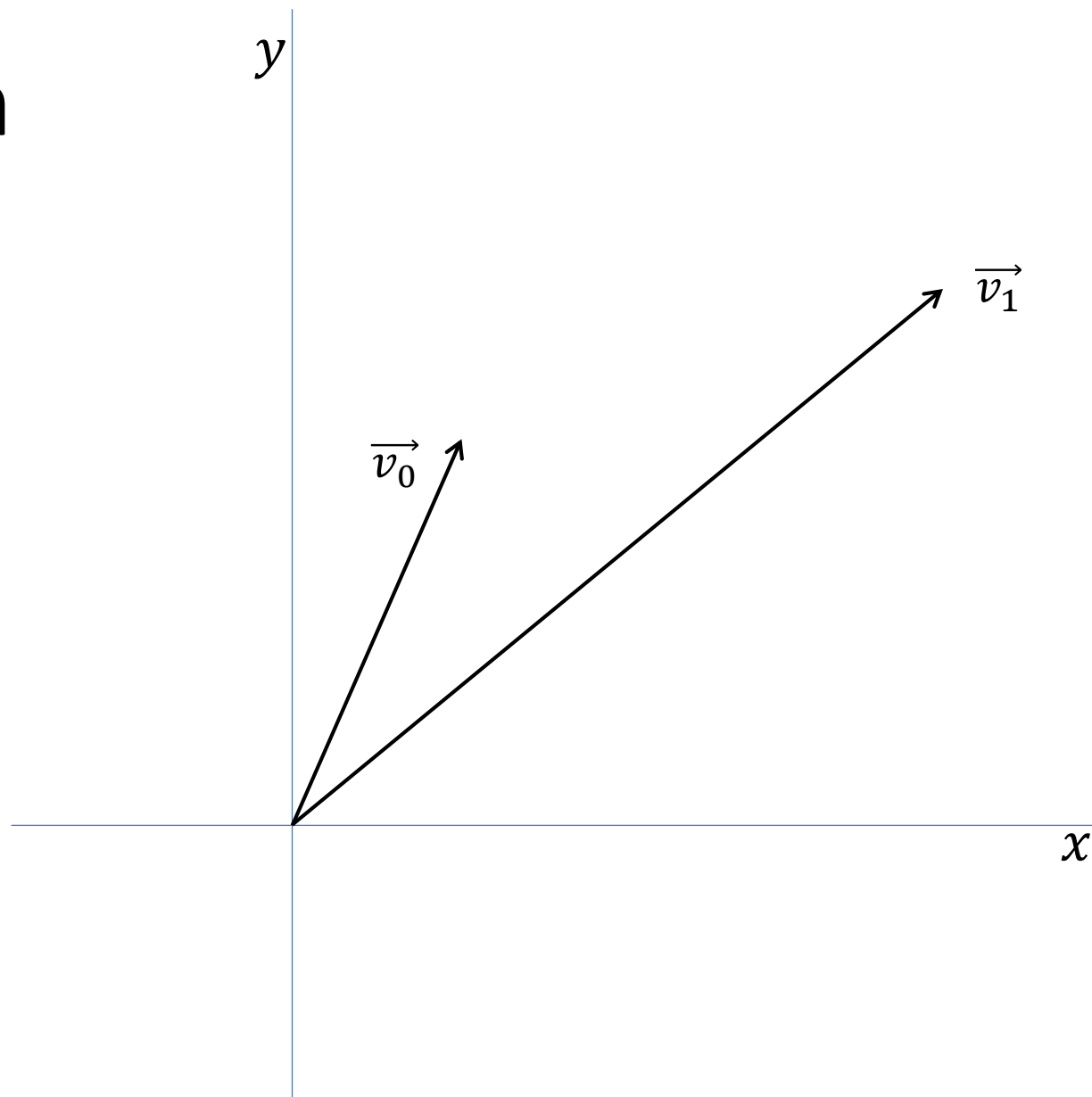
$$|\vec{w}| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$|\vec{w}| |\vec{v}| \cos \theta = \vec{v} \cdot \vec{w}$$

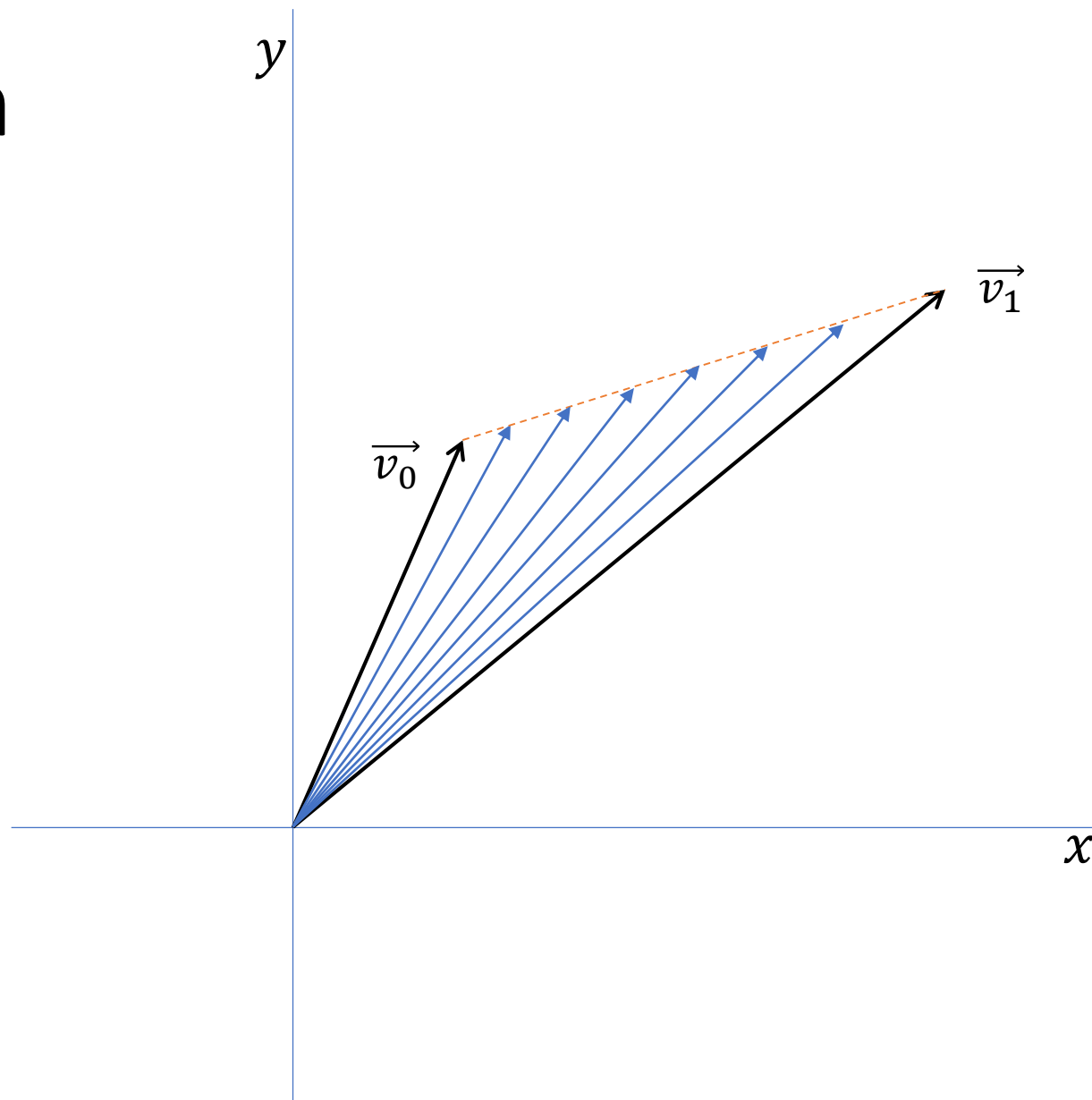
$$\frac{\vec{v} \cdot \vec{w}}{|\vec{v}| |\vec{w}|} = \frac{3}{3\sqrt{2}} = \cos 45^\circ$$



Linear interpolation

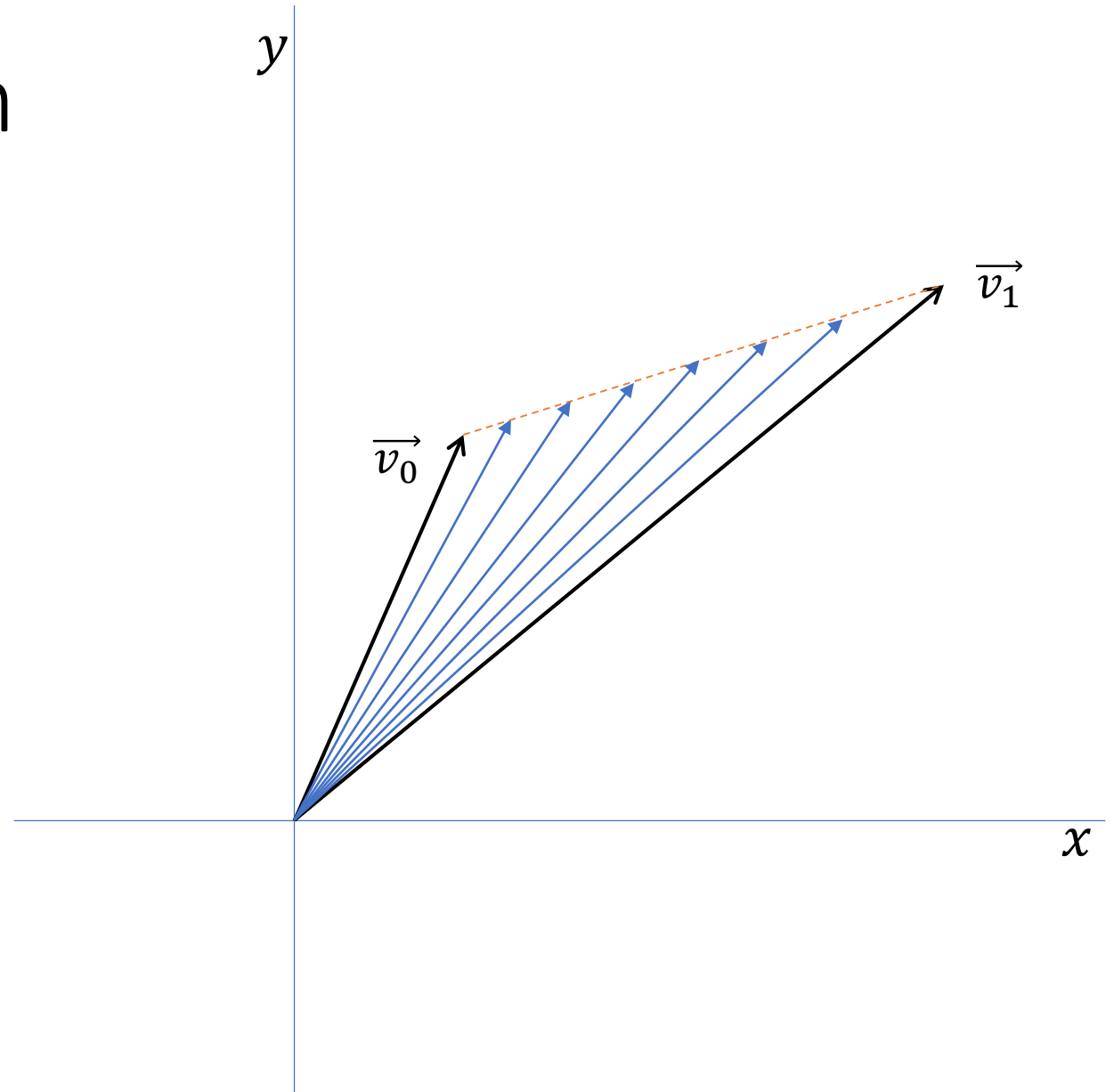


Linear interpolation



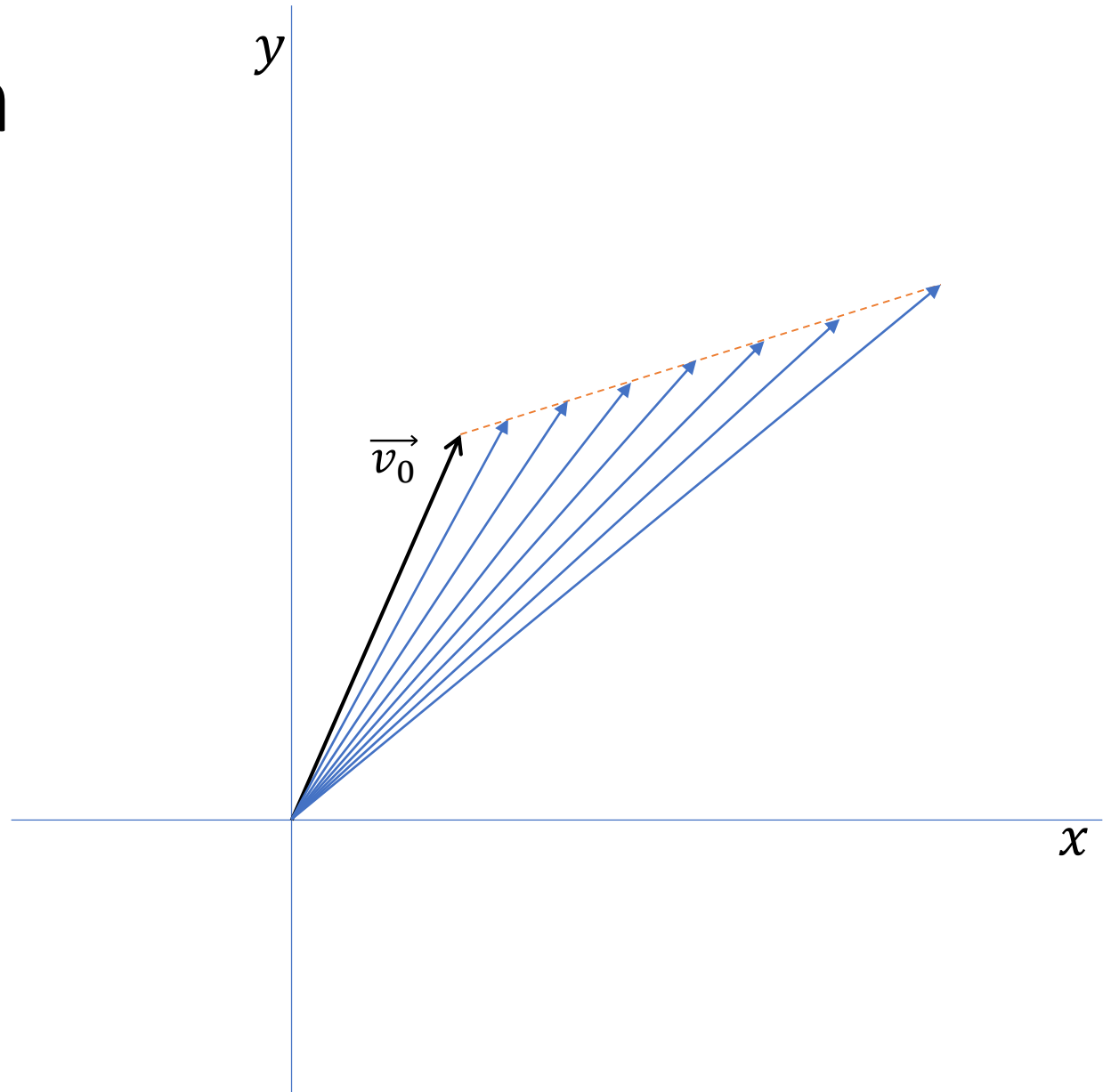
Linear interpolation

- For $t \in [0, 1]$



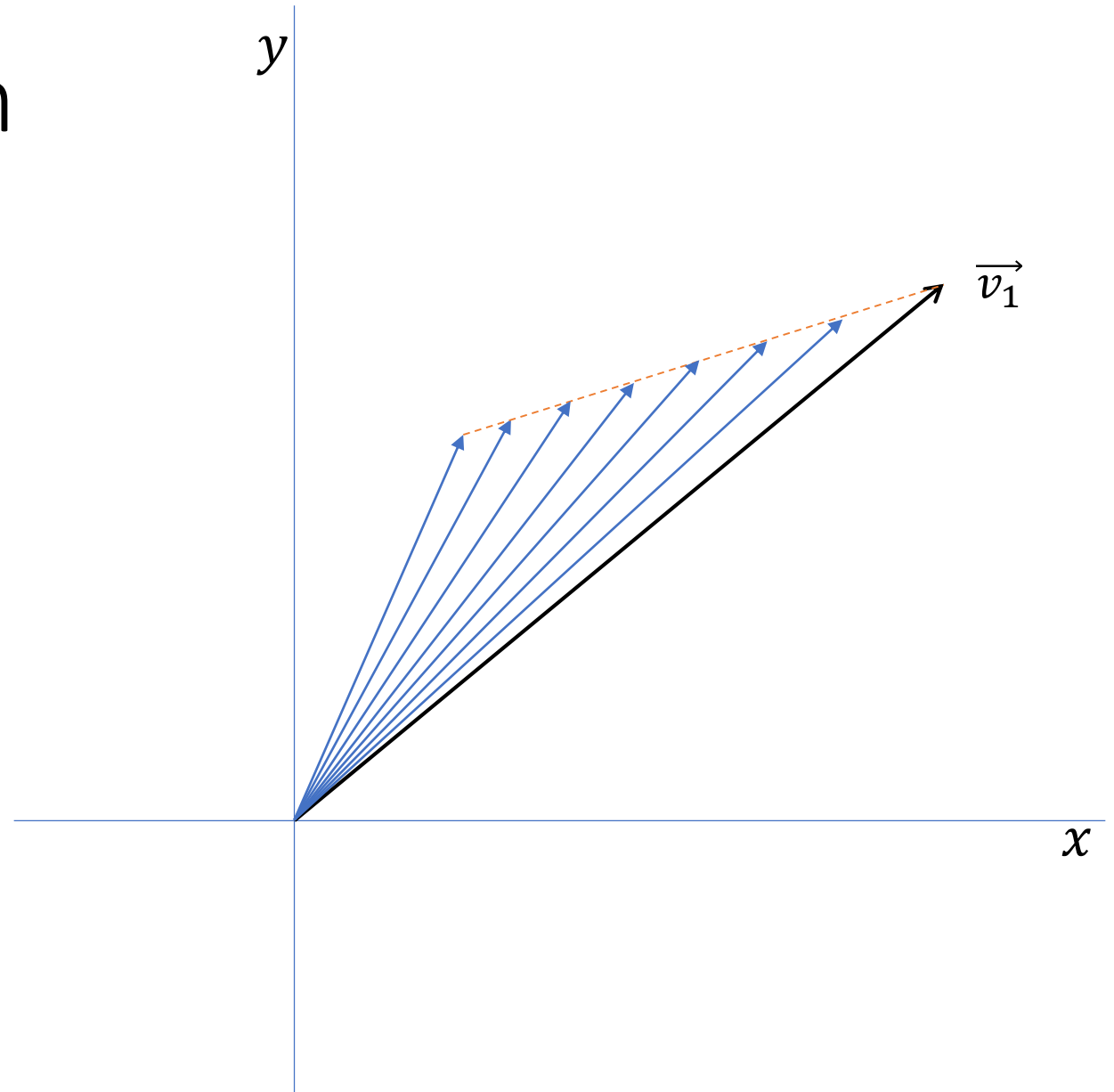
Linear interpolation

- For $t \in [0, 1]$
 - $t = 0$



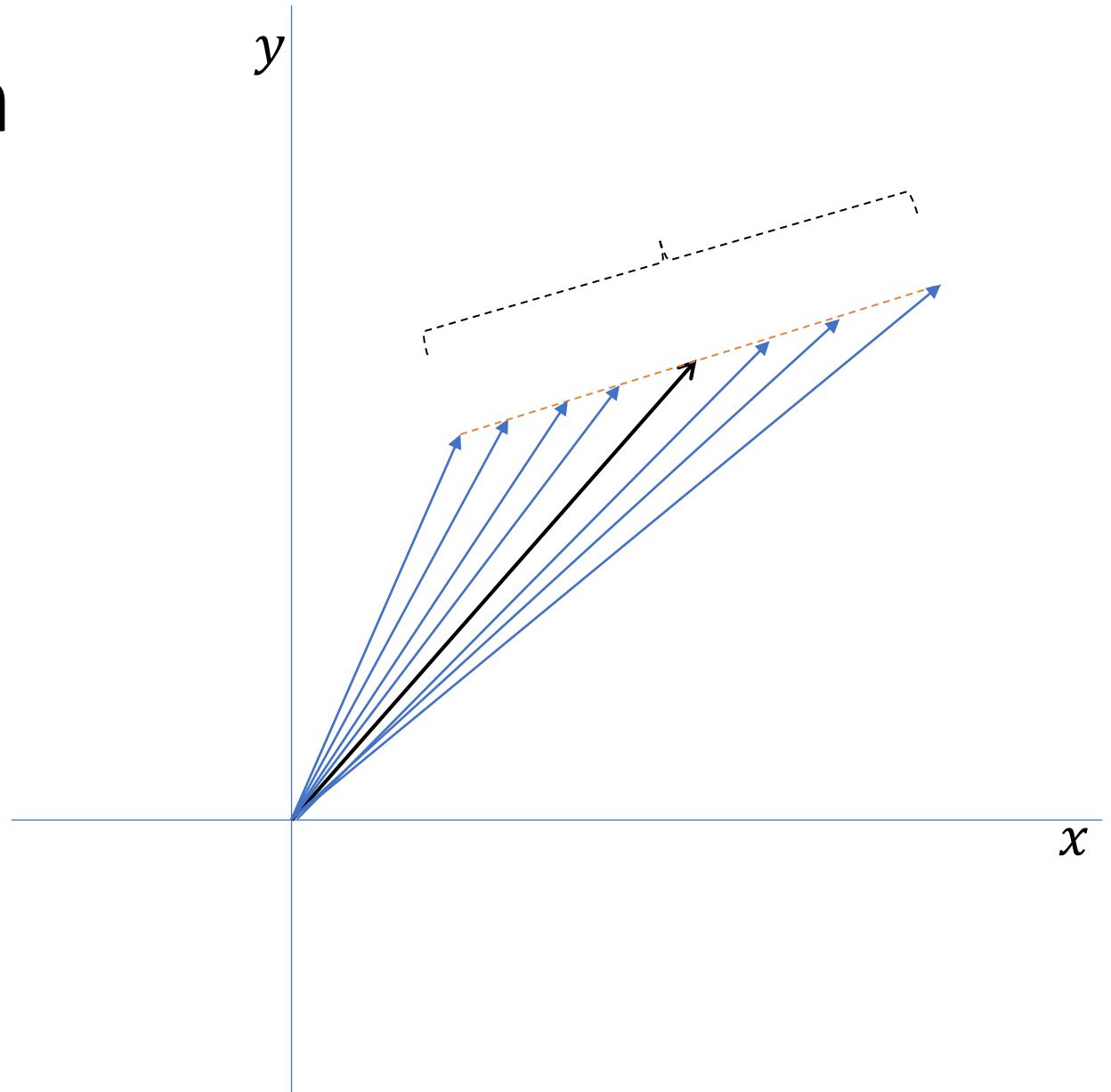
Linear interpolation

- For $t \in [0, 1]$
 - $t = 1$



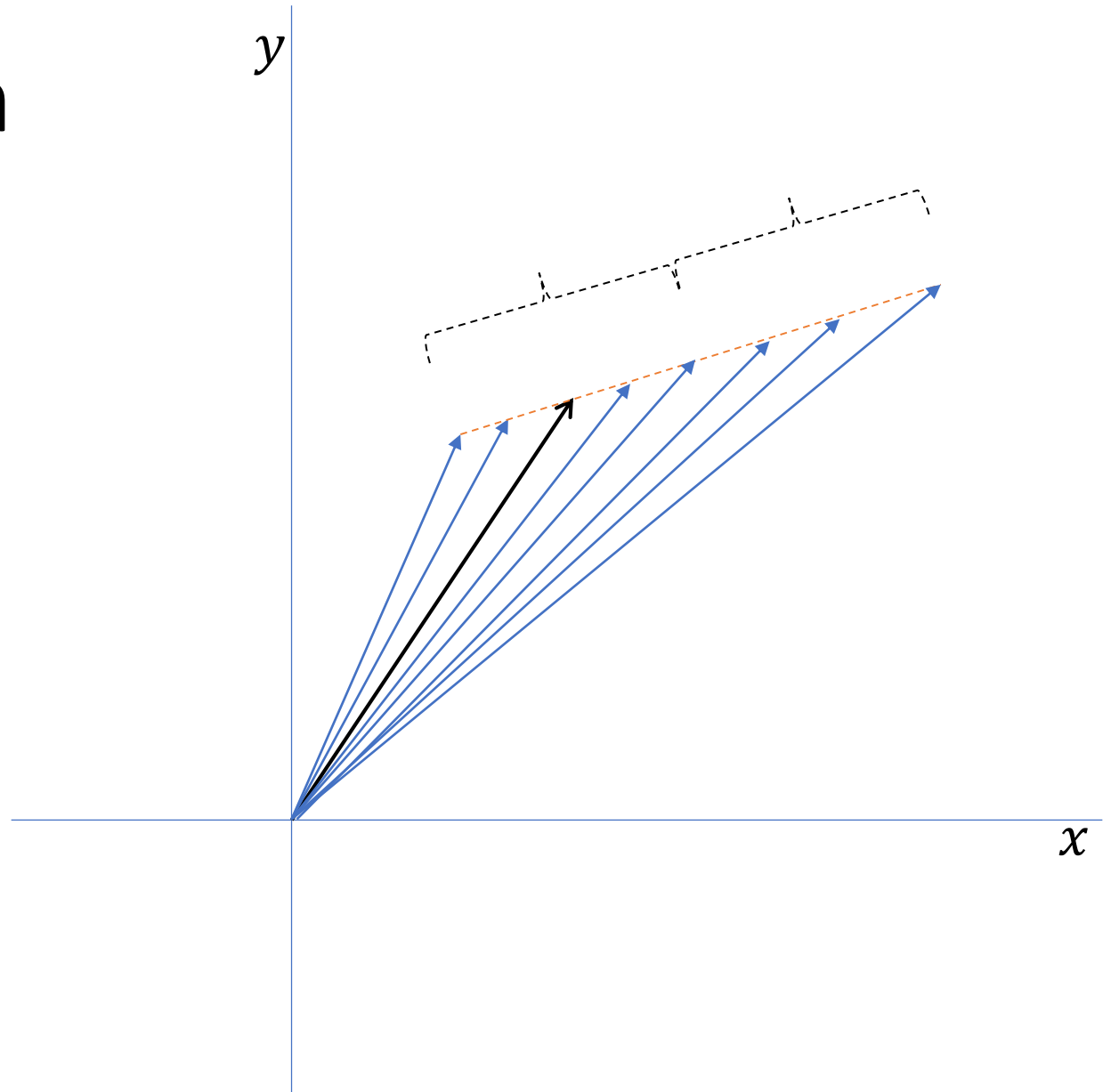
Linear interpolation

- For $t \in [0, 1]$
 - $t = \frac{1}{2}$



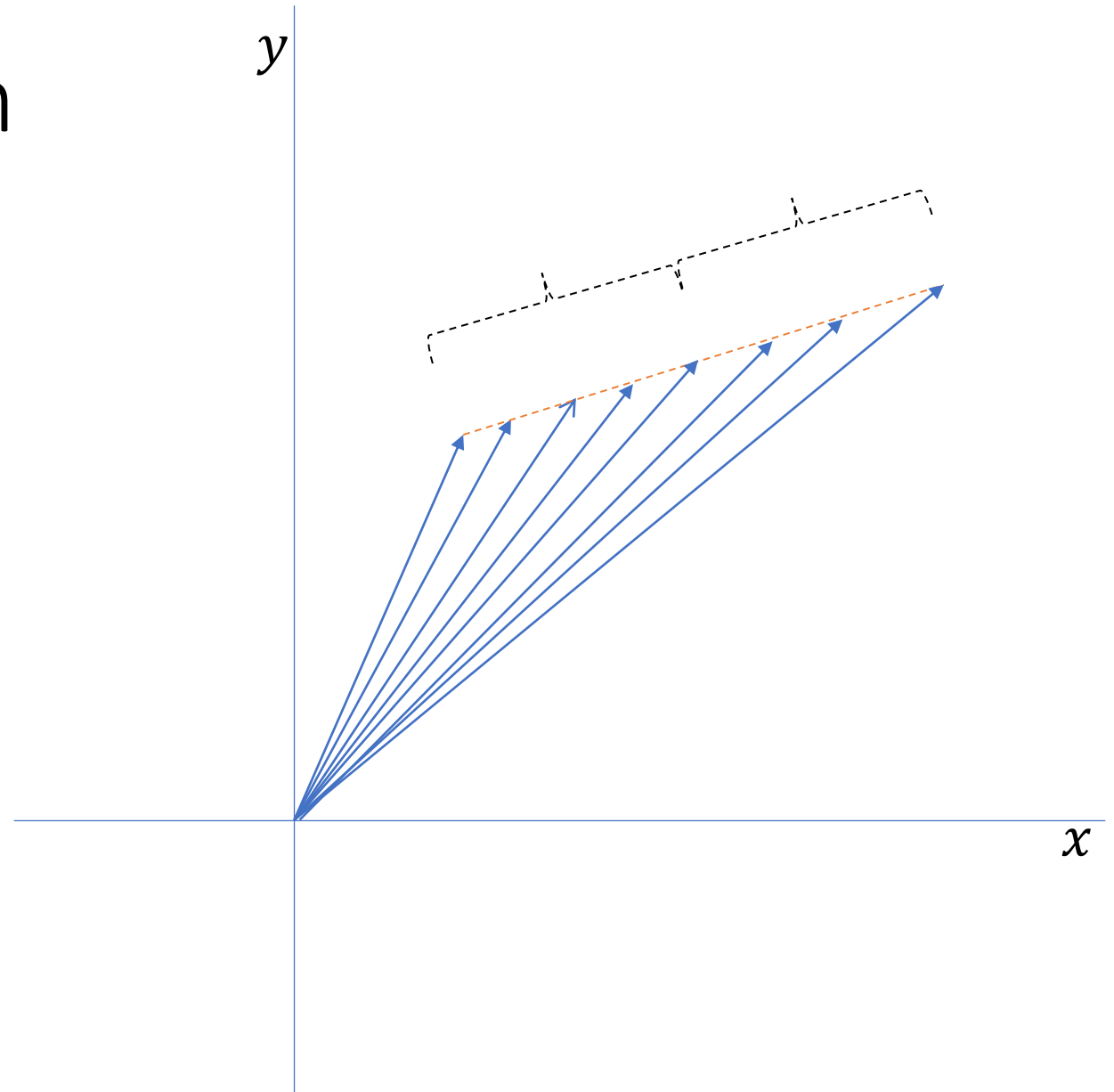
Linear interpolation

- For $t \in [0, 1]$
 - $t = \frac{1}{4}$



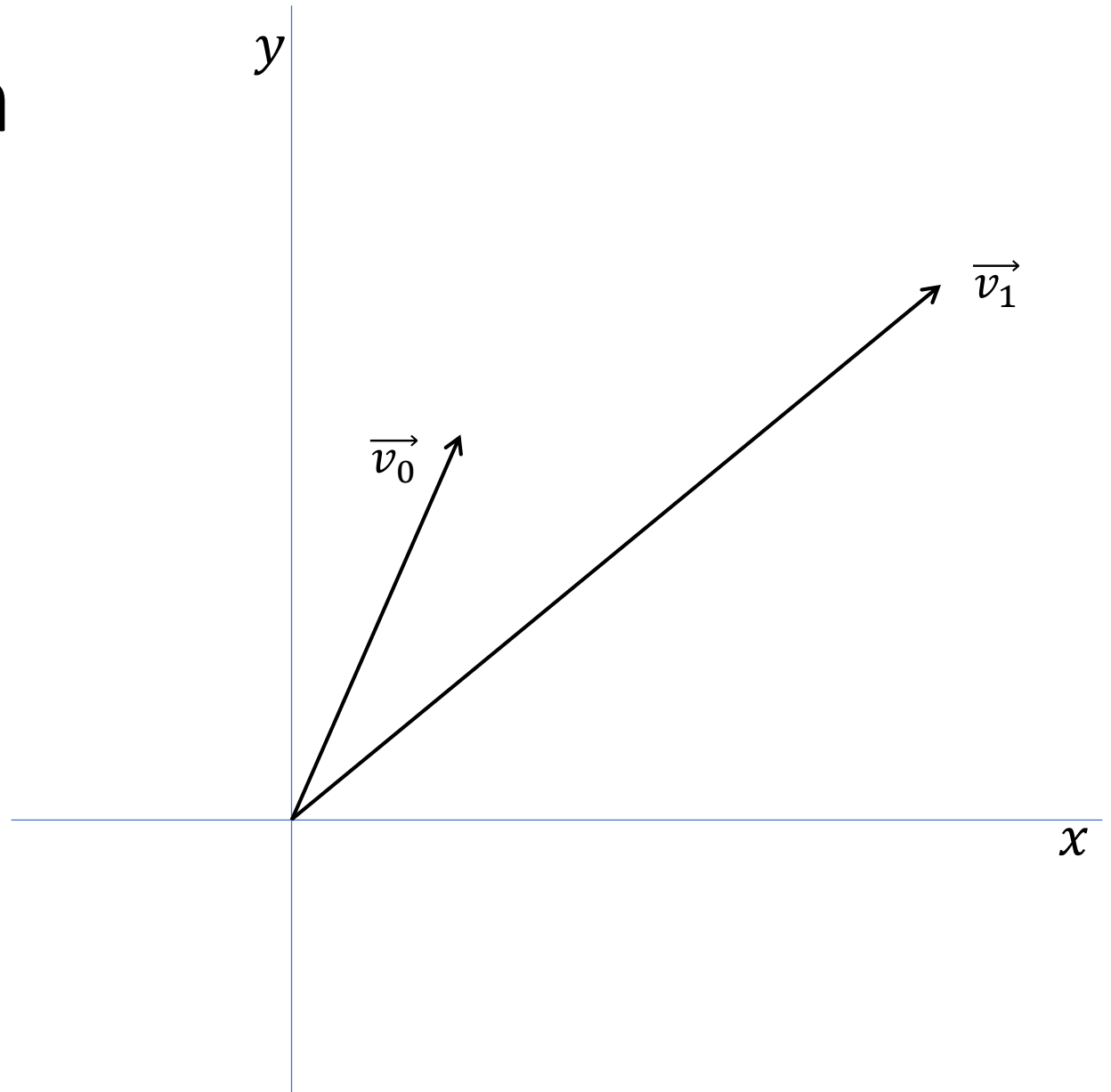
Linear interpolation

- For $t \in [0, 1]$
 - $t = \frac{3}{4}$



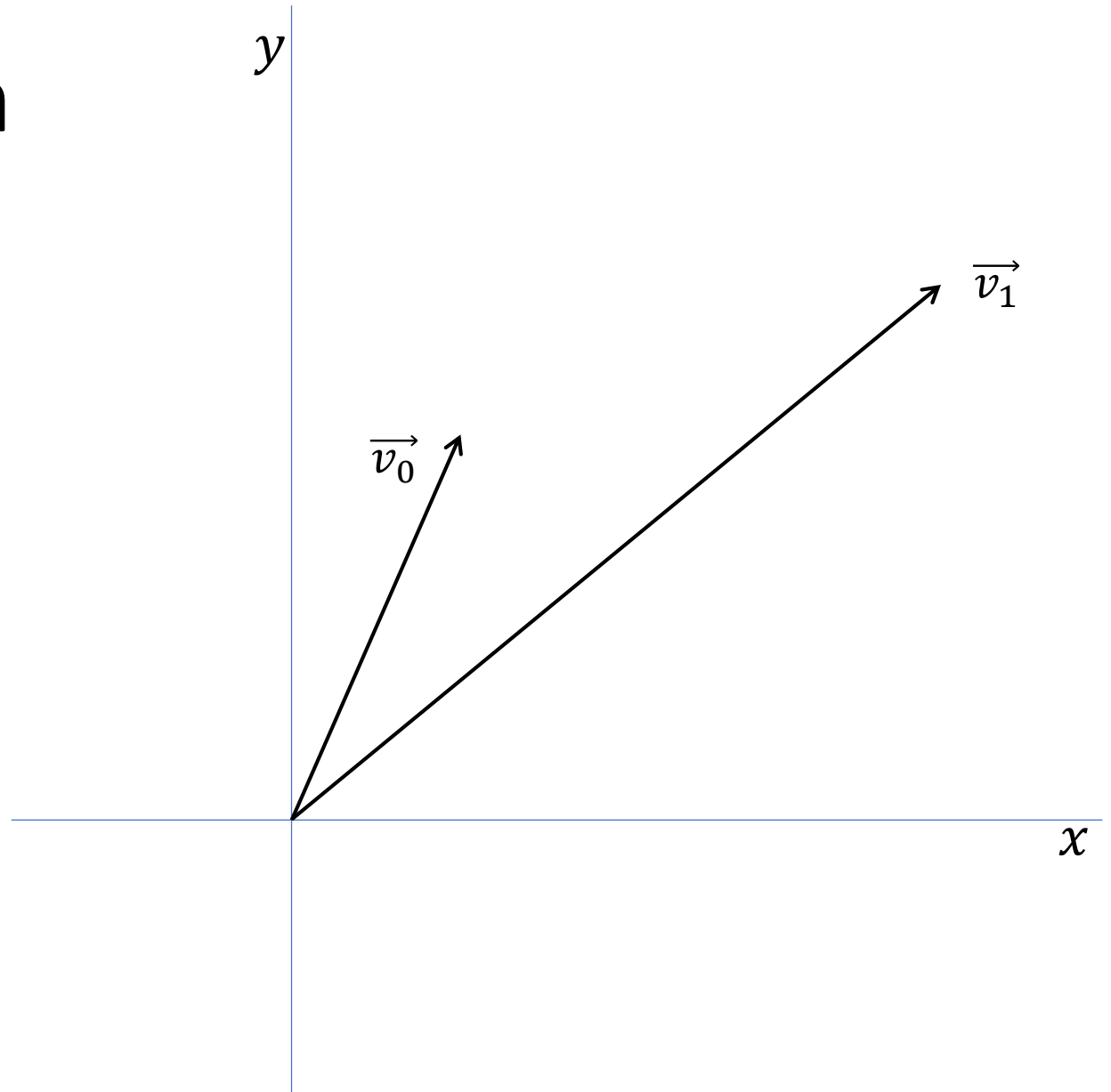
Linear interpolation

- For $t \in [0, 1]$
 - $t = 0 \Rightarrow 1\vec{v}_0 + 0\vec{v}_1$



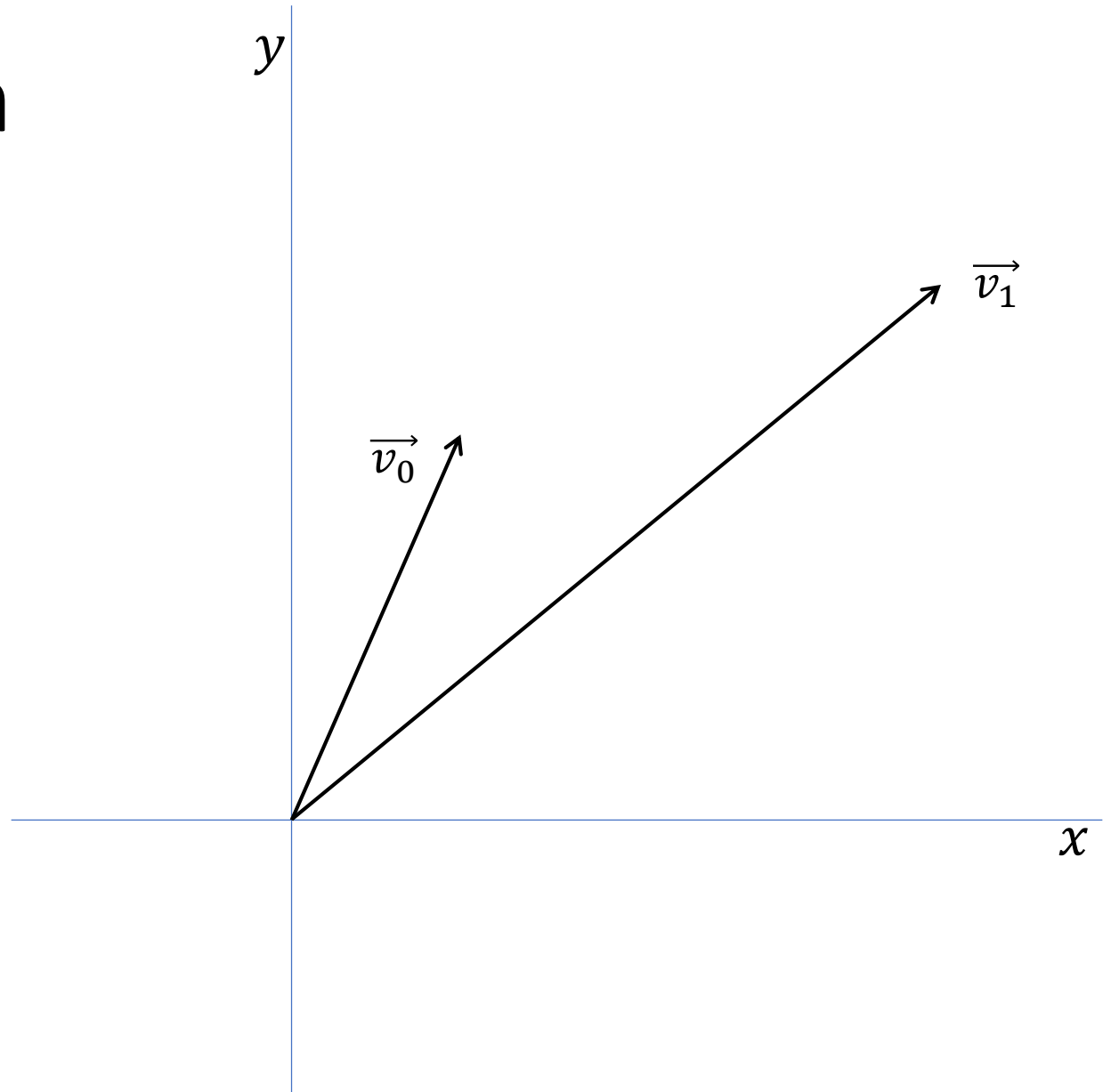
Linear interpolation

- For $t \in [0, 1]$
 - $t = 0 \Rightarrow 1\vec{v}_0 + 0\vec{v}_1$
 - $t = 1 \Rightarrow 0\vec{v}_0 + 1\vec{v}_1$



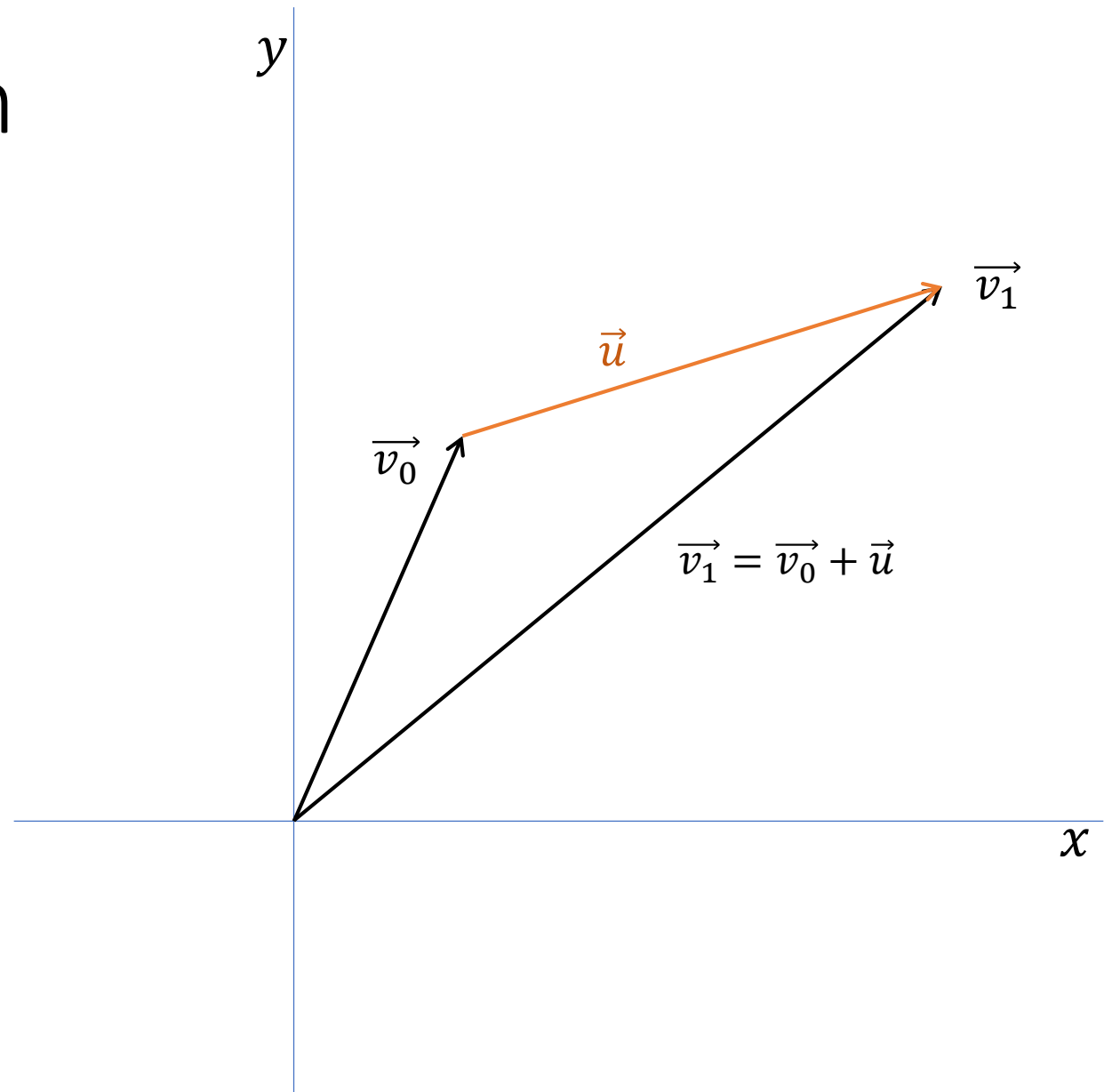
Linear interpolation

- For $t \in [0, 1]$
 - $t = 0 \Rightarrow 1\vec{v}_0 + 0\vec{v}_1$
 - $t = 1 \Rightarrow 0\vec{v}_0 + 1\vec{v}_1$
 - $a\vec{v}_0 + b\vec{v}_1$



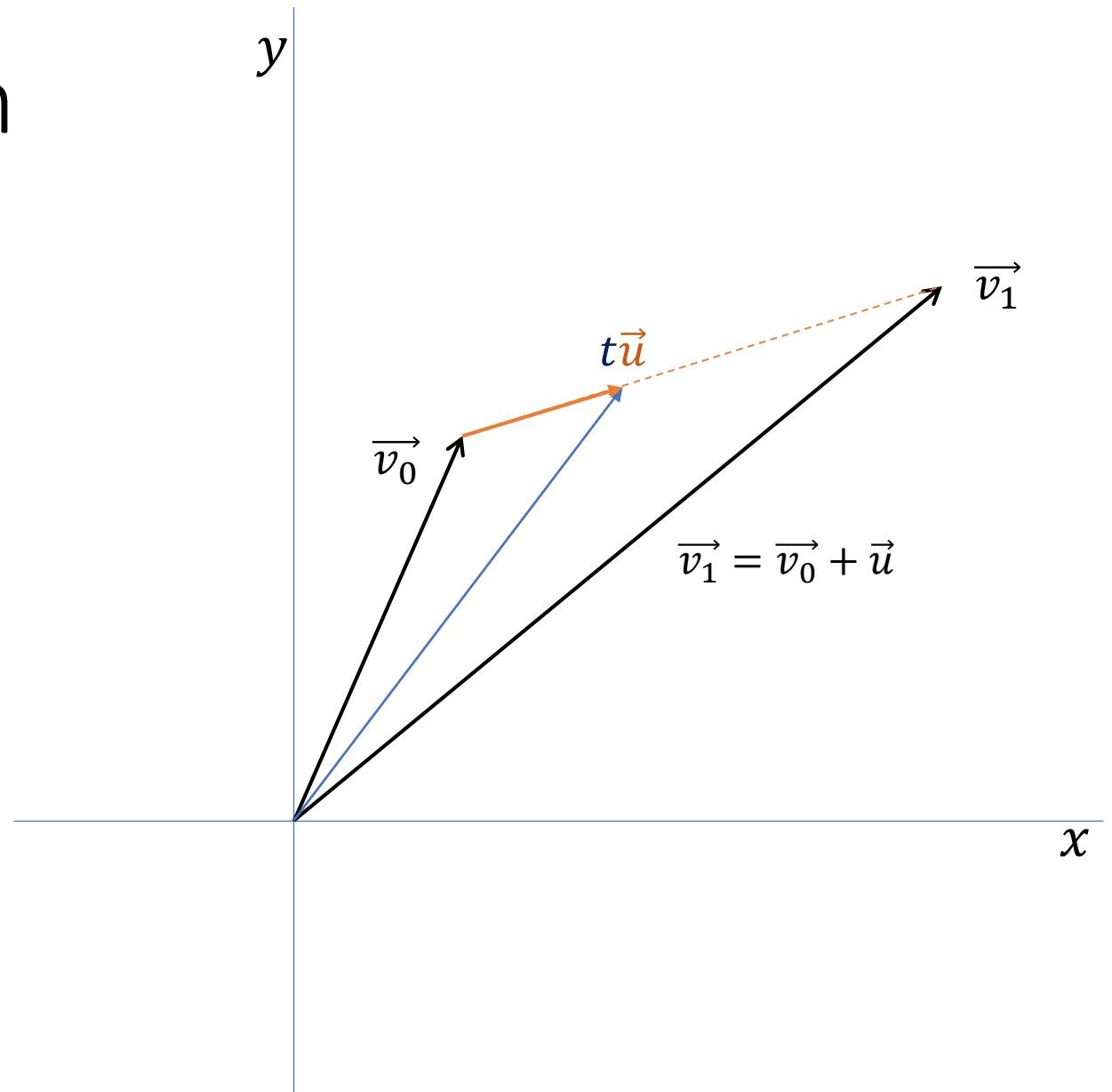
Linear interpolation

- For $t \in [0, 1]$
 - $a\vec{v}_0 + b\vec{v}_1$



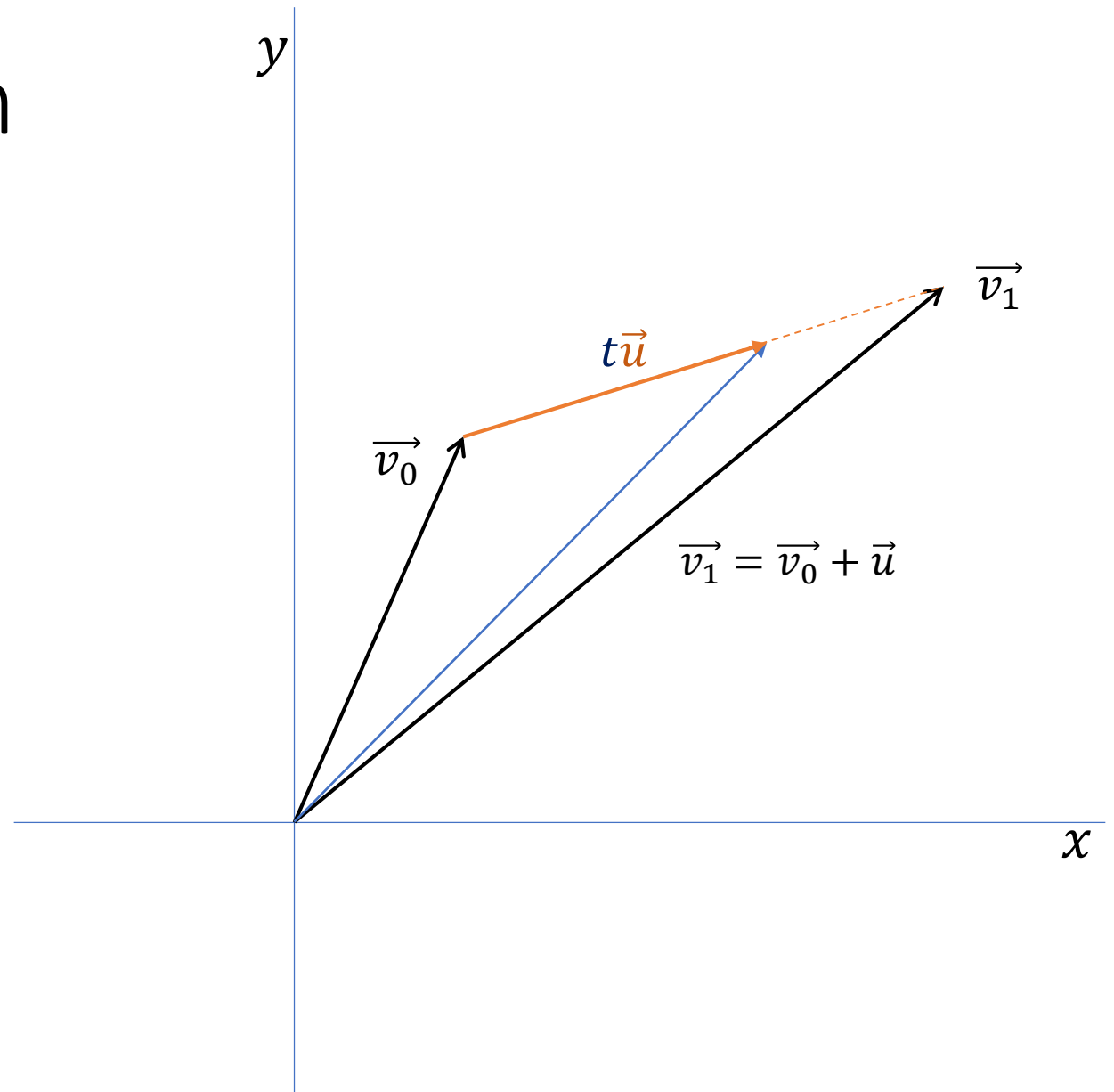
Linear interpolation

- For $t \in [0, 1]$
 - $a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_0 + t\vec{u}$



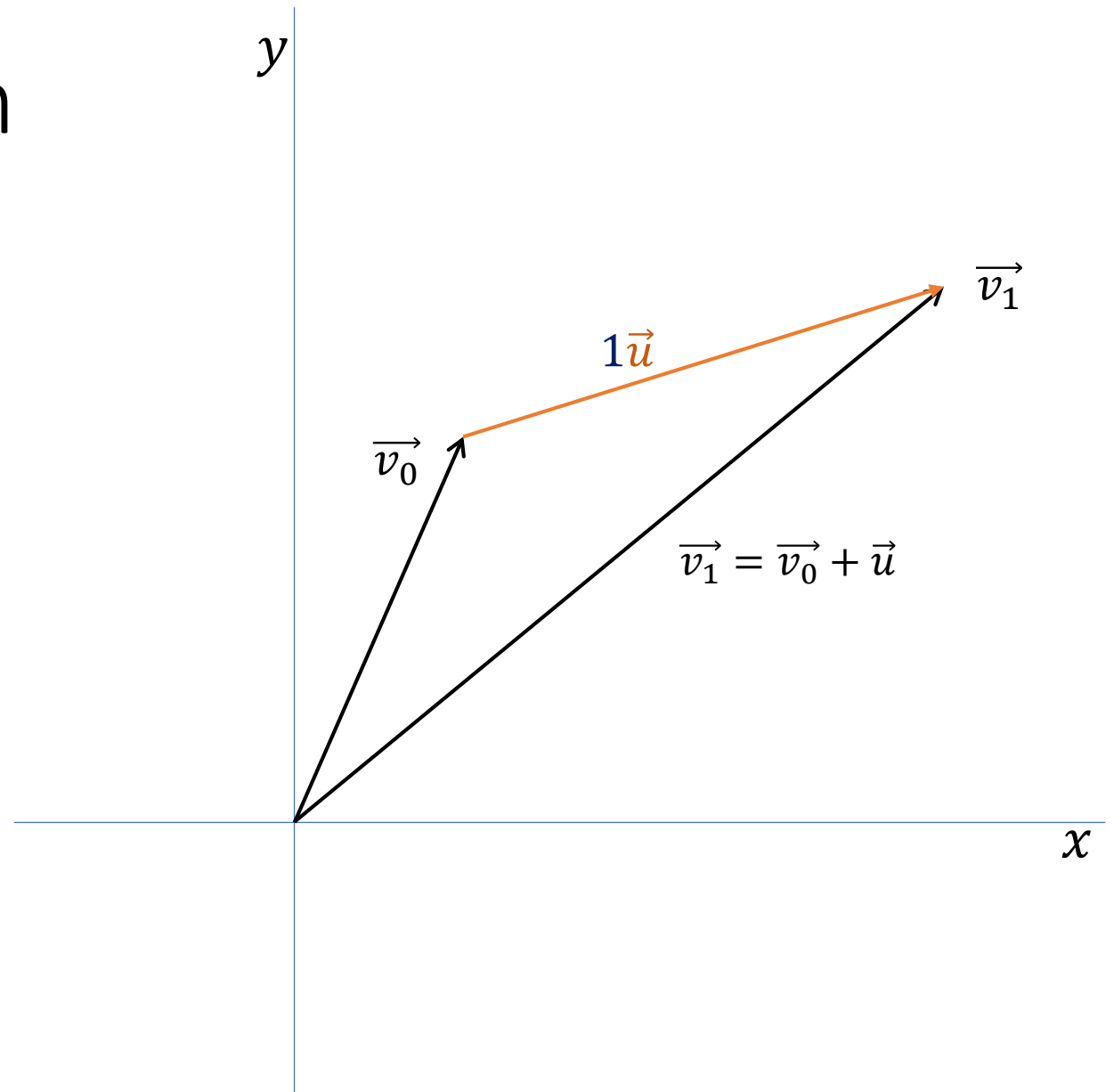
Linear interpolation

- For $t \in [0, 1]$
 - $a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_0 + t\vec{u}$



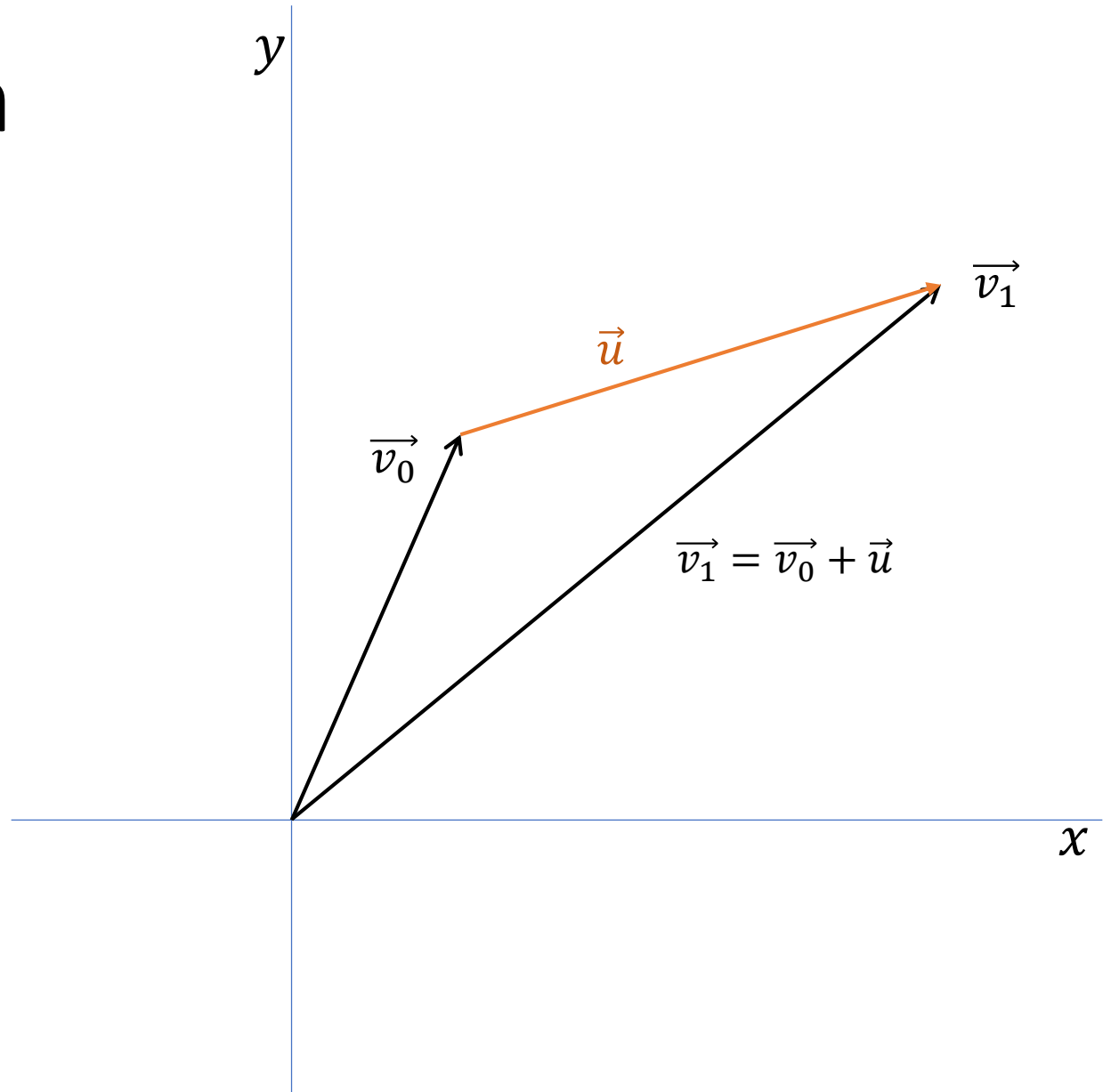
Linear interpolation

- For $t \in [0, 1]$
 - $a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_0 + 1\vec{u}$



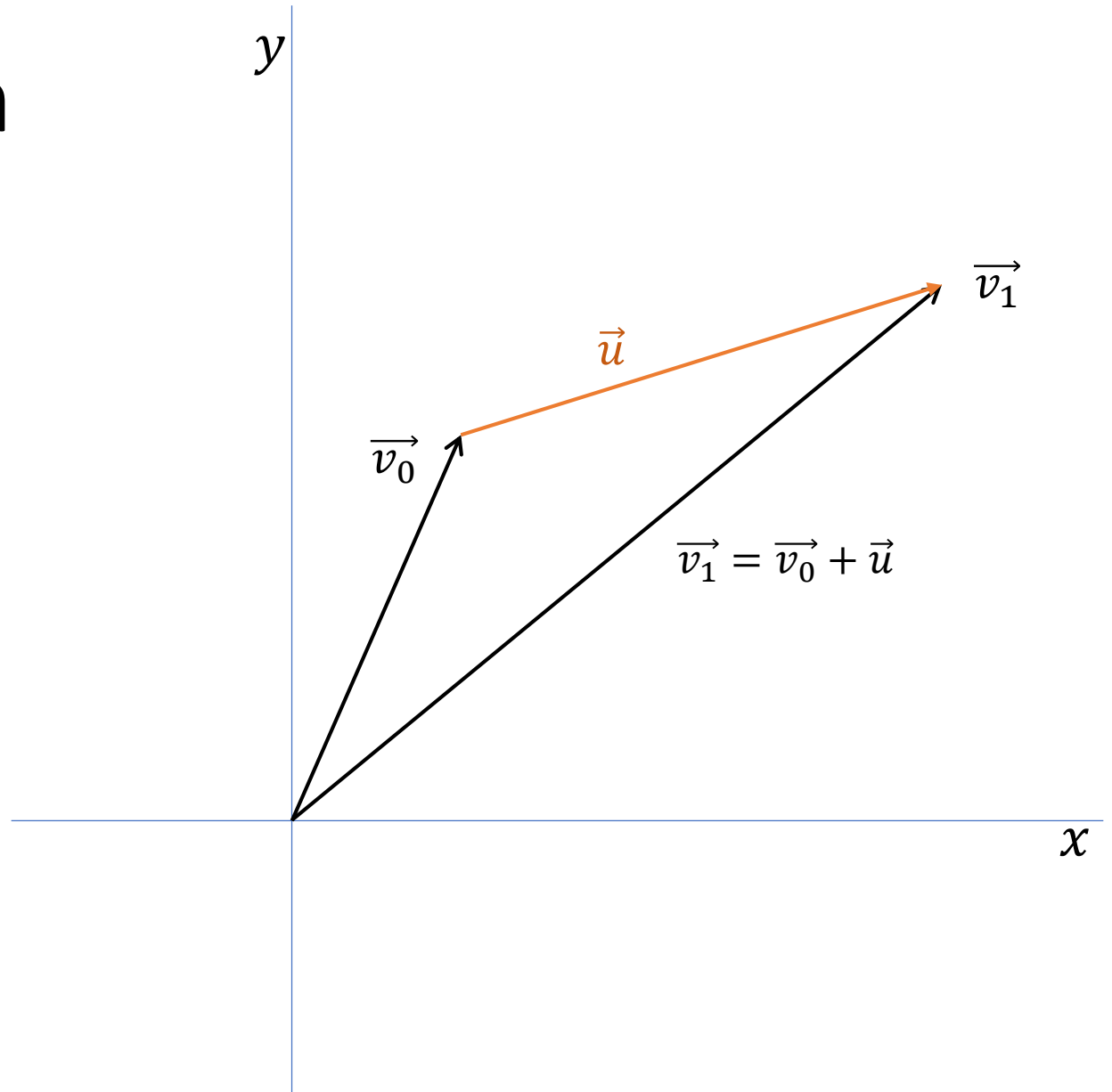
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$



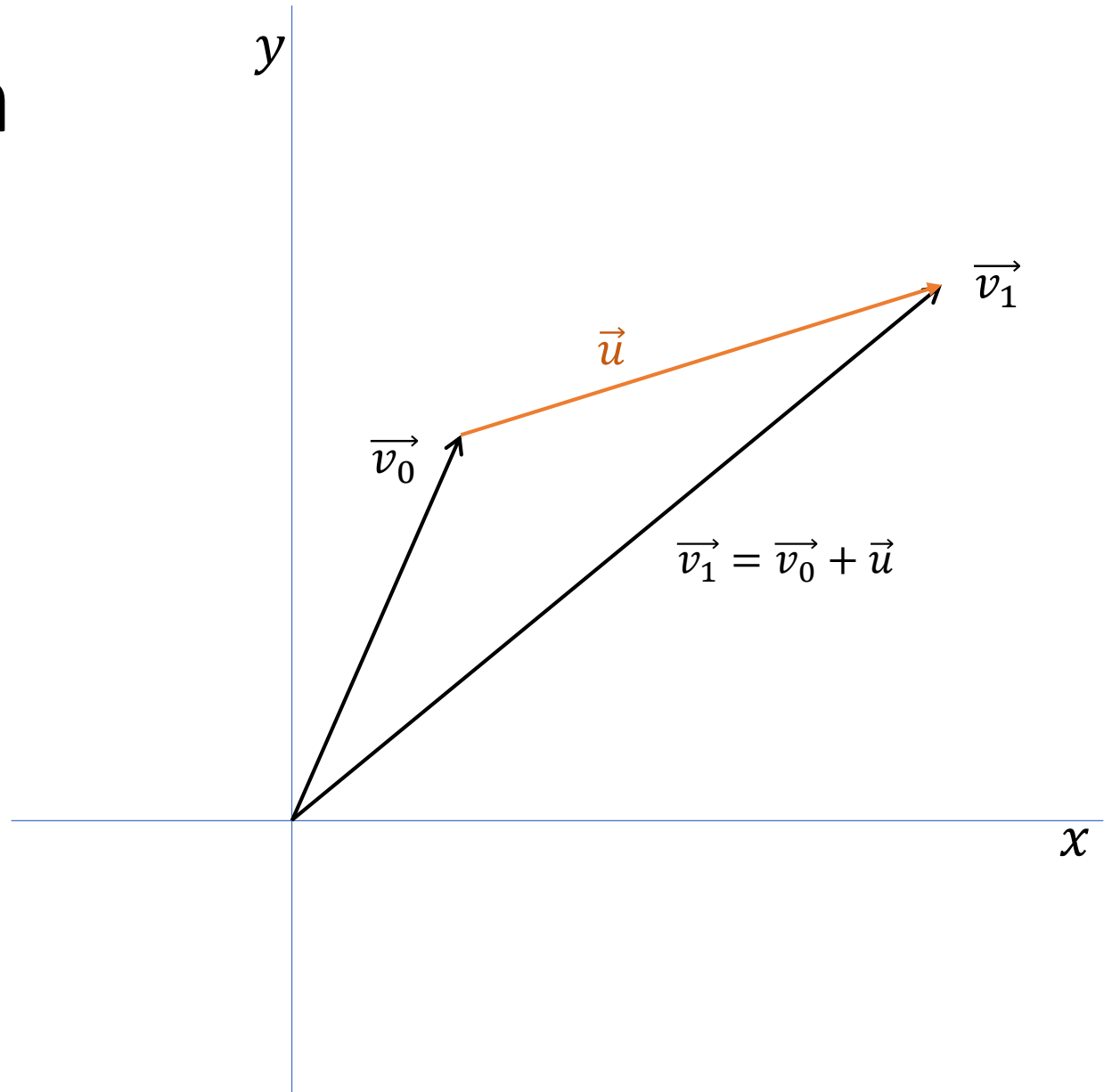
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$



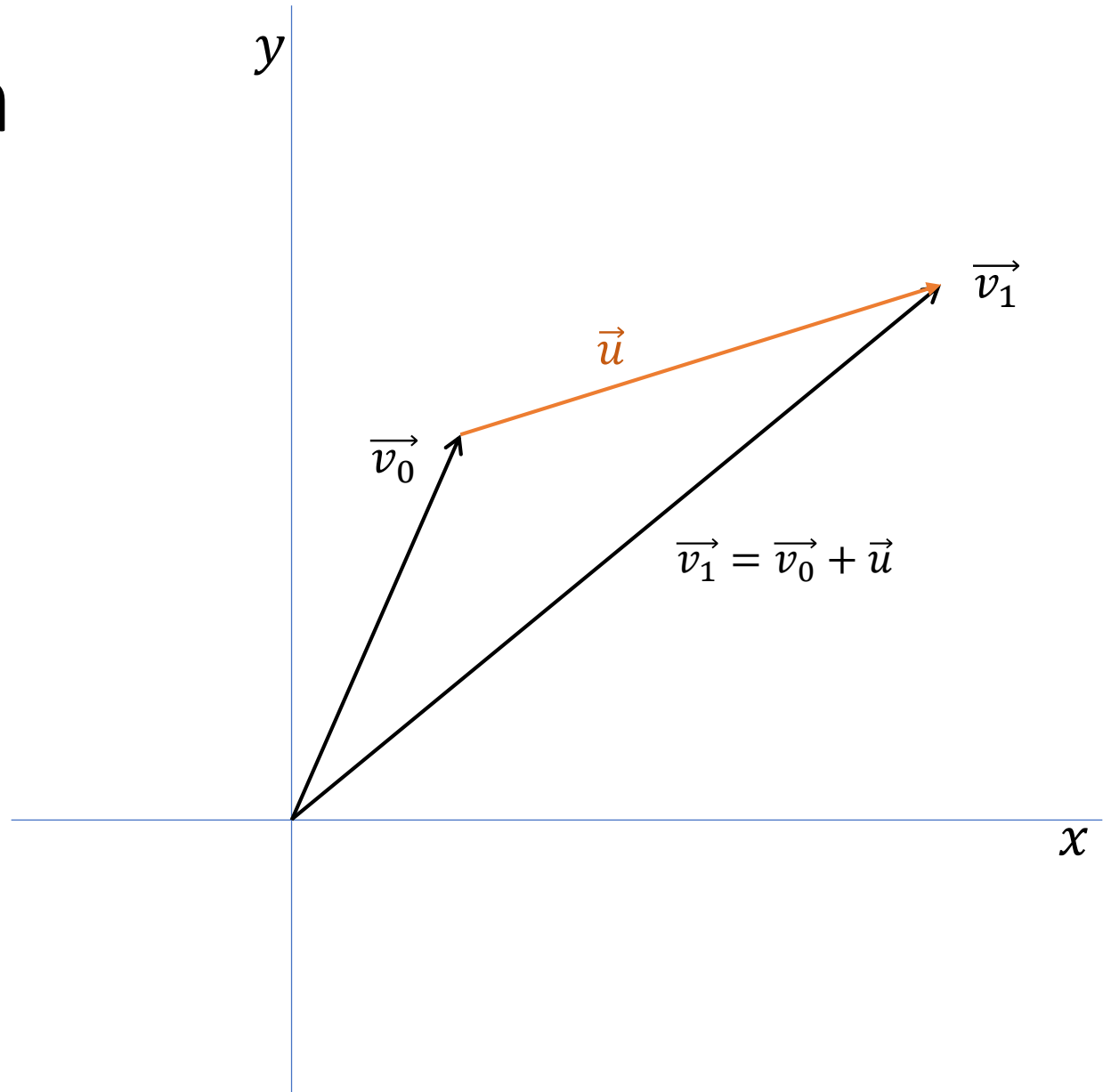
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 + t\vec{u}$



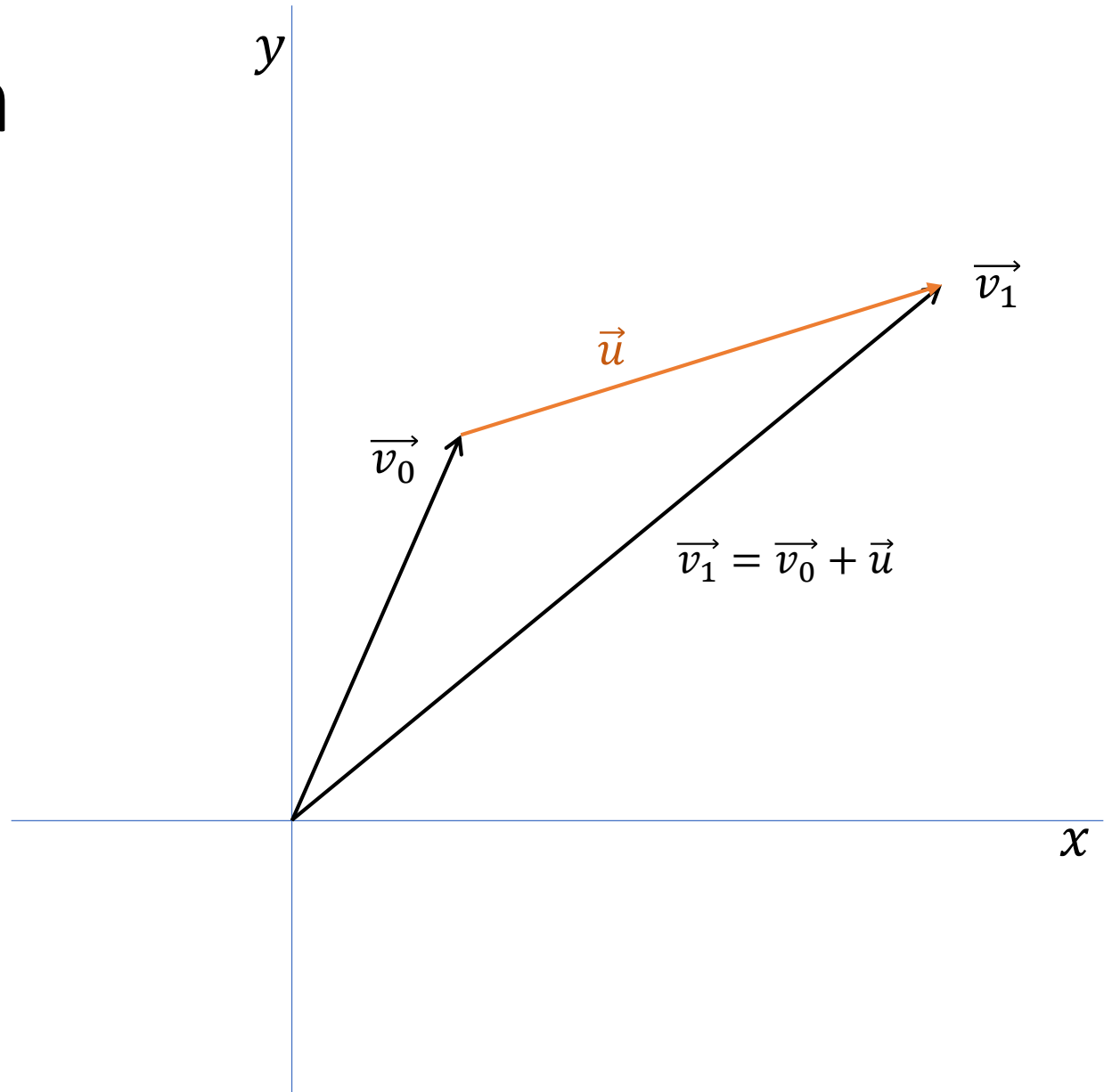
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 + t(\vec{v}_1 - \vec{v}_0)$



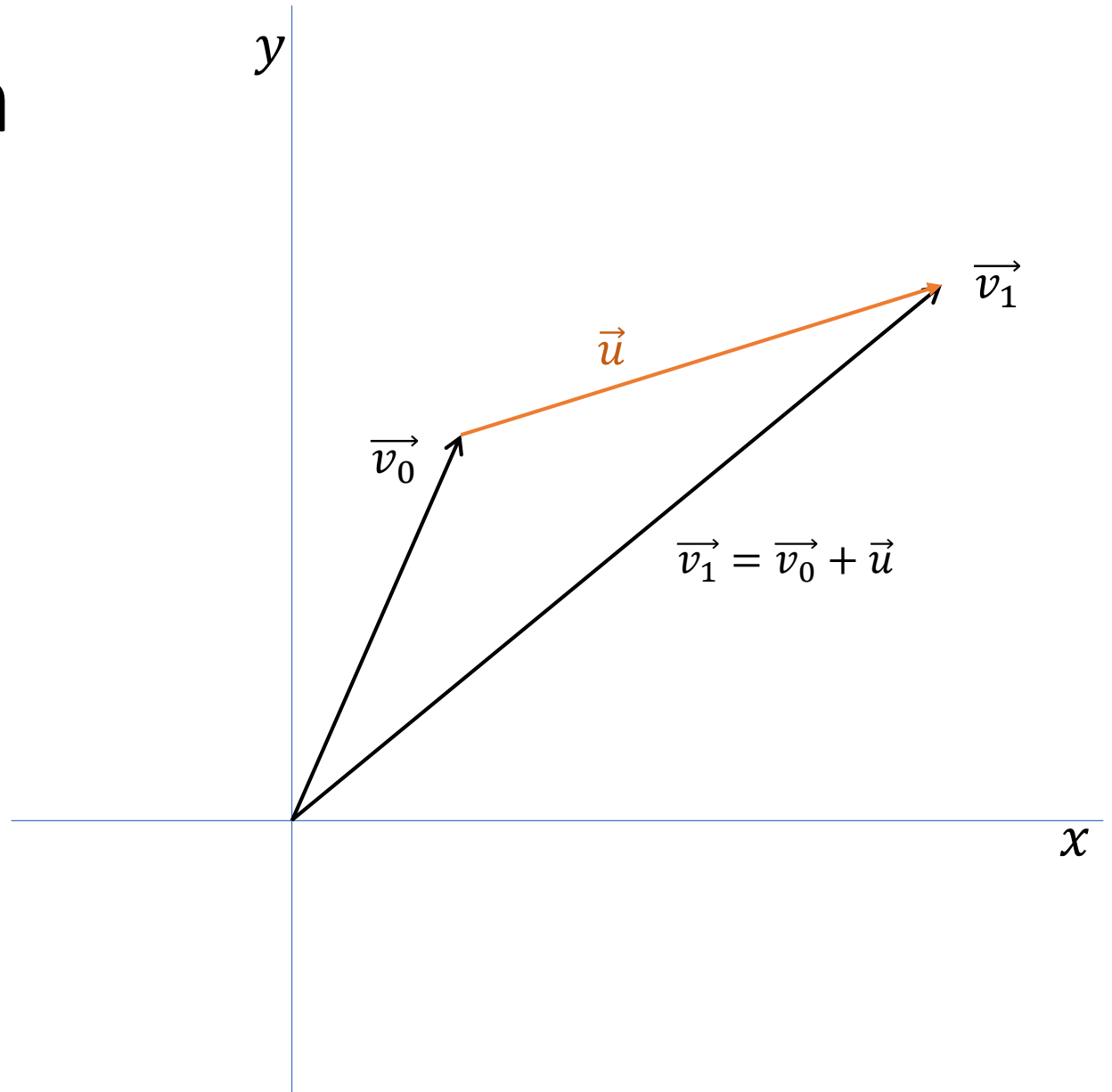
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 + t(\vec{v}_1 - \vec{v}_0)$
 - $\vec{v}_t = \vec{v}_0 + t\vec{v}_1 - t\vec{v}_0$



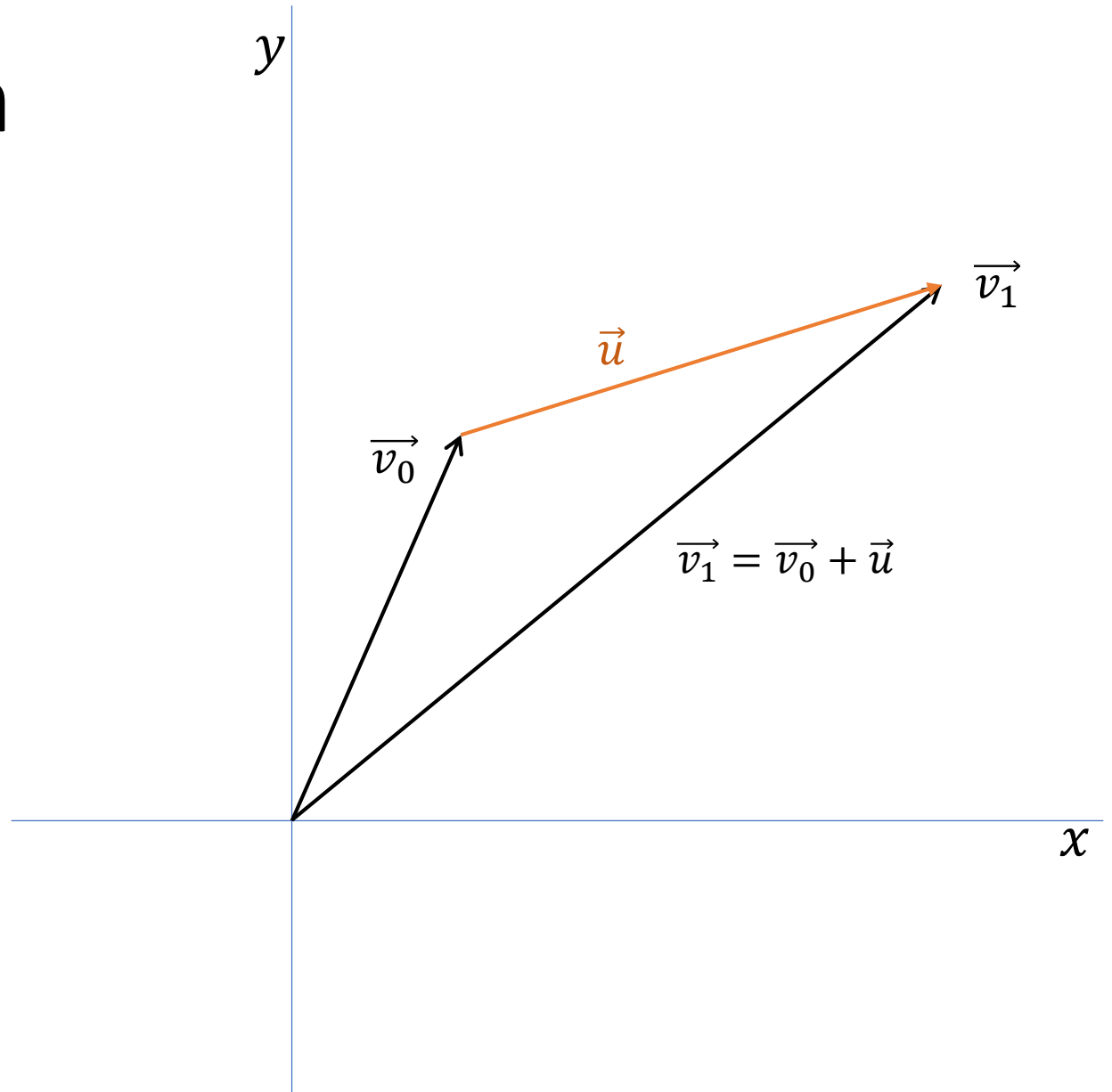
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 + t(\vec{v}_1 - \vec{v}_0)$
 - $\vec{v}_t = \vec{v}_0 + t\vec{v}_1 - t\vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 - t\vec{v}_0 + t\vec{v}_1$



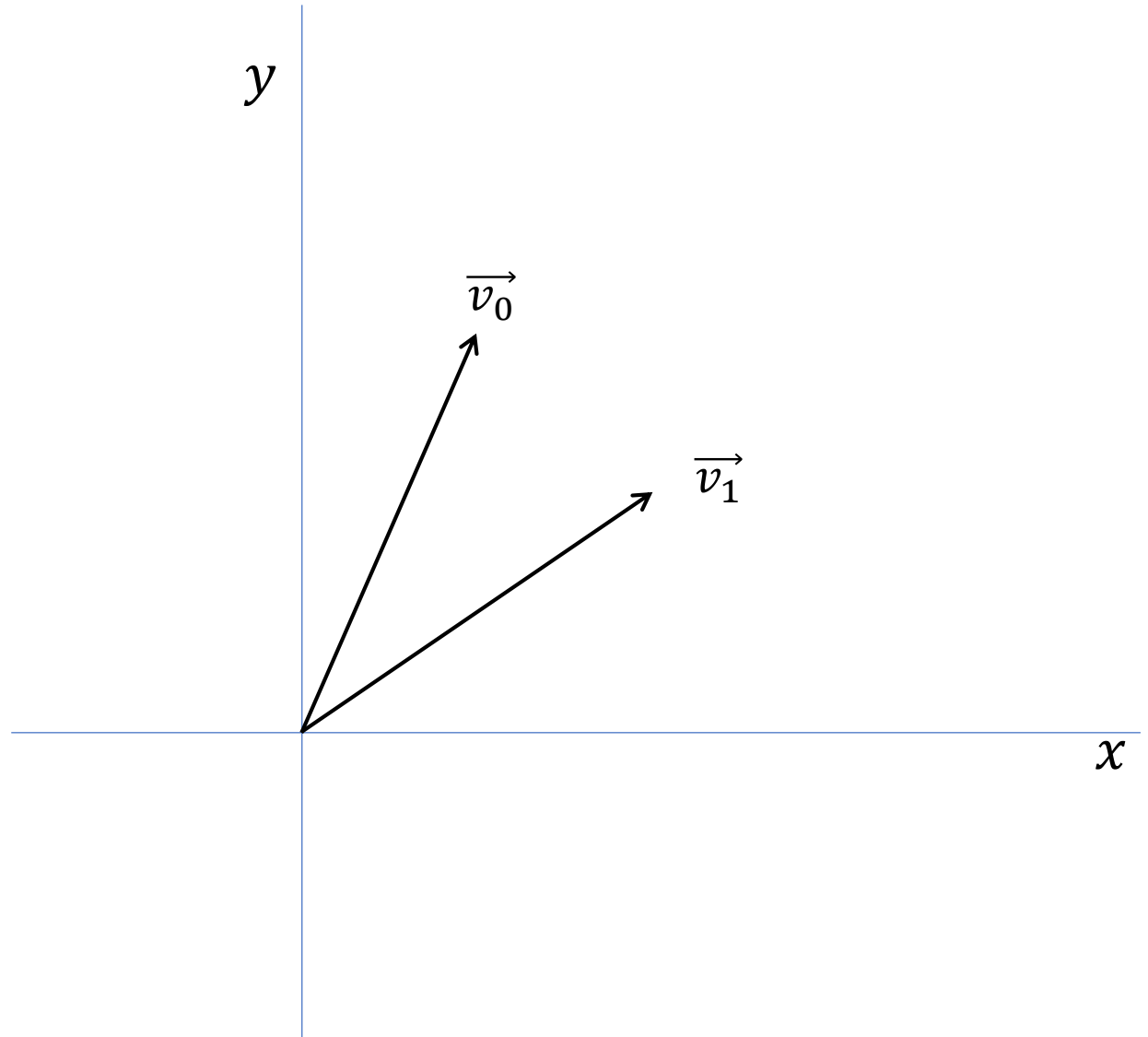
Linear interpolation

- For $t \in [0, 1]$
 - $\vec{v}_t = a\vec{v}_0 + b\vec{v}_1$
 - $\vec{v}_1 = \vec{v}_0 + \vec{u}$
 - $\vec{u} = \vec{v}_1 - \vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 + t(\vec{v}_1 - \vec{v}_0)$
 - $\vec{v}_t = \vec{v}_0 + t\vec{v}_1 - t\vec{v}_0$
 - $\vec{v}_t = \vec{v}_0 - t\vec{v}_0 + t\vec{v}_1$
 - $\vec{v}_t = (1 - t)\vec{v}_0 + t\vec{v}_1$



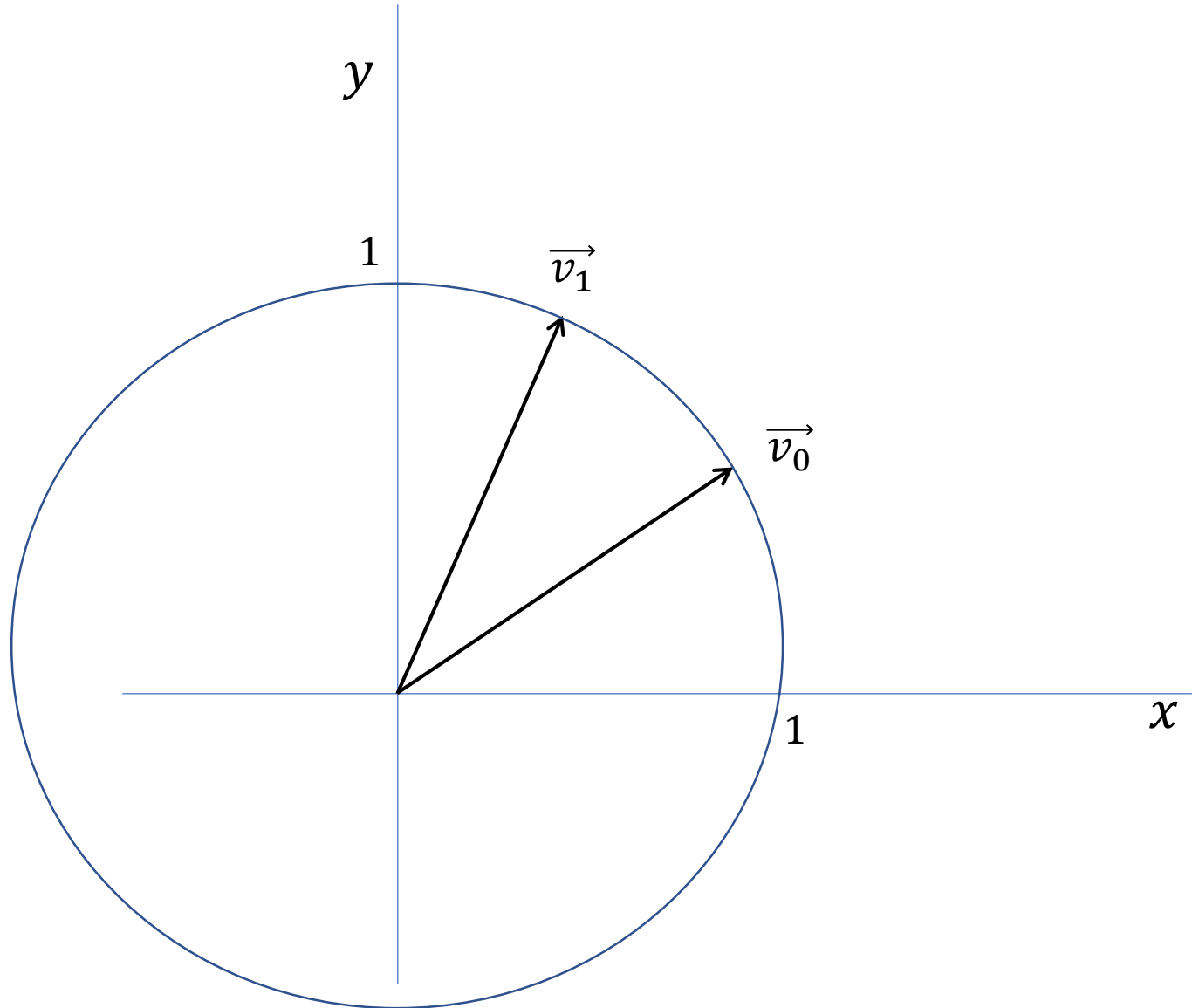
Spherical linear interpolation

- For $t \in [0, 1]$



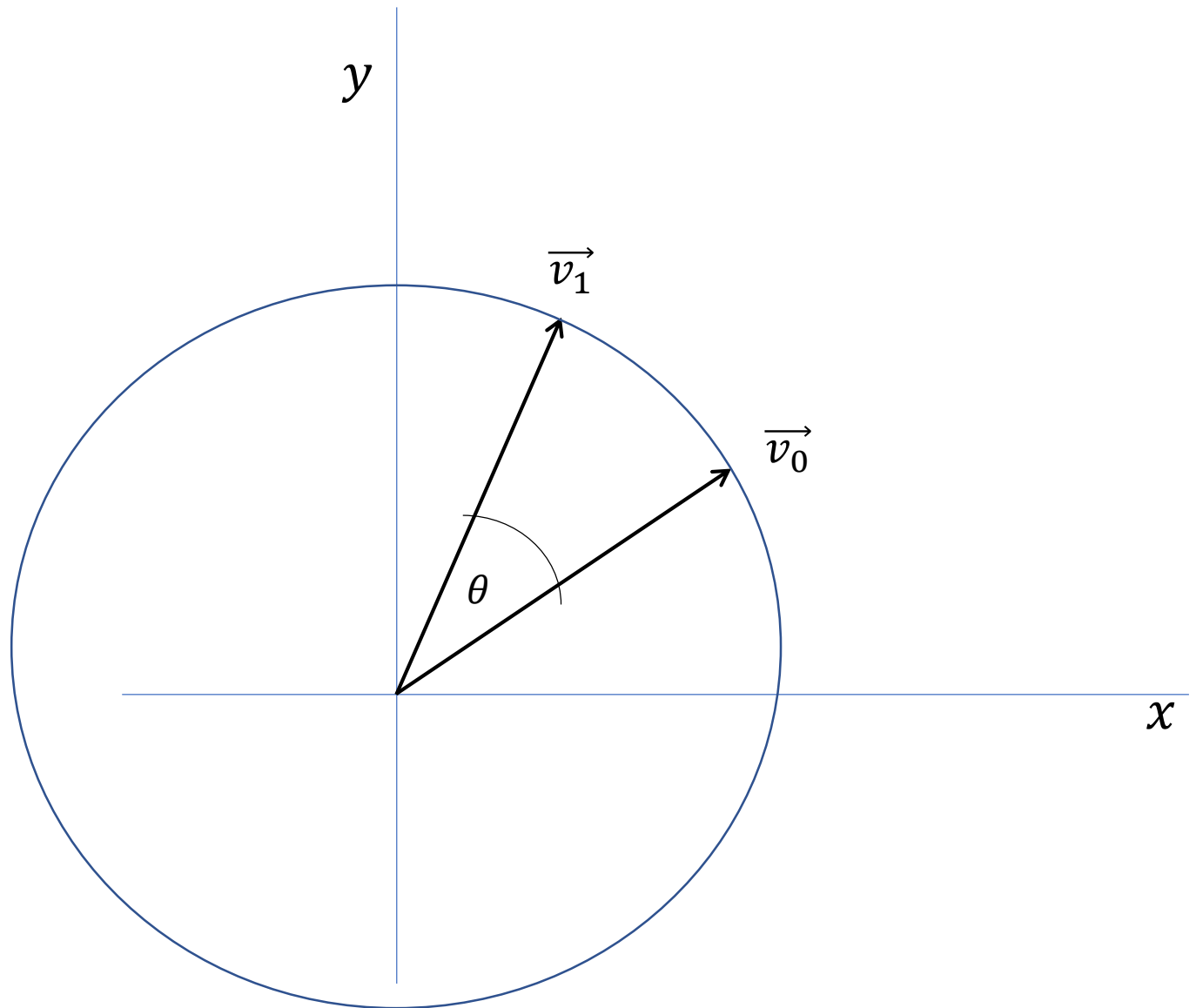
Spherical linear interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$



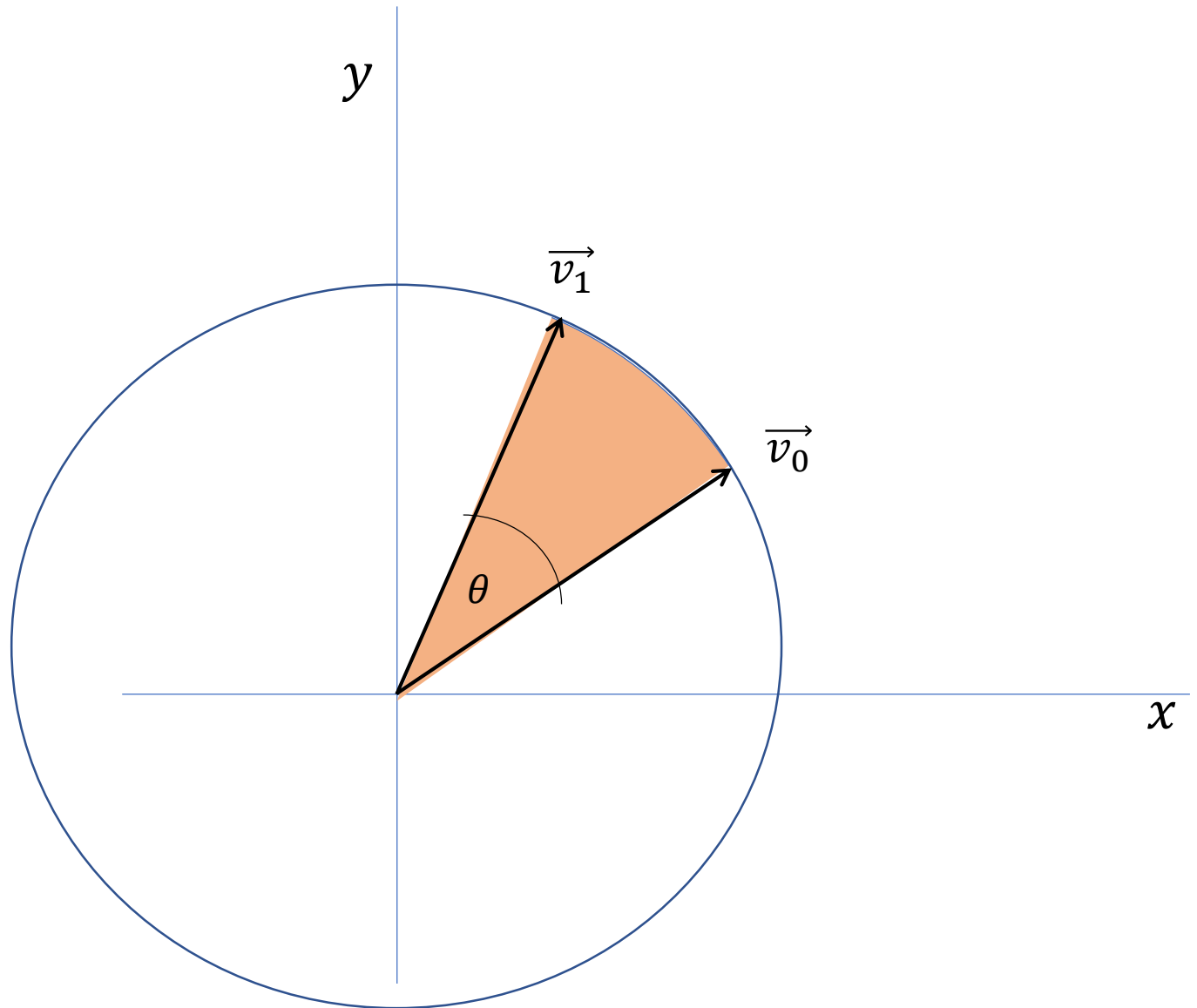
Spherical linear interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$



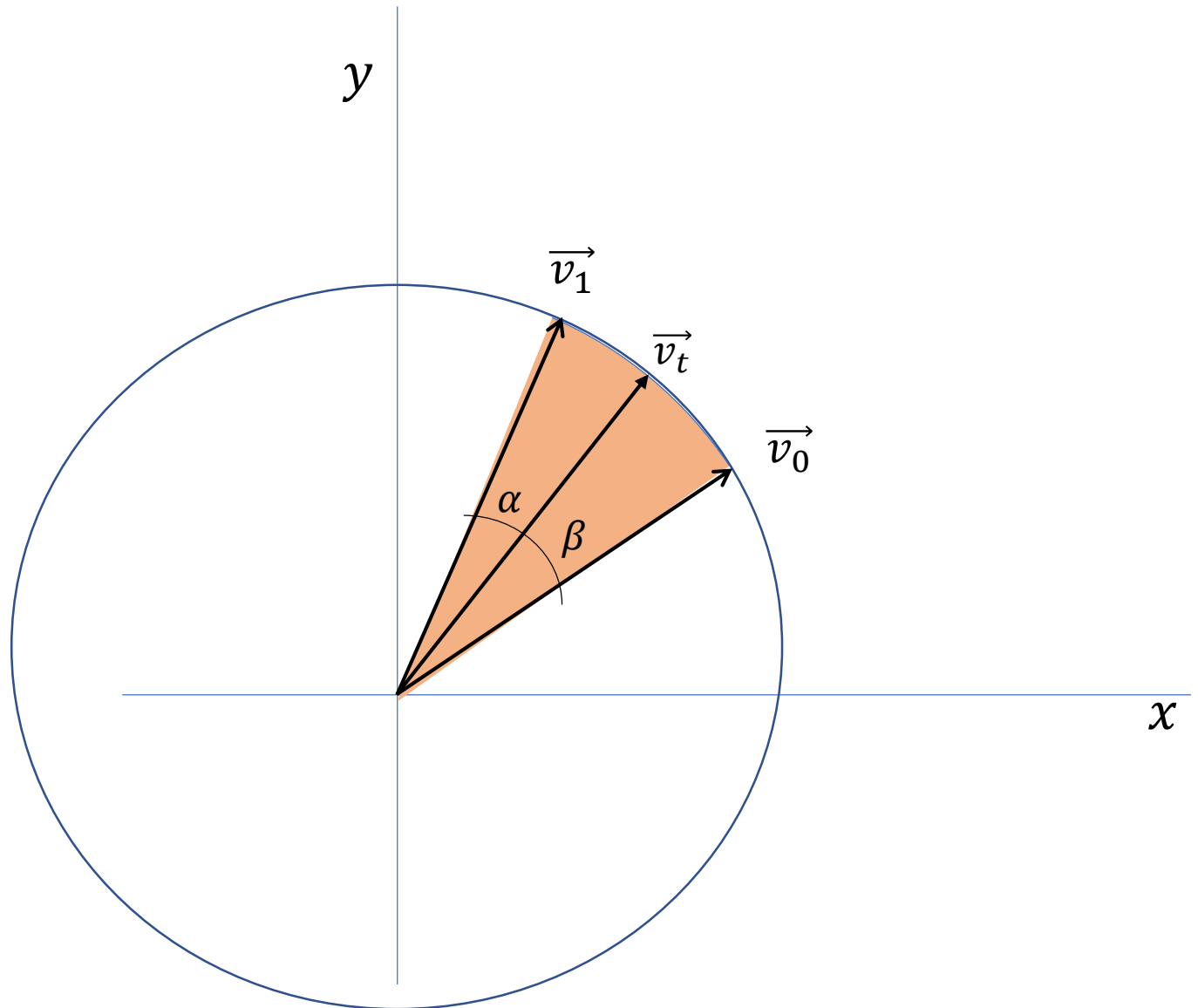
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$



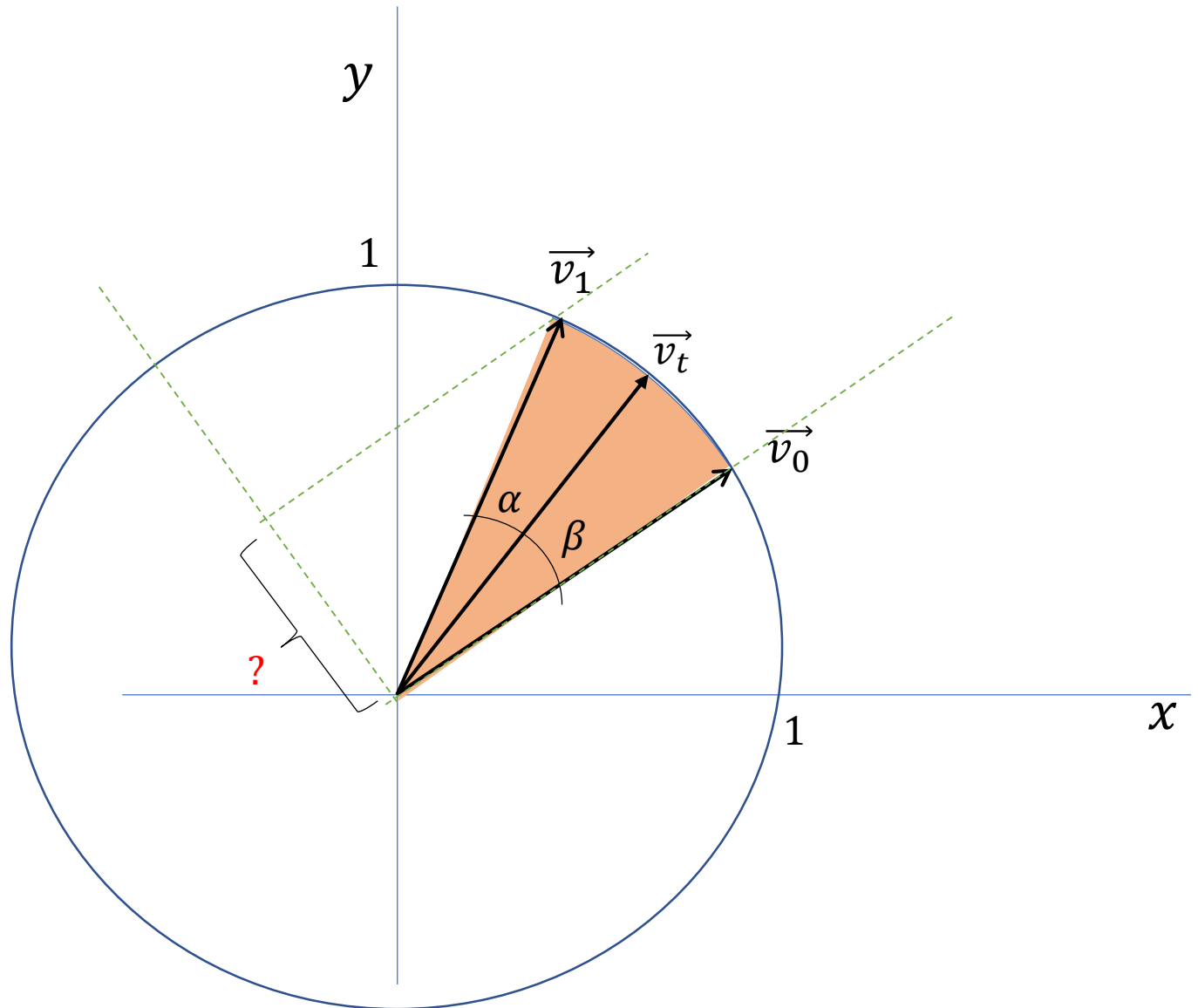
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$



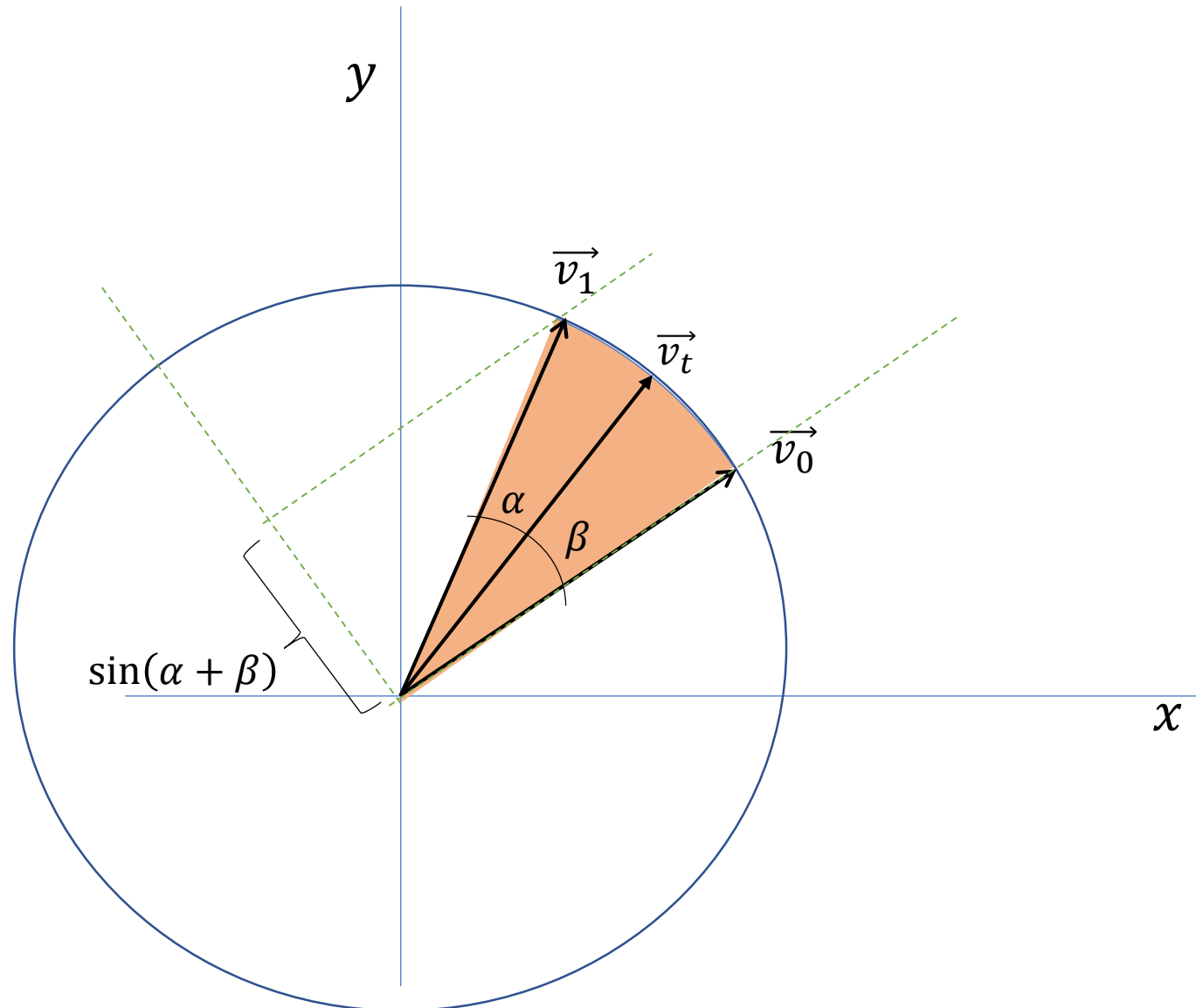
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$



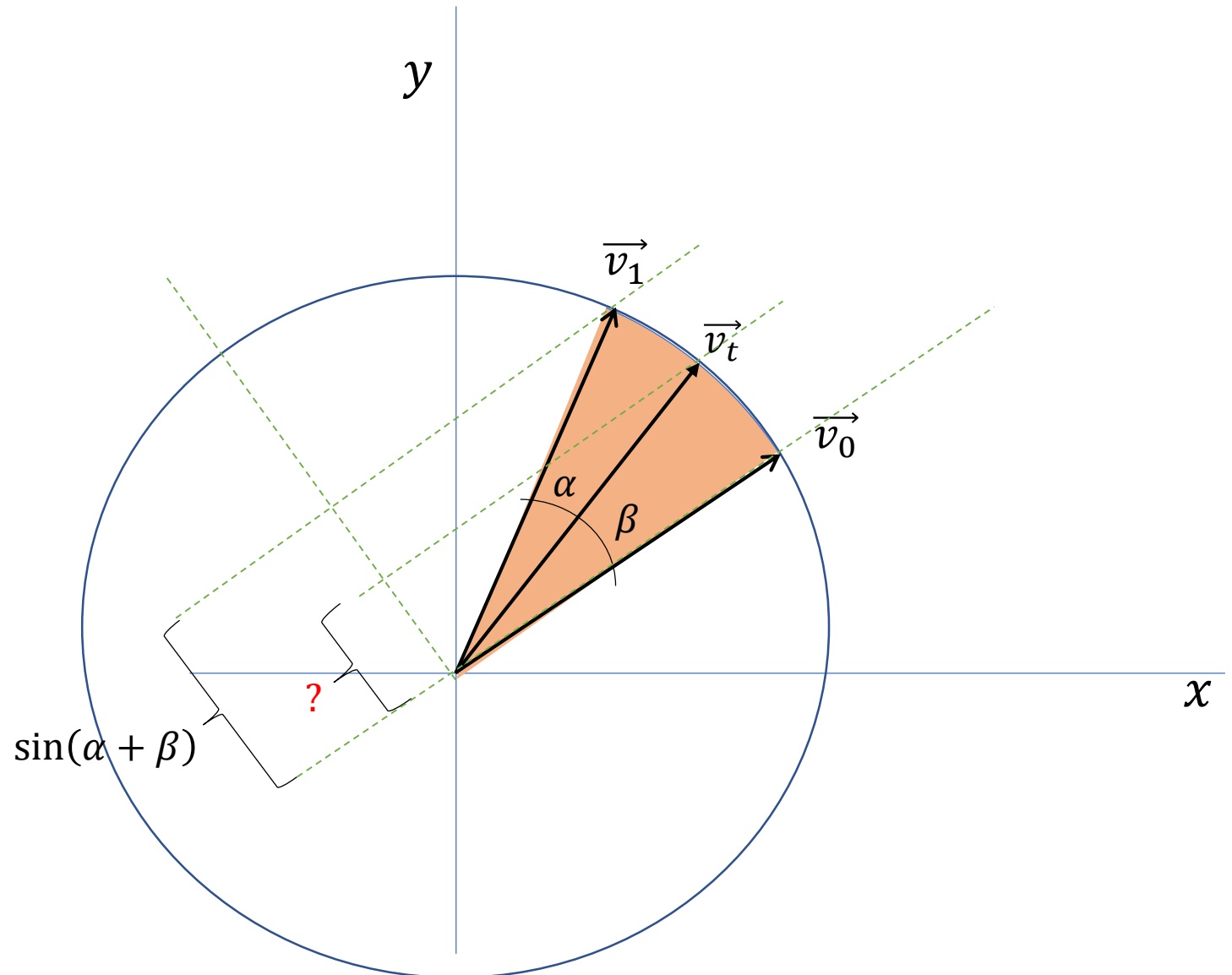
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$



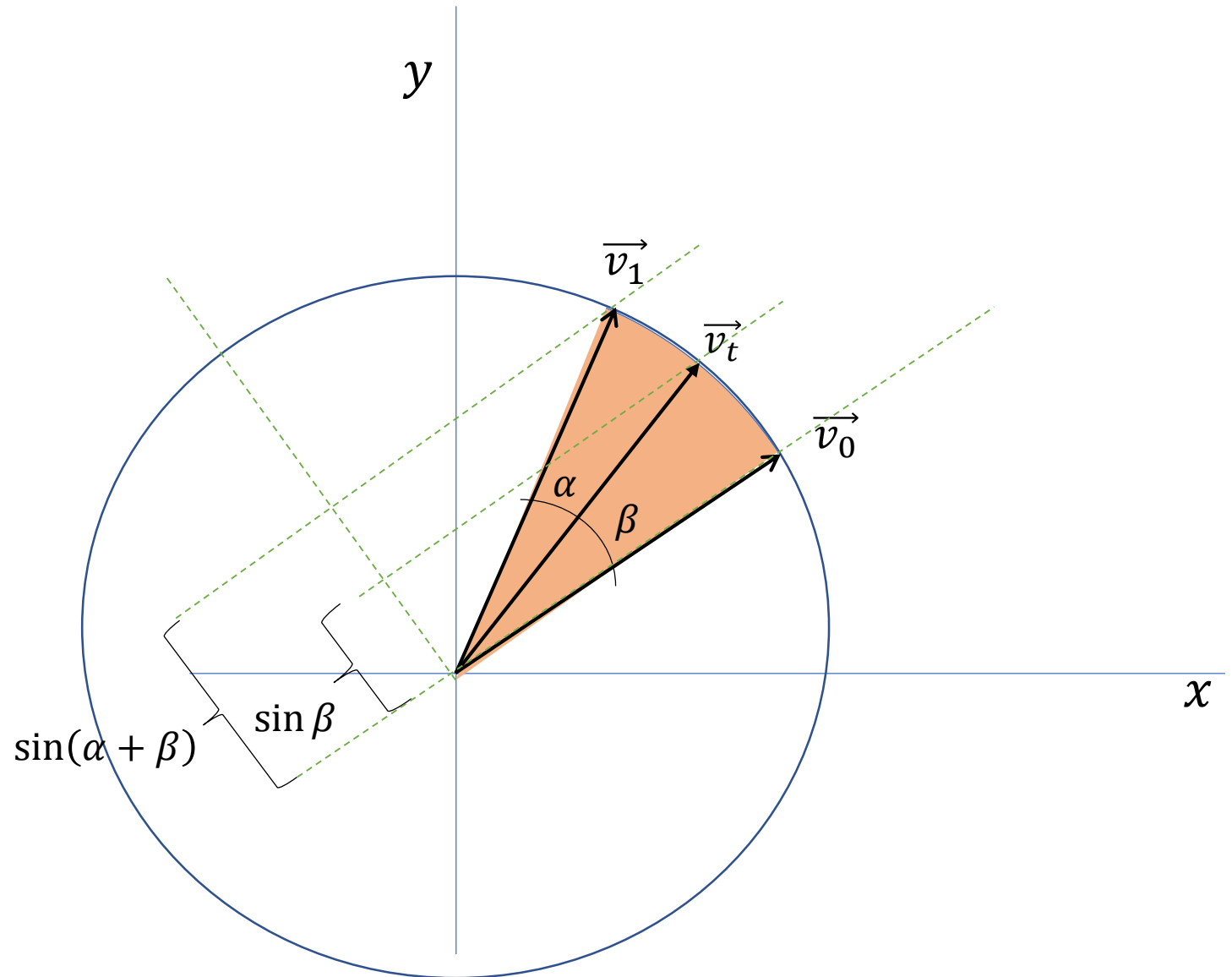
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$



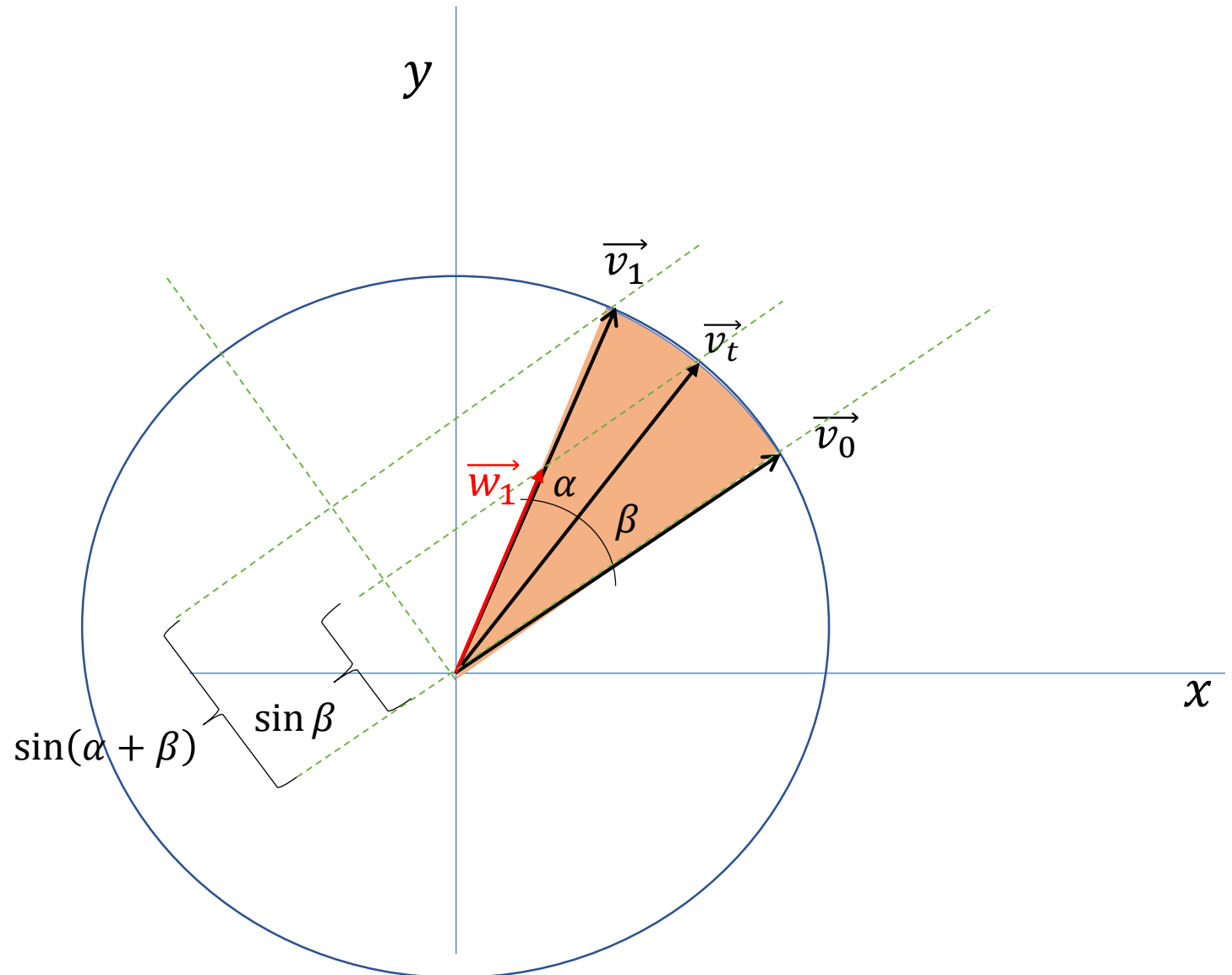
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$



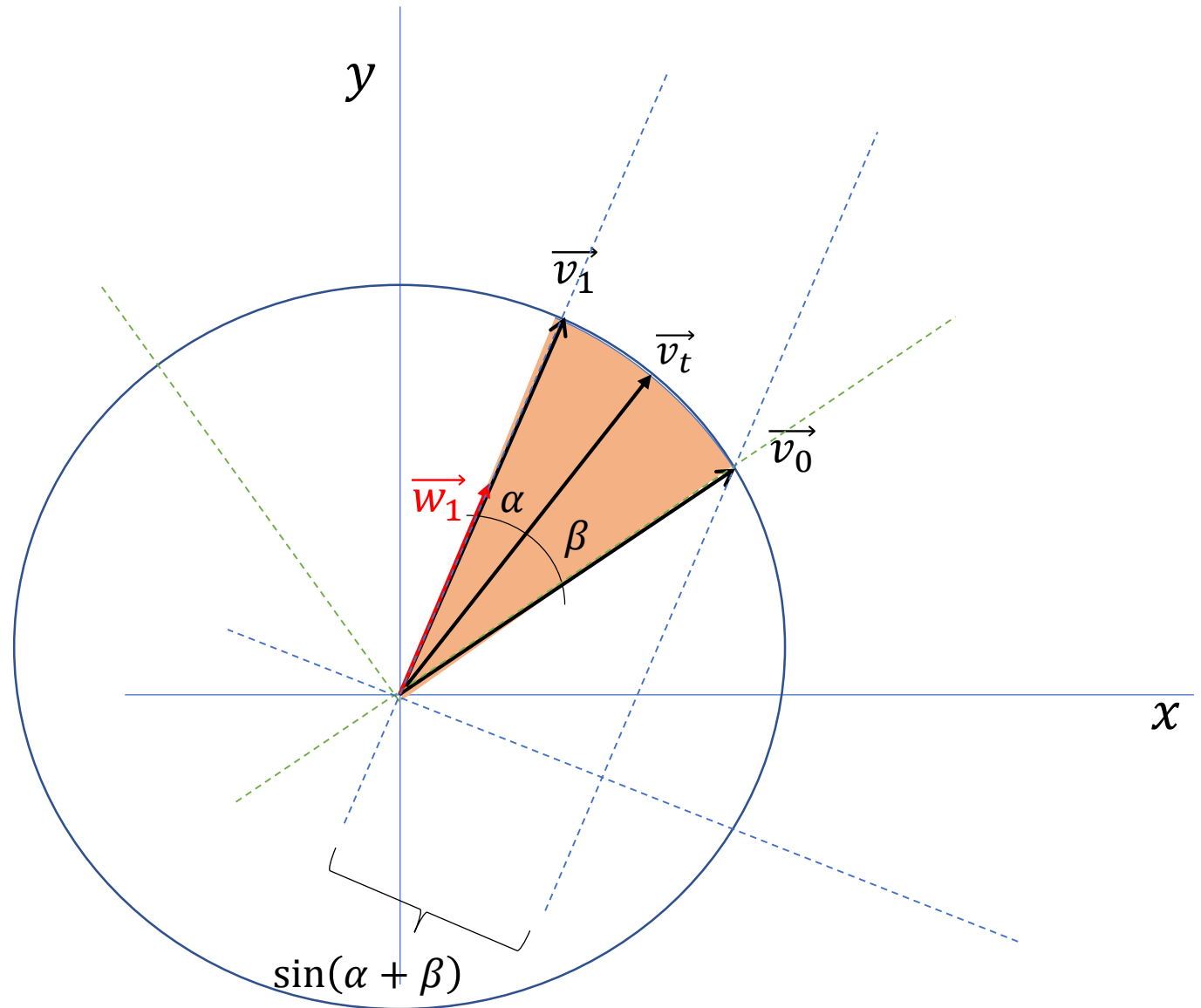
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$



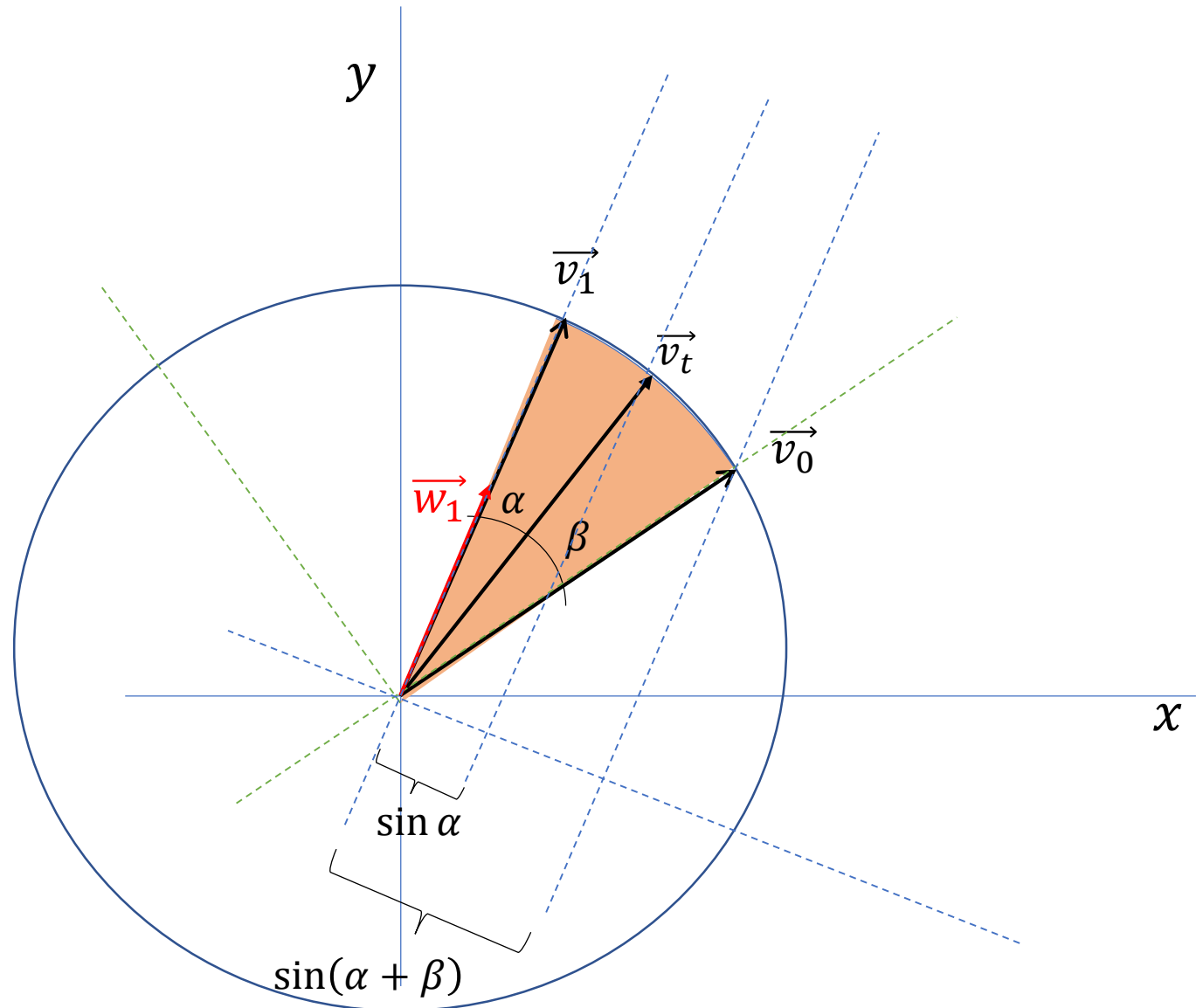
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$



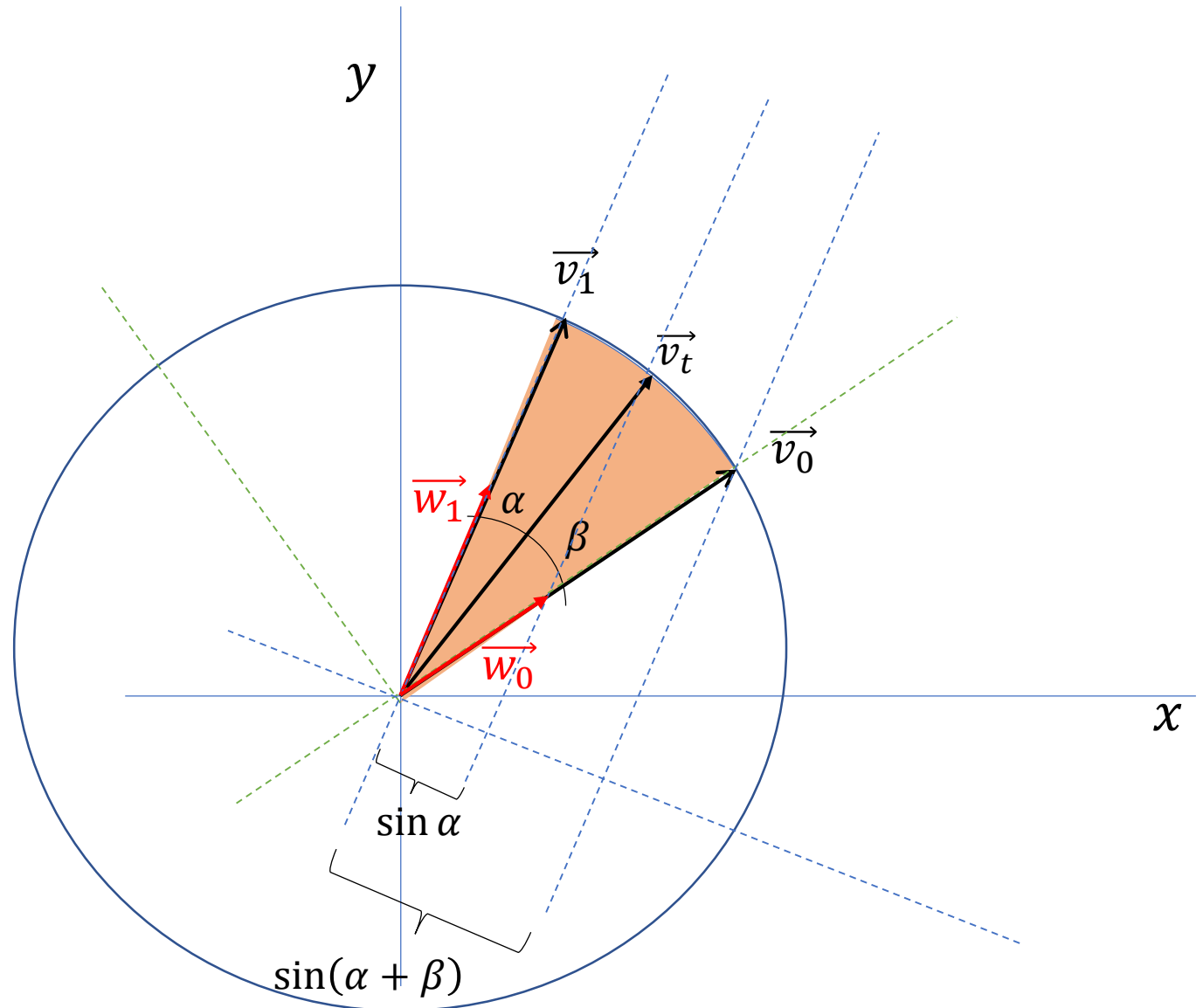
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$



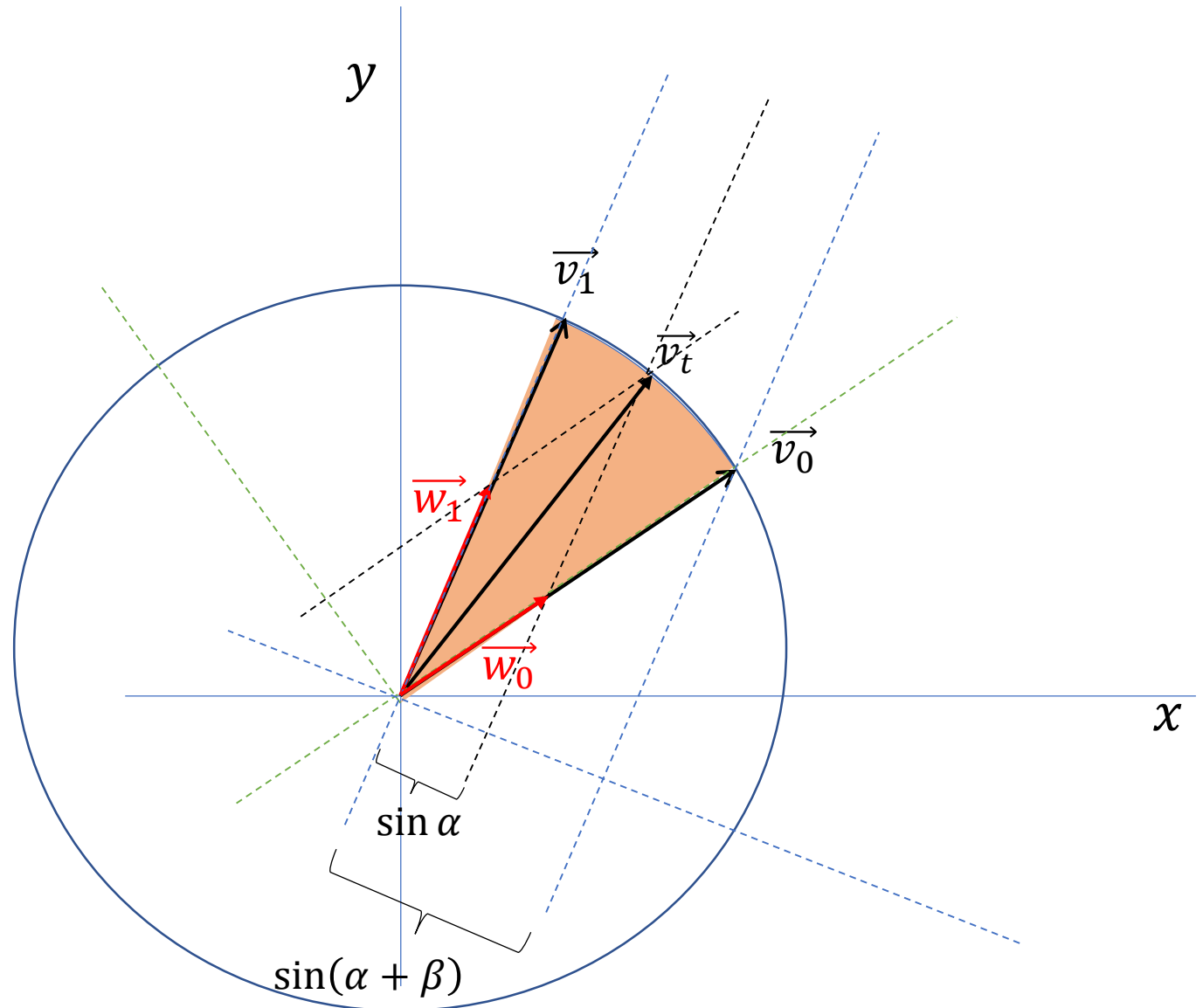
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$
 - $\vec{w}_0 = \frac{\sin \alpha}{\sin(\alpha + \beta)} \vec{v}_0$



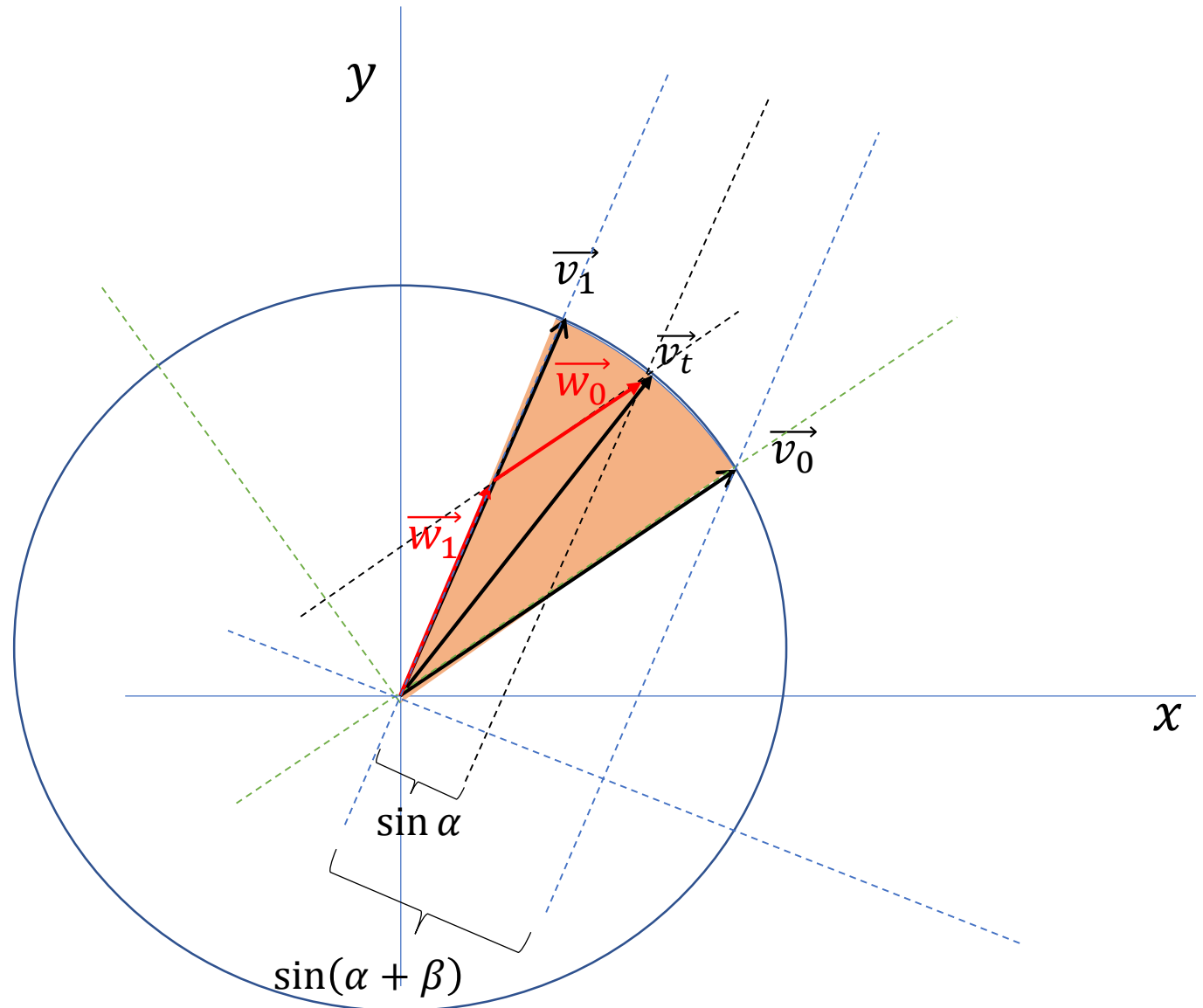
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$
 - $\vec{w}_0 = \frac{\sin \alpha}{\sin(\alpha + \beta)} \vec{v}_0$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{w}_1 = \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$
 - $\vec{w}_0 = \frac{\sin \alpha}{\sin(\alpha + \beta)} \vec{v}_0$
 - $\vec{v}_t = \vec{w}_0 + \vec{w}_1$



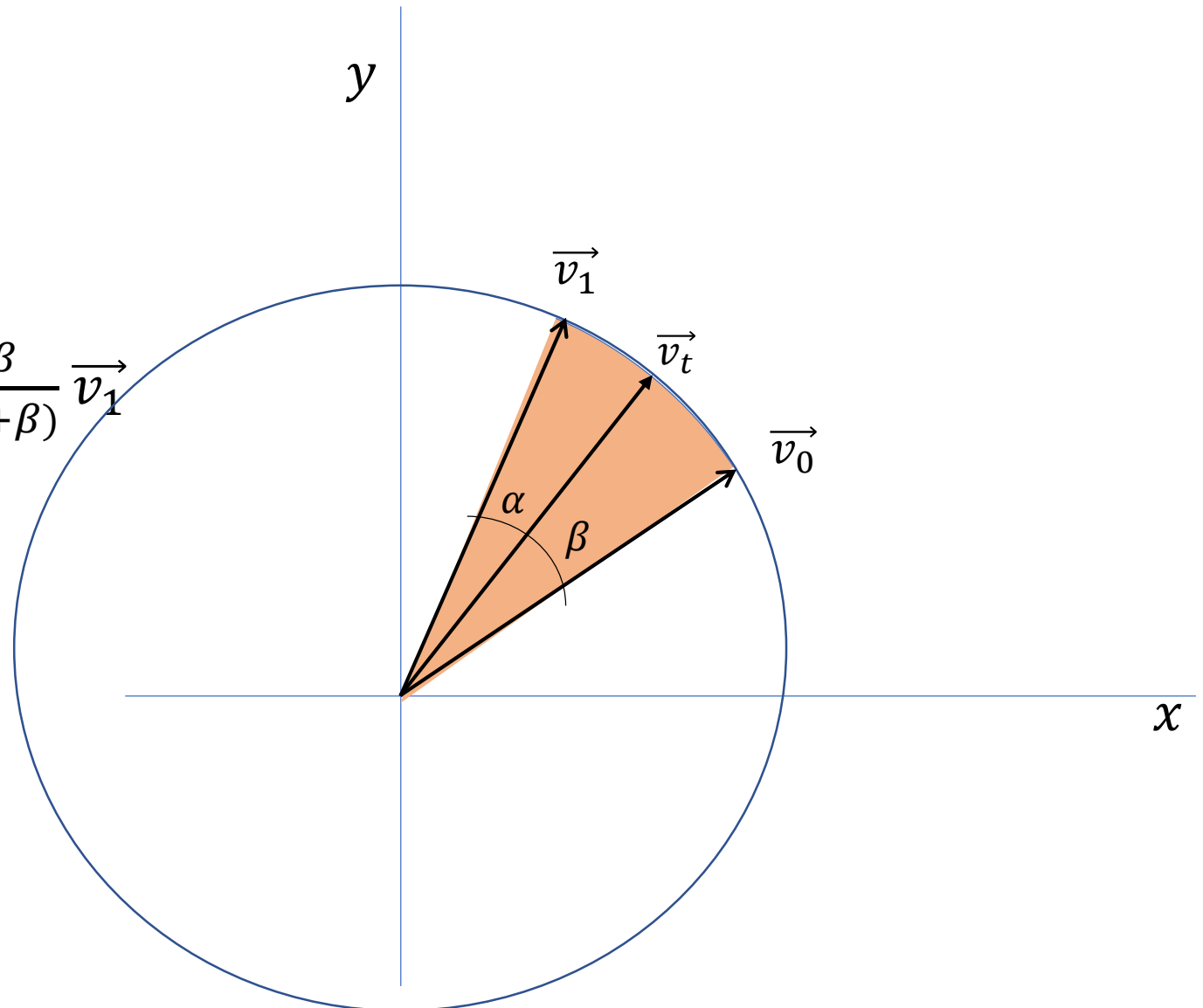
Spherical linear interpolation interpolation

- For $t \in [0, 1]$

- $|\vec{v}_0| = |\vec{v}_1|$

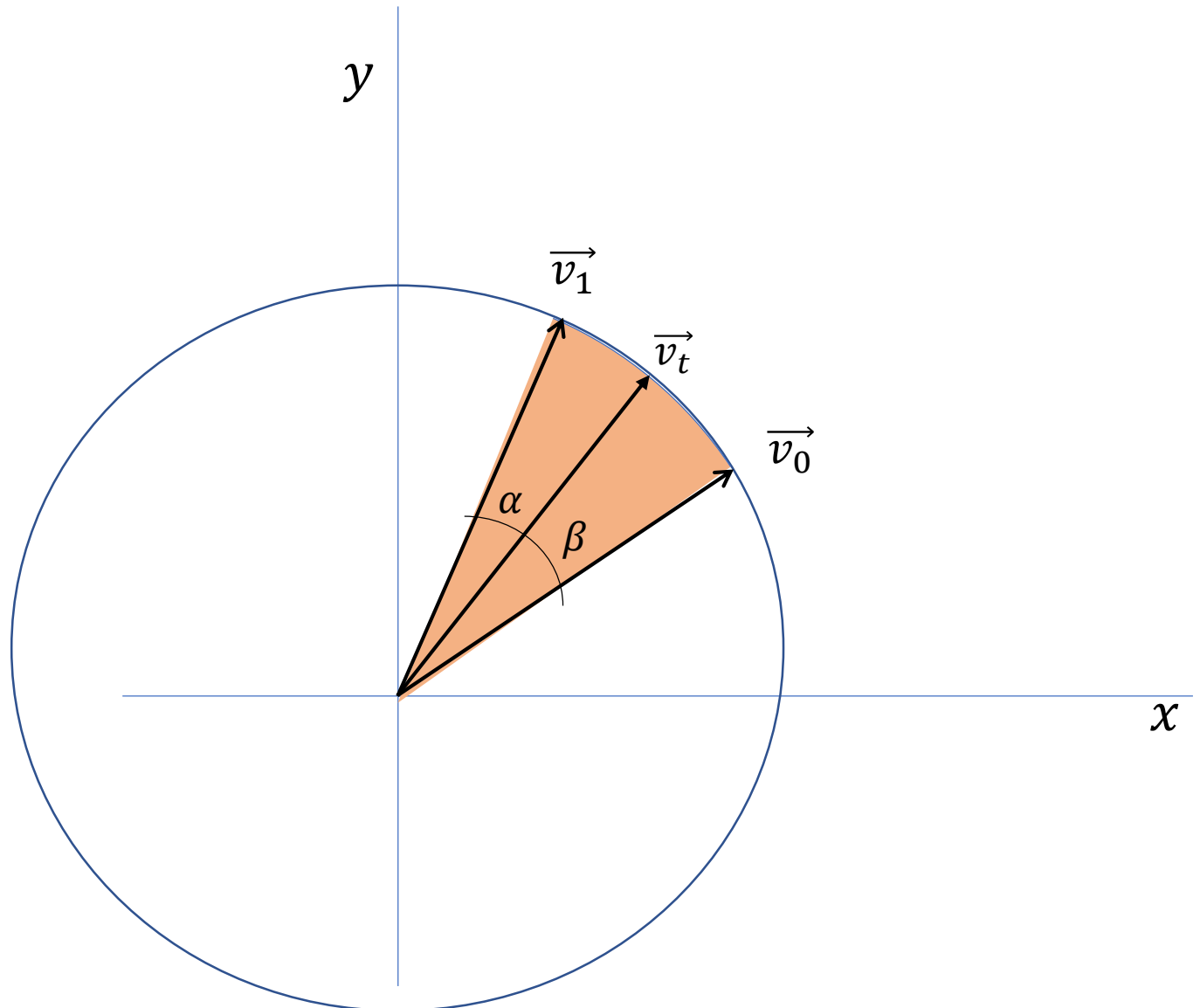
- $\theta = \alpha + \beta$

- $\vec{v}_t = \frac{\sin \alpha}{\sin(\alpha + \beta)} \vec{v}_0 + \frac{\sin \beta}{\sin(\alpha + \beta)} \vec{v}_1$



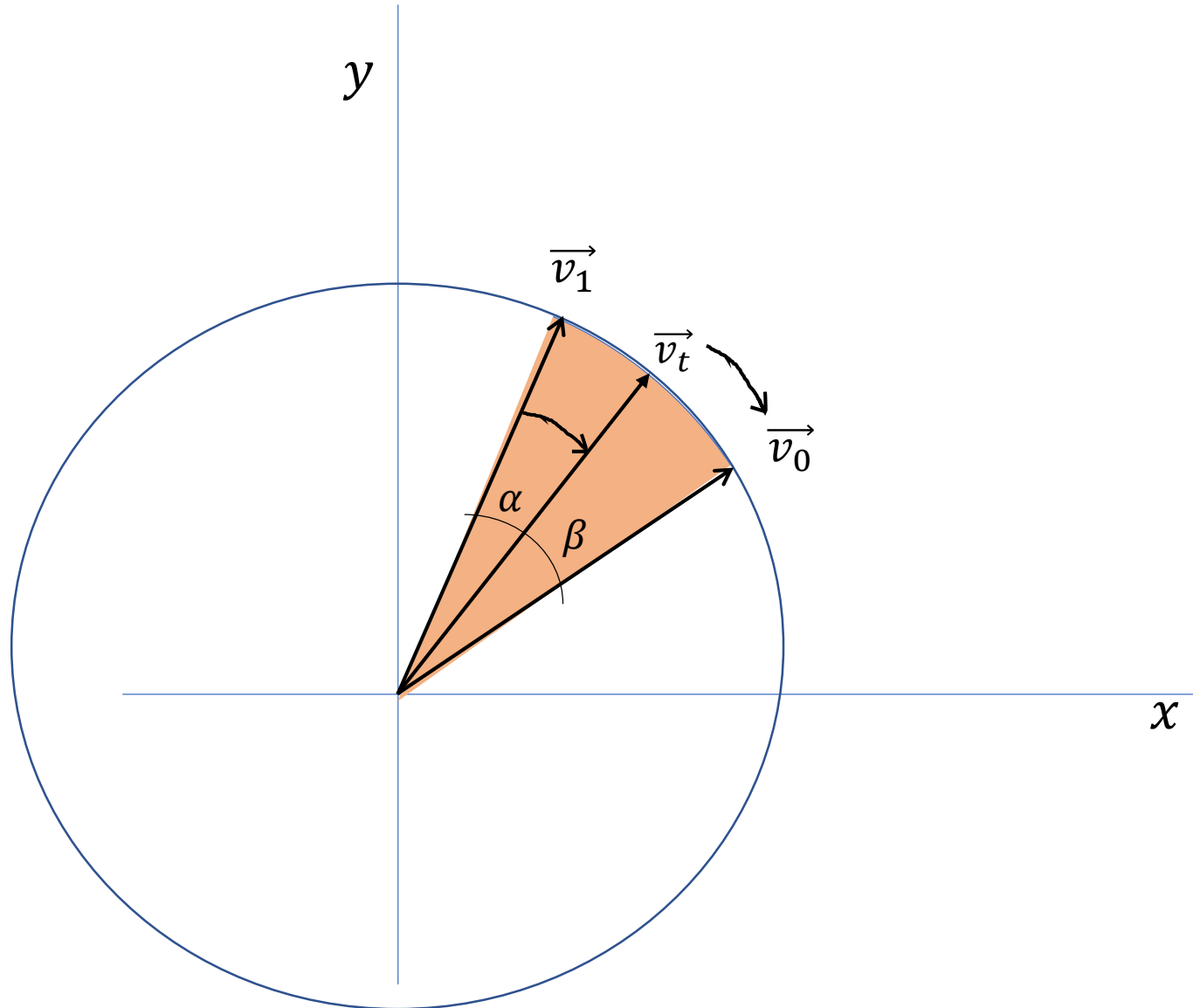
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $\vec{v}_t = \frac{\sin \alpha}{\sin \theta} \vec{v}_0 + \frac{\sin \beta}{\sin \theta} \vec{v}_1$



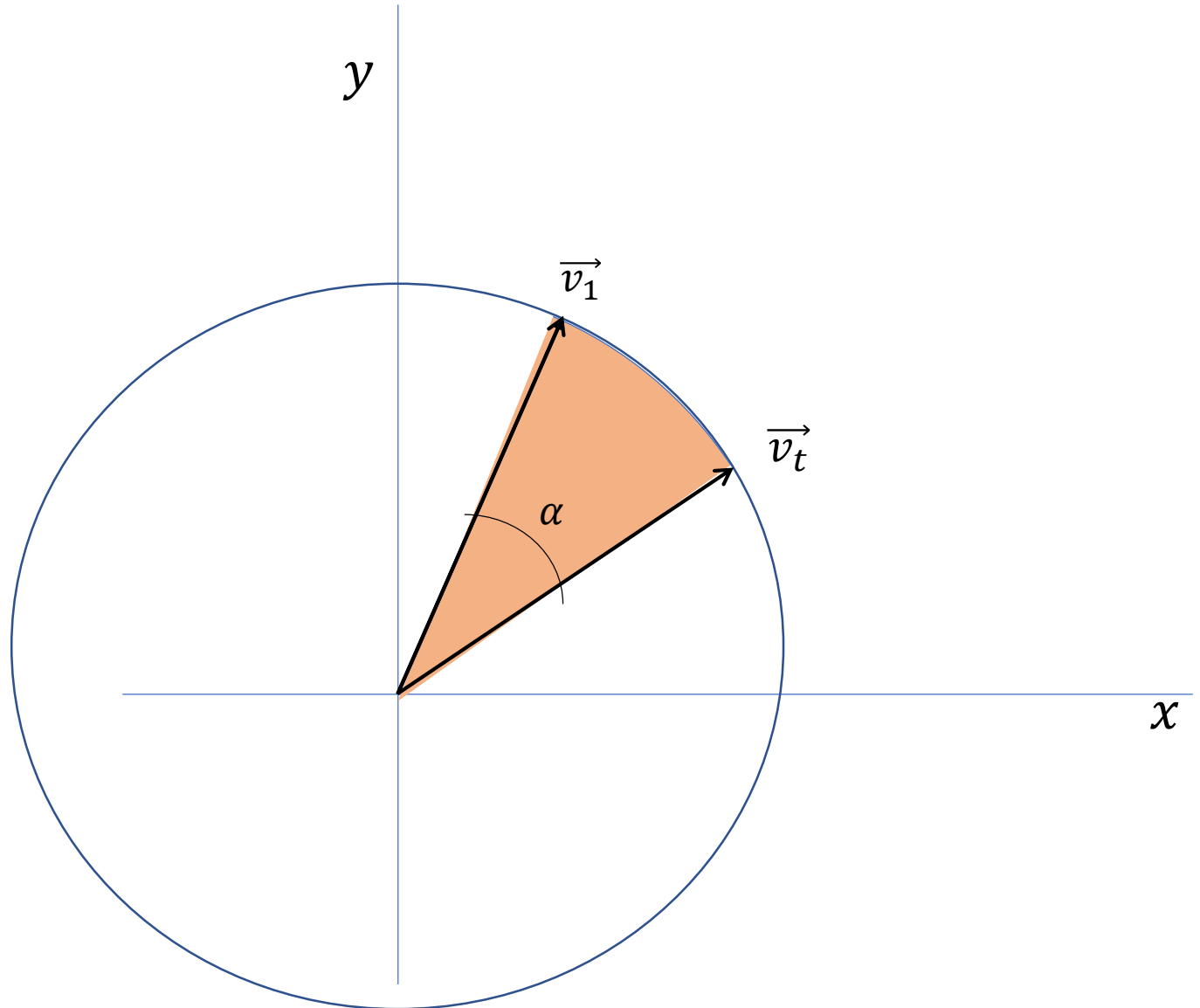
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $t = 0 \Rightarrow \vec{v}_t = \vec{v}_0$



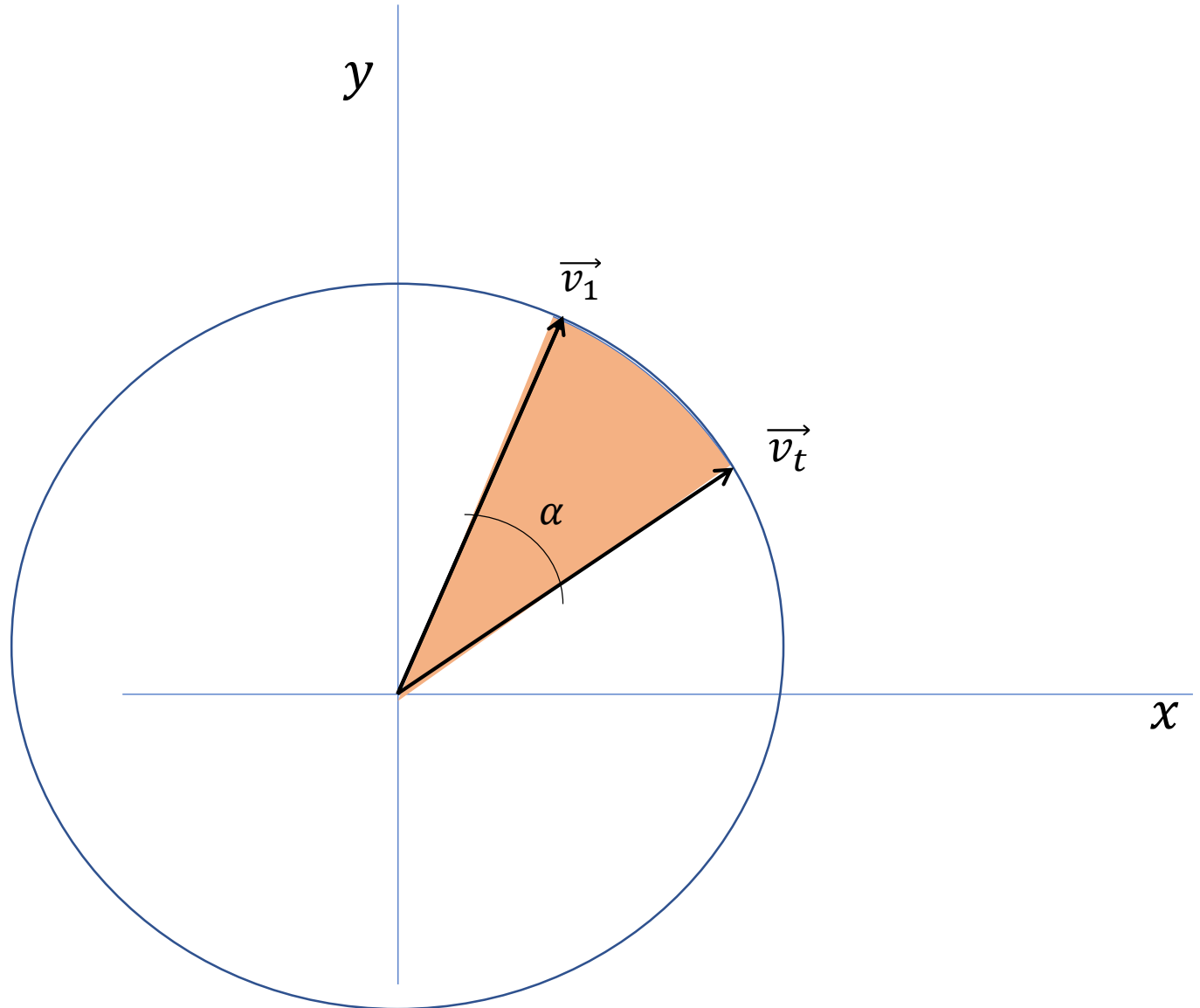
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $t = 0 \Rightarrow \theta = \alpha$
 - $\theta = \alpha \Rightarrow \beta = 0 = \theta t$
 - $\vec{v}_t = \frac{\sin \alpha}{\sin \theta} \vec{v}_0 + \frac{\sin \beta}{\sin \theta} \vec{v}_1$



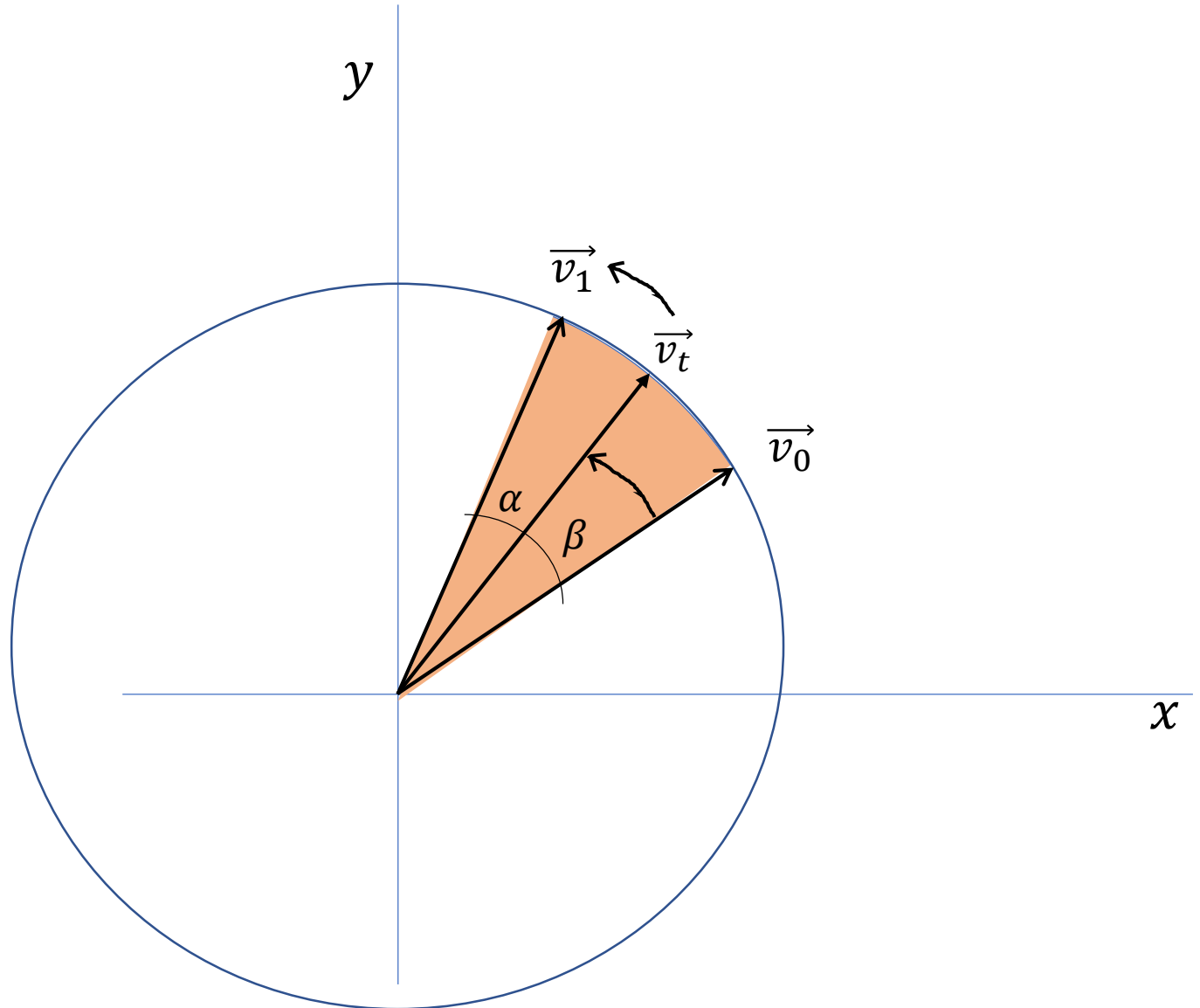
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $t = 0 \Rightarrow \theta = \alpha$
 - $\theta = \alpha \Rightarrow \beta = 0 = t\theta$
 - $\vec{v}_t = \frac{\sin \alpha}{\sin \theta} \vec{v}_0 + \frac{\sin t\theta}{\sin \theta} \vec{v}_1$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $t = 1 \Rightarrow \vec{v}_t = \vec{v}_1$
 - $\vec{v}_t = \frac{\sin \alpha}{\sin \theta} \vec{v}_0 + \frac{\sin t\theta}{\sin \theta} \vec{v}_1$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$

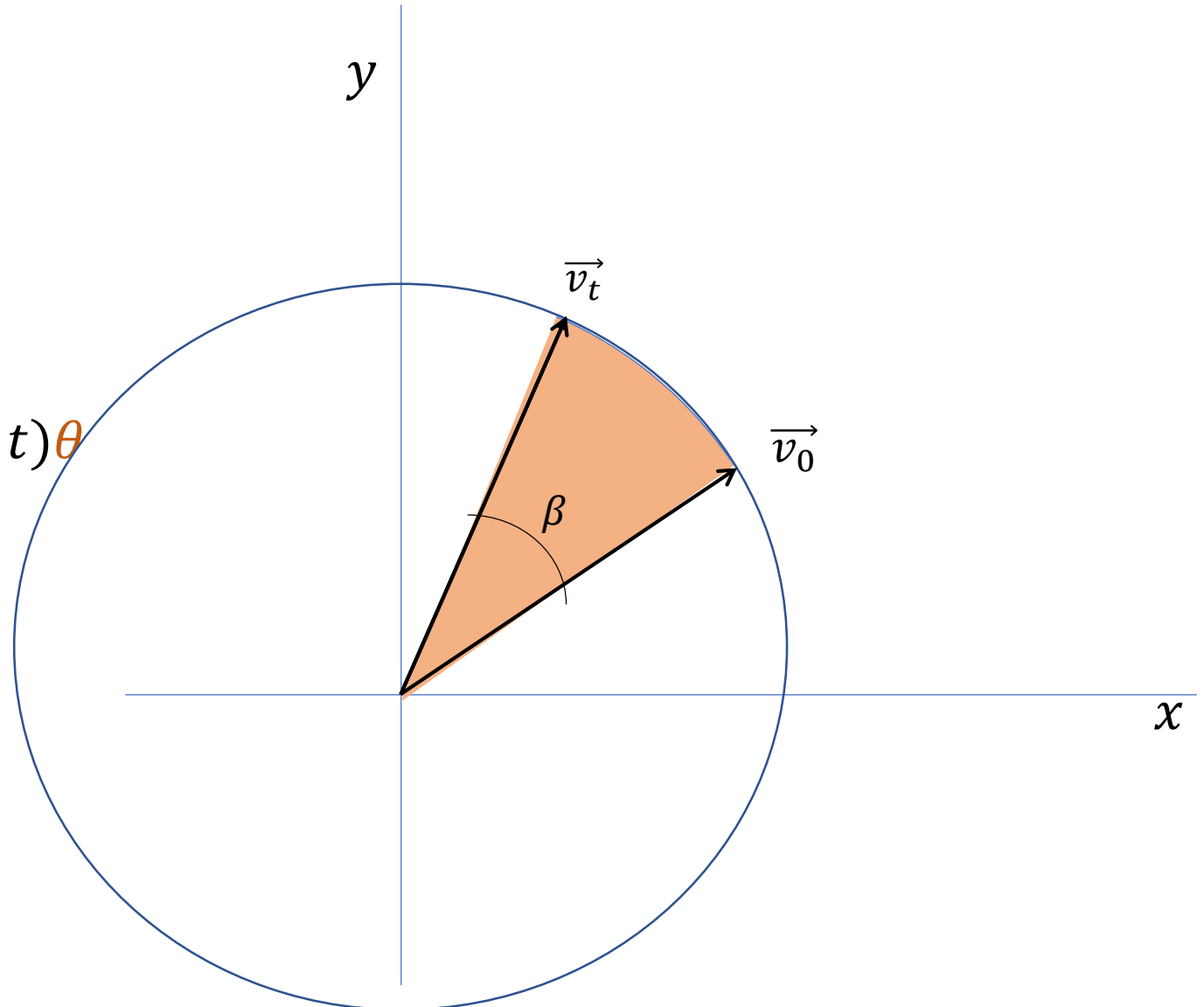
- $|\vec{v}_0| = |\vec{v}_1|$

- $\theta = \alpha + \beta$

- $t = 1 \Rightarrow \theta = \beta$

- $\theta = \beta \Rightarrow \alpha = 0 = (1 - t)\theta$

- $\vec{v}_t = \frac{\sin \alpha}{\sin \theta} \vec{v}_0 + \frac{\sin t\theta}{\sin \theta} \vec{v}_1$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$

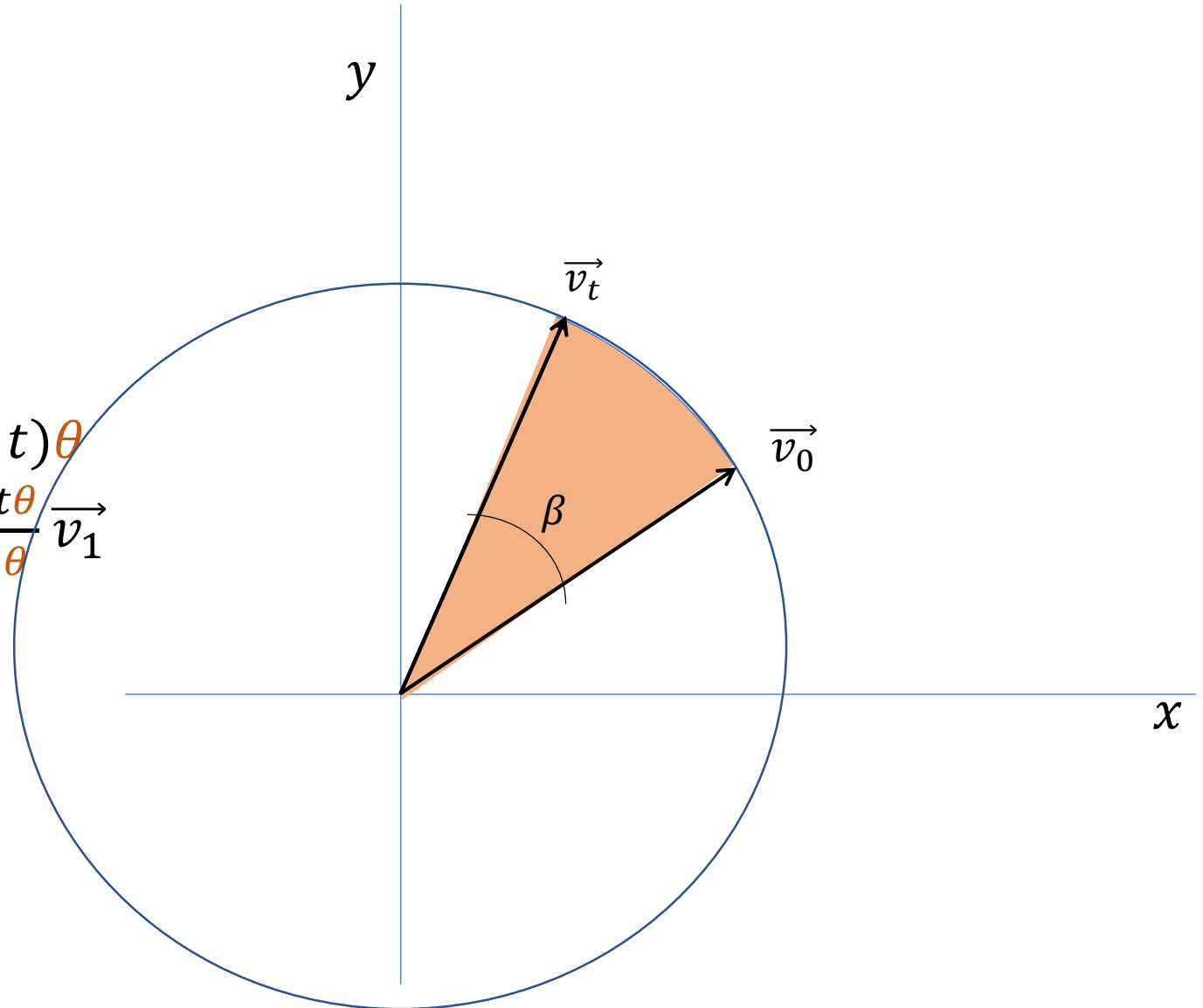
- $|\vec{v}_0| = |\vec{v}_1|$

- $\theta = \alpha + \beta$

- $t = 1 \Rightarrow \theta = \beta$

- $\theta = \beta \Rightarrow \alpha = 0 = (1 - t)\theta$

- $\vec{v}_t = \frac{\sin((1-t)\theta)}{\sin \theta} \vec{v}_0 + \frac{\sin t\theta}{\sin \theta} \vec{v}_1$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$
 - $|\vec{v}_0| = |\vec{v}_1|$
 - $\theta = \alpha + \beta$
 - $t = \frac{1}{2} \Rightarrow \alpha = \beta$
 - $\alpha = \beta \Rightarrow \alpha = \frac{\theta}{2} \wedge \beta = \frac{\theta}{2}$
 - $\vec{v}_t = \frac{\sin \frac{\theta}{2}}{\sin \theta} \vec{v}_0 + \frac{\sin \frac{\theta}{2}}{\sin \theta} \vec{v}_1$

