Computer Graphics

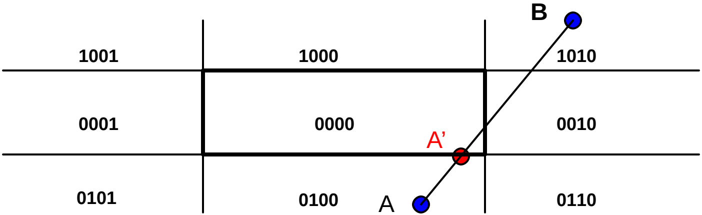
- Rasterization -

Overview

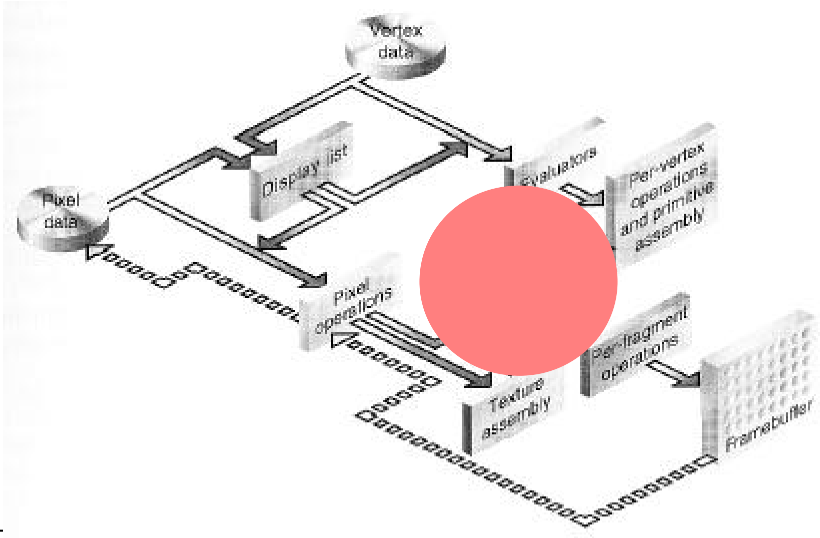
- **So far:**
  - Clipping
- **Today:**
  - Drawing 1D shapes
    - speed
    - quality
    - consistency
  - Filling 2D shapes
    - Finding inside pixels
    - Ambiguities
- **Next:**
  - RC presentation, computer graphics arts

Cohen-Sutherland revisited

- Unknown case: How to decide against which plane to clip
  1. Take one endpoint outside window (outcode ≠0000)
  2. Set outcode bits correspond to actual clipping planes
  3. From left to right (or right to left): intersect line with set-bit plane, assign intersection point as new end point
  4. Switch corresponding bit to 0
  5. Trivial accept / reject ? No: repeat from 3. for next set-bit plane



You are here ...


Vertex data
Pixel data
Display list
Evaluators
Per-vertex operations and primitive assembly
Pixel operations
Per-fragment operations
Texture assembly
Framebuffer

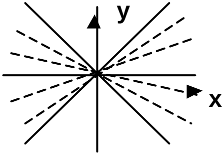
Shapes to Draw

- **Shapes to draw**
  - Lines
  - Circles, ellipses
  - Spline curves
  - …

- ***Rasterization* is the process of deciding which pixels to fill**
  - Term comes form the regular *raster* grid pattern for pixels
- **Necessity of pixel displays**
  - Line is infinitely thin, pixel is not
  - Want to draw best approximation to ideal line
  - Want to be efficient

Drawing a Line

- **Assumption**
  - Pixels are sample points on a 2D-integer-grid
    - OpenGL: integer-coordinate bottom left; X11, Foley: in the middle
  - Simple raster operations
    - setting of binary pixels
    - antialiasing later
  - End points at pixel coordinates
    - simple generalization
  - On straight lines: gradient $|m| \leq 1$
    - separate handling of horizontal and vertical lines
    - otherwise exchange of x & y: $|1/m| \leq 1$
  - Line width is one pixel
    - $|m| \leq 1$: 1 pixel per column (X-driving axis)
    - $|m| > 1$: 1 pixel per row (Y-driving axis)



$\Rightarrow$ Jaggies, aliasing !

Lines: As Function

- **Specification**
  - end points: $(x_0, y_0), (x_e, y_e)$
  - functional form: $y = mx + B$
- **Goal**
  - find pixels whose distance to the line is smallest
- **Brute-Force-Algorithm**
  - it is assumed that +X is the driving axis

```
for $x_i = x_0$ to $x_e$
 $y_i = m * x_i + B$
 setpixel($x_i$, Round($y_i$))
       // Round($y_i$)=Floor($y_i$+0.5)
```

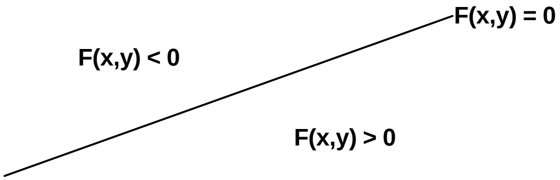
- **Comments**
  - m and y_i must be calculated with floating-point precision
  - expensive operations per pixel

Lines: DDA Algorithm

- **DDA: Digital Differential Analyzer**
  - Origin: solvers for simple incremental differential equations (the Euler method)
    - per step in time: `x´ = x + dx/dt, y´ = y + dy/dt`
- **Incremental algorithm**
  - Per pixel
    - $x_{i+1} = x_i + 1$
    - $y_{i+1} = m\ (x_i + 1) + B = y_i + m$
    - $\mathrm{setpixel}(x_{i+1},\ \mathrm{Round}(y_{i+1}))$
- **Remark**
  - Utilization of line coherence through incremental calculation
    - avoids multiplication
  - Cumulative error
    - usually negligible for short lines
    - double precision is recommended
  - Still floating point operations necessary

Lines: Midpoint Line Algorithm

- **Bresenham (´63)**
  - Also incremental, but integer arithmetic only
  - Uses a decision variable instead of the actual line equation
  - Presented for slope between 0 and 1, others can be done by symmetry
  - Implicit definition of line function: $F(x,y) := ax+by+c = 0$

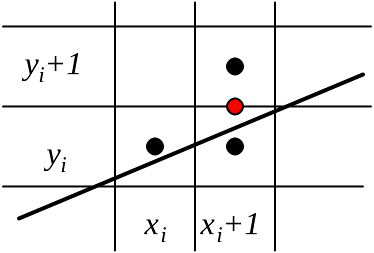


Bresenham Algorithm: Overview

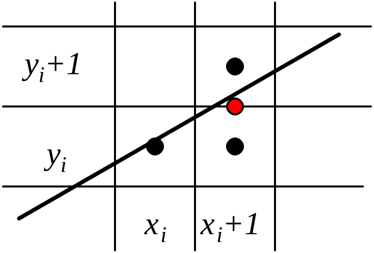
- **Goal: For each *x*, plot the pixel whose *y*-value is closest to the line**
  - Given (x_i, y_i), must choose from either (x_i+1, y_i+1) or (x_i+1, y_i)
- **Idea: compute a *decision variable***
  - Value that will determine which pixel to draw
  - Easy to update from one pixel to the next
- **Bresenham algorithm: *midpoint algorithm* for lines**
  - Other midpoint algorithms for conic sections (circles, ellipses)

Midpoint Method

- **Consider the midpoint between (x_i+1,y_i+1) and (x_i+1,y_i)**
- **If it's above the line, we choose (x_i+1,y_i), otherwise we choose (x_i+1,y_i+1)**



Choose (x_i+1,y_i)



Choose (x_i+1,y_{i+1})

Midpoint Decision Variable

- **Write the line in *implicit form:***
  - *Dx=x2-x1, Dy=y2-y1*

$$F(x,y) = ax + by + c = \Delta y \cdot x - \Delta x \cdot y + (\Delta x \cdot y_1 - \Delta y \cdot x_1)$$

- **The value of *F(x,y)* tells us where pixels are with respect to the line**
  - *F(x,y)=0*: the point is on the line
  - *F(x,y)<0*: The point is above the line
  - *F(x,y)>0*: The point is below the line
- **The decision variable is the value of**

$$d_i = 2F(x_i+1, y_i+0.5)$$

  - The factor of two makes the math easier: eliminates fraction

What Can We Decide?

$$d_i = 2\Delta y(x_i + 1) - 2\Delta x y_i + \Delta x(2c - 1)$$

- **d_i negative => next point at (x_i+1, y_i)**
- **d_i positive => next point at (x_i+1, y_i+1)**
- **At each point, we compute d_i and decide which pixel to draw**
- **How do we update it? What is d_{i+1}?**

Updating The Decision Variable

- **d_{k+1} is the old value, d_k, plus an increment:**

$$d_{k+1} = d_k + (d_{k+1} - d_k)$$

- **If we chose $y_{i+1}=y_i+1$:**

$$d_{k+1} = d_k + 2\Delta y - 2\Delta x$$

- **If we chose $y_{i+1}=y_i$:**

$$d_{k+1} = d_k + 2\Delta y$$

- **What is d_1 (assuming integer endpoints)?**

$$d_1 = 2\Delta y - \Delta x$$

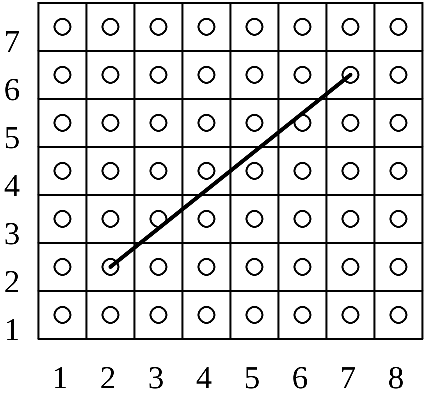
- **Notice that we don’t need *c* any more**

Bresenham Algorithm

- **For integers, slope between 0 and 1:**
  - $x=x_1$, $y=y_1$, $d=2dy - dx$, draw (x, y)
  - until $x=x_2$
    - $x=x+1$
    - If $d>0$ then { $y=y+1$; draw (x, y); $d=d+2Dy - 2Dx$; }
    - If $d<0$ then { $y=y$; draw (x, y); $d=d+2Dy$; }
- **Compute the constants ($2Dy$-$2Dx$ and $2Dy$) once at the start**
  - Inner loop does only adds and comparisons
- **Floating point has slightly more difficult initialization, but is otherwise the same**
- **Care must be taken to ensure that it doesn't matter which order the endpoints are specified in (make a uniform decision if $d==0$)**

Example: (2,2) to (7,6)

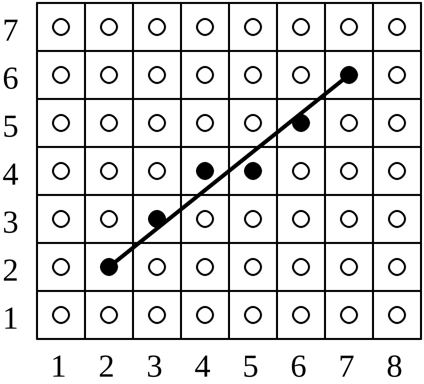
```
x=x1, y=y1, d1=2dy – dx
        If d>0 then { y=y+1; draw (x, y); d=d+2Dy - 2Dx; }
        If d<0 then { y=y; draw (x, y); d=d+2Dy; }
```



$\Delta x=5, \Delta y=4$

x	y	d

Example: (2,2) to (7,6)

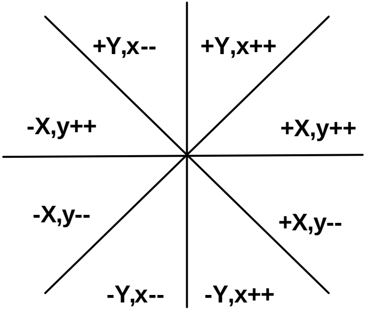


$\Delta x=5, \Delta y=4$

x	y	d
2	2	3
3	3	1
4	4	-1
5	4	7
6	5	5
7	6	3

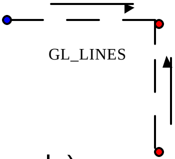
Lines: Arbitrary Directions

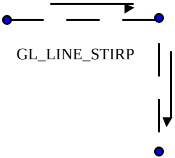
- **8 different cases**
  - driving (active) axis: ±X or ±Y
  - Increment/decrement of y or x, respectively



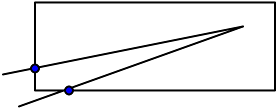
Lines: Some Remarks

- **Reversed end point order – consistency of pixel choices**
  - m > 0: (d ≤ 0)?
  - m < 0: (d ≥ 0)?



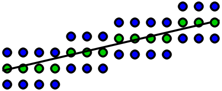


- **Dashed lines**
  - glLineStipple(Factor, 16-BitSample)
  - if (BitSample[(n++/Factor)%16]) then setpixel(...)
  - consistent continuation of dashing for line strips and loops
- **Weaker intensity of diagonal lines**
  - Same number of pixel on a larger distance (up to 41%)
- **Subpixel-precision**
  - Clipping, subpixel-coordinates
  - Correct initialization of the decision variable



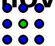
Thick Lines

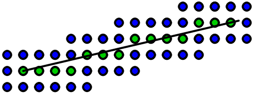
- **Pixel replication**



  - problems with even-numbered widths,
  - varying the intensity of a line as a function of slope

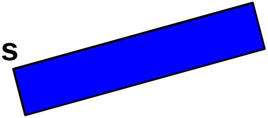
- **The moving pen**





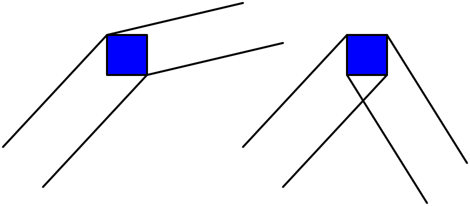
  - for some pen footprints the thickness of a line might change as a function of its slope

- **Filling areas between boundaries**



Line Joints

- **End point handling**



- **Avoid multiple drawings**
  - Local bitmap with already set pixels

Drawing Circles

- **Square roots and multiplication and trigonometry. Yuck.**
- **Symmetry. Yay.**
- **Similar to line scan conversion. Fine.**

Midpoint Circle Algorithm

- **Look at top right eighth of circle**
- **$d = F(x,y) = x^2 + y^2 - R^2$**
- **d = 0 on circle, < 0 under circle, > 0 over circle**

- **When have value at (x,y), choose next pixel by calculating d=F(x+1, y-.5)**
- **Initial d derivation, assuming start point is (0,R):**

  $F(1, R-.5) = 1 + (R^2 - R + .25) - R^2$

  $= 1.25 - R$

- **Eliminate float:**

  Define $h = d - .25$ and substitute h + .25 for d

  Initialize $h = 1 - R$ and check for h<-.25 instead of d<0

  Since h is always an integer, can just check for h<0

Midpoint Circle Algorithm

- **How to get next value of d incrementally:**
  - If didn't go down one line (same y, next x)

    $$\begin{aligned} d &= F(x+2, y-.5) = (x+2)^2 + (y-.5)^2 - R^2 \\ &= x^2+4x+4 + (y-.5)^2 - R^2 \\ &= x^2+2x+1 + (y-.5)^2 - R^2 + 2x + 3 \\ &= (x+1)^2 + (y-.5)^2 - R^2 + (2x + 3) \\ &= F(x+1, y-.5) + (2x + 3) \end{aligned}$$

    So new d is previous d plus $(2x + 3)$

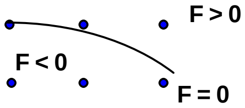
  - If did go down one line, similar derivation shows

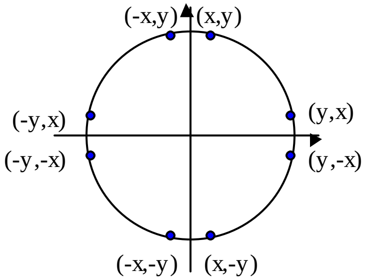
    new d is previous d plus $(2x - 2y + 5)$

Bresenham: Circle

- **Eight different cases**
  here +X, y—

```
Initialization: $x=0$, $y=R$
    $F(x,y)=x^2+y^2-R^2$
    $d=F(x+1, y-1/2)$
    $d < 0$:
        $d=F(x+2,y-1/2)$
    $d > 0$:
        $d=F(x+2,y-3/2)$
        $y=y-1$
    $x=x+1$
```





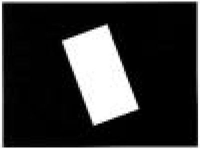
- **Eight-way symmetry: only one 45^{O} segment is needed to determine all pixels in a full circle**

Bresenham: More General

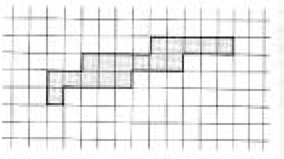
- **Midpoint method works well for ellipses and other implicitly definable curves**
    - Parabolas, hyperbolas, ...

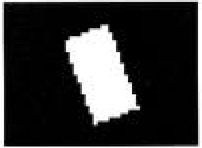
Anti-Aliasing

- **Supersampling**
  - Calculates solution in virtual screen space
    - higher resolution
  - Downsampling to real screen space
    - Grey values for partially covered pixels
  - Leaves rendering methodology unaltered



(a) Simulation of a perfect line

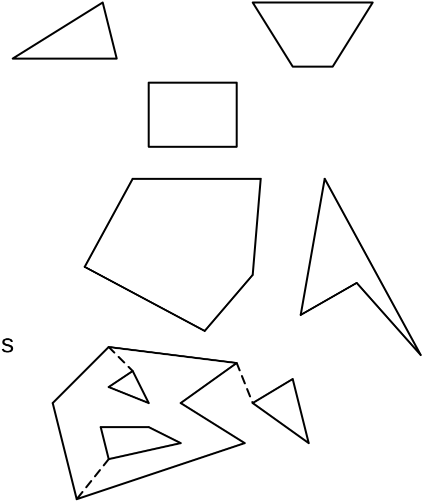




(c) Simulation of a jagged line

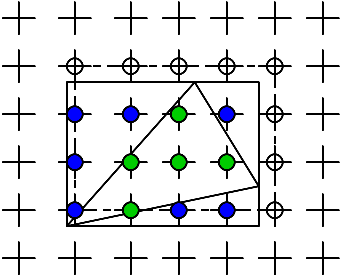
Polygons

- **Types**
  - triangles
  - trapezoids
  - rectangles
  - convex polygons
  - concave polygons
  - arbitrary polygons
    - holes
    - non-coherent
- **Two approaches**
  - polygon tessellation into triangles
    - edge-flags for internal edges
  - direct scan-conversion



Triangle Filling

- **Possible approaches**
  - *first bounding-box, then triangle*
  - *First triangle, then bounding-box*
- **Brute-Force algorithm**



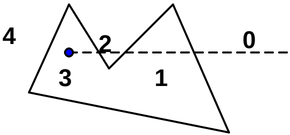
```
Raster3_box(vertex v[3])
{
  int x, y;
  bbox b;
  bound3(v, &b);
  for (y= b.ymin; y < b.ymax; y++)
    for (x= b.xmin; x < b.xmax; x++)
      if (inside(v, x, y))
       fragment(x,y);
}
```

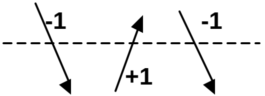
Filling Polygons

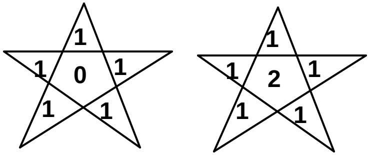
- **Sampling polygons:**
  - When is a pixel inside a polygon?
  - Given a pixel, which polygon does it lie in? *Point location*
- **Polygon representation:**
  - Polygon defined by a list of edges
    - each edge is a pair of vertices
  - All vertices are inside the view volume and map to valid pixels (clipping is behind us now)
  - Let’s assume integer window coordinates
    - to simplify things for now

Inside-Outside Tests

- **What is the interior of a polygon?**
  - Jordan curve
    - A planar curve homeomorphic to a circle is called Jordan curve. A Jordan curve separates a plane in two connected components, one of which is bounded.
  - Odd-even rule (odd parity rule)
    - counting the number of edge crossings with a ray starting at the queried point **P**
    - inside, if the number of crossings is odd
  - Non-zero winding number rule
    - signed intersections with a ray
    - inside, if the number is not equal to zero

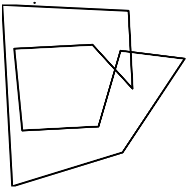




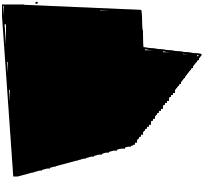


Inside/Outside Rules

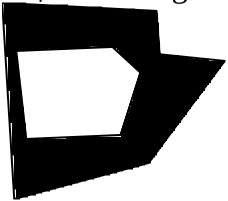
Polygon



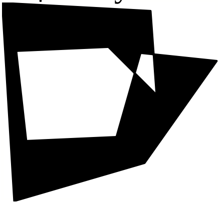
Non-exterior



Non-zero Winding No.

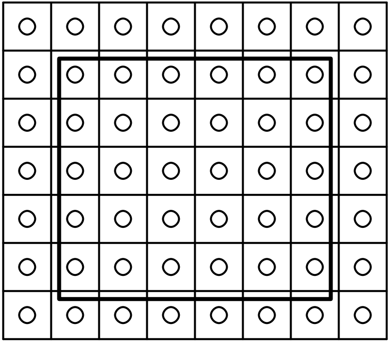


Parity



What is Inside ?

- **Assume sampling with an array of spikes**
- **If spike is inside, pixel is inside**



What is Inside ?

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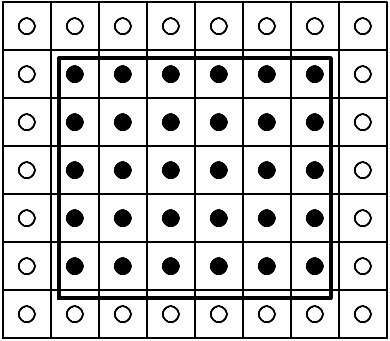
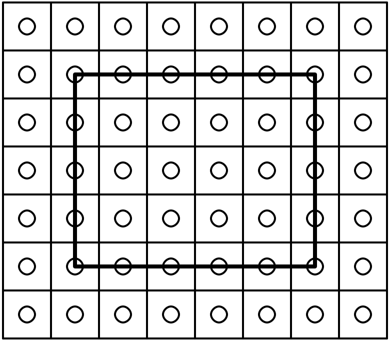


Ambiguous Cases

- **Ambiguous case: What if a pixel lies on an edge?**
  - Problem because if two polygons share a common edge, we don't want pixels on the edge to belong to both
  - Ambiguity would lead to different results if the drawing order was different
- **Rule: if $(x+e, y+e)$ is in, (x,y) is in**
- **What if a pixel is on a vertex? Does our rule still work?**

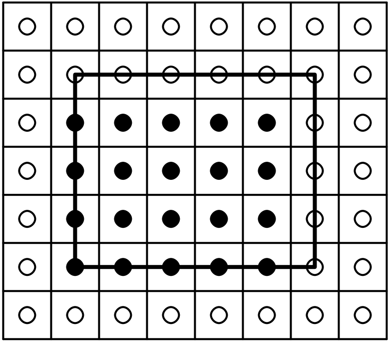
Ambiguous Case I

- **Rule:**
  - On edge?
    If $(x+e, y+e)$ is in, pixel is in
  - Which pixels are colored?
    - OpenGL origin convention !



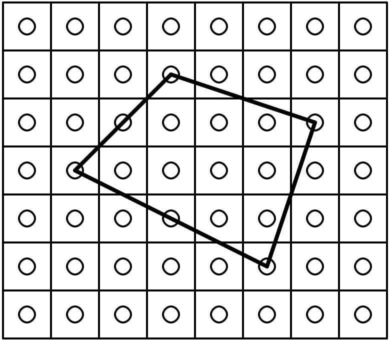
Ambiguous Case I

- **Rule:**
  - Keep left and bottom edges
  - Assuming y increases in the up direction
  - If rectangles meet at an edge, how often is the edge pixel drawn?

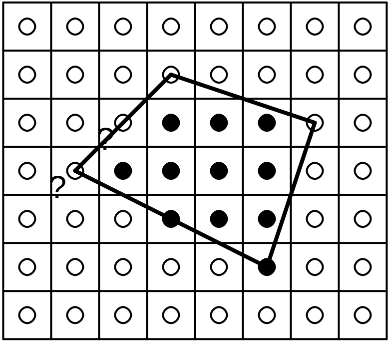


Ambiguous Case II

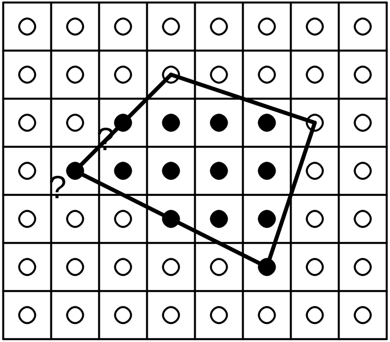
- **Rule:**
  - On edge?
    If $(x+e, y+e)$ is in, pixel is in
  - What happens for diagonal edges ?



Ambiguous Case II

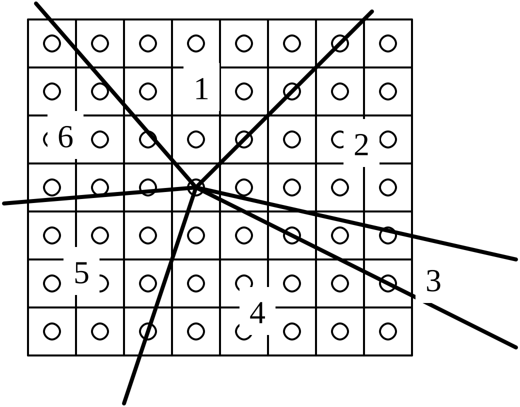


or



Really Ambiguous

- **We will accept ambiguity in such cases**
  - The center pixel may end up colored by one of two polygons in this case
  - Which two?
- **Might be solvable using *(x+e, y+e^2) (?)***
  - Arbitrarily small, irrational slope
  - Rule stays the same

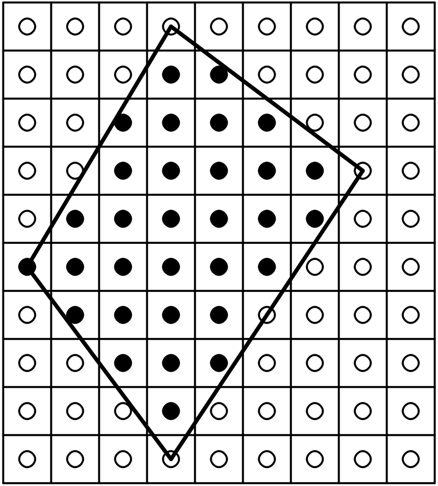


Scanline Conversion

- **Fill pixel area inside polygon edges**
- **Exploiting Coherence when filling a polygon**
  - Several contiguous pixels along a row tend to be in the polygon - a *span* of pixels
    - *Scanline* coherence
  - **Consider whole spans, not individual pixels**
  - Pixel number and position don’t vary much from one span to the next
    - *Edge coherence*
  - **Incrementally update span endpoints**

Spans

- **Process - fill the bottom horizontal span of pixels; move up and keep filling**
- **Have x_{min}, x_{max} for each span**
- **Define:**
  - floor(x): largest integer < x
  - ceiling(x): smallest integer >=x
- **Fill from ceiling(x_{min}) up to floor(x_{max})**
- **Consistent with convention**



Algorithm

- **For each row in the polygon:**
  - Throw away irrelevant edges
  - Obtain newly relevant edges
  - Fill span
  - Update current edges
- **Issues:**
  - How do we update existing edges?
  - When is an edge relevant/irrelevant?
- **All can be resolved by referring to our convention about what polygon the pixel belongs to**