Answer Set Programming

Extensions

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Outline

1. Constraints
   - Stable Models
2. Default Negation in the Heads
   - Stable Models
3. Explicit Negation
   - Motivation
   - Queens

Constraints

Definition (Stable Model)

An interpretation I is a *stable model* of a logic program Π with a set of constraints C if I is a stable model of Π and satisfies C.

Example (Generate and Test)

```
color(X, r) v color(X, g) v color(X, b) :- node(X).

:- color(X, C), color(Y, C), edge(X, Y).
```

Generalized Disjunctive Logic Programs

Definition (Generalized Disjunctive Logic Program)

A *generalized disjunctive logic program* is a set of rules

$$L_1 \vee \cdots \vee L_m \leftarrow L_{m+1} \wedge \cdots \wedge L_n$$

where $1 \leq m \leq n$ and $L_i, 1 \leq i \leq n$ are literals.

Stable Models

Definition (Program Reduct)

Let I be an interpretation. A *reduct* of a generalized disjunctive logic program Π (denoted by Π^I) is a positive disjunctive logic program with constraints obtained from Π by deleting

- rules containing a default literal $L, I \not\models L$ in the body
- rules containing a default literal $L, I \models L$ in the head
- default literals $L, I \models L$ in the bodies of remaining rules
- default literals $L, I \not\models L$ in the heads of remaining rules

Definition (Stable Model)

An interpretation I is a *stable model of* a generalized disjunctive logic program Π if I is a minimal model of Π^I.

Open vs. Closed World Assumption

Employee	
Name	...
George	
Janet	

Movie	
Title	...
Harry Potter	
Jánošík	

`? employee(Bob, ...)`

`? movie(Short Movie, ...)`

Reasoning with Incomplete Knowledge

`not train`

`train`　　$\emptyset$　　`-train`

`not -train`

$$\begin{aligned} cross &\leftarrow \sim train \\ cross &\leftarrow \neg\, train \end{aligned}$$

Extended Logic Program

Definition (Literal)

An *objective literal* is either an atom or an atom preceded by the symbol $\neg$. A *default literal* is an objective literal preceded by the symbol $\sim$. A *literal* is either an objective or a default literal.

Definition (Interpretation)

An *extended Herbrand base* is the set of all ground objective literals. A set of ground objective literals is *coherent* if it does not contain A and $\sim A$ for any ground atom A. A *Herbrand interpretation* is a coherent subset of the extended Herbrand base.

Definition (Satisfiability)

An interpretation I *satisfies* an objective literal L if $L \in I$.

Example

Example

```
 fly(X) :- bird(X), not ab(X).
  ab(X) :- penguin(X).
bird(X) :- penguin(X).
```

Example

```
 fly(X) :- bird(X), not -fly(X).
-fly(X) :- penguin(X).
bird(X) :- penguin(X).
```

Queens

Example (Domain specification)

```
% domain specification
queen(1..n).
row(1..n).
col(1..n).

% placing queens
at(Q, R, C) v -at(Q, R, C).
```

Queens

Example (Enumeration)

```
% queen is placed in at most one location
:- at(Q, R1, C1), at(Q, R2, C2), R1 <> R2.
:- at(Q, R1, C1), at(Q, R2, C2), C1 <> C2.

% queen is placed in at least one location
placed(Q) :- at(Q, R, C).
:- not placed(Q).

% no two queens are placed in the same location
:- at(Q1, R, C), at(Q2, R, C), Q1 <> Q2.
```

Queens

Example (Elimination)

```
% no two queens are placed in the same row
:- at(Q1, R, C1), at(Q2, R, C2), C1 <> C2.

% no two queens are placed in the same column
:- at(Q1, R1, C), at(Q2, R2, C), R1 <> R2.

% no two queens are placed in the same diagonal
:- at(Q1, R1, C1), at(Q2, R2, C2),
   abs(R1-R2) = abs(C1-C2).
```