# Homework 1

Let us assume the right-handed global coordinate system as depicted in the Fig. 1. There is a camera located in the origin looking in the direction of the $x$ axis. The camera will subsequently rotate around the $y$ axis (see Fig. 1). Using the Catmull-Rom interpolation, we will rotate[1] the camera around the $y$ axis as a function of the parameter $t \in [0, 1]$. For $t = 0$ the camera is looking in the direction of the $x$ axis (Fig. 1 a)), for $t = 1$ the camera is looking in the direction of the $z$ axis (Fig. 1 b)). Rotations in the 3D space will be represented by the quaternions $q_0$, $q_1$, $q_2$ and $q_3$. The quaternion $q0 = q1$ will represents the camera rotation from its initial orientation into the orientation[2] where the camera view direction is in the direction of the $x$ axis (Fig. 1 a)). Quaternion $q_2$ will represent a clockwise rotation from the initial orientation into the orientation where the camera view direction is in the direction of the $z$ axis (Fig. 1 b)). Quaternion $q_3$ will represent a clockwise rotation from the initial orientation into the orientation where the camera view direction is in the negative direction of the $x$ axis (Fig. 1 c)).

Using the Catmull-Rom method interpolate the quaternions[3] $q_0$, $q_1$, $q_2$ and $q_3$ according to the particular parameter $t$ and then calculate the normalized camera view direction vector rotated into the resulting orientation. Define the parameter $t$ as $t = \frac{1}{d+m}$, where $m$ is the number of the month in your birthday date, while $d$ is the day number.

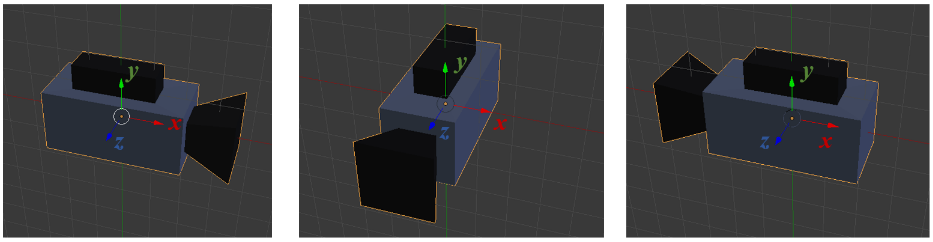
a) Express defined camera rotations (Fig. 1) around the $y$ axis in the form of the unit quaternions $q_0$, $q_1$, $q_2$ and $q_3$.
b) Compute the quaternion $q_t$ using Catmull-Rom interpolation for your parameter $t = \frac{1}{d+m}$. In each step of the computation verify, if the resulting quaternion is unit.
c) Compute the inverse of the quaternion $q_t$ to perform the rotation of the camera view vector. Express camera view direction in initial orientation as an unit vector $v$.

[1] Note that we are using the right-handed coordinate system while we are rotating clockwise.

[2] Note that at the beginning we have the camera already rotated in the orientation where the camera is looking in the direction of the $x$ axis.

[3] We can imagine normalized quaternions as a "points" on the 4D unit sphere. Catmull-Rom interpolation will compute new "points" between $q_1$ and $q_2$ on the sphere surface depending on the parameter $t$.

Fig. 1: The camera orientations.



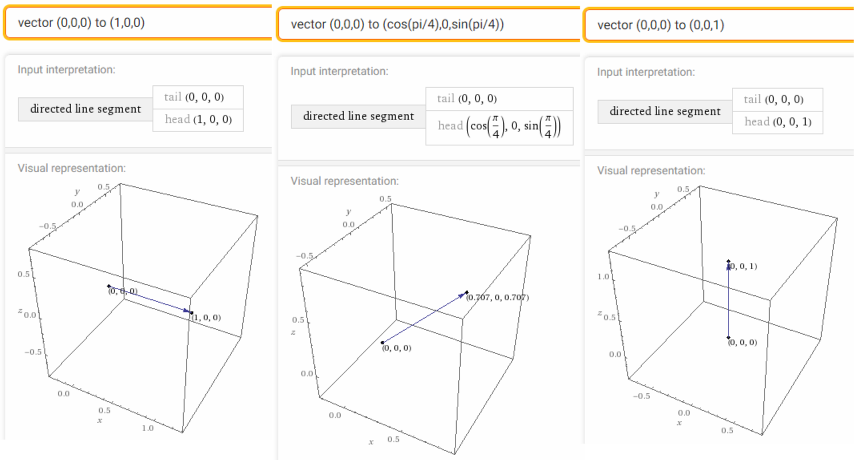

a) b) c)

d) Rotate the vector $v$ in order to obtain the view direction in the camera orientation satisfying your parameter $t = \frac{1}{d+m}$. Express rotated view direction as a unit vector $v_t$.

e) Interpolate the position of the camera over the smooth curve starting at the point $p_0$ (where the parameter $t = 0$) and ending ($t = 1$) at the point $p_3$. $p_0 = (0, 0, 0)$, $p_1 = (m, d, 0)$, $p_2 = (m/d, 10, 10)$ and $p_3 = (100, 100, 100)$. Plot the graph of the curve according to the parameter $t$ with the increment $\Delta t = 0.1$, plot the points $p_i$, plot all control points in spline.

f) Plot three camera view directions for $t = 0$, $t = \frac{1}{d+m}$ and $t = 1$ (see Fig. 2).

Fig. 2: Example of three subsequent view directions satisfying camera orientations for the particular parameter $t$. Examples are plotted using WolframAlpha.




**Explain in detail each calculation step.**