

Credulous and Skeptical Argument Games for Complete Semantics in Conflict Resolution based Argumentation *

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Abstract

Argumentation is one of the most popular approaches of defining a non-monotonic formalism and several argumentation based semantics were proposed for defeasible logic programs. Recently, a new approach based on notions of conflict resolutions was proposed, however with declarative semantics only. This paper gives a more procedural counterpart by developing skeptical and credulous argument games for complete semantics and soundness and completeness theorems for both games are provided. After that, distribution of defeasible logic program into several contexts is investigated and both argument games are adapted for multi-context system.

Introduction

Argumentation is successfully applied as an approach of defining non-monotonic formalisms. The main advantage of semantics based on formal models of argumentation is its closeness to real humans discussions. Therefore, the semantics can be explained also for people not trained in formal logic or mathematics.

To capture the knowledge, a logical language is needed. Usually the language of Defeasible Logic Programming (DeLP) is considered, where two kinds for rules are distinguished. Strict rules represent deductive reasoning: whenever their preconditions hold, we accept the conclusion. On the other hand, defeasible rules formalize tentative knowledge that can be defeated. Several semantics based on argumentation were proposed for defeasible logic programs (Prakken and Sartor 1997), (García and Simari 2004), (Caminada and Amgoud 2007), (Prakken 2010), (Modgil and Prakken 2011), (Baláž, Frtús, and Homola 2013). However, as Caminada and Amgoud (Caminada and Amgoud 2007) pointed out, careless design of semantics may lead to very unintuitive results, such as inconsistency of the system (justification for both an atom A and its negation $\neg A$ is provided) or unsatisfying of strict rules (system justifies all preconditions, but not the conclusion of a strict rule).

In this paper we take the approach by Baláž et al. (Baláž, Frtús, and Homola 2013) as the starting point, since it both respects intuitions of logic programming and satisfies desired semantical properties. In (Baláž, Frtús, and Homola 2013) notion of conflict resolutions and new methodology of justification of arguments is introduced, however only in a declarative way. Our main goal, in this paper, is to give a more procedural counterpart. This is especially useful when dealing with algorithms and implementations. We adapt skeptical and credulous argument games for complete semantics and prove soundness and completeness for both of them, what is the main contribution of this paper. Then we are investigating with distribution of defeasible logic program into several contexts (programs) and both argument games are adapted for distributed computing. This can be useful in ambient intelligence environments, where distributed and contextual defeasible reasoning is heavily applied.

The paper is structured as follows: first preliminaries of Dung's abstract argumentation frameworks and defeasible logic programming are introduced. Then the declarative conflict resolution based semantics introduced in (Baláž, Frtús, and Homola 2013) is recapitulated. Argument games are developed and their properties are proved in the next section. The last section is devoted to contextualization of defeasible logic programs.

Preliminaries

Argumentation Framework

Definition 1 (Abstract Argumentation Framework (Dung 1995)). An *abstract argumentation framework* is a pair $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where

1. \mathcal{A} is a set of *arguments*, and
2. $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is an *attack* relation on \mathcal{A} .

An argument A *attacks* an argument B if $(A, B) \in \mathcal{R}$. A set of arguments S *attacks* an argument A if an argument in S attacks A . A set of arguments S is *attack-free*¹ if S does not attack an argument in S .

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¹Note that we will use the original term “conflict-free” in slightly different context.

A set of arguments S *defends* an argument A if each argument attacking A is attacked by S . An attack-free set of arguments S is *admissible* iff S defends each argument in S . The *characteristic function* F_{AF} of an argumentation framework $AF = (\mathcal{A}, Def)$ is a mapping $F_{AF}: 2^{\mathcal{A}} \mapsto 2^{\mathcal{A}}$ where for all $S \subseteq \mathcal{A}$, $F_{AF}(S)$ is defined as $\{a \in \mathcal{A} \mid S \text{ defends } a\}$.

Definition 2 (Extension (Dung 1995)). An admissible set of arguments S is

1. a *complete extension* iff S contains each argument defended by S .
2. the *grounded extension* iff S is the least complete extension.
3. a *preferred extension* iff S is a maximal complete extension.
4. a *stable extension* iff S attacks each argument which does not belong to S .

We will prove following lemma, which will be used in procedural formalization of the grounded semantics. Its intuitive meaning is that an argument x to be in the grounded extension, it can not be defended only by itself.

Lemma 1. *Given an argumentation framework (\mathcal{A}, Def) and a finite ordinal i , argument $A \in F^{i+1}$ iff for each argument Y defeating A , there is an argument $Z \in F^i$ such that $(Z, Y) \in Def$ and $Z \neq A$.*

Proof. Induction on i :

1. $i = 0$.
Let $A \in F^{i+1}$. Then $A \in F(\emptyset)$ and consequent holds trivially.
Let for each Y defeating A , there is an argument $Z \in F^i$ such that $(Z, Y) \in Def$ and $Z \neq A$. Since $F^i = \emptyset$, previous holds only if A is not defeated by any argument. So $A \in F^1 = F^{i+1}$.
2. $i > 0$.
Suppose that inductive hypothesis holds (IH). That is:
Argument $A \in F^{i+1}$ iff for each argument defeating A , there is an argument $Z \in F^i$ such that $(Z, Y) \in Def$ and $Z \neq A$.
holds for $i \geq 0$. We want to prove it for $i + 1$.
Let $A \in F^{i+2}$. Then $A \in F(F^{i+1})$. If $A \in F^{i+1}$, the consequence follows from IH. Suppose that $A \notin F^{i+1}$. From $A \in F(F^{i+1})$ we have: for each Y defeating A there is an argument $Z \in F^{i+1}$ defeating Y . Now since $A \notin F^{i+1}$, it is the case that $Z \neq A$. So the consequence holds.
Let for each Y defeating A there is an argument $Z \in F^{i+1}$ such that $(Z, Y) \in Def$ and $Z \neq A$. Then directly from definition of F we have $A \in F(F^{i+1}) = F^{i+2}$.

□

Defeasible Logic Program

An *atom* is a propositional variable. A *classical literal* is either an atom or an atom preceded by classical negation \neg . A *default literal* is a classical literal preceded by default negation \sim . A *literal* is either a classical or a default literal. By definition $\neg\neg A$ equals to A and $\sim\sim L$ equals to L , for an atom A and a classical literal L . By \mathcal{D} we will denote the set of all default literals. By convention $\sim S$ equals to $\{\sim L \mid L \in S\}$ for any set of literals S .

A *strict rule* is an expression of the form $L_1, \dots, L_n \rightarrow L_0$ where $0 \leq n$, each L_i , $1 \leq i \leq n$, is a literal, and L_0 is a classical literal. A *defeasible rule* is an expression of the form $L_1, \dots, L_n \Rightarrow L_0$ where $0 \leq n$, each L_i , $1 \leq i \leq n$, is a literal, and L_0 is a classical literal. A *defeasible logic program* \mathcal{P} is a finite set of strict rules Π and defeasible rules Δ . In the following text we use the symbol \rightsquigarrow to denote either strict or defeasible rule.

Conflict Resolution based Semantics

Existing argumentation formalisms (Prakken 2010; García and Simari 2004; Prakken and Sartor 1997) are usually defined through five steps. At the beginning, some underlying logical *language* is chosen for describing knowledge. The notion of an *argument* is then defined within this language. Then *conflicts* between arguments are identified. The resolution of conflicts is captured by an *attack* relation among conflicting arguments. The *status* of an argument is then determined by the attack relation.

The conflict resolution based approach (Baláž, Frtús, and Homola 2013) diverge from this methodology. Instead of attacking a conflicting argument, one of the weaker building blocks (called vulnerabilities) used to construct the argument is attacked. Specifically, the resolution of a conflict is either a default assumption or a defeasible rule. The status of an argument does not depend on attack relation between arguments but on attack relation between conflict resolutions.

Conflict resolution based semantics for the DeLP consists of five steps:

1. Construction of arguments on top of the language of defeasible logic programs.
2. Identification of conflicts between arguments.
3. Proposing a conflict resolution strategy.
4. Instantiation of Dung's AFs with conflict resolutions.
5. Determination of the status of default assumptions, defeasible rules, and arguments with respect to successful conflict resolutions.

A vulnerability is a part of an argument that may be defeated to resolve a conflict. It is either a defeasible rule or a default literal.

Definition 3 (Vulnerability). Let \mathcal{P} be a defeasible logic program. A *vulnerability* is a defeasible rule in \mathcal{P} or a default literal in \mathcal{D} . By $\mathcal{V}_{\mathcal{P}}$ we will denote the set of all vulnerabilities of \mathcal{P} .

Two kinds of arguments are usually be constructed in the language of defeasible logic programs. Default arguments correspond to default literals. Deductive arguments are constructed by chaining of rules. The following is a slightly more general definition, where a knowledge base \mathcal{K} denotes literals for which no further backing is needed.

Definition 4 (Argument). Let $\mathcal{P} = (\Pi, \Delta)$ be a defeasible logic program. An argument A for a literal L over a knowledge base \mathcal{K} is

1. $[L]$, where $L \in \mathcal{K}$

$$\begin{aligned} \text{CONC}(A) &= L \\ \text{VULS}(A) &= \{L\} \cap \mathcal{D} \end{aligned}$$

2. $[A_1, \dots, A_n \rightsquigarrow L]$ where each A_i , $0 \leq i \leq n$, is an argument for a literal L_i , $r: L_1, \dots, L_n \rightsquigarrow L$ is a rule in \mathcal{P} .

$$\begin{aligned} \text{CONC}(A) &= L \\ \text{VULS}(A) &= \text{VULS}(A_1) \cup \dots \cup \text{VULS}(A_n) \cup (\{r\} \cap \Delta) \end{aligned}$$

By $\mathcal{A}_{\mathcal{P}}$ we will denote the set of all arguments of \mathcal{P} .

The typical example of knowledge base within the language of defeasible logic programming is the set of default literals \mathcal{D} and we will not specify \mathcal{K} until the section about contextual DeLP. Therefore, whenever the \mathcal{K} is left unspecified, it is implicitly set to \mathcal{D} . Arguments created by chaining of rules will be called *deductive*.

Example 1. Consider the following defeasible logic program \mathcal{P} :

$$\begin{array}{ll} \Rightarrow a & \Rightarrow c \\ \Rightarrow b & \Rightarrow d \\ a, b \rightarrow h & c, d \rightarrow \neg h \end{array}$$

Six deductive arguments can be constructed from \mathcal{P}

$$\begin{array}{ll} A_1 = [\Rightarrow a] & A_4 = [\Rightarrow c] \\ A_2 = [\Rightarrow b] & A_5 = [\Rightarrow d] \\ A_3 = [A_1, A_2 \rightarrow h] & A_6 = [A_3, A_4 \rightarrow \neg h] \end{array}$$

Vulnerabilities of arguments A_3 are A_6 are $\text{VULS}(A_3) = \{\Rightarrow a, \Rightarrow b\}$ and $\text{VULS}(A_6) = \{\Rightarrow c, \Rightarrow d\}$.

Two kinds of conflicts among arguments may arise, each corresponds to one type of negation.

Definition 5 (Conflict). Let \mathcal{P} be a defeasible logic program. Arguments $A, B \in \mathcal{A}_{\mathcal{P}}$ are *conflicting* iff A rebuts or undercuts B where

1. A *rebuts* B iff A and B are deductive arguments and $\text{CONC}(A) = \neg \text{CONC}(B)$,
2. A *undercuts* B iff A is a deductive argument, B is a default argument, and $\text{CONC}(A) = \sim \text{CONC}(B)$.

The set $C = \{A, B\}$ is called a *conflict*. The first kind is called a *rebutting* conflict and the second kind is called an *undercutting* conflict. By $\mathcal{C}_{\mathcal{P}}$ we will denote the set of all conflicts of \mathcal{P} .

Conflicts are resolved by defeating one of the building blocks of conflicting arguments. Each default assumption or defeasible rule used to construct a conflicting argument is a possible resolution. Strict rules can not be used as a resolution of any conflict because they have to be always satisfied.

Definition 6 (Conflict Resolution). Let \mathcal{P} be a defeasible logic program. A vulnerability $V \in \mathcal{V}_{\mathcal{P}}$ is a *resolution* of a conflict $C \in \mathcal{C}_{\mathcal{P}}$ if $V \in \text{VULS}(C)$. The pair $R = (C, V)$ is called a *conflict resolution*. By $\mathcal{R}_{\mathcal{P}}$ we will denote the set of all conflict resolutions of \mathcal{P} .

In general, each conflict may have more resolutions. Some of them may be more preferred than others. The choice of preferred conflict resolutions is always domain dependent. Some vulnerabilities can be defeated in one domain, but they may as well stay undefeated in another. Therefore we allow the user to choose any conflict resolution strategy she might prefer.

Definition 7 (Conflict Resolution Strategy). Let \mathcal{P} be a defeasible logic program. A *conflict resolution strategy* is a finite subset σ of $\mathcal{R}_{\mathcal{P}}$. We say that a vulnerability $V \in \mathcal{V}_{\mathcal{P}}$ is a σ -resolution of a conflict $C \in \mathcal{C}_{\mathcal{P}}$ if $(C, V) \in \sigma$. A conflict resolution strategy σ is *total* iff for each conflict $C \in \mathcal{C}_{\mathcal{P}}$ there exists a σ -resolution of C .

In existing approaches various conflict resolution strategies are applied. Examples of default, last-link and weakest-link conflict resolution strategies are presented in (Baláz, Frtús, and Homola 2013).

Example 2 (Continuation of Example 1). The only conflict in the defeasible logic program \mathcal{P} is the $C = \{A_3, A_6\}$. Consider following six conflict resolutions.

$$\begin{array}{ll} R_1 = (C, \Rightarrow a) & R_3 = (C, \Rightarrow c) \\ R_2 = (C, \Rightarrow b) & R_4 = (C, \Rightarrow d) \end{array}$$

Then $\sigma = \{R_1\}$, $\sigma' = \{R_i \mid 1 \leq i \leq 4\}$, $\sigma'' = \emptyset$ are examples of conflict resolution strategies for \mathcal{P} . We can see that strategies σ, σ' are total.

To determine in which way conflicts will be resolved, Dung's AF is instantiated with conflict resolutions. The intuitive meaning of a conflict resolution (C, V) is "the conflict C will be resolved by defeating the vulnerability V ". The conflict resolution based semantics is built on three levels of attacks: attacks on the vulnerabilities, attacks on the arguments, and attacks on the conflict resolutions. Such an approach is necessary: if a vulnerability is defeated, so should be all arguments built on it, and consequently all conflict resolutions respective to the argument.

Definition 8 (Attack). A conflict resolution $R = (C, V)$ *attacks*

- a vulnerability V' iff $V' = V$.
- an argument A iff R attacks a vulnerability in $\text{VULS}(A)$.
- a conflict resolution $R' = (C', V')$ iff either
 1. $V \neq V'$ and R attacks an argument in C' or

2. $V = V'$ and R attacks all arguments in C' .

A set of conflict resolutions $S \subseteq \mathcal{R}_{\mathcal{P}}$ attacks a vulnerability $V \in \mathcal{V}_{\mathcal{P}}$ (resp. an argument $A \in \mathcal{A}_{\mathcal{P}}$ or a conflict resolution $R \in \mathcal{R}_{\mathcal{P}}$) iff a conflict resolution in S attacks V (resp. A or R).

Intuitively, it should not happen that both a conflict resolution $R = (C, V)$ and a vulnerability V are accepted. Therefore, if R is accepted, V and all arguments constructed on top of it should be defeated. The notion of attack between conflict resolutions formalizes the ideas that there may be more alternatives how to resolve a conflict and a conflict resolution may resolve other conflicts as well, thus causing other conflict resolutions to be irrelevant. The distinction between two kinds of attacks between conflict resolutions is necessary to achieve the intended semantics when dealing with self-conflicting arguments. The interested reader is kindly referred to (Baláz, Frtús, and Homola 2013) for demonstrative examples.

Definition 9 (Instantiation). The instantiation for a conflict resolution strategy σ is an abstract argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where

- $\mathcal{A} = \sigma$
- \mathcal{R} is the attack relation on σ from the Definition 8.

Now thanks to the instantiation we can use the Dung's semantics in order to compute which vulnerabilities (resp. arguments, conflict resolutions) are undefeated (status IN), defeated (status OUT), or undecided (status UNDEC).

Definition 10 (Defense). Let σ be a conflict resolution strategy for a defeasible logic program \mathcal{P} . A set of conflict resolutions $S \subseteq \sigma$ *defends* a vulnerability $V \in \mathcal{V}_{\mathcal{P}}$ (resp. an argument $A \in \mathcal{A}_{\mathcal{P}}$ or a conflict resolution $R \in \sigma$) iff each conflict resolution in σ attacking V (resp. A or R) is attacked by S .

Definition 11 (Status). Let σ be a conflict resolution strategy for a defeasible logic program \mathcal{P} and \mathcal{E} be a complete extension of the instantiation for σ . The *status* of a vulnerability $V \in \mathcal{V}_{\mathcal{P}}$ (resp. an argument $A \in \mathcal{A}_{\mathcal{P}}$ or a conflict resolution $R \in \sigma$) with respect to \mathcal{E} is

- IN if \mathcal{E} defends V (resp. A or R),
- OUT if V (resp. A or R) is attacked by \mathcal{E} ,
- UNDEC otherwise.

Let $s \in \{\text{IN}, \text{UNDEC}, \text{OUT}\}$. By $\mathcal{A}_{\mathcal{P}}^s(\mathcal{E})$ we denote the set of all arguments with the status s with respect to a complete extension \mathcal{E} .

The following definitions define actual semantics of the DeLP program \mathcal{P} and entailment relation between a program \mathcal{P} and a literal L .

Definition 12 (Output). Let σ be a conflict resolution strategy for a defeasible logic program \mathcal{P} and \mathcal{E} be a complete extension of the instantiation for σ . The *output* of \mathcal{E} is a set of literals $\text{OUTPUT}_{\mathcal{P}}(\mathcal{E}) = \{L \in \mathcal{L} \mid \mathcal{A}_{\mathcal{P}}^{\text{IN}}(\mathcal{E})$ contains an argument for $L\}$.

Note that we will omit default literals in output to improve the legibility.

Definition 13 (Entailment). Let σ be a conflict resolution strategy for a defeasible logic program \mathcal{P} and \mathcal{F} be the instantiation for σ . Defeasible logic program \mathcal{P} *skeptically* (resp. *credulously*) *entails* a literal L , $\mathcal{P} \models_{sk} L$ (resp. $\mathcal{P} \models_{cr} L$) iff for each (resp. at least one) complete extension \mathcal{E} of \mathcal{F} , $L \in \text{OUTPUT}_{\mathcal{P}}(\mathcal{E})$.

Example 3 (Continuation of Example 2). Consider the conflict resolution strategy σ' from Example 2. The instantiation for σ' is on the Figure 1.

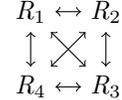


Figure 1: The instantiation for the conflict resolution strategy σ' .

All conflict resolutions are now exclusive, since to resolve the conflict, it is sufficient to reject only one of the defeasible rules. Therefore σ' induces the complete graph.

There are five complete extensions $\{R_1\}$, $\{R_2\}$, $\{R_3\}$, $\{R_4\}$, $\{\}$ of the instantiation and each of them determine one program output $\{b, c, d, \neg h\}$, $\{a, c, d, \neg h\}$, $\{a, b, d, h\}$, $\{a, b, c, h\}$, $\{\}$.

Procedural Semantics

In the previous section we recapitulated (Baláz, Frtús, and Homola 2013) conflict resolution based semantics in the original declarative way. Although this declarative approach is very elegant and provides nice algebraic investigations, the more procedural style of semantics is appropriate when dealing with algorithms and implementations. One can see a parallel in a mathematical logic where we are similarly interested in a logical calculi (proof theory) which is sound and complete with respect to defined model-theoretic semantics. In this section our goal is to define skeptical and credulous argument games for complete semantics.

For a conflict resolution $R = (\{A, B\}, V)$ we define auxiliary functions which will be frequently used.

$$\begin{aligned} con(R) &= \{A, B\} \\ res(R) &= V \\ vuls(R) &= (\text{VULS}(A) \setminus \{V\}) \cup (\text{VULS}(B) \setminus \{V\}) \cup \\ &\quad (\text{VULS}(A) \cap \text{VULS}(B) \cap \{V\}) \end{aligned}$$

$con(R)$ denotes the conflict and $res(R)$ the resolution of a conflict resolution R . The meaning of the set of vulnerabilities $vuls(R)$ can be explained as following: suppose R is in a conflict resolution strategy σ and \mathcal{E} is a complete extension of instantiation for σ . If $R \in \mathcal{E}$ and all the vulnerabilities in $vuls(R)$ have the status IN, then in order to resolve the conflict $con(R)$, the status of the vulnerability $res(R)$ is OUT.

Now we characterize the attack between conflict resolutions in terms of aforementioned functions. This will be useful in proofs for soundness and completeness of argument games.

Proposition 1. *Let \mathcal{P} be a defeasible logic program, σ a conflict resolution strategy and $R = (C, V)$, $R' = (C', V') \in \sigma$ are conflict resolutions. Then R attacks R' iff $res(R) \in vuls(R')$.*

Proof. Let $C = \{A, B\}$ and $C' = \{A', B'\}$.

Suppose R attacks R' . There are two cases to consider:

- $V \neq V'$ and R attacks an argument in C' . Then there is an argument $X \in C'$ with $V \in \text{VULS}(X)$. Since $res(R) = V$ and $V \neq V'$, $res(R) \in vuls(R')$.
- $V = V'$ and R attacks all arguments in C' . Then $V \in \text{VULS}(A') \cap \text{VULS}(B')$. From the previous and $V = V'$ we have $V \in vuls(R')$.

Suppose $res(R) \in vuls(R')$. There are two cases to consider:

- $V = V'$. Since $V \notin (\text{VULS}(A') \setminus \{V'\}) \cup (\text{VULS}(B') \setminus \{V'\})$, it is the case that $V \in \text{VULS}(A') \cap \text{VULS}(B')$ and R attacks both arguments A' and B' and consequently R attacks R' .
- $V \neq V'$. Then $V \in (\text{VULS}(A') \setminus \{V'\}) \cup (\text{VULS}(B') \setminus \{V'\})$. Consequently there is an argument $X \in C'$ with $V \in \text{VULS}(X)$ and therefore R attacks R' .

□

Argumentation can be seen and thus also formalized as a discussion of two players. The aim of the first player (called *proponent* PRO) is to prove an initial argument. The second player is an *opponent* (OPP), what means that her goal is to prevent proponent to prove the initial argument. Hence a dispute essentially is a sequence of moves where each player gives a counterargument to the last stated.

Proof theory of argumentation is well studied area and argument games for various semantics were proposed (Modgil and Caminada 2009), (Prakken and Sartor 1997). The process of proving a literal L via an argument game, in conflict resolution based setting, considered in this paper, takes two steps:

1. Find an argument A with conclusion L .
2. Justify all vulnerabilities in $\text{VULS}(A)$.

Intuitively, a move (pl, R, \mathcal{V}) is a triple denoting: player pl claims that the set of vulnerabilities \mathcal{V} is true and resolution R is a reason for the other player why her set of vulnerabilities is not justified.

Definition 14 (Move). Let σ be a conflict resolution strategy for a defeasible logic program \mathcal{P} . A *move* is a triple $\mu = (pl, R, \mathcal{V})$, where $pl \in \{\text{OPP}, \text{PRO}\}$ denotes the player, $R \in \sigma$ is a resolution and $\mathcal{V} \subseteq \mathcal{V}_{\mathcal{P}}$ is a set of vulnerabilities.

Now since the very first move in a dialogue does not counter argue any of the previous move, the resolution R will be left unspecified and in such case we will write $(pl, -, \mathcal{V})$. Convention $\overline{\text{PRO}} = \text{OPP}$ and $\overline{\text{OPP}} = \text{PRO}$ will be used for denoting the opposite players. We say that a move (pl, R, \mathcal{V}) attacks a move $(\overline{pl}, R', \mathcal{V}')$ iff $res(R) \in \mathcal{V}'$.

Definition 15 (Argument Dialogue). A *dialogue* is a finite nonempty sequence of moves μ_1, \dots, μ_n , $1 \leq i < n$ where:

- $pl_i = \text{PRO}$ (OPP) iff i is odd (even)
- μ_{i+1} attacks μ_i

Intuitively, for a given argument, there can be more than one counterargument. This leads to a tree representation of discussion. Now, since the burden of proof is on the player PRO, proponent proves an initial argument if she wins all disputes. On the other hand, the burden of attack is on the player OPP, meaning that opponent must “play” all possible counterarguments, against PRO’s last argument, forming new branches in a discussion tree.

Definition 16 (Argument Game). Let σ be a conflict resolution strategy for a defeasible logic program P . An *argument game for an argument A* is a finite tree such that:

- $(\text{PRO}, -, \text{VULS}(A))$ is the root,
- all branches are dialogues,
- if move μ played by PRO is a node in the tree, then every move $(\text{OPP}, R, vuls(R))$ defeating μ is a child of μ .
- if μ, μ' are any moves played by PRO in T then μ does not defeat μ' .

A player wins a dispute if the counterpart can not make any move (give a counterargument). This can roughly be paraphrased as “the one who has the last word laughs best”. Since the burden of proof is on the proponent, PRO, in order to win, has to win all branches in the game. On the other hand, for opponent to win an argument game, it is sufficient to win at least one branch of the game.

Definition 17 (Winner). A player pl *wins a dialogue* iff she plays the last move in it. Player PRO (resp. OPP) *wins an argument game T* iff she wins all (resp. at least one of the) branches in the argument game T . An argument game is *successful* iff it is won by PRO.

Definition 18 (Proved Literal). Let σ be a conflict resolution strategy for a defeasible logic program P . A literal L is :

- *proved in an argument game T* iff T is a successful argument game for an argument A with $\text{CONC}(A) = L$.
- *proved* iff there is an argument game T proving L .

Now we propose two particular argument games and prove their soundness and completeness with respect to declarative semantics defined in the previous section.

Argument Game for Skeptical Complete Semantics

First we will investigate with skeptical complete semantics which corresponds to the grounded semantics. Since the grounded semantics gives the highest burden of proof on membership of the extension it defines, the opponent is allowed to repeat her moves and proponent is not.

Definition 19 (Skeptical Game). An argument game T is called *skeptical* iff in each branch of T holds: if $(\text{PRO}, R, \mathcal{V})$, $(\text{PRO}, R', \mathcal{V}')$ are two moves played by PRO, then $R \neq R'$.

Argument game for skeptical complete semantics is sound and complete with respect to declarative conflict resolution based grounded semantics.

Proposition 2. *Let P be a defeasible logic program and L be a literal. $P \models_{sk} L$ iff L is skeptically proved².*

Proof. Let σ be a conflict resolution strategy for P .

Suppose $P \models_{sk} L$. Then there is an argument A such that $\text{CONC}(A) = L$ and the set of vulnerabilities $\text{VULS}(A)$ is defended by every complete extension of instantiation for σ . Then it is also defended by the least complete (grounded) extension \mathcal{E} . Now we will create the skeptical argument game T for A . The T will satisfy following specification:

- if $(\text{PRO}, -, \mathcal{V})$ is the root of T then $\mathcal{V} \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$.
- if $\mu \in T$ is a move played by PRO such that $\text{vuls}(\mu) \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$ and μ' is a child of μ , then there is a move μ_2 with $\text{vuls}(\mu_2) \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$ such that μ_2 is a child of μ' .

Procedure creating argument game T for an argument A :

1. The root of T is $(\text{PRO}, -, \text{VULS}(A))$.
2. If $\mu = (\text{PRO}, R, \mathcal{V})$ is a node in T , then every move $(\text{OPP}, R', \text{vuls}(R'))$ defeating μ will be added as a child of μ .
3. If $\mu' = (\text{OPP}, R, \mathcal{V})$ is a node in T such that μ' is a child of a move μ played by PRO and $\text{vuls}(\mu) \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$, then \mathcal{E} defends all vulnerabilities in $\text{vuls}(\mu)$ and consequently there is a conflict resolution $R_2 \in \mathcal{E}$ such that move $\mu_2 = (\text{PRO}, R_2, \mathcal{V}_2)$ defeats μ' and μ_2 does not occur in the path from the root to μ' . Note that such conflict resolution R_2 is guaranteed to exist by Lemma 1 and an assumption that \mathcal{E} is the grounded extension defending $\text{VULS}(A)$. Add such move μ_2 as a child of μ' .

From the construction of T we have $\text{vuls}(\mu) \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$ for each μ played by PRO. It is not hard to see that these three steps induce a skeptical argument game T for argument A and T satisfies the specification above. Note that the property that no moves

²A literal L is skeptically proved iff there is an skeptical argument game T such that L is proved in T .

played by PRO are defeating follows from the fact that $\text{vuls}(\mu) \subseteq \mathcal{V}_P^{\text{IN}}(\mathcal{E})$ for each μ played by PRO.

Now the fact that T is successful follows from the assumption that \mathcal{E} is a complete extension.

Suppose L is skeptically proved. Then there is an argument A such that $\text{CONC}(A) = L$ and there is a successful skeptical argument game T for A . Consider the set of all conflict resolutions \mathcal{E} played by PRO in T ,

$$\mathcal{E} = \{R \mid (\text{PRO}, R, \mathcal{V}) \in T\}$$

We should now argue that \mathcal{E} is defeat-free and defends all its members. It is easy to see that defeat-freeness of \mathcal{E} directly follow from the definition of argument game.

Now we show admissibility of \mathcal{E} . Let $R \in \mathcal{E}$ and R' be a conflict resolution defeating R . Let $\mu = (\text{PRO}, R, \text{vuls}(R)) \in T$. Then $(\text{OPP}, R', \text{vuls}(R'))$ is a child of μ . Since T is successful, in every branch PRO plays the last move. Therefore there is a conflict resolution $R_2 \in \mathcal{E}$ defeating R' .

Now we will extend \mathcal{E} in order it is complete extension. Recall that we assume only finite defeasible logic programs with propositional atoms. Consequently the conflict resolution strategy is a finite set $\sigma = \{R_1, \dots, R_n\}$, $n \geq 0$, which can be iterated. Let us create the following sequence \mathcal{E}_i , $0 \leq i < n$, of sets of conflict resolutions:

$$\begin{aligned} \mathcal{E}_0 &= \mathcal{E} \\ \mathcal{E}_{i+1} &= \begin{cases} \mathcal{E}_i \cup \{R_i\} & \text{if } \mathcal{E}_i \cup \{R_i\} \text{ is admissible} \\ \mathcal{E}_i & \text{otherwise} \end{cases} \end{aligned}$$

It is easy to see that \mathcal{E}_n is complete extension. Now we prove that \mathcal{E}_n is the least complete extension.

Assume by contradiction there is a complete extension $\mathcal{E}' \subset \mathcal{E}_n$. First we will argue that $\mathcal{E} \subseteq \mathcal{E}'$. Let $c(\mu)$ denote the set of all children of μ in T . Realize that $\bigcup c(c(\mu))$ then denotes the set of all moves, played by the same player as μ , defeating some move in $c(\mu)$. Now for each move $\mu \in T$ played by PRO we assign the number $n(\mu)$ defined as

$$n(\mu) \begin{cases} 1 & \text{if } \mu \text{ is a leaf in } T \\ 1 + \max(\{n(\mu') \mid \mu' \in \bigcup c(c(\mu))\}) & \text{otherwise} \end{cases}$$

The fact that $\mathcal{E} \subseteq \mathcal{E}'$ now can be proved by induction on $n(\mu)$.

We argued that $\mathcal{E} \subseteq \mathcal{E}'$. From $\mathcal{E}' \subset \mathcal{E}_n$ it follows there is a conflict resolution in $\mathcal{E}_n \setminus \mathcal{E}'$. Let i , $1 \leq i \leq n$, n is the number of conflict resolutions, be the least number such that $R_i \in \mathcal{E}_n \setminus \mathcal{E}'$. Then $\mathcal{E}' \supseteq \mathcal{E}_{i-1}$, where \mathcal{E}_{i-1} defends R_i . But from monotonicity of characteristic function we have \mathcal{E}' defends R_i as well. Then since $R_i \notin \mathcal{E}'$ it must be the case that $\mathcal{E}' \cup \{R_i\}$ is not defeat-free. Then \mathcal{E}' defeats R_i . Since \mathcal{E}' defends R_i , \mathcal{E}' defeats itself. Contradiction that \mathcal{E}' is the grounded extension! Hence \mathcal{E}_n is the grounded extension.

Finally, since the grounded extension defends $\text{VULS}(A)$, $P \models_{sk} L$ holds. \square

Let demonstrate the skeptical argument game in example.

Example 4. Consider the following defeasible logic program $\mathcal{P} = \{\Rightarrow a, \Rightarrow \neg a\}$ with conflict resolution strategy $\sigma = \{R_1, R_2\}$. There are two deductive arguments A_1, A_2 , one conflict C and two conflict resolutions R_1, R_2 .

$$\begin{array}{ll} A_1 & = [\Rightarrow a] & A_2 & = [\Rightarrow \neg a] \\ C & = \{A_1, A_2\} & R_2 & = (C, \Rightarrow \neg a) \\ R_1 & = (C, \Rightarrow a) & & \end{array}$$

We would like to skeptically prove literal a . The skeptical argument game for argument A_1 is on the Figure 2.

$$\begin{array}{c} \mu_1 = (\text{PRO}, -, \{\Rightarrow a\}) \\ | \\ \mu_2 = (\text{OPP}, R_1, \{\Rightarrow \neg a\}) \\ | \\ \mu_3 = (\text{PRO}, R_2, \{\Rightarrow a\}) \\ | \\ \mu_4 = (\text{OPP}, R_1, \{\Rightarrow \neg a\}) \end{array}$$

Figure 2: The skeptical argument game for argument A_1 .

Proponent cannot repeat her move μ_3 and therefore she loses the game.

Argument Game for Credulous Complete Semantics

Credulous complete semantics corresponds to the preferred semantics, where an argument can be defended by itself. Therefore, in credulous game, proponent is allowed to repeat her moves and opponent is not.

Definition 20 (Credulous Game). An argument game T is called *credulous* iff in each branch of T holds: if $(\text{OPP}, R, \mathcal{V}), (\text{OPP}, R', \mathcal{V}')$ are two moves played by OPP, then $R \neq R'$.

Argument game for credulous complete semantics is sound and complete with respect to declarative conflict resolution based preferred semantics.

Proposition 3. Let \mathcal{P} be a defeasible logic program and L be a literal. $\mathcal{P} \models_{cr} L$ iff L is credulously proved³.

Proof. Let σ be a conflict resolution strategy for \mathcal{P} .

Suppose $\mathcal{P} \models_{cr} L$. Then there is an argument A such that $\text{CONC}(A) = L$ and the set of vulnerabilities $\text{VULS}(A)$ is defended by at least one complete extension \mathcal{E} of instantiation for σ . We will create the credulous argument game T for A in the following way:

1. The root of T is $(\text{PRO}, -, \text{VULS}(A))$.

³A literal L is credulously proved iff there is an credulous argument game T such that L is proved in T .

2. If $\mu = (\text{PRO}, R, \mathcal{V})$ is a node in T , then every move $(\text{OPP}, R', \mathcal{V}')$ attacking μ and not yet occurring in the path from the root to μ , will be added as a child of μ .
3. If $\mu = (\text{OPP}, R, \mathcal{V})$ is a node in T , then since R attacks some R' in \mathcal{E} , there is a conflict resolution $R_2 \in \mathcal{E}$ such that move $\mu_2 = (\text{PRO}, R_2, \mathcal{V}_2)$ attacks μ . Add such move μ_2 as a child of μ .

It is not hard to see three steps above induce a credulous argument game for argument A . Now the fact that PRO wins all the branches follows from the assumption that \mathcal{E} is a complete extension of instantiation for σ defending all vulnerabilities in $\text{VULS}(A)$.

Suppose L is credulously proved. Then there is an argument A such that $\text{CONC}(A) = L$ and a credulous argument game T for A won by PRO. Consider the set of all conflict resolutions \mathcal{E} played by PRO in T ,

$$\mathcal{E} = \{R \mid (\text{PRO}, R, \mathcal{V}) \in T\}$$

We should now argue that \mathcal{E} is an attack-free and defends all its members. Both properties can be shown that directly follow from the assumption that T is successful (i.e. won by PRO). Let \mathcal{E}_g be the grounded extension of instantiation for σ . Attack-freeness of $\mathcal{E} \cup \mathcal{E}_g$ follows from the Proposition 2 and the assumption that T is a successful game for an argument A . The completeness of \mathcal{E} is now a consequence of the Fundamental Lemma (Dung 1995). Since there is a complete extension defending $\text{VULS}(A)$, $\mathcal{P} \models_{cr} L$ holds. \square

Now we will consider the defeasible logic program \mathcal{P} and conflict resolution strategy σ from Example 4 and try to prove literal a credulously.

Example 5 (Continuation of Example 4). We would like to credulously prove literal a . The credulous argument game for argument A_1 is on the Figure 3.

$$\begin{array}{c} \mu_1 = (\text{PRO}, -, \{\Rightarrow a\}) \\ | \\ \mu_2 = (\text{OPP}, R_1, \{\Rightarrow \neg a\}) \\ | \\ \mu_3 = (\text{PRO}, R_2, \{\Rightarrow a\}) \end{array}$$

Figure 3: The credulous argument game for argument A_1 .

Opponent cannot repeat her move μ_2 and therefore the game is successful.

In (Governatori et al. 2004; Billington et al. 2010) several variants of defeasible logics with procedural semantics are proposed. Repeating an argument for PRO in our approach corresponds to the Δ proof tag of (Billington et al. 2010) and repeating an argument by OPP in our approach corresponds to the σ proof tag of (Billington et al. 2010).

Contextual DeLP

In the previous section we developed a procedural semantics based on argument games, now we will generalize these ideas to a distributive setting, where not only one, but the whole set of defeasible logic programs is assumed. Each of these programs may be viewed as a context (i.e. agent), which describes the world within its own language (i.e. propositional symbols). Contexts are interconnected into multi-context system through non-monotonic bridge rules, which import knowledge (foreign literals) from other contexts.

Our goal is to adapt the argument games to multi-context systems and satisfy following requirements:

- To minimize the necessary communication complexity between contexts. The conflict between arguments can be decided in other context, but the structure of arguments should not be communicated.
- Contexts provide just distributive computing, they should not change the semantics. Hence if we look at multi-context system as a monolithic program, the output should be the same as in distributive case.

Note that the distributed reasoning is a very complex task involving also issues of communication protocols and information security. In this chapter we abstract from this and focus only on the reasoning part.

Distributed computing of semantics is a hot topic in the area of multi-agent systems, for García and Simari's (García and Simari 2004) DeLP a distributed argumentation framework was proposed in (Thimm and Kern-Isberner 2008). Contextual defeasible reasoning is also applied in environment of Ambient Intelligence (Bikakis and Antoniou 2010), where devices, software agents and services are supposed to integrate and cooperate in support of human objectives.

A vocabulary V is a set of propositional variables. We say that a literal is *local* if its propositional variable is in V , otherwise it is *foreign*. A *local rule* contains only local literals. A *mapping rule* contains local literal in the head and at least one foreign literal in the body. A *contextual defeasible logic program* is a set of local strict rules, and local or mapping defeasible rules.

Sometimes we will denote the context pertaining to a foreign literal. For example $2: a, c \Rightarrow b$ means that foreign literal a is imported from the second context.

Definition 21 (Context). A *context* is a triple $C = (V, P, \sigma)$ where V is a set of propositional variables, P is a contextual defeasible logic program and σ is a conflict resolution strategy.

Since, within the one context we do not know the structure of an argument supporting some foreign literal, foreign literals cannot be used as resolutions of conflicts (their set of vulnerabilities is empty).

Contextual argument is an argument, where some of the literals (foreign) do not need a further backing and are considered as an import of the knowledge from the other context.

Definition 22 (Contextual Argument). Given a context $C = (V, P, \sigma)$ and the set of foreign literals F , a *contextual argument* is an argument over a knowledge base $\sim V \cup F$. The set of all foreign literals contained by an argument in a set of arguments \mathcal{A} will be denoted $F(\mathcal{A})$.

Contextual argument is *foreign* if it is of the form $[L]$, where L is a foreign literal.

Following proposition means that foreign literals cannot incorporate a conflict.

Proposition 4. *Given a context $C = (V, P, \sigma)$ and the set of foreign literals F , a foreign argument A cannot be in conflict with by any contextual argument from context C .*

Proof. Follows directly from the fact that for a non-foreign contextual argument A and a foreign argument B , $\text{CONC}(A)$ is based upon V , $\text{CONC}(B)$ is based upon F and $F \cap V = \emptyset$. \square

Definition 23 (Multi-Context System). A *multi-context system*⁴ is a finite nonempty set of contexts $\mathcal{C} = \{C_1, \dots, C_n\}$ where $0 < n$, each $C_i = (V_i, P_i, \sigma_i)$, $1 \leq i \leq n$, is a context and $\{V_1, \dots, V_n\}$ is a partition of the set of all propositional variables in $\bigcup_{i=1}^n P_i$.

A multi context system \mathcal{C} is *cyclic* iff there are contexts C_1, C_2, \dots, C_n , $n \geq 2$ such that context C_i , $1 \leq i < n$, contains a mapping rule with a foreign literal from the context C_{i+1} and C_n , contains a mapping rule with a foreign literal from the context C_1 . A multi context system is *acyclic* iff it is not cyclic.

Sometimes it is useful to look at a multi-context system as a monolithic defeasible logic program and vice versa. We say that a multi-context system $\mathcal{C} = \{C_1, \dots, C_n\}$ is a *contextualization* of a defeasible logic program \mathcal{P} and conflict resolution strategy σ iff $\mathcal{P} = \bigcup_{i=1}^n P_i$ and $\sigma = \bigcup_{i=1}^n \sigma_i$. The idea of contextualization of a program or an argument is illustrated in the following example.

Example 6. Consider the following multi-context system consisting of two contexts

$C_1 = (\{a, d, h\}, P_1, \sigma_1)$	$C_2 = (\{b, c\}, P_2, \sigma_2)$
$\Rightarrow a$	$\Rightarrow b$
$\Rightarrow d$	$\Rightarrow c$
$2: b, a \rightarrow h$	
$2: c, d \rightarrow \neg h$	
$\sigma_1 = \{(\{A_3^1, A_6^1\}, \Rightarrow a)\}$	$\sigma_2 = \emptyset$

Six contextual arguments can be constructed in P_1

$A_1^1 = [\Rightarrow a]$	$A_4^1 = [c]$
$A_2^1 = [\bar{b}]$	$A_5^1 = [\Rightarrow d]$
$A_3^1 = [A_1^1, A_2^1 \rightarrow h]$	$A_6^1 = [A_4^1, A_5^1 \rightarrow \neg h]$

⁴Note that symbol C was originally used to denote a conflict and symbol \mathcal{C} for denoting the set of all conflicts. However, the denotation of symbols will always be clear from the actual text.

Two contextual arguments can be constructed in P_2

$$A_1^2 = [\Rightarrow b] \quad A_2^2 = [\Rightarrow c]$$

We can see that \mathcal{C} is a contextualization of defeasible logic program \mathcal{P} and conflict resolution strategy σ from Example 2. Similarly, we will define a notion of *contextual version of argument* by examples: arguments $A_1^1, A_3^1, A_5^1, A_6^1$ are (in order) contextual versions of arguments A_1, A_3, A_5, A_6 , but A_2^1, A_4^1 are not contextual versions of arguments A_2, A_4 in Example 1.

The process of proving a literal L via an argument game in contextual setting is still consisting of two steps:

1. Find a contextual argument A with conclusion L .
2. Justify all vulnerabilities in $\text{VULS}(A)$ and send acceptance queries to contexts pertaining to foreign literals $F(\{A\})$.

The second step means that whenever a player pl plays in a dialogue a move μ , not only all vulnerabilities of μ but also all foreign literals occurring in μ must be justified in order to pl will be the winner.

It is not hard to see that support dependency through foreign literals may be cyclic in a multi-context system. For example context C_1 may use a foreign literal from context C_2 and vice versa. Therefore we have to take care of termination of the queries to other contexts.

Example 7. Consider the following multi-context system consisting of two contexts, each using foreign literal from the other context.

Context 1	Context 2
$\Rightarrow a$	1: $a \Rightarrow \neg b$
2: $b \Rightarrow \neg a$	$\Rightarrow b$
$\sigma_1 = \{R_1 = (C_1, \Rightarrow a)\}$	$\sigma_2 = \{R_2 = (C_2, \Rightarrow b)\}$

Where conflict $C_1 = \{[\Rightarrow a], [2: b \Rightarrow \neg a]\}$ and conflict $C_2 = \{[1: a \Rightarrow \neg b], [\Rightarrow b]\}$.

Consider now query about credulous acceptance of literal a . There is only one rule deriving a and the only conflict resolution R_1 defeating it. Recall the intuitive meaning of conflict resolution in distributive setting: If the vulnerability $\{b \Rightarrow \neg a\}$ and foreign literal b are accepted, rule $\Rightarrow a$ is defeated. Defeasible rule $b \Rightarrow \neg a$ is not a resolution of any conflict so its trustworthiness is not a subject of dispute. Now the query about acceptance of the foreign literal b is given to the Context 2. The process of proving b in Context 2 is similar, therefore we skip details and only remark that query about acceptance of the foreign literal a is given back to the Context 1. We can see that naive adaptation of argument games may lead to infinite sending of queries between contexts which have cyclic support dependency.

To overcome problem illustrated in the previous example, from now on in this paper we investigate with acyclic multi-context systems only and more general cases are left for the future work.

Now we will define notions for contextual proving and argument games. Contextual argument game is

an argument game T accompanied with a query function \mathcal{Q} defining queries for every move in T . Intuitively, a query is a foreign literal that needs to be proved in other context.

Definition 24 (Contextual Argument Game). Let C be a context and μ be a move (pl, R, V) . A *contextual argument game for a contextual argument A* is a pair (T, \mathcal{Q}) , where T is an argument game for A and \mathcal{Q} is a query function

$$\mathcal{Q}(\mu) = \begin{cases} F(\{A\}) & \text{if } \mu \text{ is the root of the tree} \\ F(\text{con}(R)) & \text{otherwise} \end{cases}$$

assigning queries for each move.

We say that a contextual argument game for a literal L is a contextual argument game for a contextual argument A with $\text{CONC}(A) = L$. Given a query function \mathcal{Q} , the set of all foreign literals, played by a player pl in a contextual argument game (T, \mathcal{Q}) , will be denoted by $\mathcal{Q}(pl)$.

Contextual skeptical and credulous games respect conditions of move repetitions. That is, in contextual skeptical (credulous) game, opponent (proponent) is allowed to repeat her moves and proponent (opponent) is not. However, since parts of the argument game can be queried to another contexts, we have to take care that requirements of (non)repetitions of moves are satisfied also there. Realize that each time a query about foreign literal F to other context C' is sent from a move (pl, R, \mathcal{V}) in an argument game T , no matter whether pl is proponent or opponent, the argument game for F in context C' will be started by proponent. Therefore, if pl is PRO, the semantics of argument game in context C' does not change. On the other hand, if pl is OPP, the semantics of argument game in context C' will switch in order to keep the requirements of (non)repetitions of moves.

This leads into two mutually recursive definitions of skeptical and credulous contextual argument games. Note however that the recursion is well-founded (always terminates) since we are considering multi-context systems with acyclic support dependency only.

Definition 25 (Contextual Skeptical Game). Let μ be a move (pl, R, \mathcal{V}) . A contextual argument game (T, \mathcal{Q}) is called *skeptical* iff

- T is skeptical game and
- for each move in T with $\mathcal{Q}(\mu) \neq \emptyset$ there is a $\text{sem}(\mu)$ contextual argument game, where

$$\text{sem}(\mu) = \begin{cases} \text{skeptical} & \text{if } pl = \text{PRO} \\ \text{credulous} & \text{otherwise} \end{cases}$$

defines the acceptance semantics for queries.

Definition 26 (Contextual Credulous Game). Let μ be a move (pl, R, \mathcal{V}) . A contextual argument game (T, \mathcal{Q}) is called *credulous* iff

- T is credulous game and

- for each move in T with $\mathcal{Q}(\mu) \neq \emptyset$ there is a $sem(\mu)$ contextual argument game, where

$$sem(\mu) = \begin{cases} \text{credulous} & \text{if } pl = \text{PRO} \\ \text{skeptical} & \text{otherwise} \end{cases}$$

defines the acceptance semantics for queries.

Recall that player pl , in order to be the winner, has to justify not only all the vulnerabilities played by her, but also all pl 's queries have to be successful. Hence, although player does not play the last move in a dialogue, she can still be a winner if a query of the second player is not justified.

Again, the definition is recursive but the assumption of acyclicity guarantees its termination.

Definition 27 (Contextual Winner). Let (T, \mathcal{Q}) be a contextual argument game. A player pl wins a dialogue in contextual argument game (T, \mathcal{Q}) iff

- all contextual argument games for literals in $\mathcal{Q}(pl)$ are successful and
- at least one of the following holds:
 - pl plays the last move in the dialogue, or
 - at least one of the contextual argument game for literals in $\mathcal{Q}(\overline{pl})$ is not successful.

A player PRO (resp. OPP) wins a contextual argument game iff she wins all (resp. at least one of the) branches in the contextual argument game. A contextual argument game is *successful* iff it is won by PRO.

Definition 28 (Contextually Proved Literal). Let \mathcal{C} be a multi-context system and $C \in \mathcal{C}$ be a context. A literal L is (skeptically, resp. credulously) *proved* in:

- a contextual argument game (T, \mathcal{Q}) iff there is a contextual argument A with $\text{CONC}(A) = L$, T is an (skeptical, resp. credulous) argument game for A and (T, \mathcal{Q}) is successful.
- a context C iff $C = (V, P, \sigma)$, $L \in V$ and there is a contextual argument game (skeptically, resp. credulously) proving L .
- a multi-context system \mathcal{C} iff there is a context C such that L is (skeptically, resp. credulously) proved in C .

One of our goals was that contextualization of a program provides just a distributive computing and should not change its output. The following proposition claims that we are successful by achieving it.

Proposition 5. Let \mathcal{C} be an acyclic contextualization of a defeasible logic program P and L be a literal.

1. $P \models_{sk} L$ iff L is skeptically proved in \mathcal{C} .
2. $P \models_{cr} L$ iff L is credulously proved in \mathcal{C} .

Proof. Observe that from Proposition 2 and Proposition 3 it is sufficient to prove

1. L is skeptically proved iff L is skeptically proved in \mathcal{C} .
2. L is credulously proved iff L is credulously proved in \mathcal{C} .

We will show the equivalences through transformations of the skeptical (resp. credulous) argument game into the contextual skeptical (resp. credulous) game and vice versa.

Let P be a defeasible logic program, a multi-context system \mathcal{C} its contextualization, T a successful argument game (either skeptical or credulous) for an argument A and $C = (V, P', \sigma)$ be the context such that A' is an contextual argument in P' and it is a contextual version of A . Appropriate contextual argument game (T', \mathcal{Q}) will be created in the following way:

1. $\mu = (\text{PRO}, -, \text{VULS}(A'))$ is the root of T' . Set query function $\mathcal{Q}(\mu) = F(\{A\})$.
2. If $\mu = (pl, R, \mathcal{V})$ is a move in T' , then for each conflict resolution $R' \in \sigma$ such that $res(R') \in \mathcal{V}$, create a move $\mu' = (\overline{pl}, R', vuls(R'))$. Now add μ' as a child of μ in T' iff
 - T is skeptical game and one of the following holds:
 - $pl = \text{PRO}$; or
 - $pl = \text{OPP}$ and μ' is not yet occurring in the path from the root to μ .
 - T is credulous game and one of the following holds:
 - $pl = \text{OPP}$; or
 - $pl = \text{PRO}$ and μ' is not yet occurring in the path from the root to μ .
Set $\mathcal{Q}(\mu') = F(\text{con}(R'))$.

Now recursively repeat this process for each literal in $\mathcal{Q}(\mu)$ and each move $\mu \in T'$. Note, that this procedure always terminates, since we are considering multi-context systems with acyclic support dependency only.

It is straightforward to verify that if T is skeptical (resp. credulous) argument game, then (T', \mathcal{Q}) is contextual skeptical (resp. credulous) argument game.

Suppose L is skeptically (resp. credulously) proved. By an induction on number of all queries in T' (i.e. the cardinality of $\bigcup_{\mu \in T'} \mathcal{Q}(\mu)$) it can be showed that T' is successful. Note that the trivial case means T and T' are identical.

Let (T', \mathcal{Q}) be a successful contextual argument (either skeptical or credulous) game for a contextual argument A' , which is a contextual version of A . Appropriate argument game T for A will be created in the following way:

1. $(\text{PRO}, -, \text{VULS}(A') \cup \mathcal{V})$ is the root of T , where \mathcal{V} is the union of all vulnerabilities in the roots of contextual argument games of foreign literals in $F(A)$.
2. If $\mu = (pl, R, \mathcal{V})$ is a move in T , then for each conflict resolution $R' \in \sigma$ such that $res(R') \in \mathcal{V}$, create a move $\mu' = (\overline{pl}, R', vuls(R'))$. Now add μ' as a child of μ in T iff
 - T' is skeptical game and one of the following holds:
 - $pl = \text{PRO}$; or
 - $pl = \text{OPP}$ and μ' is not yet occurring in the path from the root to μ .
 - T' is credulous game and one of the following holds:

- $pl = \text{OPP}$; or
- $pl = \text{PRO}$ and μ' is not yet occurring in the path from the root to μ .

It is straightforward to verify that if T' is a skeptical (resp. credulous) argument game, then T is a skeptical (resp. credulous) argument game. Suppose L is skeptically (resp. credulously) proved in \mathcal{C} . By an induction on the depth of T it can be showed that T is successful as well. \square

Distribution of argument games is demonstrated in example.

Example 8. Consider the following multi-context system consisting of two contexts

$C_1 = (\{a\}, P_1, \sigma_1)$	$C_2 = (\{b\}, P_2, \sigma_2)$
$\Rightarrow a$	$\Rightarrow b$
$2: b \Rightarrow \neg a$	$\Rightarrow \neg b$
$\sigma_1 = \{(\{A_1^1, A_3^1\}, \Rightarrow a)\}$	$\sigma_2 = \{(\{A_1^2, A_2^2\}, \Rightarrow b)\}$

Three contextual arguments can be constructed in P_1

$$\begin{array}{ll} A_1^1 = [\Rightarrow a] & A_2^1 = [b] \\ A_3^1 = [A_2^1 \Rightarrow \neg a] & \end{array}$$

Two contextual arguments can be constructed in P_2

$$A_1^2 = [\Rightarrow b] \quad A_2^2 = [\Rightarrow \neg b]$$

The contextual argument game T (both skeptical and credulous) is on the Figure 4, the contextual game T' for query b is on the Figure 5.

$$\begin{array}{c} \mu_1^1 = (\text{PRO}, -, \{\Rightarrow a\}) \\ | \\ \mu_2^1 = (\text{OPP}, R_1, \{2: b \Rightarrow \neg a\}), \mathcal{Q}(\mu_2^1) = \{b\} \end{array}$$

Figure 4: The contextual argument game for literal a in context C_1 .

$$\begin{array}{c} \mu_1^2 = (\text{PRO}, -, \{\Rightarrow b\}) \\ | \\ \mu_2^2 = (\text{OPP}, R_2, \{\Rightarrow \neg b\}) \end{array}$$

Figure 5: The contextual argument game for a query b in context C_2 .

Although the proponent did not play the last move in T , she is still winner, since the query about foreign literal b was not successful.

Conclusion

We have developed a procedural conflict resolution based semantics by adaptation of skeptical and credulous argument games for complete semantics. The

soundness and completeness properties for both type of games are proved, what is the main contribution of this paper. At the end we have showed how the semantics of defeasible logic program can be computed in a distributive fashion and both skeptical and credulous argument games were modified for multi-context systems. However, only multi-context systems with acyclic support dependency have been considered and the more general cases were left for the future work.

References

- Baláz, M.; Frtús, J.; and Homola, M. 2013. Conflict resolution in structured argumentation. In *Proceedings of the 19th International Conference on Logic for Programming, Artificial Intelligence, and Reasoning*.
- Bikakis, A., and Antoniou, G. 2010. Defeasible Contextual Reasoning with Arguments in Ambient Intelligence. *IEEE Transactions on Knowledge and Data Engineering* 22(11):1492–1506.
- Billington, D.; Antoniou, G.; Governatori, G.; and Maher, M. 2010. An inclusion theorem for defeasible logics. *ACM Trans. Comput. Logic* 12(1):6:1–6:27.
- Caminada, M., and Amgoud, L. 2007. On the evaluation of argumentation formalisms. *Artificial Intelligence* 171(5-6):286–310.
- Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77(2):321–357.
- García, A. J., and Simari, G. R. 2004. Defeasible logic programming: an argumentative approach. *Theory and Practice of Logic Programming* 4(2):95–138.
- Governatori, G.; Maher, M. J.; Antoniou, G.; and Billington, D. 2004. Argumentation semantics for defeasible logic. *J. Log. and Comput.* 14:675–702.
- Modgil, S., and Caminada, M. 2009. Proof theories and algorithms for abstract argumentation frameworks. In Rahwan, I., and Simari, G., eds., *Argumentation in Artificial Intelligence*. Springer Publishing Company Incorporated. 105–129.
- Modgil, S., and Prakken, H. 2011. Revisiting Preferences and Argumentation. In *Proceedings of the Twenty-Second International Joint Conference on Artificial Intelligence*, 1021–1026. AAAI Press.
- Prakken, H., and Sartor, G. 1997. Argument-based extended logic programming with defeasible priorities. *Journal of Applied Nonclassical Logics* 7(1):25–75.
- Prakken, H. 2010. An abstract framework for argumentation with structured arguments. *Argument & Computation* 1(2):93–124.
- Thimm, M., and Kern-Isberner, G. 2008. A distributed argumentation framework using defeasible logic programming. In *Proceedings of the 2008 Conference on Computational Models of Argument: Proceedings of COMMA 2008*, 381–392. Amsterdam, The Netherlands, The Netherlands: IOS Press.