Introduction to cognitive science

Session 8: Dynamical Systems

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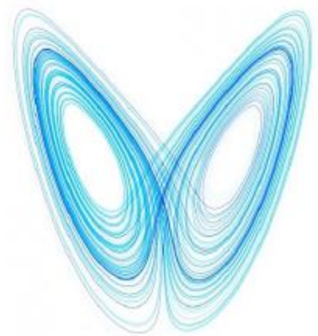
Dynamical system approach

- Alternative to the prevailing **information processing** framework
- Focusing on **process** rather than on **states**
- Good for describing **continuous temporal dynamics**
- Uses the mathematical language of **differential equations**
- Used in many different scientific areas:
  - Physics
  - Immunology
  - Biology
  - Brain sciences
  - Psychology
  - Business



Basic terms

- **System structure:** variables and relations
- **System state:** values of a set of variables at some timepoint
- **System behaviour:** how the state evolves through time (differential equations for relations, mathematical functions for variables)
- Phase space, trajectory, phase portrait
- Equilibrium
- Stability, Attractor

Example

- Spring-mass system

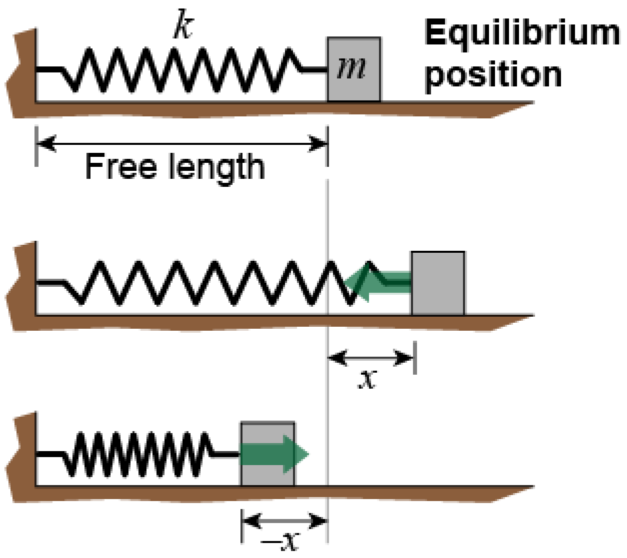


Image from: https://education.pasco.com/epub/PhysicsNGSS/BookInd-1220.html

Dynamical system

- Mathematical object that unambiguously describes how the state of some system evolves over time (Beer, 2000)
  - $<T, S, \Phi_t>$
    - Ordered time set (continuous/discrete)
    - **State (phase) space** (numerical/symbolic, continuous/discrete/hybrid, finite/infinite-dimensional)
    - Evolution operator (rule)
      - Transforms an initial state at some time to another state at another time
      - Implicit/explicit, deterministic/stochastic

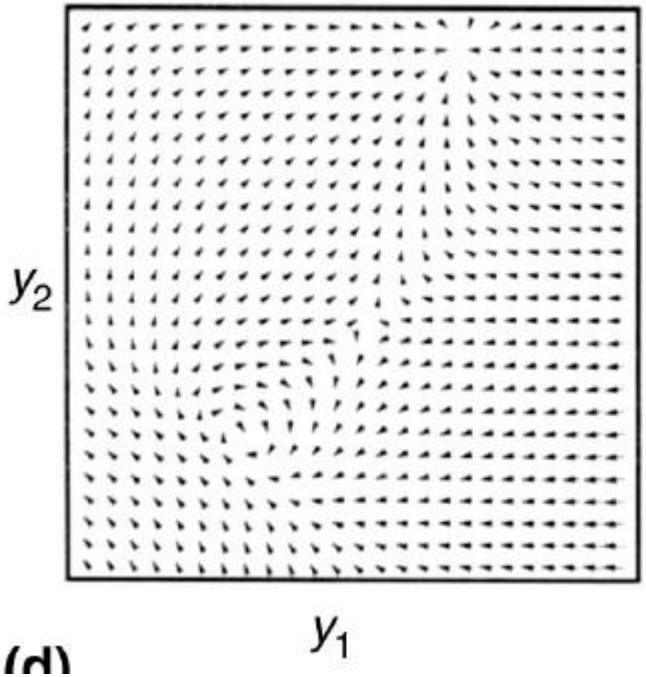
Phase portrait

- 2-D continous time dynamical system
- Φ_t:

$$\dot{y}_1 = f_1\,(y_1, y_2)$$

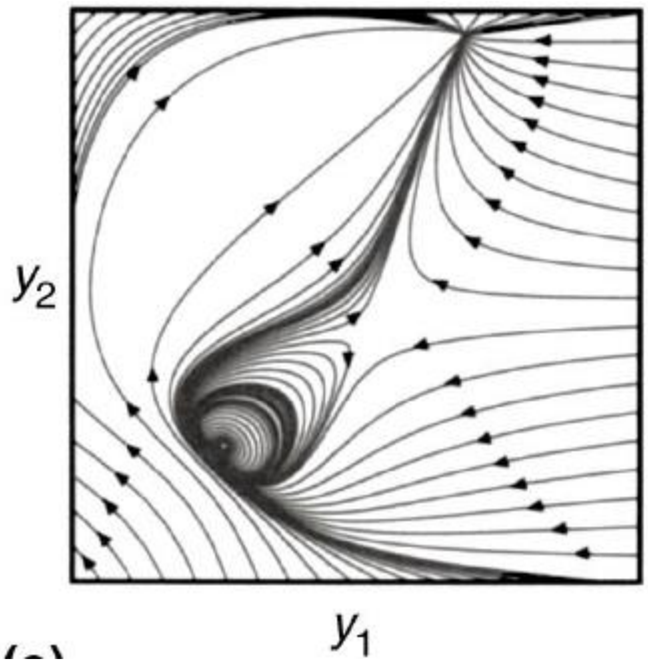
$$\dot{y}_2 = f_2\,(y_1, y_2)$$

- Define a **vector field** that assigns an instantaneous direction and magnitude of change to each point in the state space



Solution trajectory

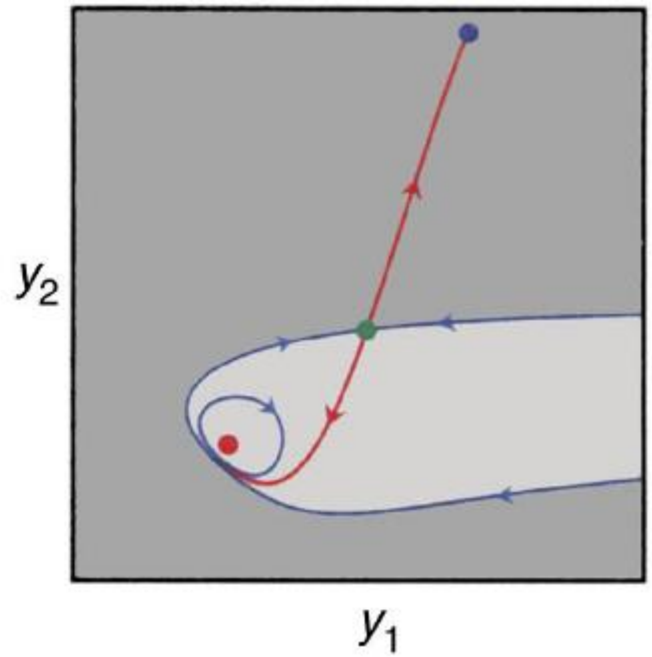
- **the sequence of states, starting from some initial state**
  - The set of all possible solution trajectories
    - **Flow**



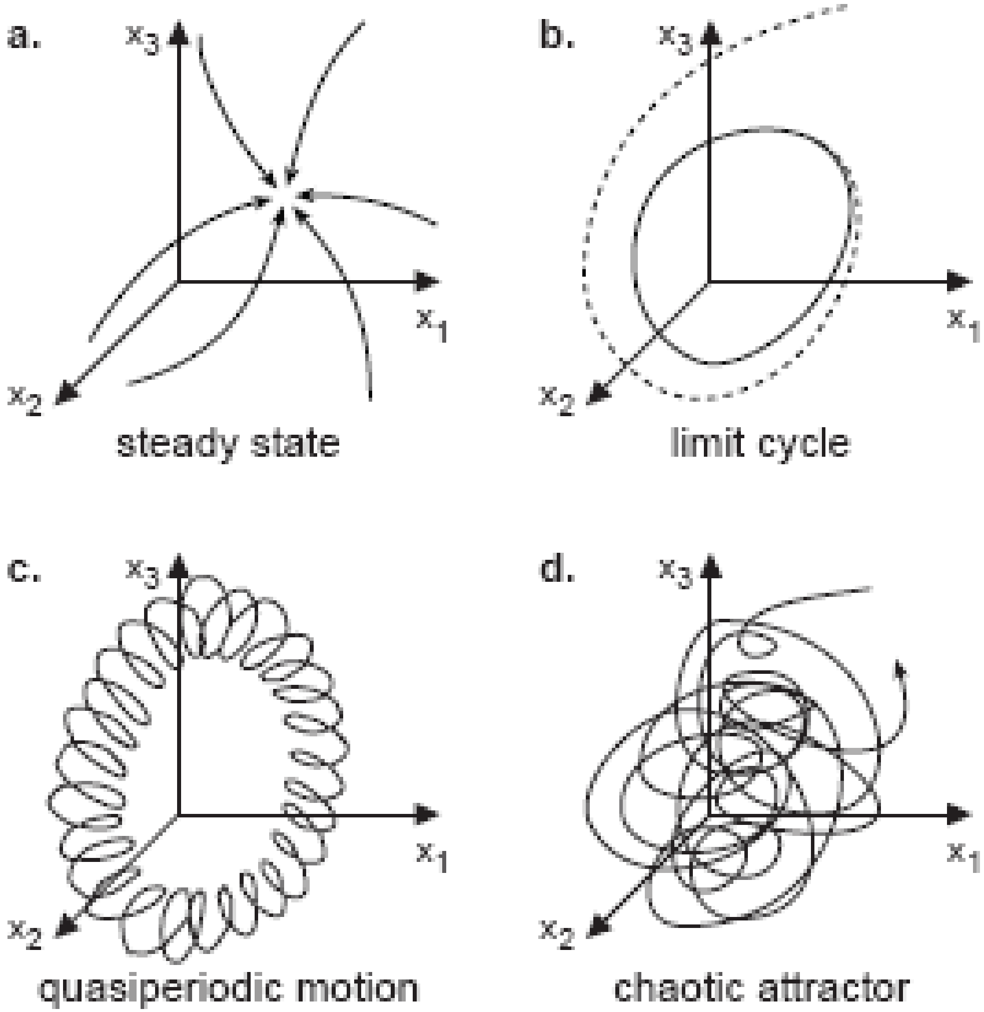
- in most cases, explicit expressions for solutions are not available
  - Still there are methods to extract information about the **flow**

Phase portrait of a DS

- Many dynamical systems eventually end up in a small subset of the **state space**
  - **Limit set**
    - Equilibrium point
      - Single point producing constant behavior
    - Limit cycles
      - Trajectory that closes on itself
      - Produce endless rhytmic behavior
  - For stable limit sets – **attractors**
    - all nearby trajectories converge to the limit set
    - Small perturbation – return to ls
  - Multiple limit sets – **basins of attraction**



Different attractors

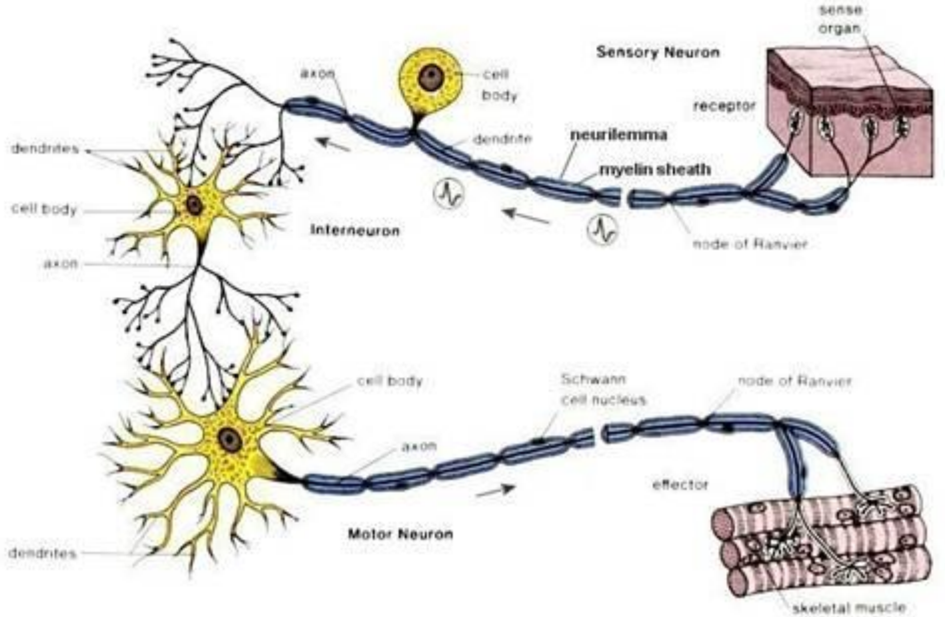


Cybernetic regulation

- Homeostasis
- Feedback loop
- Negative feedback

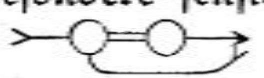
Reflex arc

- There are muscle length sensory receptors in the quadriceps muscle. When the patellar tendon is tapped, it causes a **stretch** in the quads, which stimulates the length receptors. This fires an action potential in the sensory neuron, which goes to the spinal cord, where it innervates the motor neuron to the same muscle. Assuming the stimulus was strong enough, the motor neuron also fires an action potential, and causes a **contraction** in the quads, resulting in a slight kick of the lower leg. Thus, the reflex arc is an example of negative feedback.



Negative feedback

- "nervous excitation always flows towards the stretched muscles"
- activity of the nervous system facilitates the contraction of stretched muscles and thereby counteracts and regulates the stretching of muscles

Viel häufiger findet die Kontrolle innerhalb des Körpers statt. Hier sind zwei Fälle zu unterscheiden: entweder wird die Bewegung der Effektorenmuskeln durch besondere sensible Nerven rezipiert, wie das beifolgende Schema zeigt.  Oder es wird die den effektorischen Nerven übertragene Erregung durch besondere zentrale Rezeptoren zum Teil aufgefangen und dem Merkorgan zugeführt.  Diese Rezeptoren bilden das zentrale Sinnesorgan von Helmholtz, das anatomisch noch völlig im Dunkeln liegt.

Little diagrams in the text illustrating a description of feedback and reafferent control (Uexküll 1920: 201).

Negative feedback

- Negative feedback occurs when the output of a system acts to oppose changes to the input of the system
- The result is that the changes are attenuated (weakened)
- In contrast, positive feedback is a feedback in which the system responds in the same direction as the perturbation, resulting in amplification of the original signal instead of stabilizing the signal. Both positive and negative feedback require a feedback loop to operate.

Coupling

Predator-prey model

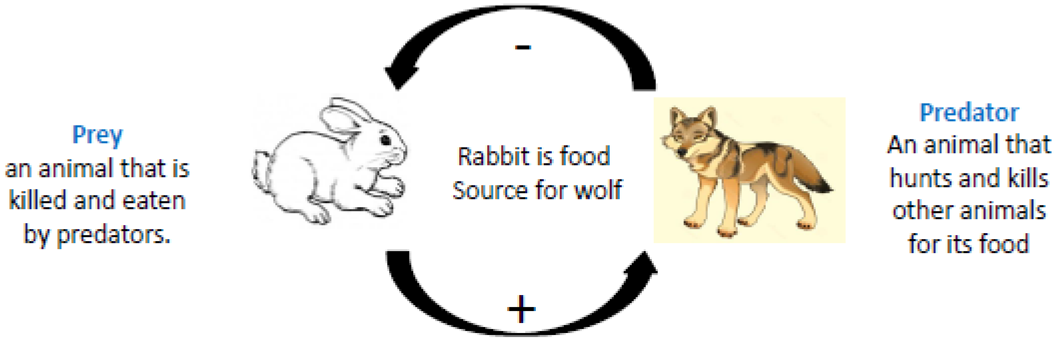
- Also known as the Lotka-Volterra model, (1910-1926)
- Used in chemistry, ecology, biomathematics, economy etc.

Problem:

Population growth of two species in which one species is the primary food source for the other. They are not competing for the same resources.

$X(t) \rightarrow$ population of rabbit at time t (prey)

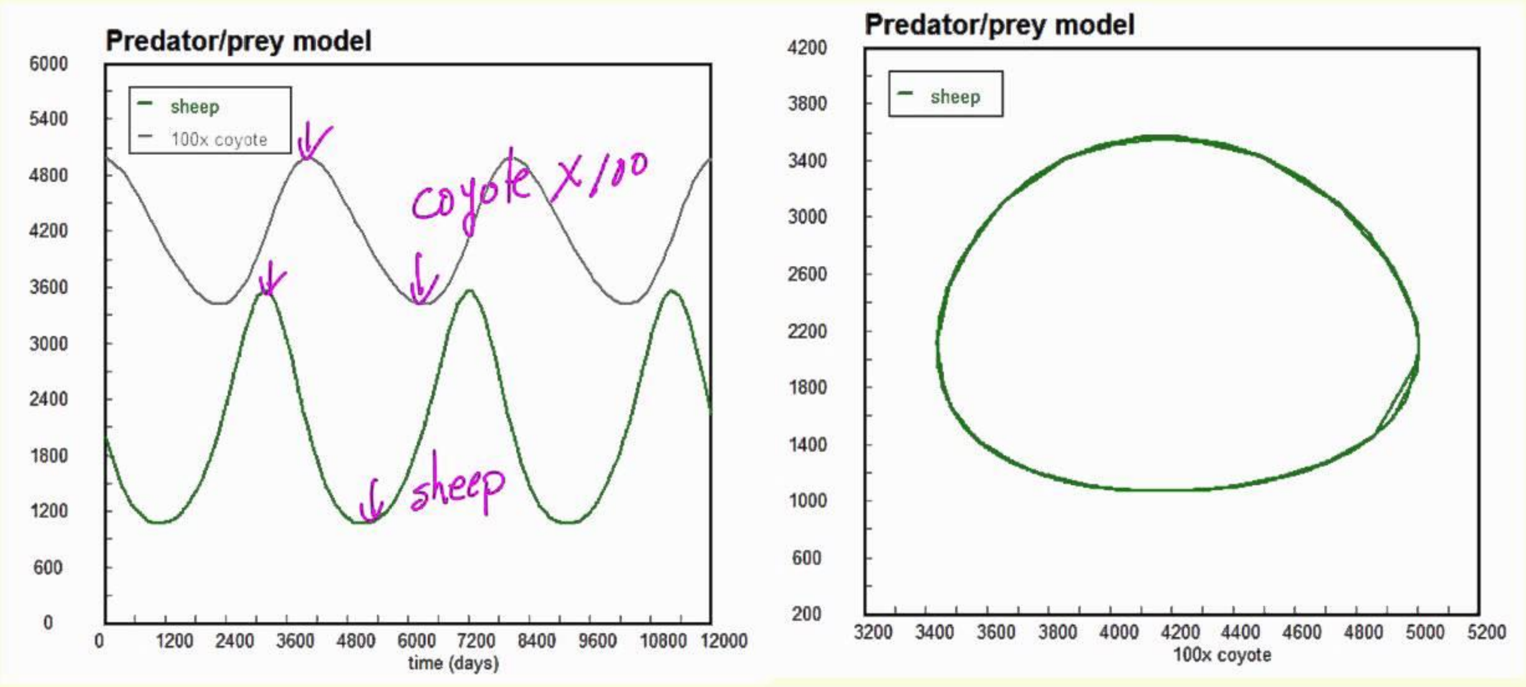
$Y(t) \rightarrow$ population of wolf at time t (predator)



Source: https://mathsfinalyear.wordpress.com/2016/05/13/predator-prey-model-lotka-volterra-model/

Predator-Prey

Initially: 2,000 sheep, 50 coyotes



National Science Foundation, Shell, CU-EEF & Department of Chemical and Biological Engineering University of Colorado Boulder

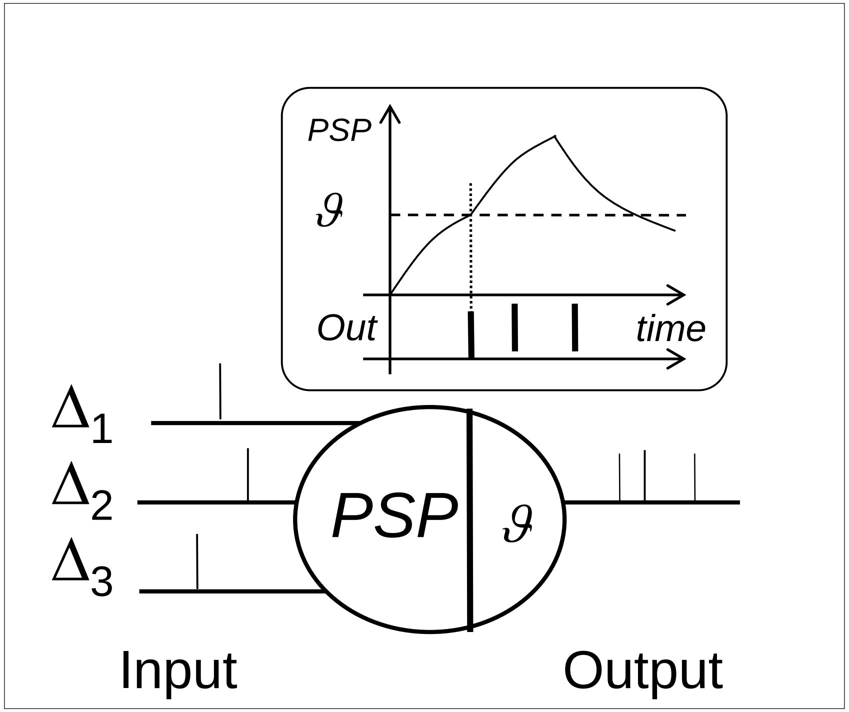
Dynamical Hypothesis in Cognitive Science

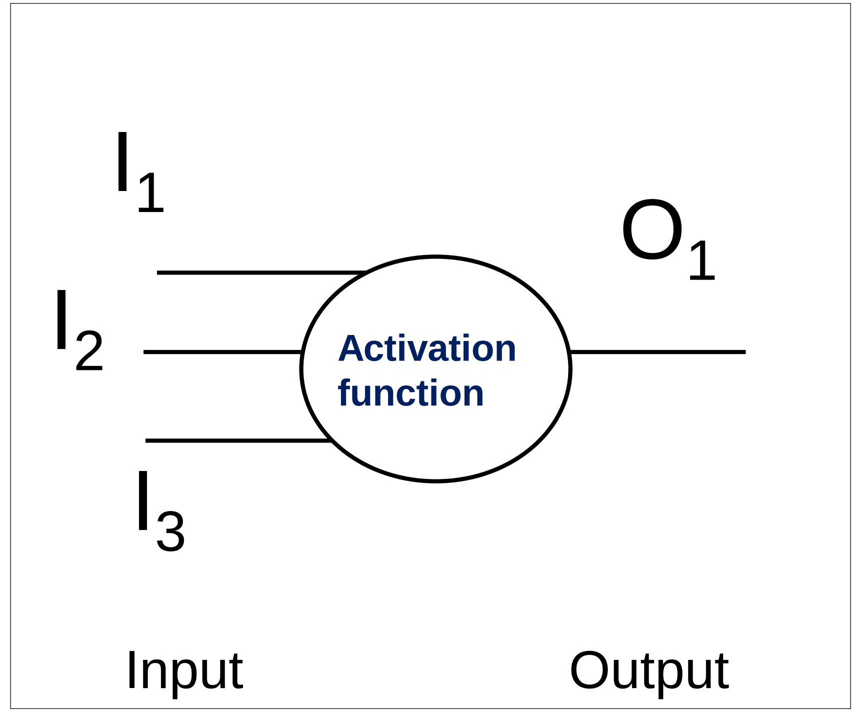
- Cognitive systems *are* dynamical systems (ontological claim)
- Cognitive systems *can be described as* dynamical systems (methodological claim)

Dynamical Hypothesis in Cognitive Science

- In fact, a cognitive system is composed of multiple interacting DS operating in parallel.
- Whenever temporal aspect (**timing**) is crucial, DS are adequate modeling choice.

Spiking versus rate neuron models





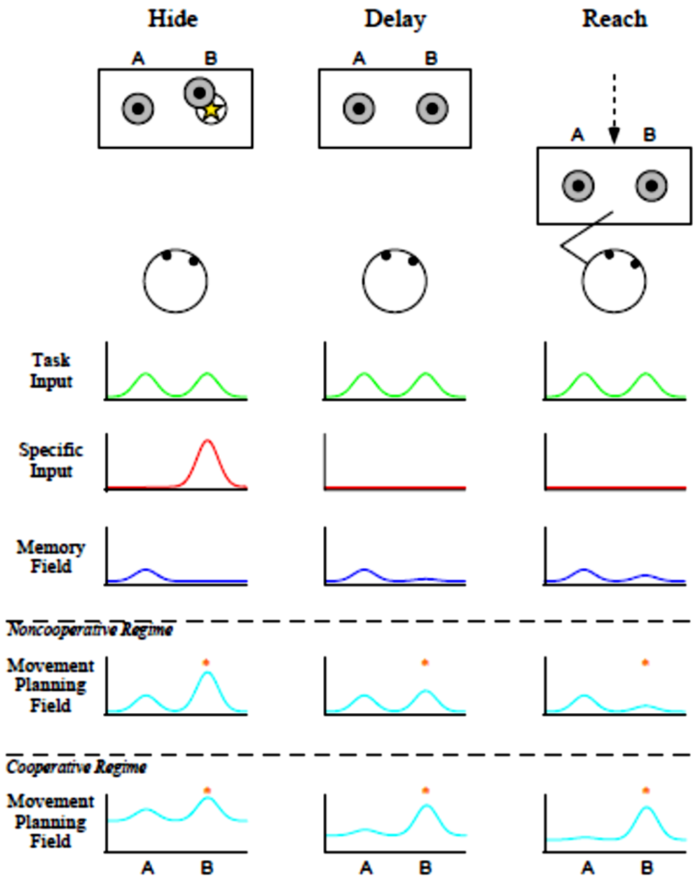
Spiking model: output depends on timing of input spikes

Rate model: output depends on the sum of input rates

Examples from human cognition

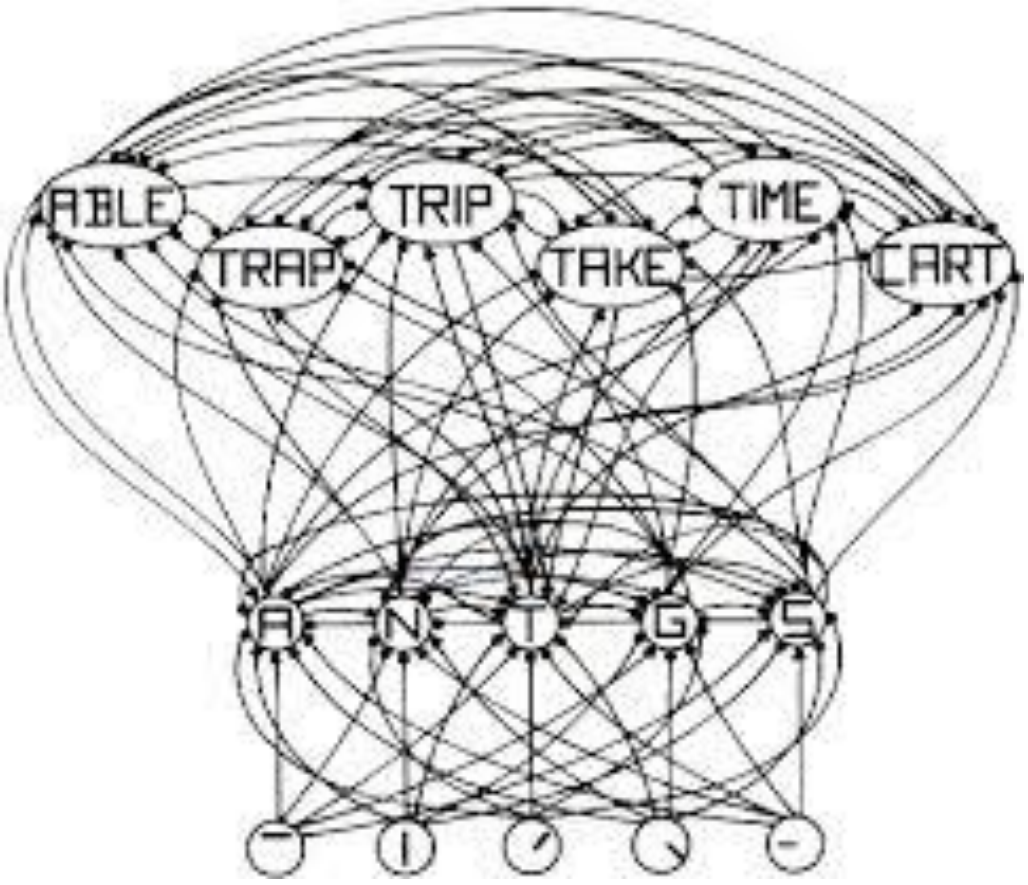
- Parallel processes of decision making: competition by lateral inhibition, until a choice reaches a threshold
- Conscious experience in Global Neuronal Workspace (Dahaene) framework
- Motor control in Dynamic Field Theory (Schöner)
- A-not-B error in infants (Thelen et al.)

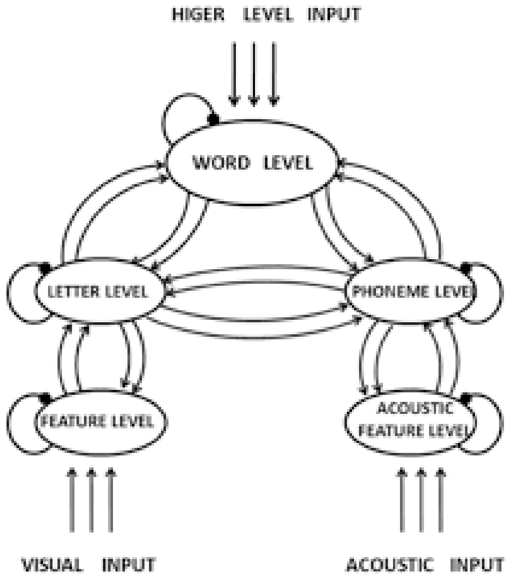
A-not-B error



Beer, R. D. (2000): Dynamical approaches to cognitive science. Trends in Cognitive Sciences 4(3), 91-99.

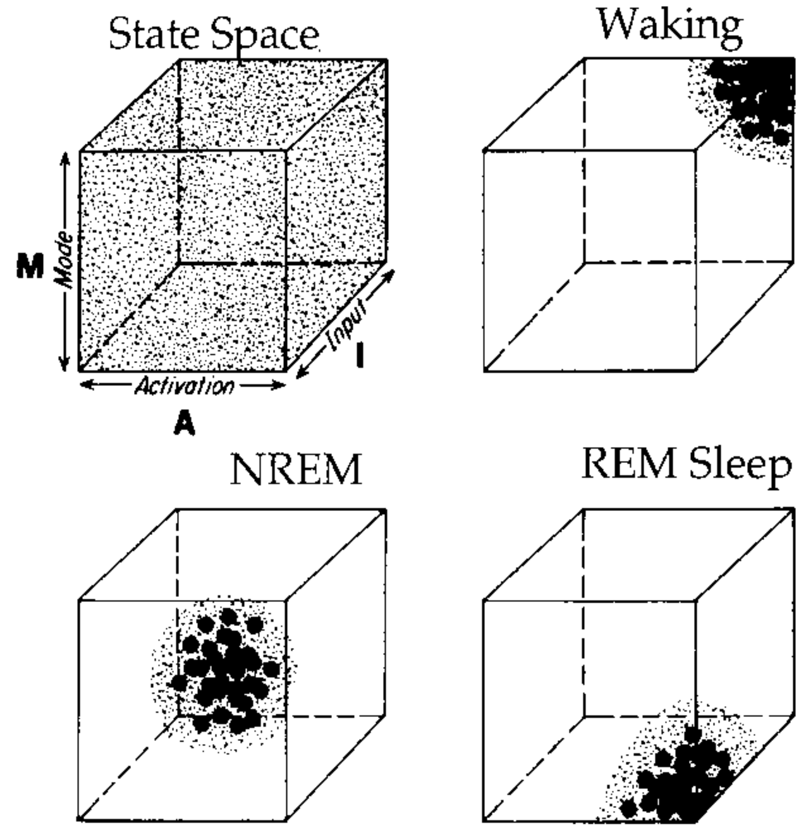
Interactive activation





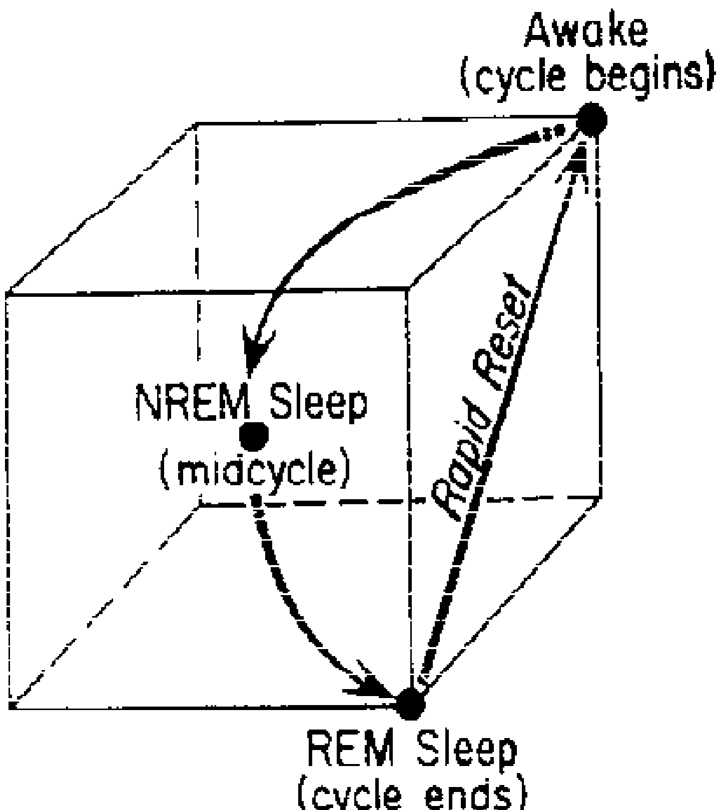
Example from human cognition

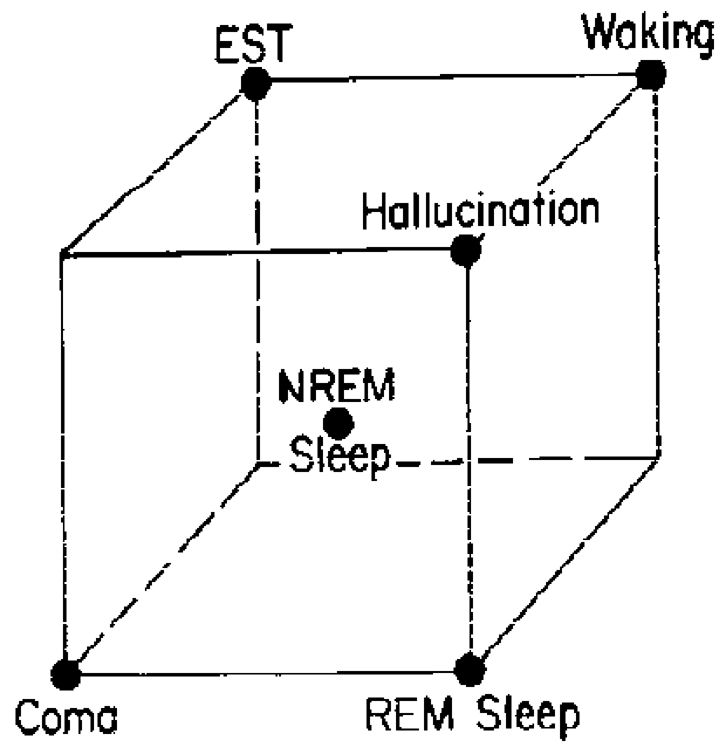
- AIM model
  - Activation
  - I/O gating
  - Modulatory factor



State space of consciousness

- Movement during the sleep cycle
- Normal and pathological states



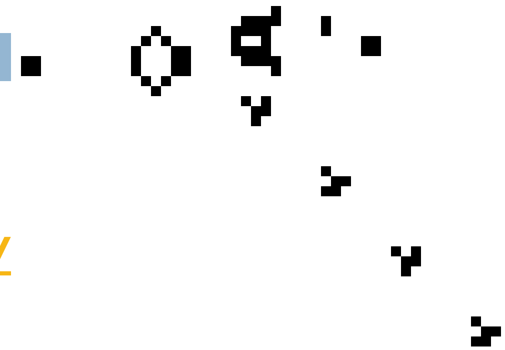


Emergence

- Very simple rules can create very complicated behavior
- **Multi-agent systems**
- **Boids** (Reynolds, 1986) – flocking behavior of birds
  - **separation:** steer to avoid crowding local flockmates
  - **alignment:** steer towards the average heading of local flockmates
  - **cohesion:** steer to move toward the average position (center of mass) of local flockmates

- http://www.youtube.com/watch?v=GUkjC-69vaw
- http://www.red3d.com/cwr/boids/

Game of life (Conway, 1970)



- Cellular automaton
- https://www.youtube.com/watch?v=C2v
- Rules
  - Any live cell with fewer than two live neighbours dies, as if caused by under-population.
  - Any live cell with two or three live neighbours lives on to the next generation.
  - Any live cell with more than three live neighbours dies, as if by overcrowding.
  - Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

Collective intelligence

- **Decentralized**
- **Self-organized**
- Simple agents interact locally with one another and with environment
  - Follow simple rules
  - No centralized control structure
  - Interactions lead to the **emergence** of intelligent behavior unknown to individual agents
- E.g. Ant colonies, bird flocking…

Questions?