Lecture 11: Abduction

2-AIN-144/2-IKV-131 Knowledge Reperesentation & Reasoning

Martin Baláž, Martin Homola

Department of Applied Informatics
Faculty of Mathematics, Physics and Informatics
Comenius University in Bratislava

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Example

Assuming a theory Γ:

$$rain \rightarrow wet\_road$$
$$rain \rightarrow wet\_grass$$
$$sun \leftrightarrow \neg rain$$
$$irrigation \rightarrow wet\_grass$$
$$sun \wedge hot\_day \rightarrow irrigation$$

Observing $\Delta = \{wet\_grass\}$, what can we conclude?

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Observing $\Delta = \{wet\_grass\}$, what can we conclude?
Nothing really. But we can sat that $A = \{rain\}$ is a possible explanation of Δ.

Abduction

Definition (Abduction)

Given two sets of formulae $\mathcal{E}$ (possible effects) and Φ (possible explanations) s.t. $\mathcal{E} \cap \Phi = \emptyset$, and given a set of formulae Γ, we say that $A \subseteq \Phi$ abductively explains $\Delta \subseteq \mathcal{E}$ w.r.t. the background theory Γ if:

- $\Gamma \cup A$ is consistent
- $\Gamma \not\models \Delta$
- $\Gamma \cup A \models \Delta$

Example (cont.)

Given $\mathcal{E} = \{wet\_road, wet\_grass\}$, $\Phi = \{rain, sun, irrigation, hot\_day\}$, and a theory Γ:

$$
\begin{aligned}
rain &\rightarrow wet\_road \\
rain &\rightarrow wet\_grass \\
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irrigation &\rightarrow wet\_grass \\
sun \wedge hot\_day &\rightarrow irrigation
\end{aligned}
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What are all possible explanations of $\Delta = \{wet\_grass\}$?

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What are all possible explanations of $\Delta = \{wet\_grass\}$?

$$A_1 = \{rain\}$$
$$A_2 = \{irrigation\}$$
$$A_3 = \{sun, hot\_day\}$$
$$A_4 = \{sun, irigation\}$$
$$A_5 = \{sun, hot\_day, irigation\}$$

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A_1 &= \{rain\} \\
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A_5 &= \{sun, hot\_day, irigation\}
\end{aligned}
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Strength of explanations

Not all explanations are equally preferred. A_2 and A_5 can be deduced from A_3 using Γ. We say that A_3 is stronger then A_2 and A_5.

Definition

Given Γ, $\Delta \subseteq \mathcal{E}$, and $A, A' \subseteq \Phi$ s.t. $A \neq A'$. We say that the explanation A' is

- stronger than A w.r.t. Γ if $\Gamma \cup A' \models A$ and $\Gamma \cup A \not\models A'$
- independent w.r.t. Γ if no stronger explanation Γ exists.

Minimality of explanations

Explanations A_1, A_3, A_4 are all independent, still A_4 can be seen as less preferred because it has a proper subset (A_2) which is an explanation too.

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Definition

Given Γ, $\Delta \subseteq \mathcal{E}$ we say that an explanation $A \subseteq \Phi$ of Δ w.r.t. Γ is minimal if there is no $A' \subsetneq A$ that is also an explanation of Δ w.r.t. Γ.

Computing explanations

A more abstract look at the abduction problem:

Definition (Abduction Problem)

An abduction problem is a quadruple $\langle \Delta, \Phi, e, < \rangle$ s.t.

- Δ is a set of observations (to be explained)
- Φ is a set of hypotheses (possible explanations)
- $e : 2^{\Phi} \to 2^{\Delta}$ is an explanation function
- $< \subseteq 2^{\Phi} \times 2^{\Phi}$, a plausibility oder on possible explanations is a partial order.

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- $< \subseteq 2^{\Phi} \times 2^{\Phi}$, a plausibility oder on possible explanations is a partial order.

Note: The explanation functions e stands for the deduction w.r.t. some background theory Γ: if $e(H) = D \subseteq \Delta$ in means that the set of hypotheses H explains the set of observations D.

The plausibility order stands for preference on hypotheses. We always prefer the more plausible hypothesis.

We can easily compute some explanation if the abduction problem is independent:

Definition (Independent Abduction Problem)

An abduction problem $\langle \Delta, \Phi, e, < \rangle$ is independent if

$$(\forall H \subseteq \Phi)\ e(H) = \bigcup_{h \in H} e(h)$$

Computing explanations (cont.)

If an explanation of an independent abduction problem $\langle \Delta, \Phi, e, < \rangle$ exists, the following greedy algorithm computes some explanation:

1. if $e(\Phi) \neq \Delta$ return "No explanation" and terminate
2. $H := \Phi$
3. for all $h \in \Phi$ do
    1. if $e(H \setminus \{h\}) = \Delta$ then $H := H \setminus \{h\}$
4. return H

Computing best explanations (cont.)

Definition (Ordered Abduction Problem)

An abduction problem $\langle \Delta, \Phi, e, < \rangle$ is ordered if if for all $h, h' \in \Phi$ s.t. $h \neq h'$ we have either $h < h'$ or $h' < h$.

Definition (Best explanations)

Given an abduction problem $\langle \Delta, \Phi, e, < \rangle$, an explanation $H \subseteq \Phi$ if Δ is best if there is no explanation $H' \subseteq \Phi$ s.t. $H < H'$.

Computing best explanations (cont.)

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Note that more than one best explanation is possible as $<$ is a partial order.

Computing best explanations (cont.)

If an explanation of an ordered independent abduction problem $\langle \Delta, \Phi, e, < \rangle$ exists, the following greedy algorithm computes some best explanation:

1. if $e(\Phi) \neq \Delta$ return "No explanation" and terminate
2. $H := \Phi$
3. for all $h \in \Phi$ ordered from lest to most plausible do
    1. if $e(H \setminus \{h\}) = \Delta$ then $H := H \setminus \{h\}$
4. return H

References:

- Šefránek, J.:Inteligencia ako výpočet. IRIS, 2000.