Variables in description logic

- **Ground descriptions** are descriptions without variable

  \[
  \text{FemaleStudent} \equiv \text{Person} \land \text{Female} \land \exists \text{studyIn}. \text{University}
  \]

- **Patterns** are concept descriptions with variables

  \[
  \text{Pattern} \equiv \exists x. \text{Y} \land \exists y. \text{U}
  \]
  
  with Y a concept variable and x a role variable

- **Variable substitutions**

  \[
  \sigma(x') = \text{Person} \land \text{Female} ; \sigma(x) = \text{studyIn}
  \]
  
  \[
  \sigma(\text{Pattern}) \equiv \text{Person} \land \text{Female} \land \exists \text{studyIn}. \text{University}
  \]

**Refreshing semantics**

- **Classical (non-refreshing) semantics**: All instances of a variable are assigned to the same value.

- **Refreshing semantics**: Instances produced by unfolding cycles may lead to different assignments.

Once unfolded **Academic twice**, the resulting definitions are obtained (indexes distinguish instances):

- **Classical semantics**:

  \[
  \text{Academic} \equiv \text{Person} \land \exists x. \text{University} \land \exists y. \text{Academic}
  \]

- **Refreshing semantics**:

  \[
  \text{Academic} \equiv \text{Person} \land \exists x_1. \text{University} \land \exists y_1. (\exists x_2. \text{University} \land \exists y_2. \text{Academic})
  \]

Note that with refreshing semantics, we may have, \( \sigma(x_1) \neq \sigma(x_2) \) or \( \sigma(y_1) \neq \sigma(y_2) \).

- These new substitution possibilities may propose new or even better solutions to matching problems.

**Introduction**

**Matching**

- Is there a substitution \( \sigma \) such that \( \sigma(P) \sqsubseteq C \) with \( P \) a pattern and \( C \) a ground description?

- **Pattern** \( \sqsubseteq \text{FemaleStudent} \) has a solution since \( \sigma(\text{Pattern}) \sqsubseteq \text{FemaleStudent} \)

**Examples**

- **TBox \( T \)**

  \[
  \begin{align*}
  \text{PhDStudent} & \equiv \text{Person} \land \exists \text{studyIn}. \text{University} \land \exists \text{supervisedBy}. \text{Doctor} \\
  \text{Doctor} & \equiv \text{Person} \land \exists \text{getPhDIn}. \text{University} \land \exists \text{formerly}. \text{PhDstudent} \\
  \text{FrenchUniversity} & \equiv \text{University} \land \exists \text{locatedIn}. \text{France} \\
  \text{FrenchPhDstudent} & \equiv \text{Person} \land \exists \text{getPhDIn}. \text{FrenchUniversity} \land \exists \text{formerly}. \text{FrenchPhDstudent} \\
  \text{FrenchPhDstudent} & \equiv \text{Person} \land \exists \text{studyIn}. \text{FrenchUniversity} \land \exists \text{supervisedBy}. \text{Doctor}
  \end{align*}
  \]

- **Let consider the following pattern Academic**

  \[
  \text{Academic} \equiv \text{Person} \land \exists x. \text{University} \land \exists y. \text{Academic}
  \]

  **Is there a substitution \( \sigma \) such that \( \sigma(\text{Academic}) \sqsubseteq \text{Doctor} \)?**

  - Under non-refreshing semantics this problem is unsolvable.
  - Under refreshing semantics, the following substitution \( \sigma \) is possible

  \[
  \begin{align*}
  \sigma(x_1) & = \text{getPhDIn} \\
  \sigma(x_2) & = \text{formerly} \\
  \sigma(y_1) & = \text{supervisedBy} \\
  \sigma(y_2) & = \text{formerly}
  \end{align*}
  \]

  \[
  \begin{align*}
  \sigma(\text{Academic}) & \equiv \text{Person} \land \exists x_1. \text{University} \land \exists y_1. (\exists x_2. \text{University} \land \exists y_2. \text{Academic})
  \end{align*}
  \]

  Note that \( \sigma(\text{Academic}) \sqsubseteq \text{Doctor} \)

  - **Conclusion**

    Some unsolvable matching problems for the non-refreshing semantics find a solution with the refreshing semantics.

- **Let consider the following pattern Acad2**

  \[
  \begin{align*}
  \text{Acad2} & \equiv \text{Person} \land \exists \text{getPhDIn}. \text{X} \land \exists \text{formerly}. (\text{Person} \land \exists \text{studyIn}. \text{X} \land \text{Acad2})
  \end{align*}
  \]

  **Is there a substitution \( \sigma \) such that \( \sigma(\text{Acad2}) \sqsubseteq \text{FrenchDoctor} \)?**

  - Under non-refreshing semantics, \( \sigma(X) = \text{FrenchUniversity} \) is a solution.
  - Under refreshing semantics, the following substitution \( \theta \) is possible

  \[
  \begin{align*}
  \theta(x_1) & = \text{FrenchUniversity} \\
  \theta(x_2) & = \text{University} \text{ for } i > 1
  \end{align*}
  \]

  Note that \( \sigma(\text{Acad2}) \sqsubseteq \theta(\text{Acad2}) \equiv \text{FrenchDoctor} \)

  - **Conclusion**

    Some matching problems find a better (i.e. closer to the targeted concept)) solution under the refreshing semantics.

**New reasoning task : Weak-subsumption**

Let \( \mathcal{T} \) be an \( \mathcal{EL} \)-TBox and let \( P, Q \) be two \( \mathcal{EL} \)-patterns (with potentially refreshing variables).

Then, \( P \) is weakly subsumed by \( Q \) iff there exist substitutions \( \psi_1 \) and \( \psi_2 \) s.t. \( \psi_1(P) \) is subsumed by \( \psi_2(Q) \).

**Preliminary Results & Perspectives**

- Our main result is showing that testing weak-subsumption in \( \mathcal{EL} \) for the greatest fix-point semantics with role variables is EXPTIME-Complete.

  The main steps of our approach are

  - Associate to each pattern an automaton which is a compact representation of all the possible instantiations.
  - Characterize weak-subsumption in terms of existential simulation between automata.
  - Devise a correct algorithm which has exponential time complexity at worst proving its optimality.

- **Futur research works**

  - Extending our framework to handle concept variables.
  - Considering additional reasoning that go beyond weak-subsumption.
  - Investigating other description logics such as \( \mathcal{FL} \) and \( \mathcal{ALN} \).

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[1] Raedler, F., Moranowski, R.: Matching with respect to general concept inclusions in the description logic \( \mathcal{EL} \).


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