

# Answer Set Programming

## Extensions

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# Outline

- 1 Constraints
  - Stable Models
- 2 Default Negation in the Heads
  - Stable Models
- 3 Explicit Negation
  - Motivation
  - Queens

# Constraints

## Definition (Stable Model)

An interpretation  $I$  is a *stable model* of a logic program  $\Pi$  with a set of constraints  $C$  if  $I$  is a stable model of  $\Pi$  and satisfies  $C$ .

## Example (Generate and Test)

```
color(X, r) v color(X, g) v color(X, b) :- node(X).  
  
:- color(X, C), color(Y, C), edge(X, Y).
```

# Generalized Disjunctive Logic Programs

## Definition (Generalized Disjunctive Logic Program)

A *generalized disjunctive logic program* is a set of rules

$$L_1 \vee \cdots \vee L_m \leftarrow L_{m+1} \wedge \cdots \wedge L_n$$

where  $1 \leq m \leq n$  and  $L_i, 1 \leq i \leq n$  are literals.

# Stable Models

## Definition (Program Reduct)

Let  $I$  be an interpretation. A *reduct* of a generalized disjunctive logic program  $\Pi$  (denoted by  $\Pi^I$ ) is a positive disjunctive logic program with constraints obtained from  $\Pi$  by deleting

- rules containing a default literal  $L, I \not\models L$  in the body
- rules containing a default literal  $L, I \models L$  in the head
- default literals  $L, I \models L$  in the bodies of remaining rules
- default literals  $L, I \not\models L$  in the heads of remaining rules

## Definition (Stable Model)

An interpretation  $I$  is a *stable model* of a generalized disjunctive logic program  $\Pi$  if  $I$  is a minimal model of  $\Pi^I$ .

# Open vs. Closed World Assumption

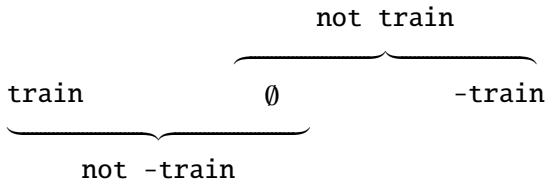
Employee	
Name	...
George	
Janet	

? employee(Bob, ...)

Movie	
Title	...
Harry Potter	
Jánošík	

? movie(Short Movie, ...)

# Reasoning with Incomplete Knowledge



*cross* ←  $\sim$  *train*

*cross* ←  $\neg$  *train*

# Extended Logic Program

## Definition (Literal)

An *objective literal* is either an atom or an atom preceded by the symbol  $\neg$ . A *default literal* is an objective literal preceded by the symbol  $\sim$ . A *literal* is either an objective or a default literal.

## Definition (Interpretation)

An *extended Herbrand base* is the set of all ground objective literals. A set of ground objective literals is *coherent* if it does not contain  $A$  and  $\sim A$  for any ground atom  $A$ . A *Herbrand interpretation* is a coherent subset of the extended Herbrand base.

## Definition (Satisfiability)

An interpretation  $I$  *satisfies* an objective literal  $L$  if  $L \in I$ .



# Example

## Example

```
fly(X) :- bird(X), not ab(X).  
ab(X) :- penguin(X).  
bird(X) :- penguin(X).
```

## Example

```
fly(X) :- bird(X), not -fly(X).  
-fly(X) :- penguin(X).  
bird(X) :- penguin(X).
```

# Queens

## Example (Domain specification)

```
% domain specification
queen(1..n).
row(1..n).
col(1..n).

% placing queens
at(Q, R, C) v -at(Q, R, C).
```

# Queens

## Example (Enumeration)

```
% queen is placed in at most one location
:- at(Q, R1, C1), at(Q, R2, C2), R1 <> R2.
:- at(Q, R1, C1), at(Q, R2, C2), C1 <> C2.

% queen is placed in at least one location
placed(Q) :- at(Q, R, C).
:- not placed(Q).

% no two queens are placed in the same location
:- at(Q1, R, C), at(Q2, R, C), Q1 <> Q2.
```

# Queens

## Example (Elimination)

```
% no two queens are placed in the same row
:- at(Q1, R, C1), at(Q2, R, C2), C1 <> C2.

% no two queens are placed in the same column
:- at(Q1, R1, C), at(Q2, R2, C), R1 <> R2.

% no two queens are placed in the same diagonal
:- at(Q1, R1, C1), at(Q2, R2, C2),
   abs(R1-R2) = abs(C1-C2).
```