Linear First Order Differential Equations

$v = s / t$

$a = dv / dt$

Linear First Order Differential Equations

$v = s / t$

$a = dv / dt$

$g = 9.78 \; m / s^2$

Linear First Order Differential Equations

v = s / t
a = dv / dt
g = 9.78 m / s^2

$$\frac{dv}{dt} = a$$

Linear First Order Differential Equations

$v = s / t$
$a = dv / dt$
$g = 9.78 \text{ m} / s^2$

$$dv = a\ dt$$

Linear First Order Differential Equations

$v = s / t$
$a = dv / dt$
$g = 9.78\ m / s^2$

$$\int dv = \int a\ dt$$

Linear First Order Differential Equations

$v = s / t$
$a = dv / dt$
$g = 9.78 \ m / s^2$

$$v = at + c$$

Linear First Order Differential Equations

v = s / t
a = dv / dt
g = 9.78 m / s^2

?

$$v = at + c$$

Linear First Order Differential Equations

v = s / t

a = dv / dt

g = 9.78 m / s^2

$$t = 0 \rightarrow s = 0$$

?

$$v = at + c$$

Linear First Order Differential Equations

$v = s / t$

$a(t) = dv / dt$

$a(t) = t - 3t^2 + t^3$

Linear First Order Differential Equations

$v = s / t$

$a(t) = dv / dt$

$a(t) = t - 3t^2 + t^3$

$$\frac{dv}{dt} = t - 3t^2 + t^3$$

Linear First Order Differential Equations

v = s / t

a(t) = dv / dt

$a(t) = t – 3t^2 + t^3$

$$dv = tdt - 3t^2dt + t^3dt$$

$v = s / t$

$a(t) = dv / dt$

$a(t) = t - 3t^2 + t^3$

$$v = \frac{t^2}{2} - t^3 + \frac{t^4}{4} + c$$

Linear First Order Differential Equations

$$t = 0 \to f(t) = 8$$

$$f'(t) = 4 + t - \frac{1}{2} f(t)$$

Linear First Order Differential Equations

$$t = 0 \rightarrow f(t) = 8$$

$$f'(t) = 4 + t - \frac{1}{2} f(t)$$

$$f'(t) + \frac{1}{2} f(t) = 4 + t$$

g(t)

a

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t) \quad \Big/ \mu(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$$

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$$

$$\mu'(t) = a\mu(t)$$

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t)f'(t) + \mu'(t)f(t) = [\mu(t)f(t)]'$$

$$\mu'(t) = a\mu(t)$$

$$\int \frac{\mu'(t)}{\mu(t)}dt = \int a\,dt$$

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\int \frac{\mu'(t)}{\mu(t)}dt = \int a dt$$

$$[ln(f(x))]' = \frac{f'(x)}{f(x)}$$

$$ln|\mu(t)| = at + b$$

Linear First Order Differential Equations

$$f'(t) + af(t) = g(t)$$

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$ln|\mu(t)| = at + b$$

$$\mu(t) = ce^{at}$$

Linear First Order Differential Equations

$$\mu(t)f'(t) + a\mu(t)f(t) = \mu(t)g(t)$$

$$\mu(t) = ce^{at}$$

Linear First Order Differential Equations

$$\mu(t) f'(t) + a\mu(t) f(t) = \mu(t) g(t)$$

$$\mu(t) = ce^{at}$$

$$ce^{at} f'(t) + ace^{at} f(t) = ce^{at} g(t)$$

Linear First Order Differential Equations

$$\mu(t) f'(t) + a\mu(t) f(t) = \mu(t) g(t)$$

$$\mu(t) = ce^{at}$$

$$\cancel{c}e^{at} f'(t) + a\cancel{c}e^{at} f(t) = \cancel{c}e^{at} g(t)$$

Linear First Order Differential Equations

$$e^{at} f'(t) + a e^{at} f(t) = e^{at} g(t)$$

$$[e^{at} f(t)]' = e^{at} g(t)$$

$$[e^{f(x)}]' = f(x)' e^{f(x)}$$

Linear First Order Differential Equations

$$[e^{at} f(t)]' = e^{at} g(t)$$

a

g(t)

$$f'(t) + \frac{1}{2} f(t) = 4 + t$$

$$[e^{\frac{1}{2}t} f(t)]' = 4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$$

Linear First Order Differential Equations

$$\left[e^{\frac{1}{2}t}f(t)\right]' = 4e^{\frac{1}{2}t} + te^{\frac{1}{2}t}$$

$$e^{\frac{1}{2}t}f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

per partes

$$\int u'(x)v(x)\,dx = u(x)v(x) - \int u(x)v'(x)\,dx$$

$$e^{\frac{1}{2}t} f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

$$t = 0 \rightarrow f(t) = 8$$

$$e^{\frac{1}{2}t} f(t) = 8e^{\frac{1}{2}t} + 2te^{\frac{1}{2}t} - 4e^{\frac{1}{2}t} + c$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

Linear First Order Differential Equations

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

Getting Numerical with Euler's Method

$$f(t+\Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$$

$$t < \xi < t + \Delta t$$

Getting Numerical with Euler's Method

$$f(t+\Delta t) = f(t) + \Delta t f'(t) + \frac{h^2}{2} f''(\xi)$$

$$t < \xi < t + \Delta t$$

$$f'(t) = 4 + t - \frac{1}{2} f(t)$$

$$t = 0 \rightarrow f(t) = 8$$

$$\Delta t = 0.1$$

Getting Numerical with Euler's Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$\Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0+0.1) = f(0) + 0.1f'(0) = 8 + 0.1(4 + 0 - \frac{8}{2}) = 8$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

$$f(0.1) = 4+2*0.1+4e^{-0.1/2} = 8.005$$

Getting Numerical with Euler's Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$\Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0+0.1) = f(0) + 0.1f'(0) = 8 + 0.1(4 + 0 - \frac{8}{2}) = 8$$

$$f(0.1+0.1) = f(0.1)+0.1f'(0.1) = 8+0.1(4+0.1-\frac{8}{2}) = 8.01$$

$$f(t) = 4 + 2t + 4e^{-t/2}$$

$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} =$$

8.0193

Getting Numerical with Runge-Kutta Method

$$
\begin{aligned}
y_{n+1} &= y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\
k_1 &= hf(x_n, y_n), \\
k_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}), \\
k_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}), \\
k_4 &= hf(x_{n+1}, y_n + k_3).
\end{aligned}
$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t)$$

$$\Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \qquad \Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$k_1 = \Delta t f'(t) = 0.1(4 + 0 - \frac{8}{2}) = 0$$

$$k_2 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_1}{2})) = 0.1(4 + 0.05 - 4) = 0.005$$

$$k_3 = 0.1(4 + (0 + \frac{0.1}{2}) - \frac{1}{2}(f(0) + \frac{k_2}{2})) = 0.1(4 + 0.05 - \frac{8.0025}{2}) = 0.004875$$

$$k_4 = 0.1(4 + (0 + 0.1) - \frac{1}{2}(f(0) + k_3)) = 0.1(4 + 0.1 - \frac{8.004875}{2}) = 0.00976$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \qquad \Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} =$$
8.00492

$$k_1 = \Delta t f'(0.1) = 0.1(4 + 0.1 - \frac{8.00492}{2}) = 0.00975$$

$$k_2 = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_1}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.004875}{2}) =$$
0.0145

$$k_3 = 0.1(4 + (0.1 + \frac{0.1}{2}) - \frac{1}{2}(f(0.1) + \frac{k_2}{2})) = 0.1(4 + 0.15 - \frac{8.00492 + 0.00726}{2}) =$$
0.0144

$$k_4 = 0.1(4 + (0.1 + 0.1) - \frac{1}{2}(f(0.1) + k_3)) = 0.1(4 + 0.2 - \frac{8.00492 + 0.0144}{2}) =$$
0.019

Getting Numerical with Runge-Kutta Method

$$f'(t) = 4 + t - \frac{1}{2}f(t) \qquad \Delta t = 0.1$$

$$t = 0 \rightarrow f(t) = 8$$

$$f(0) = 8$$

$$f(0.1) \approx f(0) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8 + \frac{2(0.005 + 0.004875) + 0.00976}{6} = 8.00492$$

$$f(0.2) \approx f(0.1) + \frac{k_1 + 2(k_2 + k_3) + k_4}{6} = 8.00492 + \frac{0.00975 + 2(0.0145 + 0.0144) + 0.019}{6} = 8.0194$$

$$f(0.1) = 4 + 2 * 0.1 + 4e^{-0.1/2} = 8.005$$

$$f(0.2) = 4 + 2 * 0.2 + 4e^{-0.2/2} = 8.0193$$