Answer Set Programming

Translation of Logic Programs

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Outline

Translation of Logic Programs

For a logic program from one class, does exists an equivalent logic program from another class?

Example

```
a v b.

b :- not a.
a :- not b.
```

Equivalence

Definition (Equivalence)

Let Π_1 and Π_2 be a logic programs.
We say that $\Pi_1 \prec \Pi_2$ if every stable model of Π_1 is a stable model of Π_2.
We say that $\Pi_1 \equiv \Pi_2$ if $\Pi_1 \prec \Pi_2$ and $\Pi_2 \prec \Pi_1$.

Example

```
a v b.

b :- not a.
a :- not b.
```

Translation

Theorem

Let Π be a generalized logic program (with constraints). Let Π' be a normal logic program with constraints containing

$$A \leftarrow body(r)$$

for all $r \in \Pi$, $head(r) = A$,

$$\leftarrow body(r) \wedge A$$

for all $r \in \Pi$, $head(r) = \sim A$, *and*

$$\leftarrow body(r)$$

for all $r \in \Pi$, $head(r) = \bot$. *Then* $\Pi \equiv \Pi'$.

Equivalence

Definition (Restricted Equivalence)

Let Π_1 and Π_2 be a logic programs and $A \subseteq \mathcal{B}_{\Pi_1} \cap \mathcal{B}_{\Pi_2}$ be a set of grounded atoms.
We say that $\Pi_1 <_A \Pi_2$ if for all stable models M_1 of Π_1 there exists a stable model M_2 of Π_2 such that $M_1 \cap A = M_2 \cap A$.
We say that $\Pi_1 \equiv_A \Pi_2$ if $\Pi_1 <_A \Pi_2$ and $\Pi_2 <_A \Pi_1$.

Example

```
in(X) v out(X).

in(X) :- not -in(X).
-in(X) :- not in(X).
```

Translation

Theorem

Let Π be a logic program with constraints and inconsistent be a new propositional variable. Let Π' be a logic program containing

$$head(r) \leftarrow body(r)$$

for all $r \in \Pi$, $head(r) \neq \bot$, and

$$inconsistent \leftarrow \sim inconsistent \wedge body(r)$$

for all $r \in \Pi$, $head(r) = \bot$. Then $\Pi \equiv_{\mathcal{B}_\Pi} \Pi'$.

Equivalence

Definition (Homomorphic Equivalence)

Let Π_1 and Π_2 be a logic programs and $h : \mathcal{B}_{\Pi_1} \mapsto \mathcal{B}_{\Pi_2}$ be a homomorphism.
We say that $\Pi_1 \prec_h \Pi_2$ if for all stable models M_1 of Π_1 holds $M_2 = h(M1) = \{h(A) \mid A \in M_1\}$ is a stable model of Π_2.
We say that $\Pi_1 \equiv_h \Pi_2$ if h is an isomorphism and $\Pi_1 \prec_h \Pi_2$ and $\Pi_2 \prec_{h^{-1}} \Pi_1$.

Example

```
in(X) v out(X).

in(X) :- not -in(X).
-in(X) :- not in(X).
```

Translation

Theorem

Let Π be an extended logic program (with constrains). Let Π' be a logic program with constraints obtained from Π by

- *replacing all objective literals $\neg A$ by $-A$*
- *adding constraints $\leftarrow A \wedge -A$ for all atoms A.*

Then $\Pi \equiv_h \Pi'$ where $h \colon \mathcal{B}_{\Pi}^{\neg} \mapsto \mathcal{B}_{\Pi'}$ is an isomorphism such that

- $h(A) = A$
- $h(\neg A) = -A$

for all $A \in \mathcal{B}_{\Pi}$.

Head Cycle Free

Definition (Literal Dependency Graph)

The *literal dependency graph* of a normal disjunctive logic program Π is a directed graph where $\mathcal{B}_\Pi$ is the set of nodes and there is an edge from A' to A if there exists a rule r with $A \in head(r)$ and $A' \in body(r)$.

Definition (Head Cycle Free Disjunctive Logic Program)

A disjunctive logic program is *head cycle free* if its literal dependency graph does not contain directed cycles that go through two literals that belong to the head of the same rule.

Translation

Theorem

Let Π be a head cycle free normal disjunctive logic program. Let Π' be a normal logic program containing

$$A_i \leftarrow body(r) \wedge \sim A_1 \wedge \cdots \wedge \sim A_{i-1} \wedge \sim A_{i+1} \wedge \sim A_m$$

for all $r \in \Pi$, $head(r) = A_1 \vee \cdots \vee A_m$ and $1 \leq i \leq m$. Then $\Pi \equiv \Pi'$.

Example

```
a v b v c.

a :- not b, not c.
b :- not a, not c.
c :- not a, not b.
```

Negative Acyclic

Definition

A generalized disjunctive logic program Π is *negative acyclic* if there is a level mapping ℓ such that

- for every $L_1 \in head^+(r)$ and $L_2 \in head^-(r)$ holds $\ell(L_1) > \ell(L_2)$
- for every $L_1 \in head^+(r)$ and $L_2 \in body^+(r)$ holds $\ell(L_1) \geq \ell(L_2)$

Example

```
a v not a.
```

Translation

Theorem (Removing $\sim$ from the Head)

Let Π be a negative acyclic generalized disjunctive logic program. Let Π' be a normal disjunctive logic program containing

$$head^{+}(r) \leftarrow \sim head^{-}(r) \wedge body(r)$$

for all $r \in \Pi$. Then $\Pi \equiv \Pi'$.

Example

```
a v not b.
b :- a.
```

$$\begin{aligned} M_1 &= \emptyset \\ M_2 &= \{a, b\} \end{aligned}$$