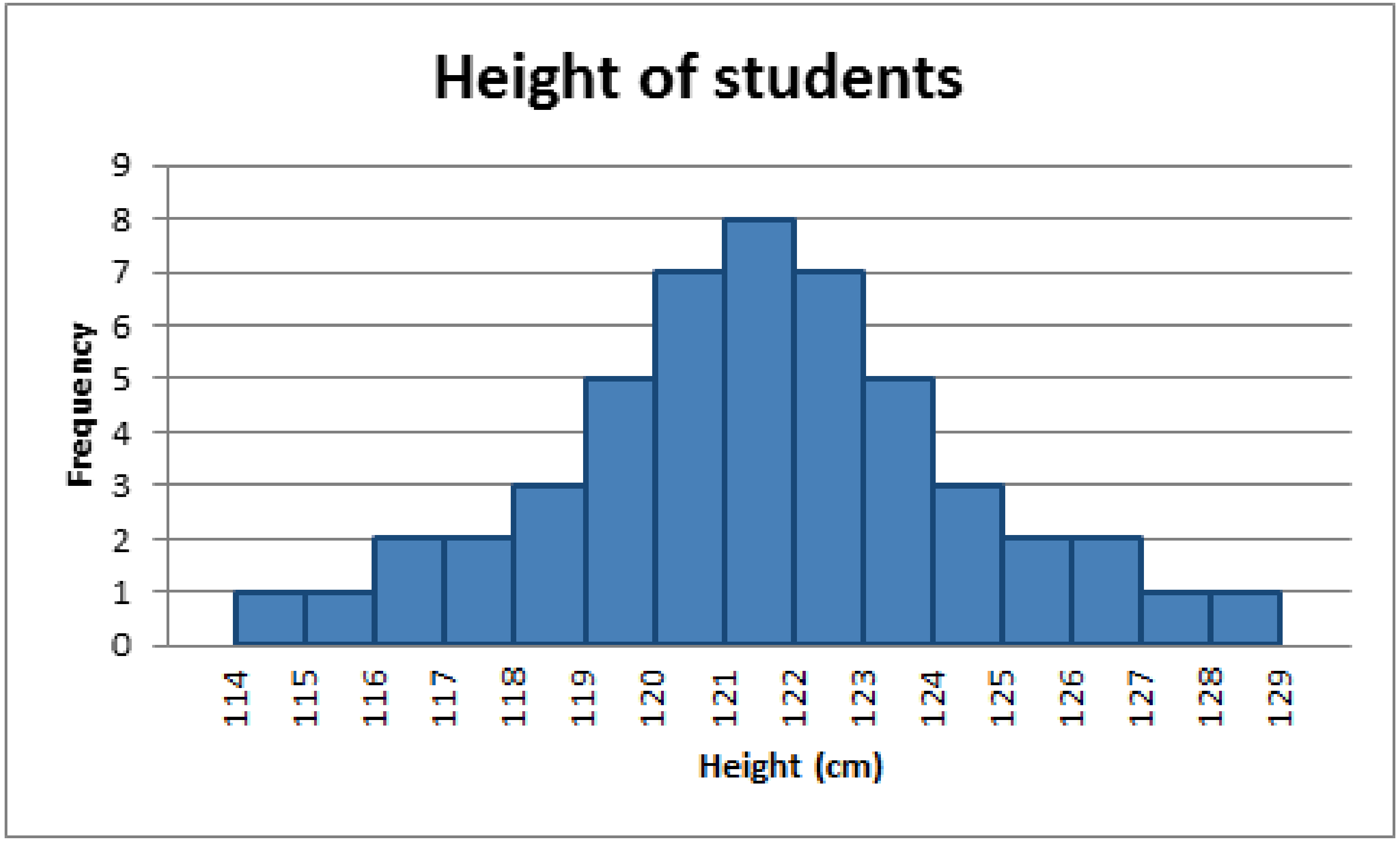
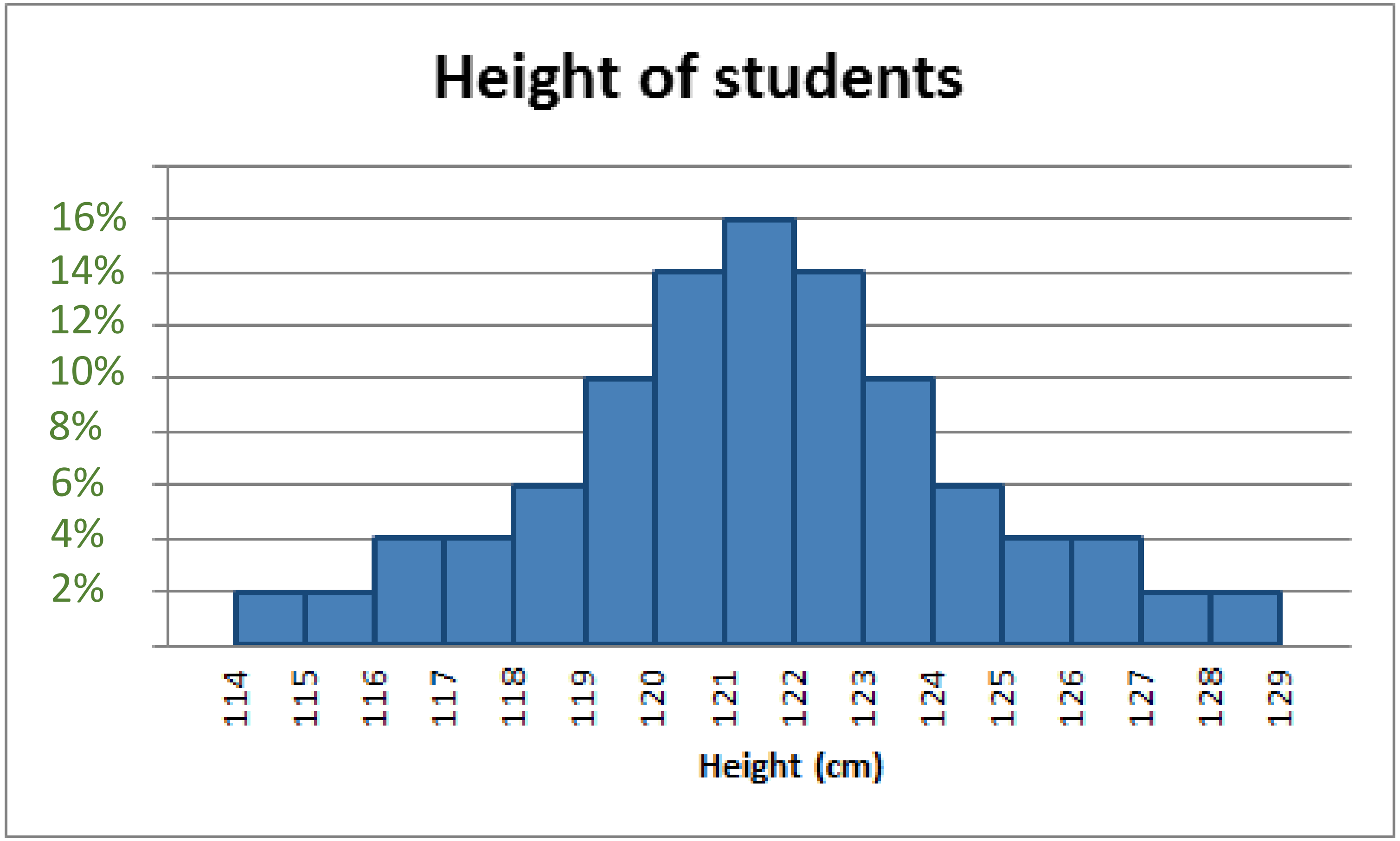
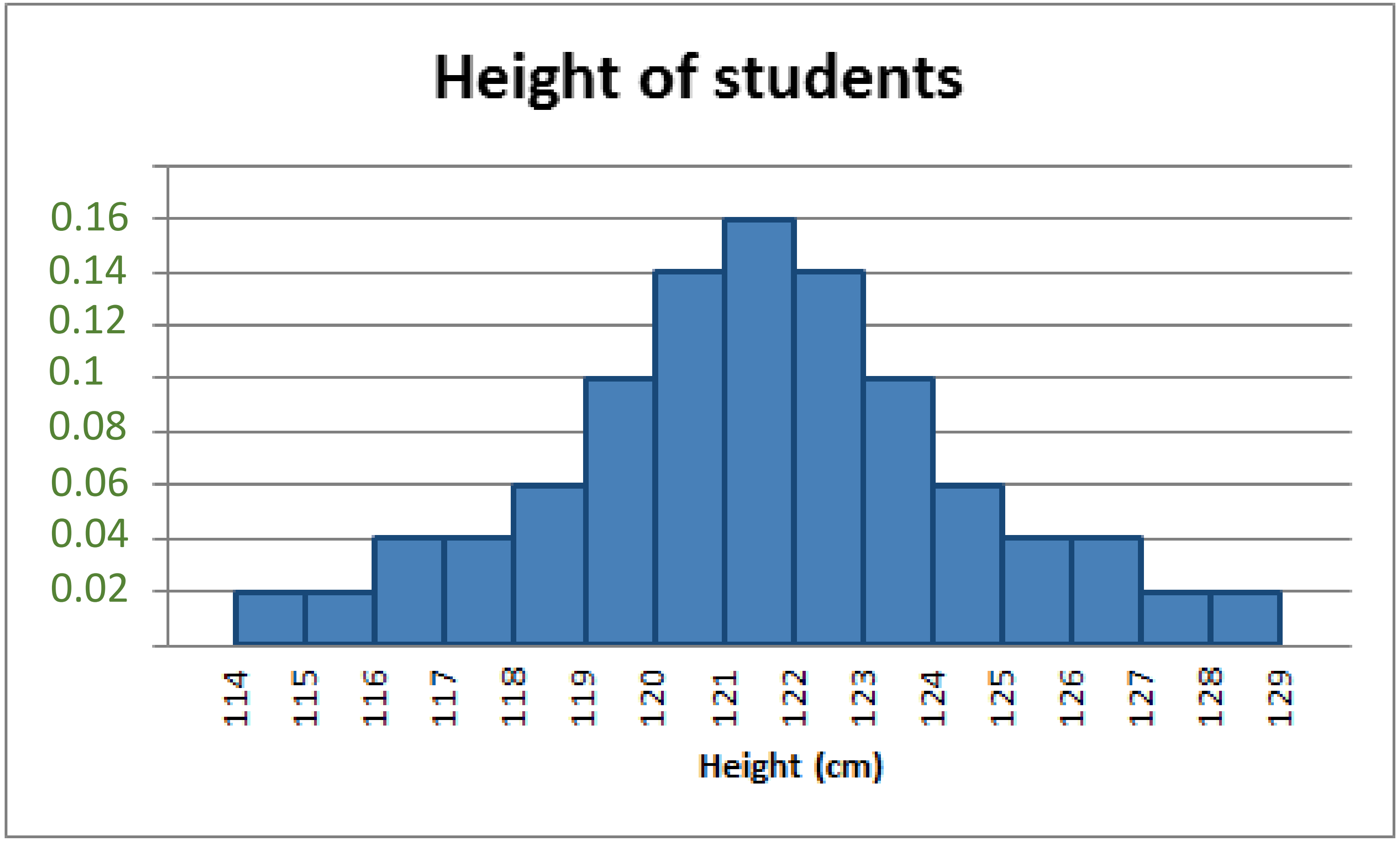
expected value of a random variable



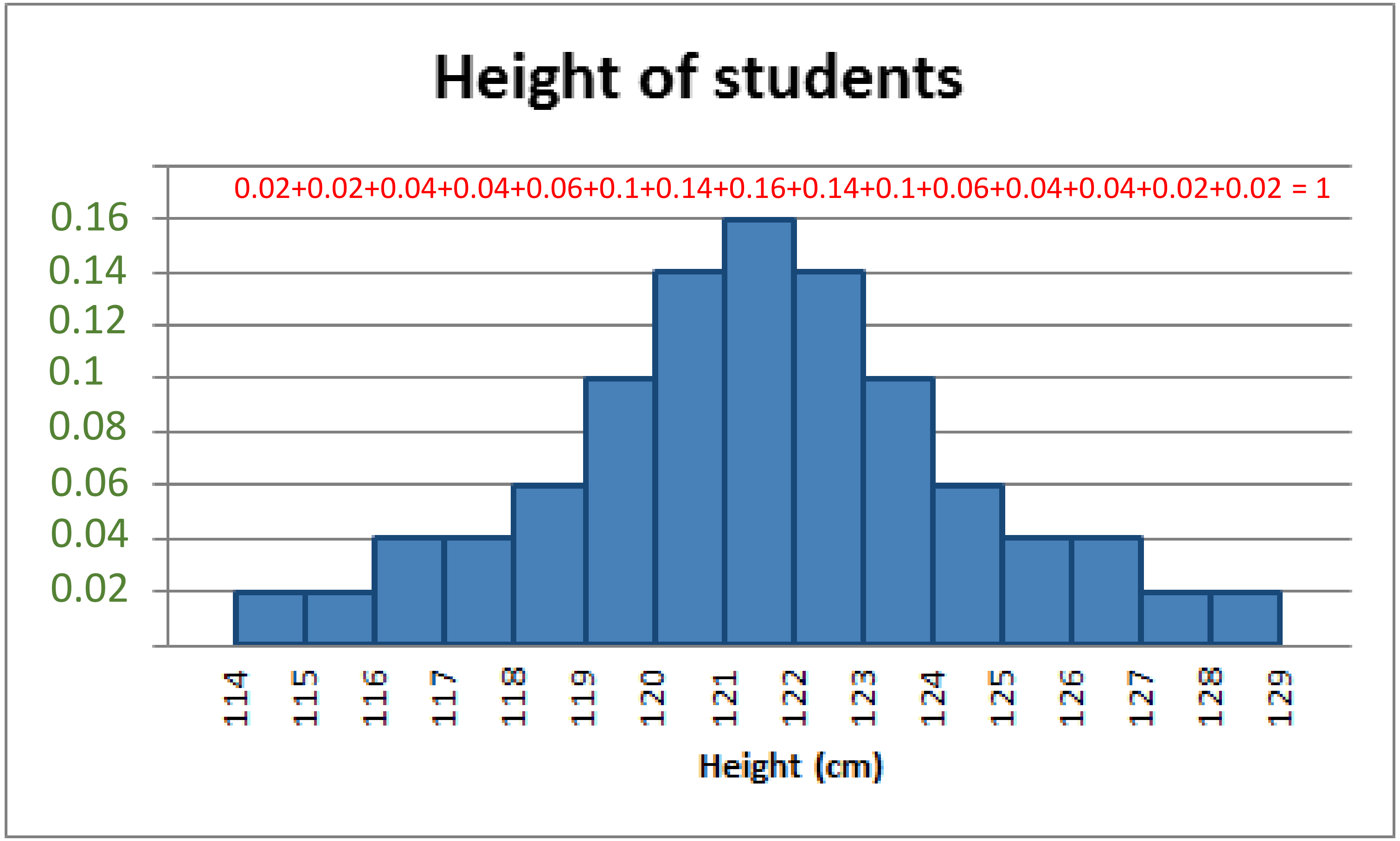
expected value of a random variable



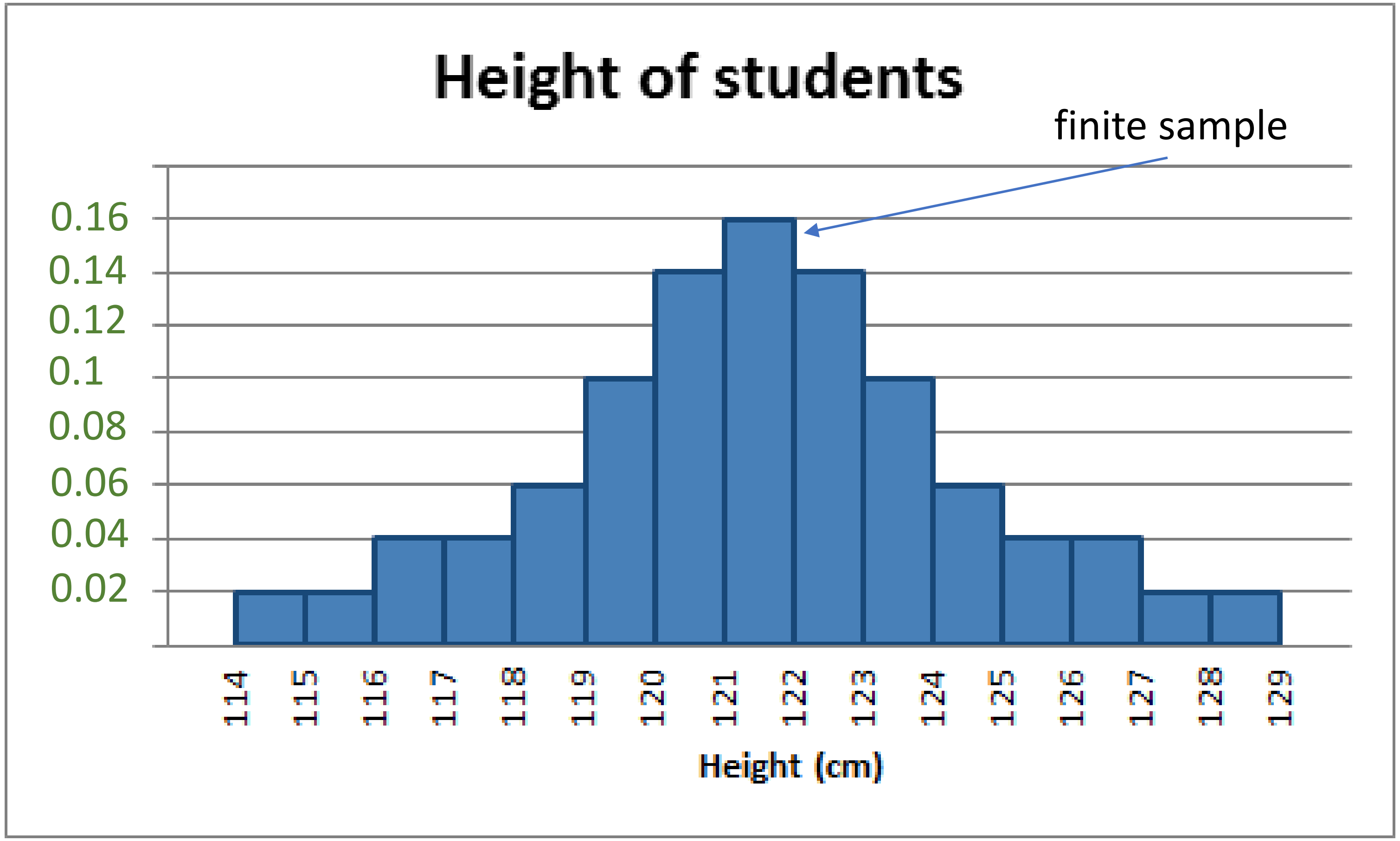
expected value of a random variable



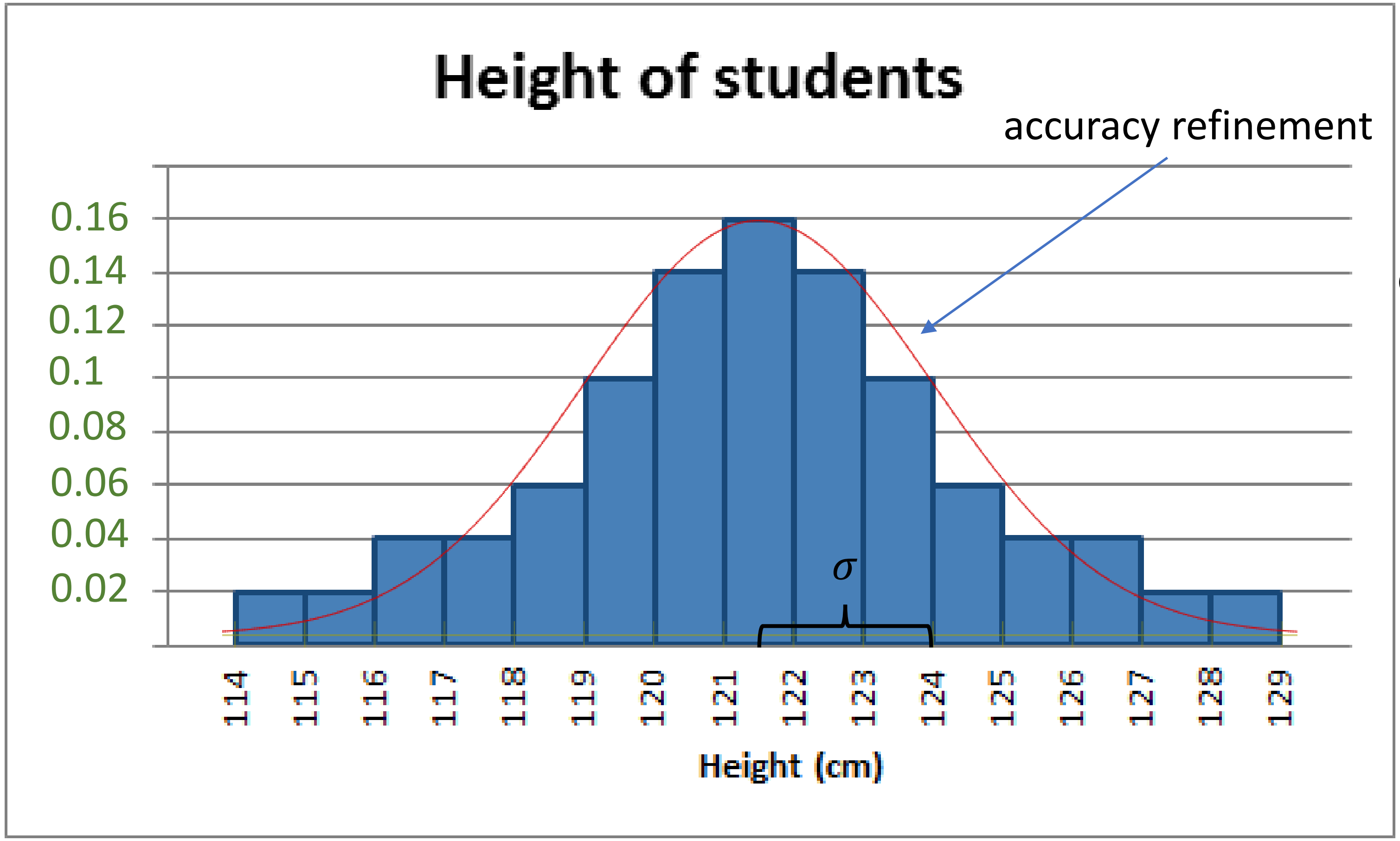
expected value of a random variable



expected value of a random variable



expected value of a random variable

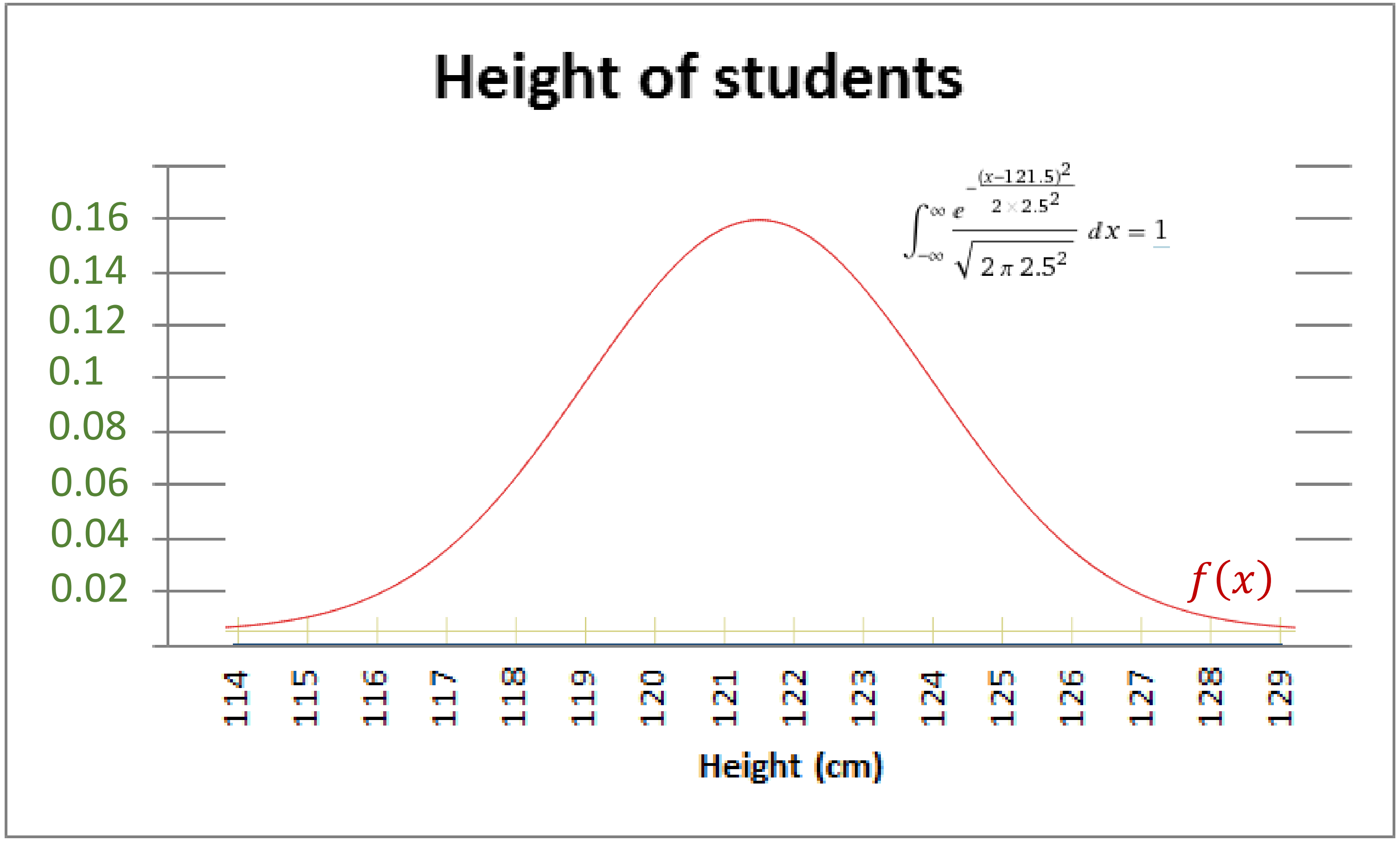


probability density function of the normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\sigma = 2.5; \mu = 121.5$

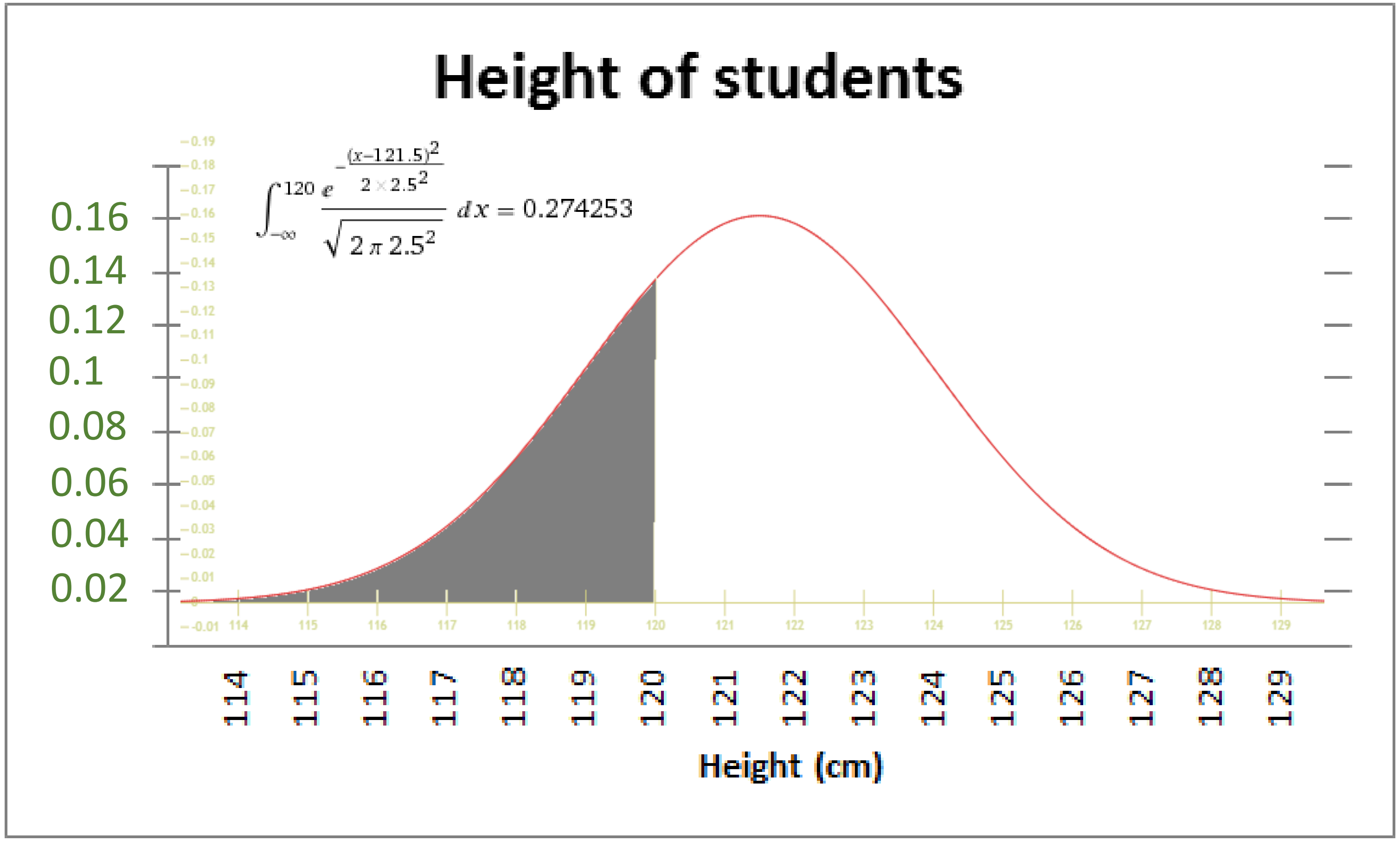
expected value of a random variable



probability density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

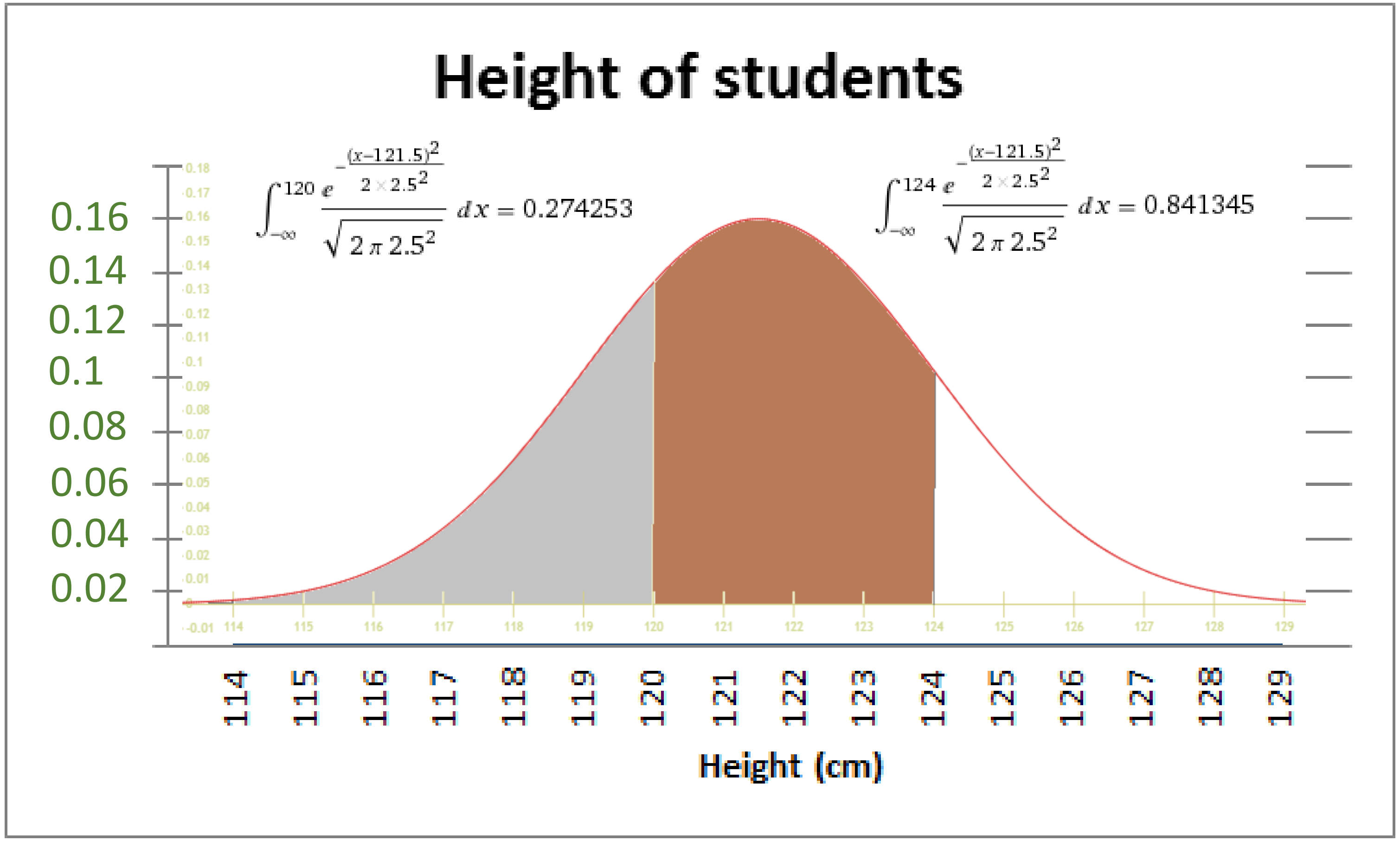
expected value of a random variable



probability density function

$$P[X \leq 120] = \int_{-\infty}^{120} f(x)\,dx$$

expected value of a random variable



probability density function

$$P[120 < X \leq 124] =$$

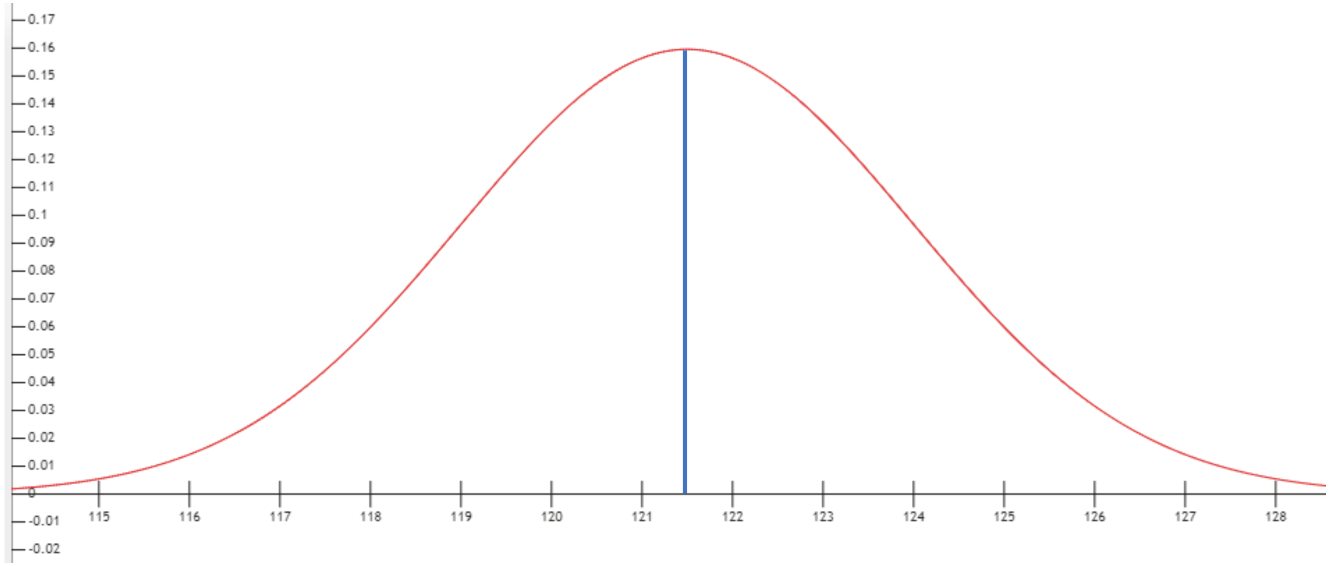
$$= \int_{120}^{124} f(x)\, dx =$$

= 0.841345 - 0.274253 =

= 0.567092

expected value of a random variable

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$



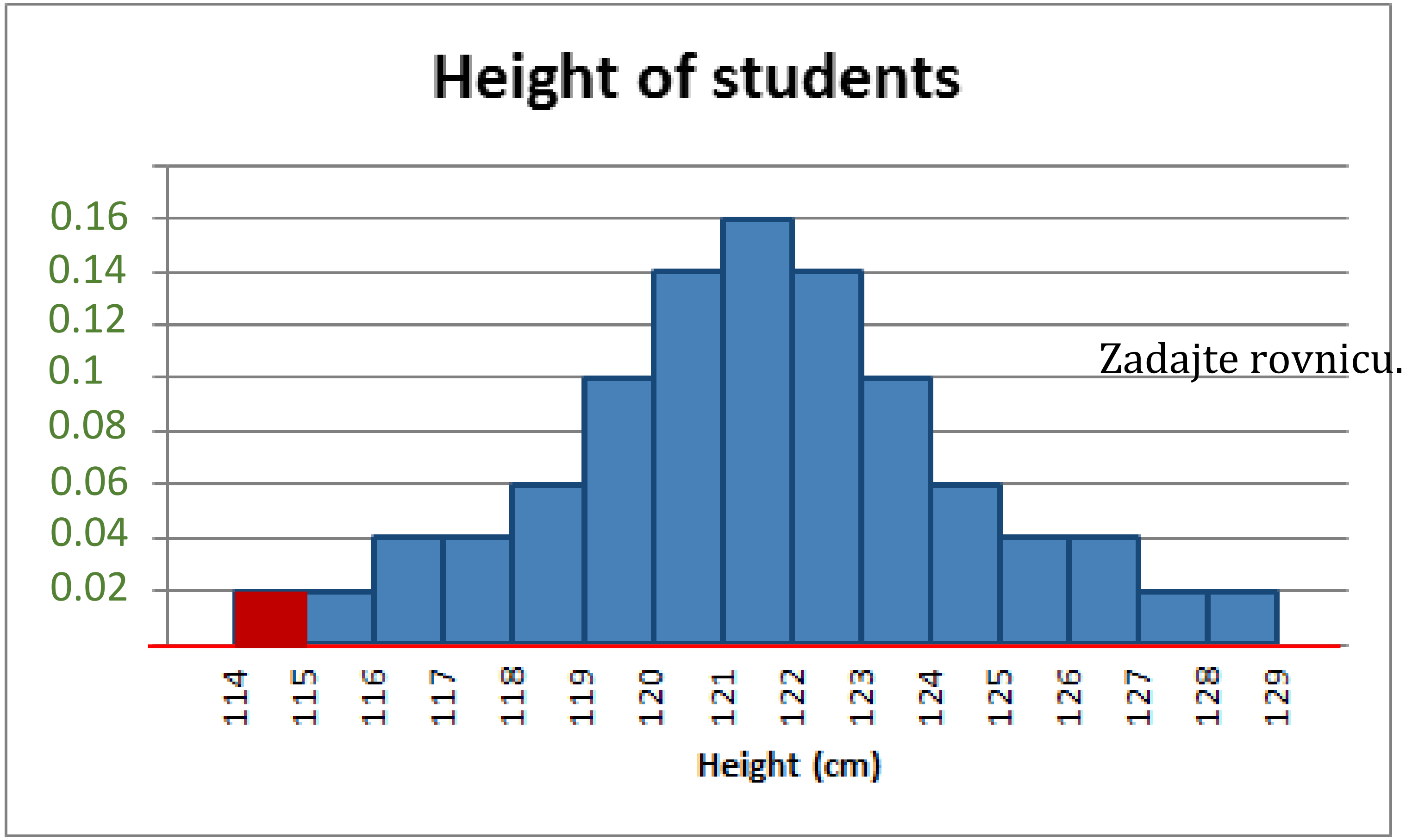
probability density function of the normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\sigma = 2.5; \mu = 121.5$$

$$\int_{-\infty}^{\infty} \frac{x\, e^{-\frac{(x-121.5)^2}{2 \times 2.5^2}}}{\sqrt{2\pi 2.5^2}}\, dx = 121.5$$

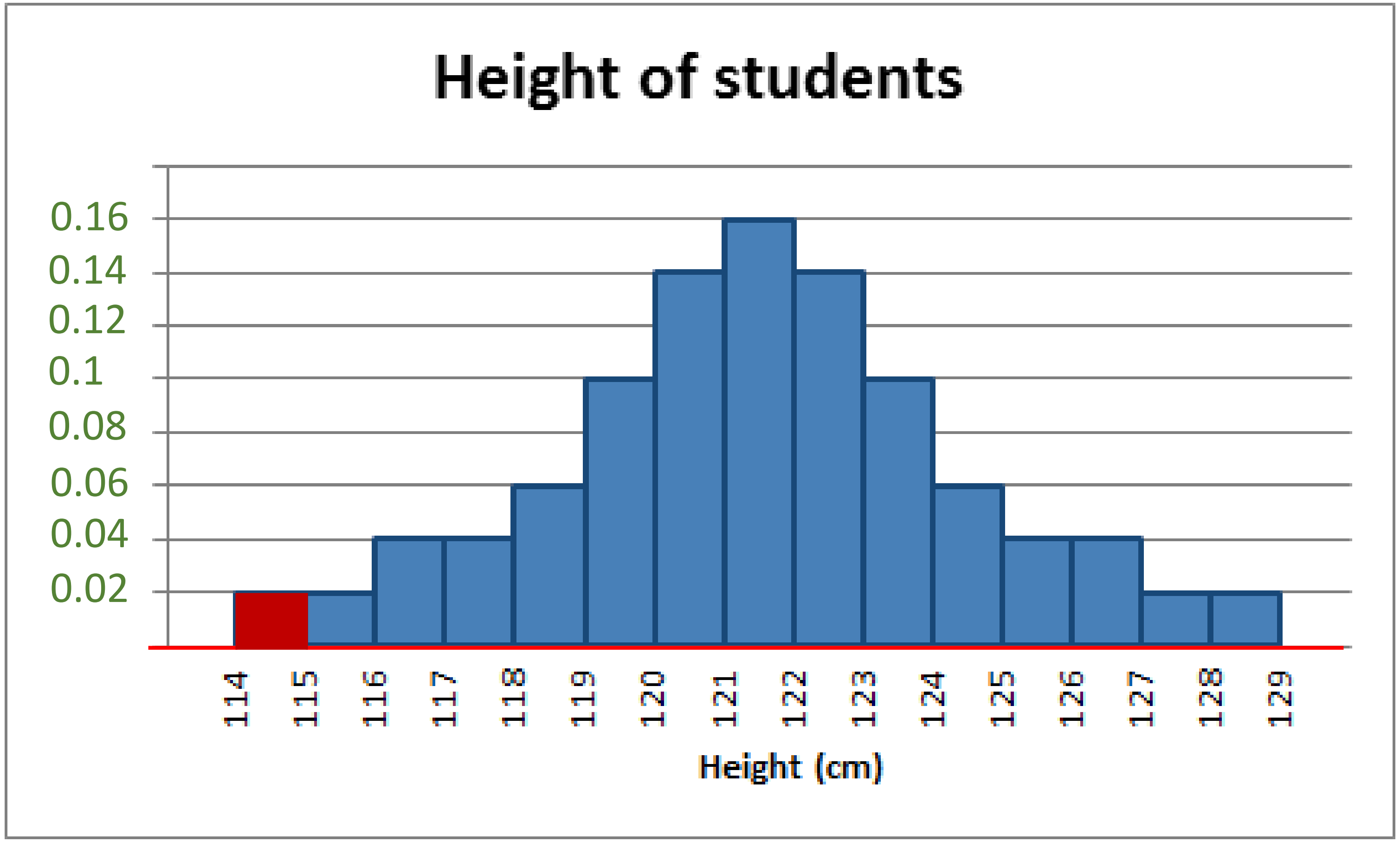
expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 114] \\ 0.02 & x \in (114,115] \\ 0 & x \in (115, \infty) \end{cases}$$

expected value of a random variable



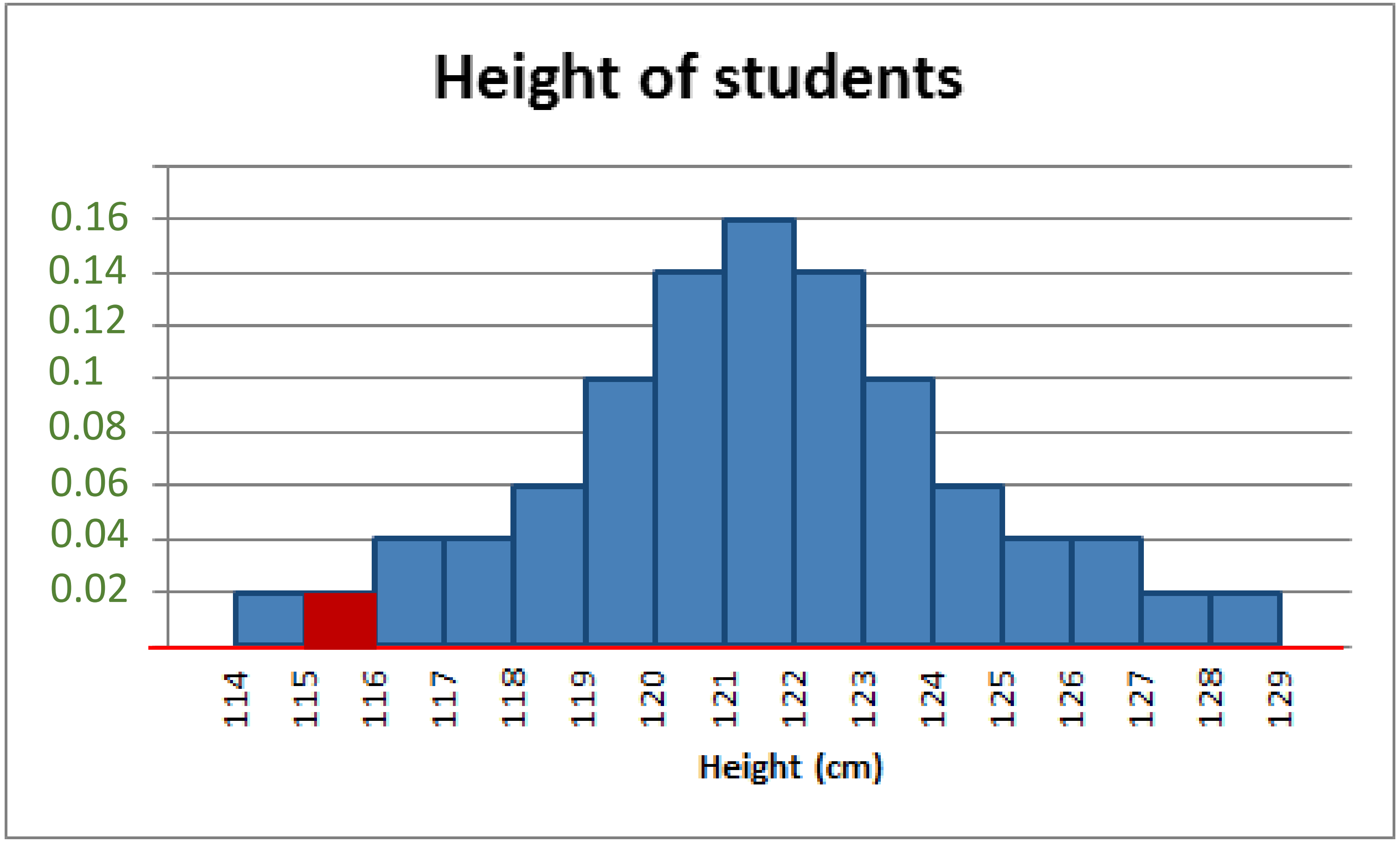
$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 114] \\ 0.02 & x \in (114,115] \\ 0 & x \in (115, \infty) \end{cases}$$

$$0.02 \int_{114}^{115} x\, dx =$$
$$= 0.02 \left[\frac{x^2}{2}\right]_{114}^{115} =$$
$$= 0.02 \frac{115^2 - 114^2}{2} =$$
$$= 0.02 \times 114.5$$

0.02×114.5

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 115] \\ 0.02 & x \in (115, 116] \\ 0 & x \in (116, \infty) \end{cases}$$

$$0.02 \int_{115}^{116} x\, dx =$$

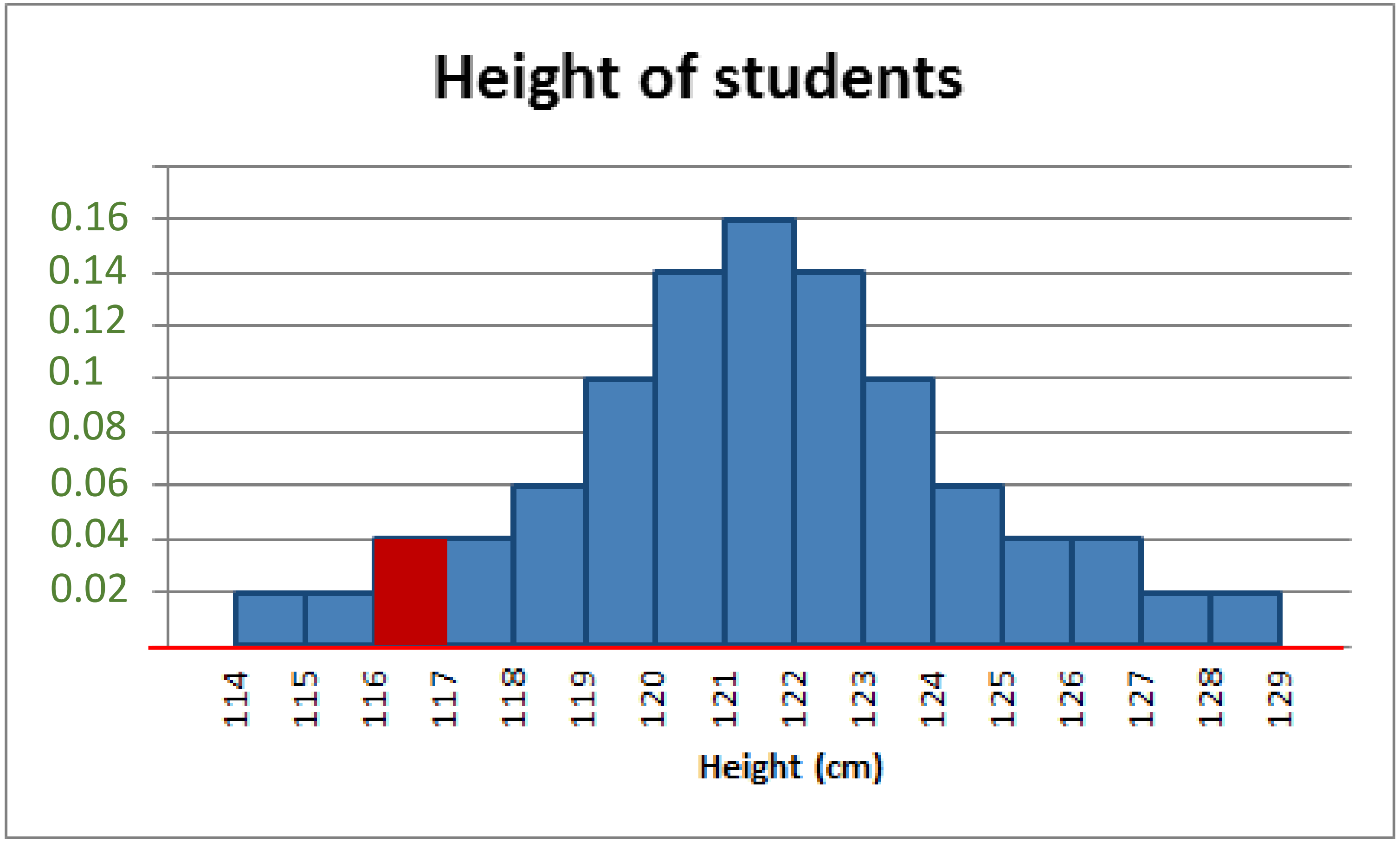
$$= 0.02 \left[\frac{x^2}{2}\right]_{115}^{116} =$$

$$= 0.02 \frac{116^2 - 115^2}{2} =$$

$$= 0.02 \times 115.5$$

$0.02 \times 114.5 + 0.02 \times 115.5$

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 116] \\ 0.04 & x \in (116, 117] \\ 0 & x \in (117, \infty) \end{cases}$$

$$0.04 \int_{116}^{117} x\, dx =$$

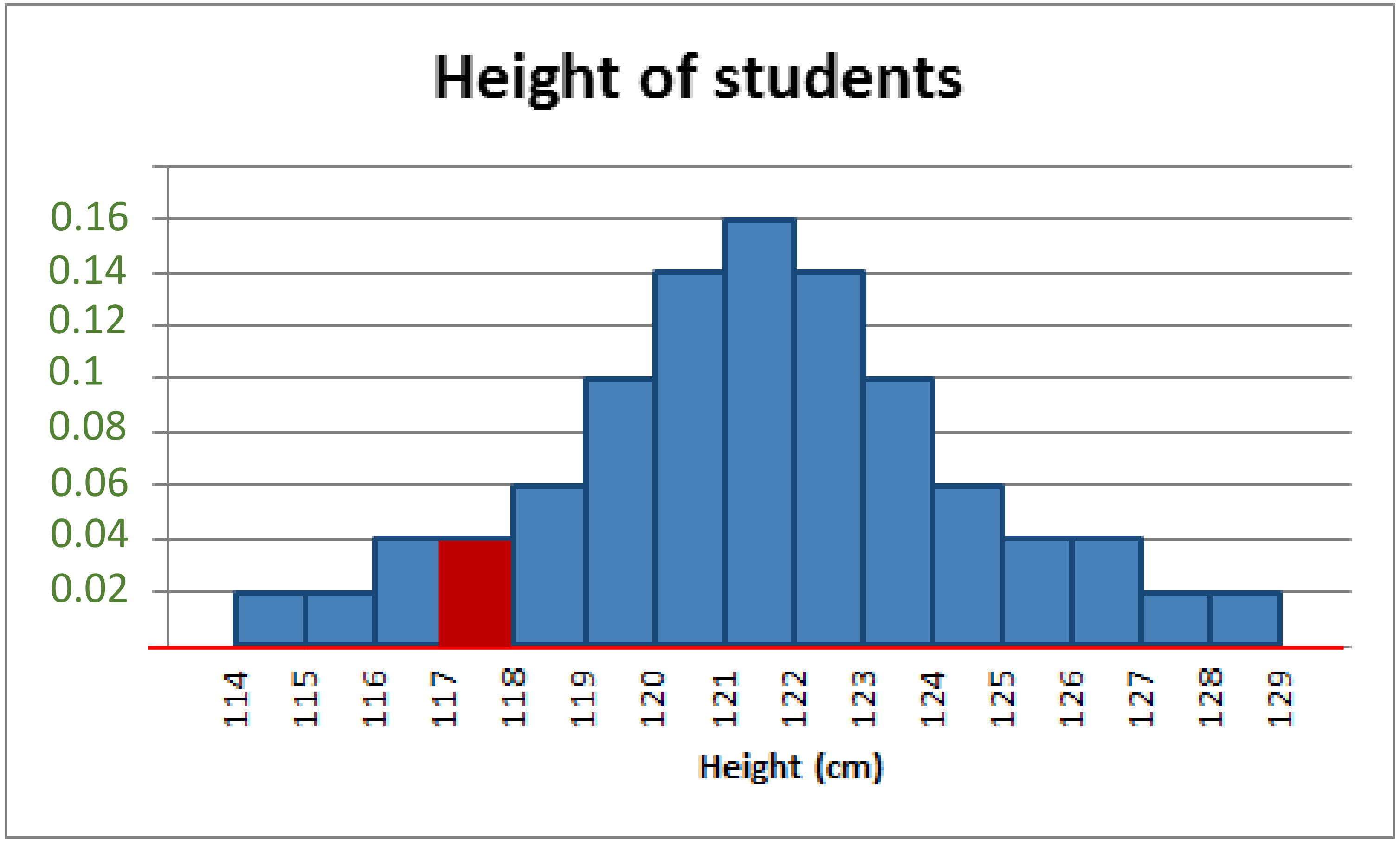
$$= 0.04 \left[\frac{x^2}{2}\right]_{116}^{117} =$$

$$= 0.04 \frac{117^2 - 116^2}{2} =$$

$$= 0.04 \times 116.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5$

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 117] \\ 0.04 & x \in (117,118] \\ 0 & x \in (118, \infty) \end{cases}$$

$$0.04 \int_{117}^{118} x\, dx =$$

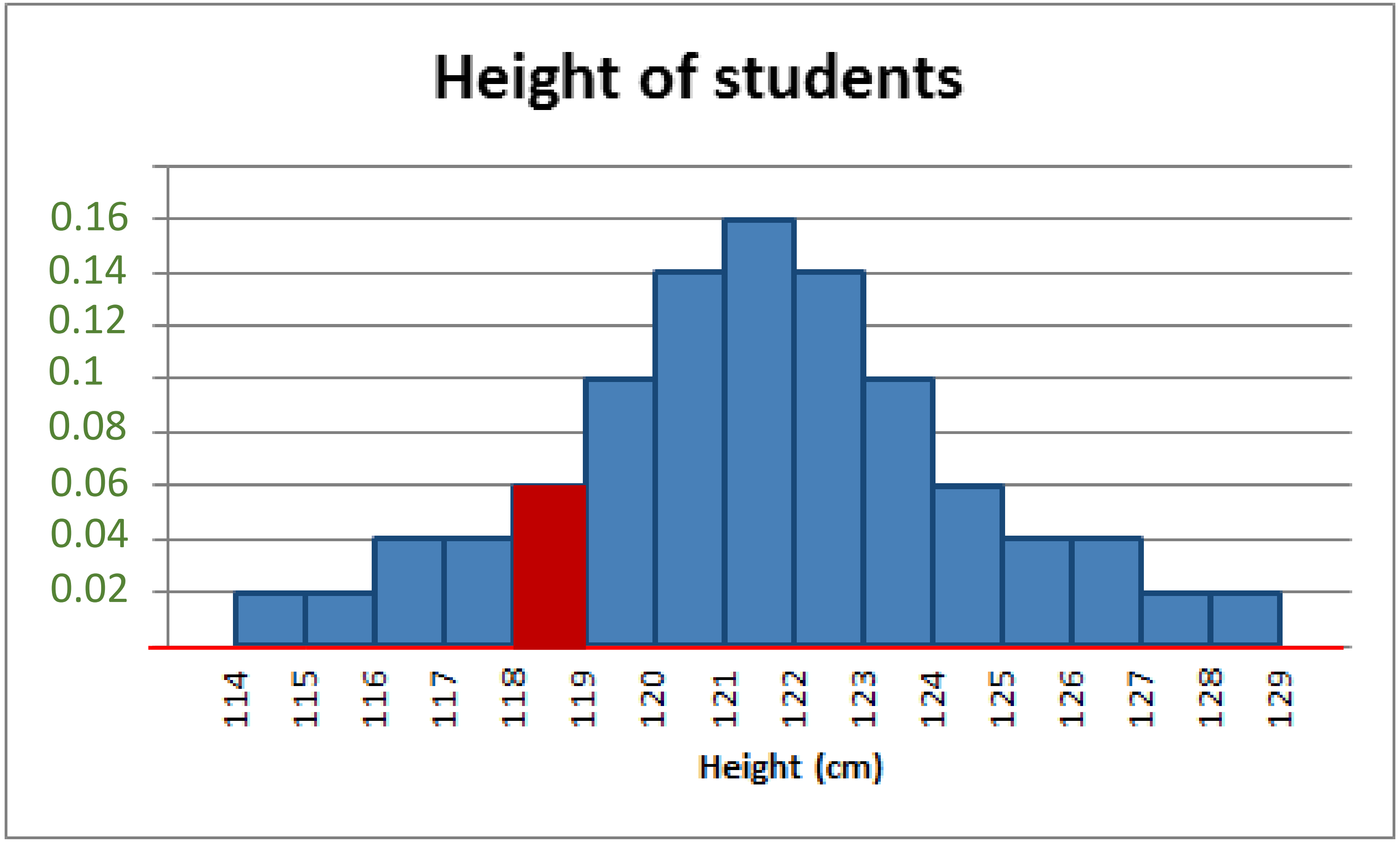
$$= 0.04 \left[\frac{x^2}{2}\right]_{117}^{118} =$$

$$= 0.04\, \frac{118^2 - 117^2}{2} =$$

$$= 0.04 \times 117.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5$

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 118] \\ 0.06 & x \in (118,119] \\ 0 & x \in (119, \infty) \end{cases}$$

$$0.06 \int_{118}^{119} x\, dx =$$

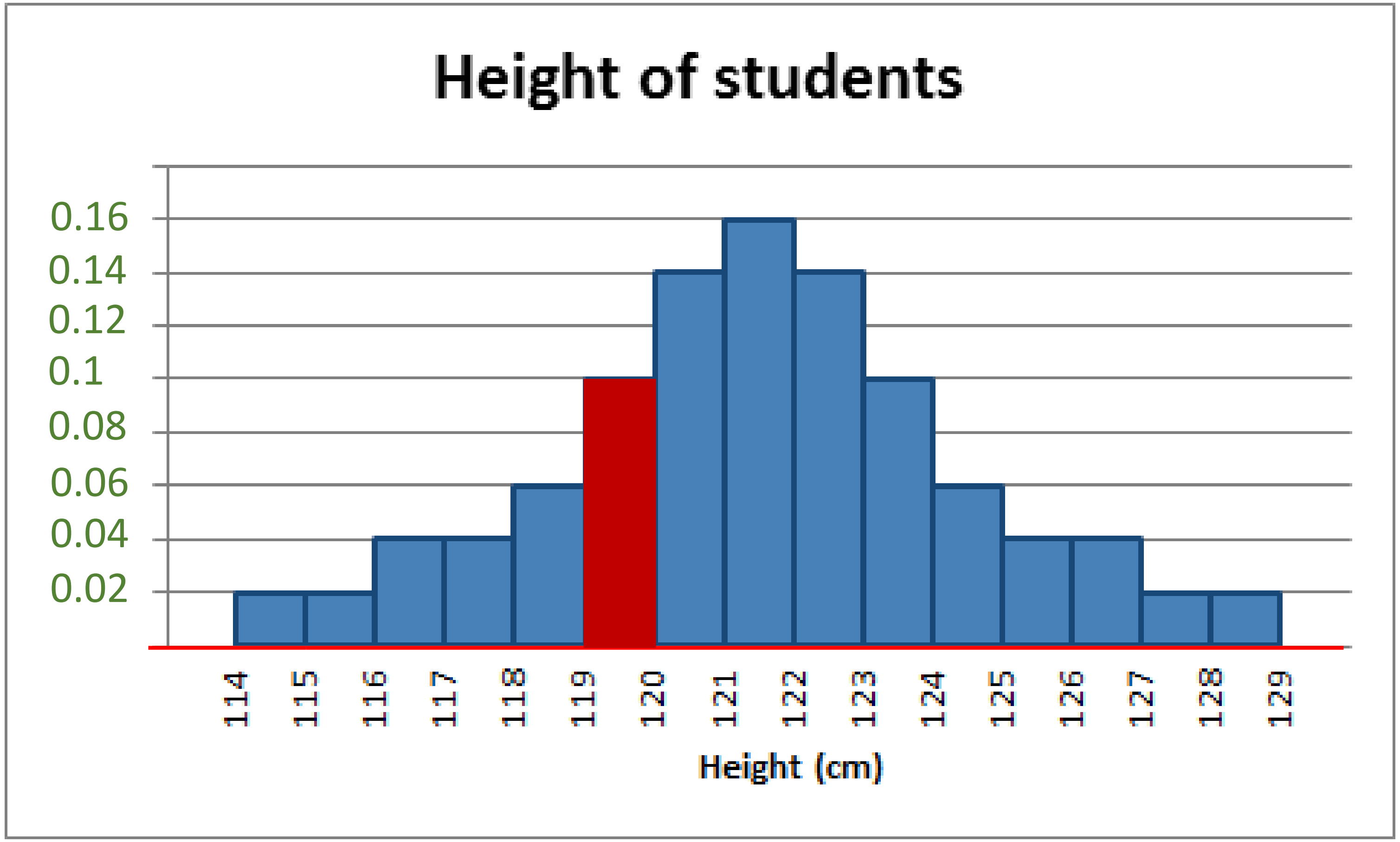
$$= 0.06 \left[\frac{x^2}{2}\right]_{118}^{119} =$$

$$= 0.06 \frac{119^2 - 118^2}{2} =$$

$$= 0.06 \times 118.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5$

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 119] \\ 0.1 & x \in (119,120] \\ 0 & x \in (120, \infty) \end{cases}$$

$$0.1 \int_{119}^{120} x\, dx =$$

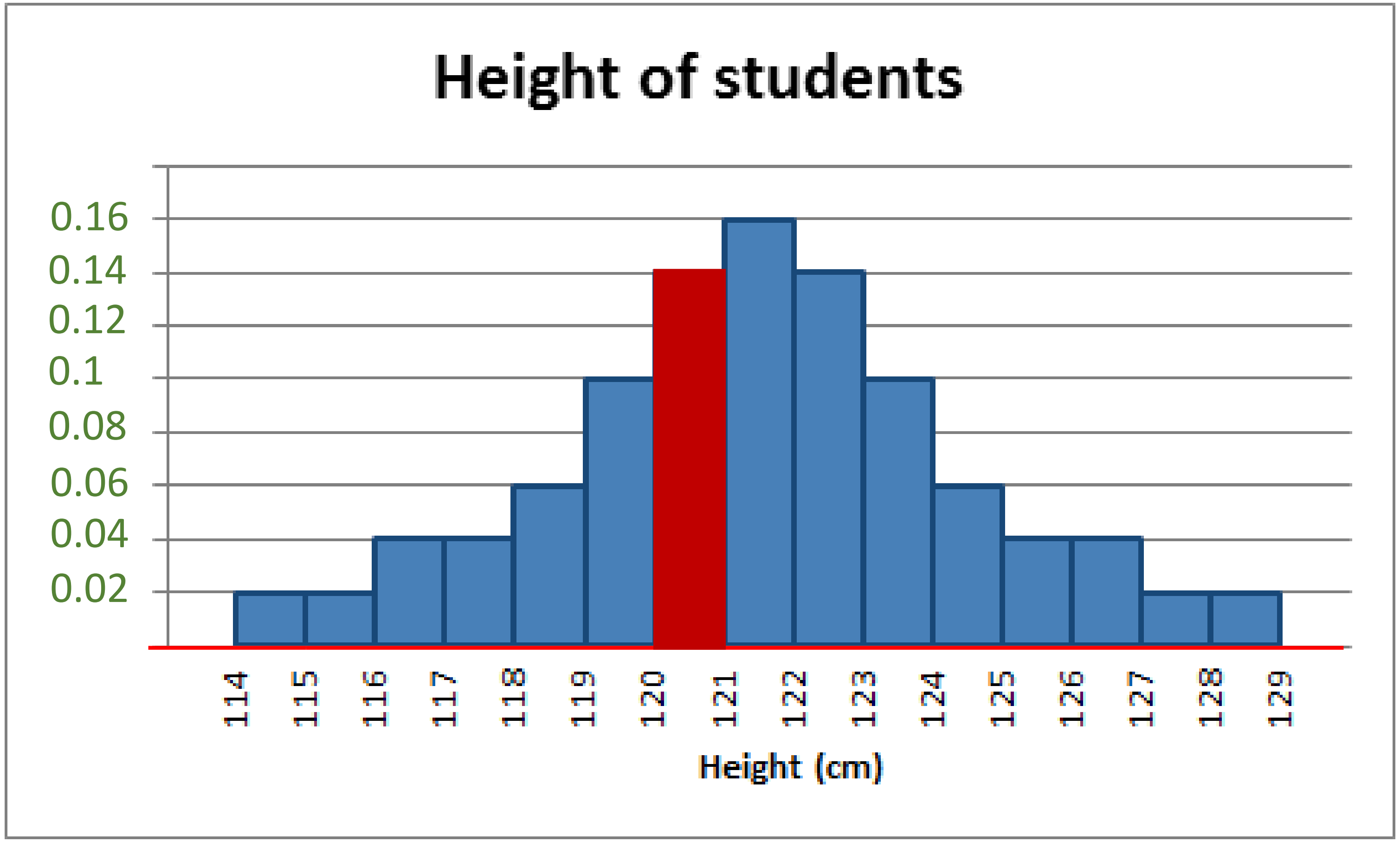
$$= 0.1 \left[\frac{x^2}{2}\right]_{119}^{120} =$$

$$= 0.1 \frac{120^2 - 119^2}{2} =$$

$$= 0.1 \times 119.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5$

expected value of a random variable



$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 120] \\ 0.14 & x \in (120,121] \\ 0 & x \in (121, \infty) \end{cases}$$

$$0.14 \int_{120}^{121} x\, dx =$$

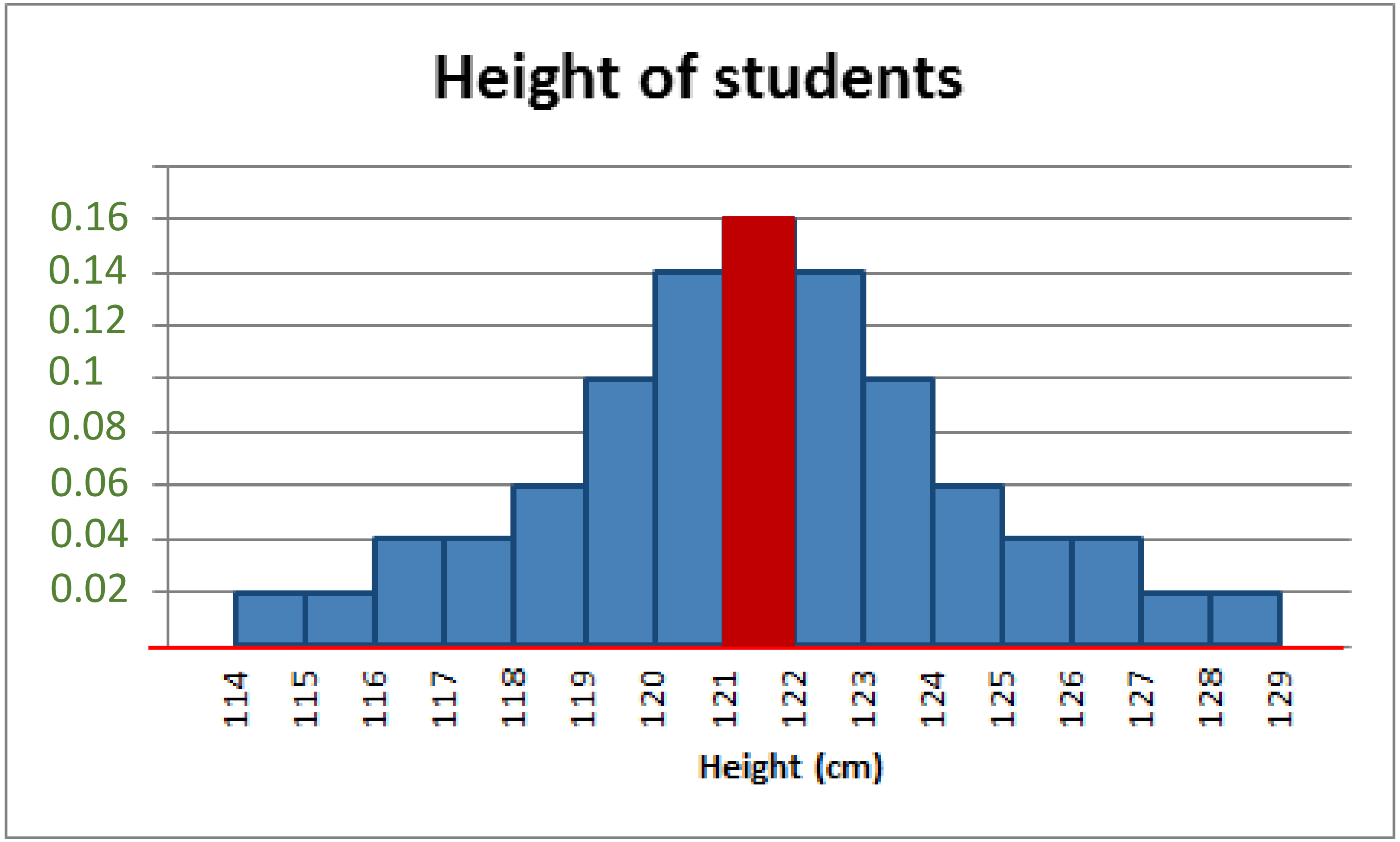
$$= 0.14 \left[\frac{x^2}{2}\right]_{120}^{121} =$$

$$= 0.14 \frac{121^2 - 120^2}{2} =$$

$$= 0.14 \times 120.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5$

expected value of a random variable



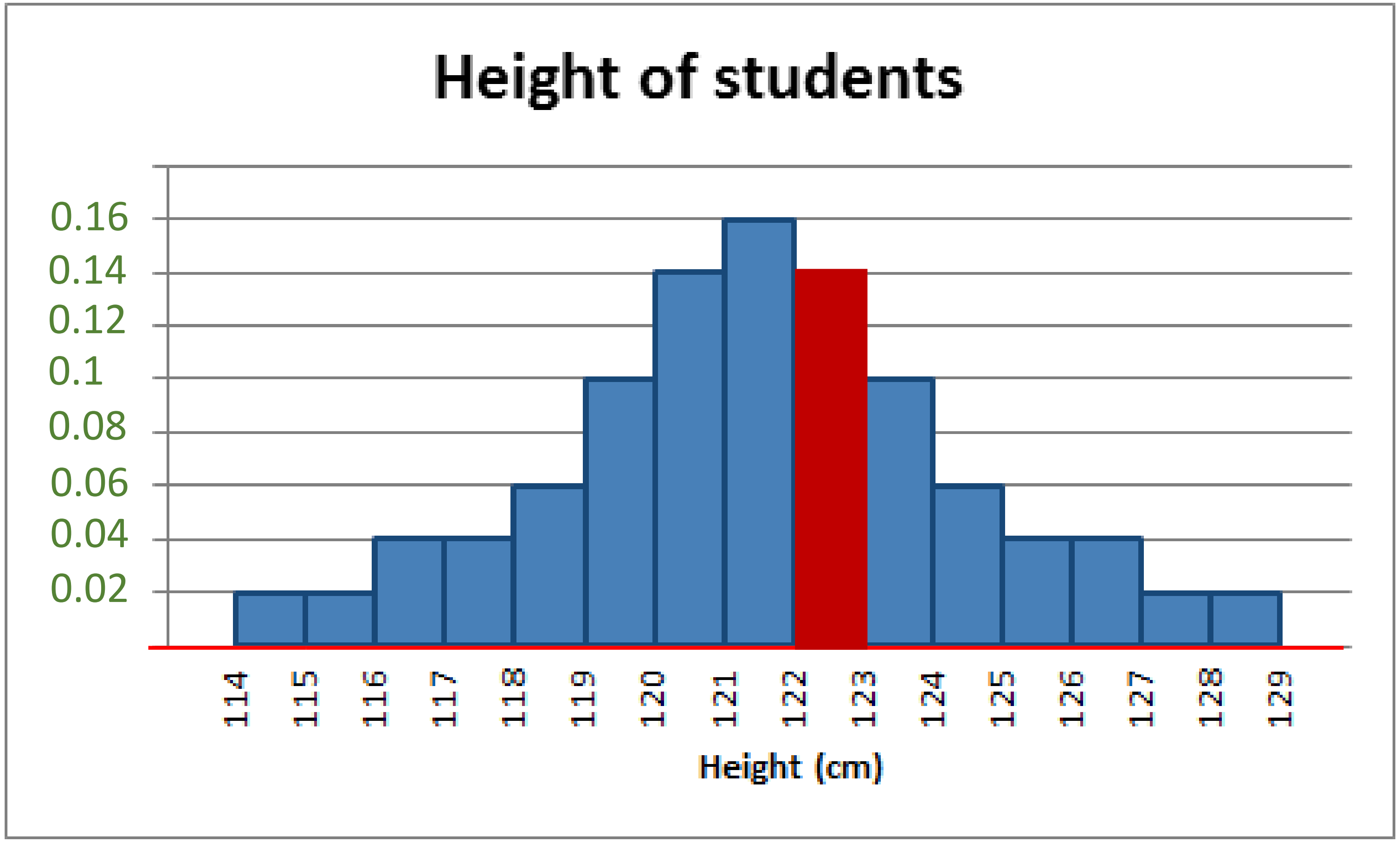
$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$f(x) = \begin{cases} 0 & x \in (-\infty, 121] \\ 0.16 & x \in (121, 122] \\ 0 & x \in (122, \infty) \end{cases}$$

$$0.16 \int_{121}^{122} x\, dx = \\ = 0.16 \times 121.5$$

$$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5$$

expected value of a random variable

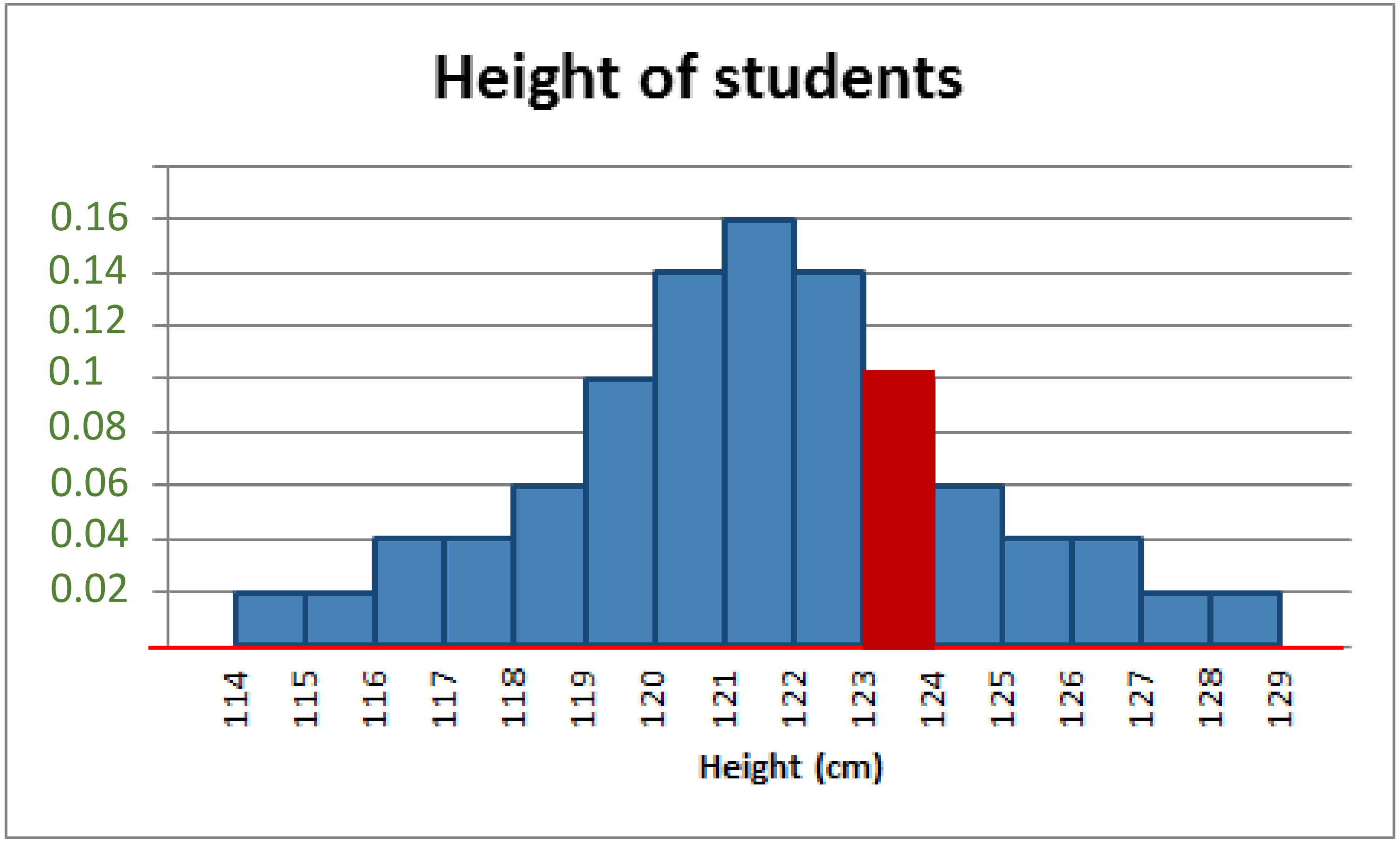


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.14 \int_{122}^{123} x\, dx =$$
$$= 0.14 \times 122.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5$

expected value of a random variable

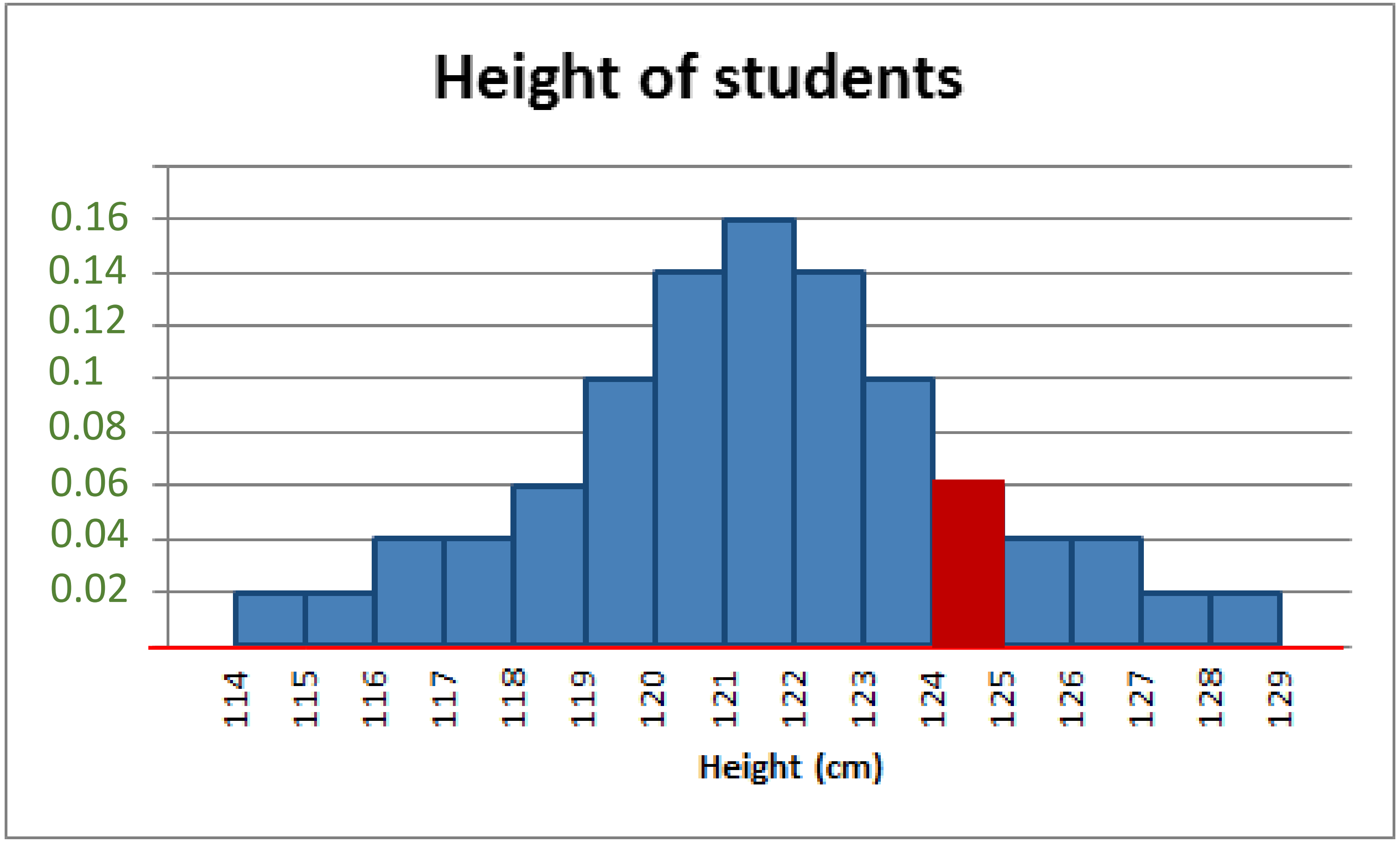


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.1 \int_{123}^{124} x\, dx = $$
$$= 0.1 \times 123.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5$

expected value of a random variable

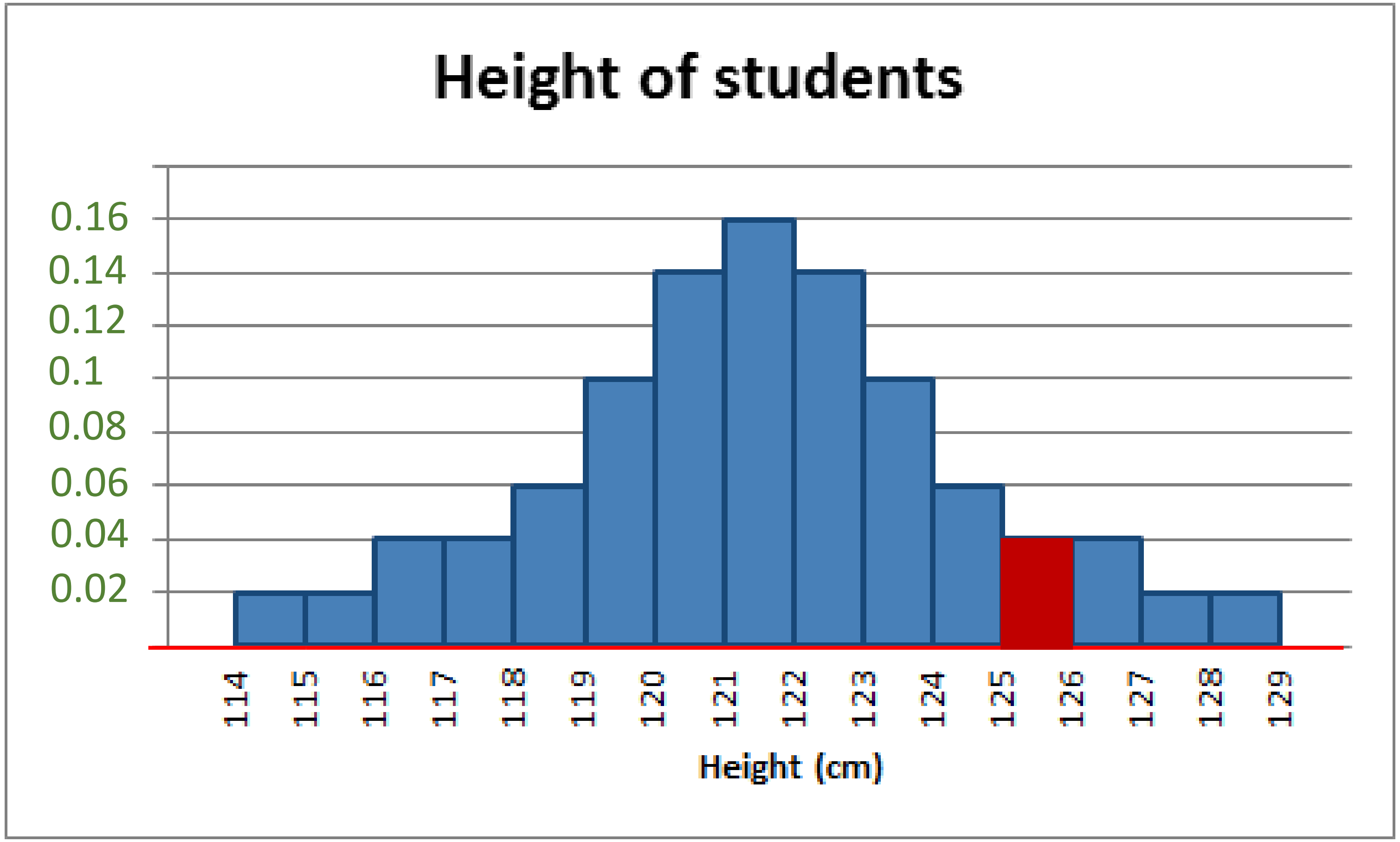


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.06 \int_{124}^{125} x\, dx =$$
$$= 0.06 \times 124.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5$

expected value of a random variable

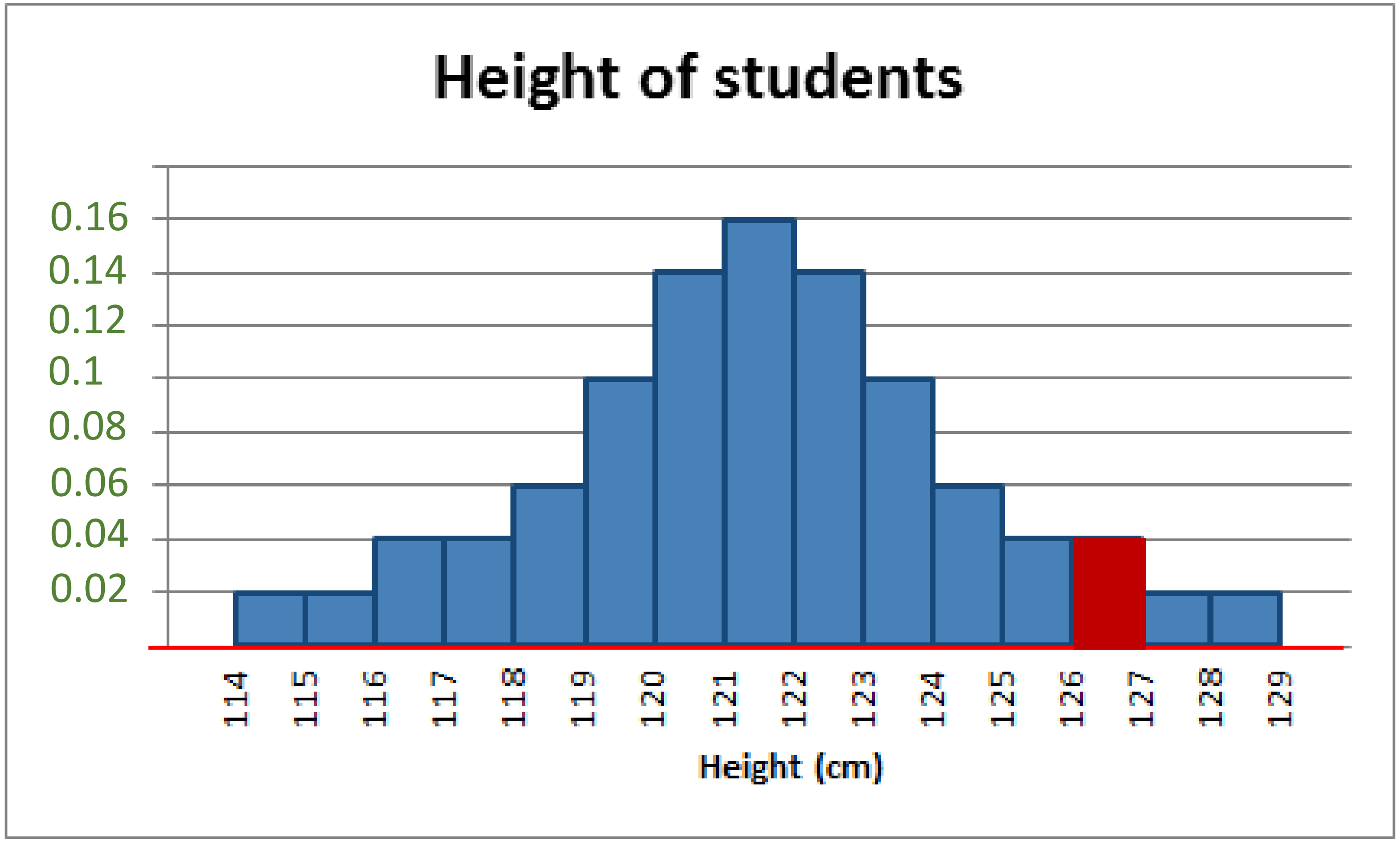


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.04 \int_{125}^{126} x\, dx = $$
$$= 0.04 \times 125.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5$

expected value of a random variable

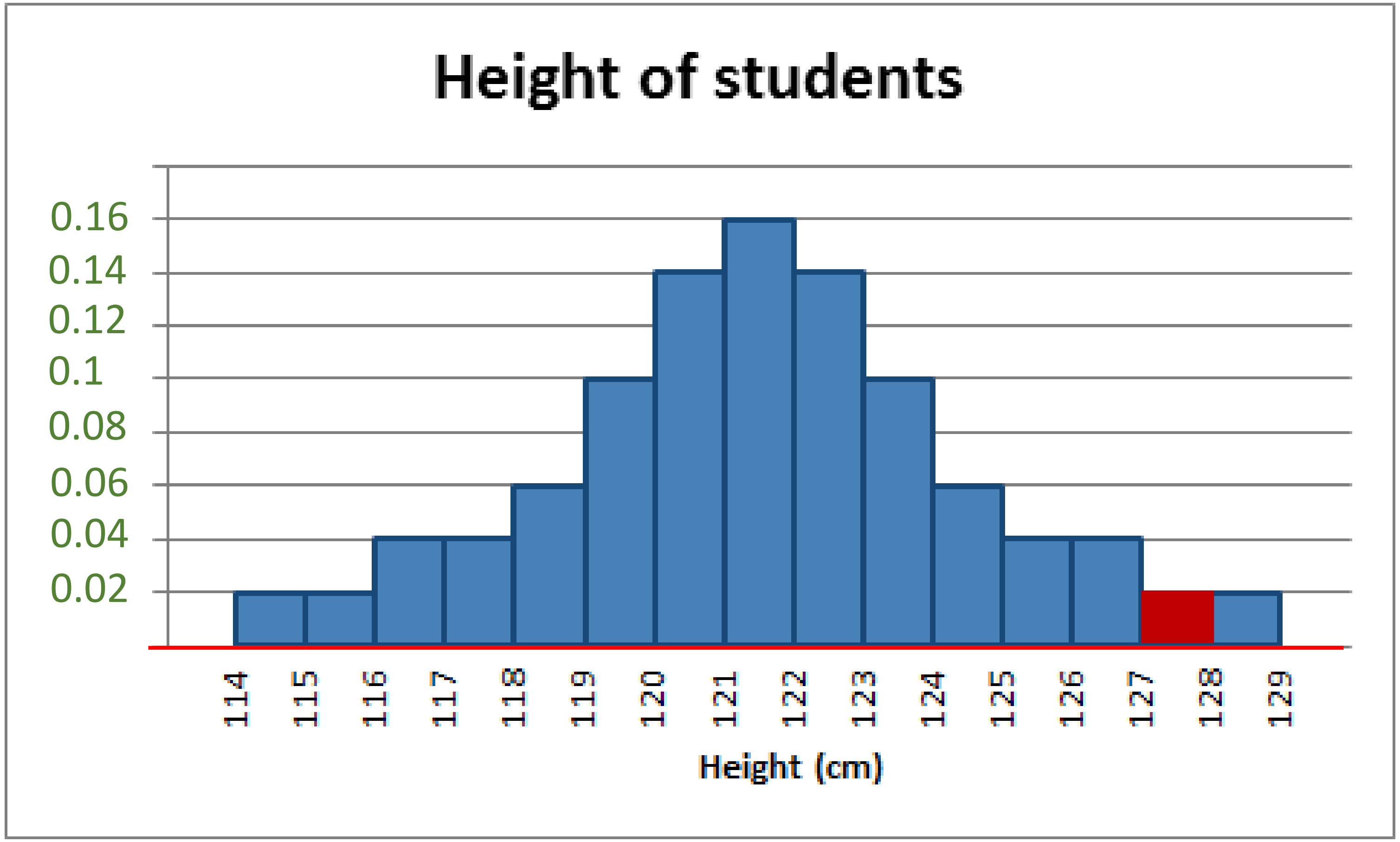


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.04 \int_{126}^{125} x\, dx =$$
$$= 0.04 \times 126.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5 + 0.04 \times 126.5$

expected value of a random variable

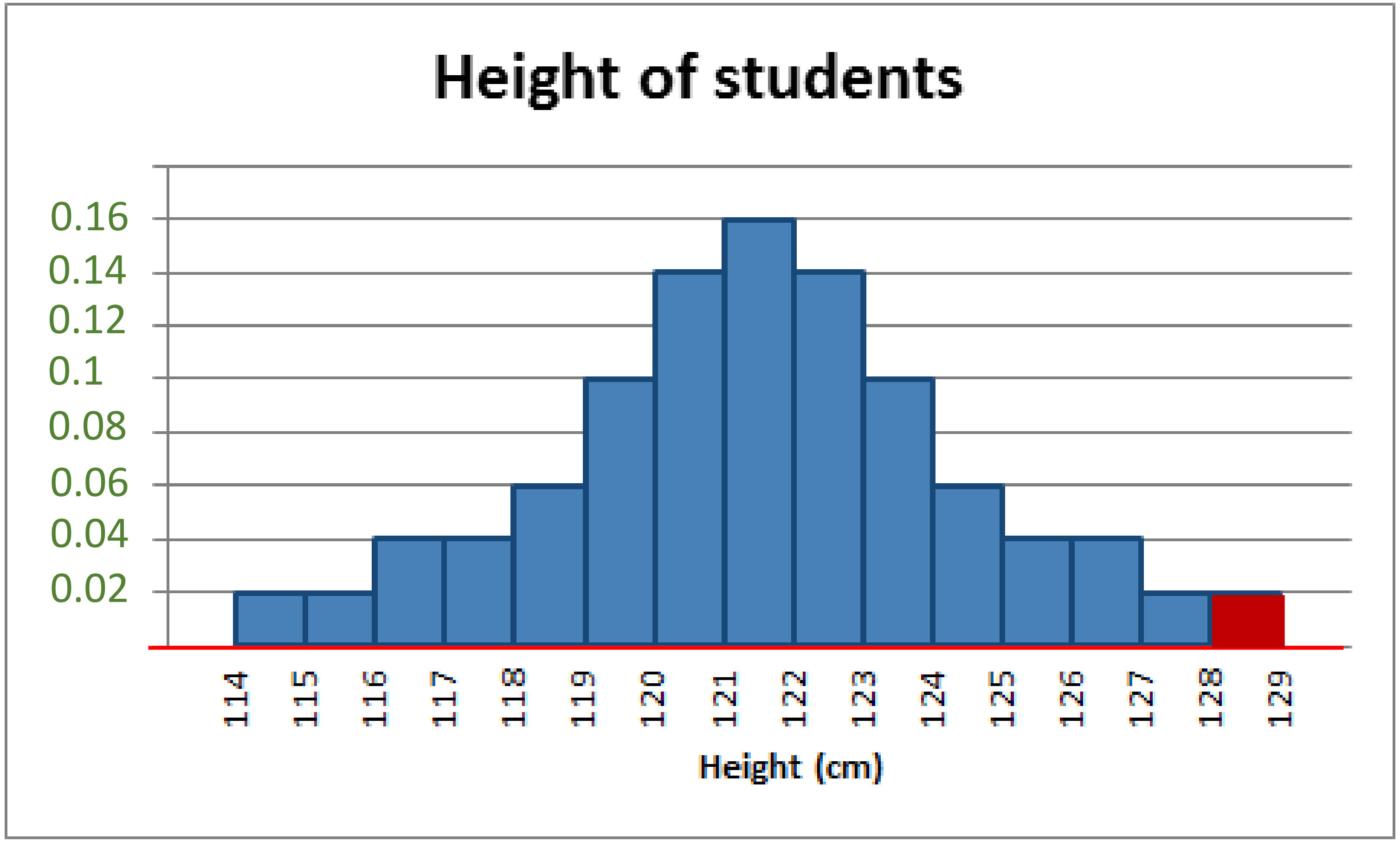


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.02 \int_{127}^{128} x\, dx = $$
$$= 0.02 \times 127.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5 + 0.04 \times 126.5 + 0.02 \times 127.5$

expected value of a random variable

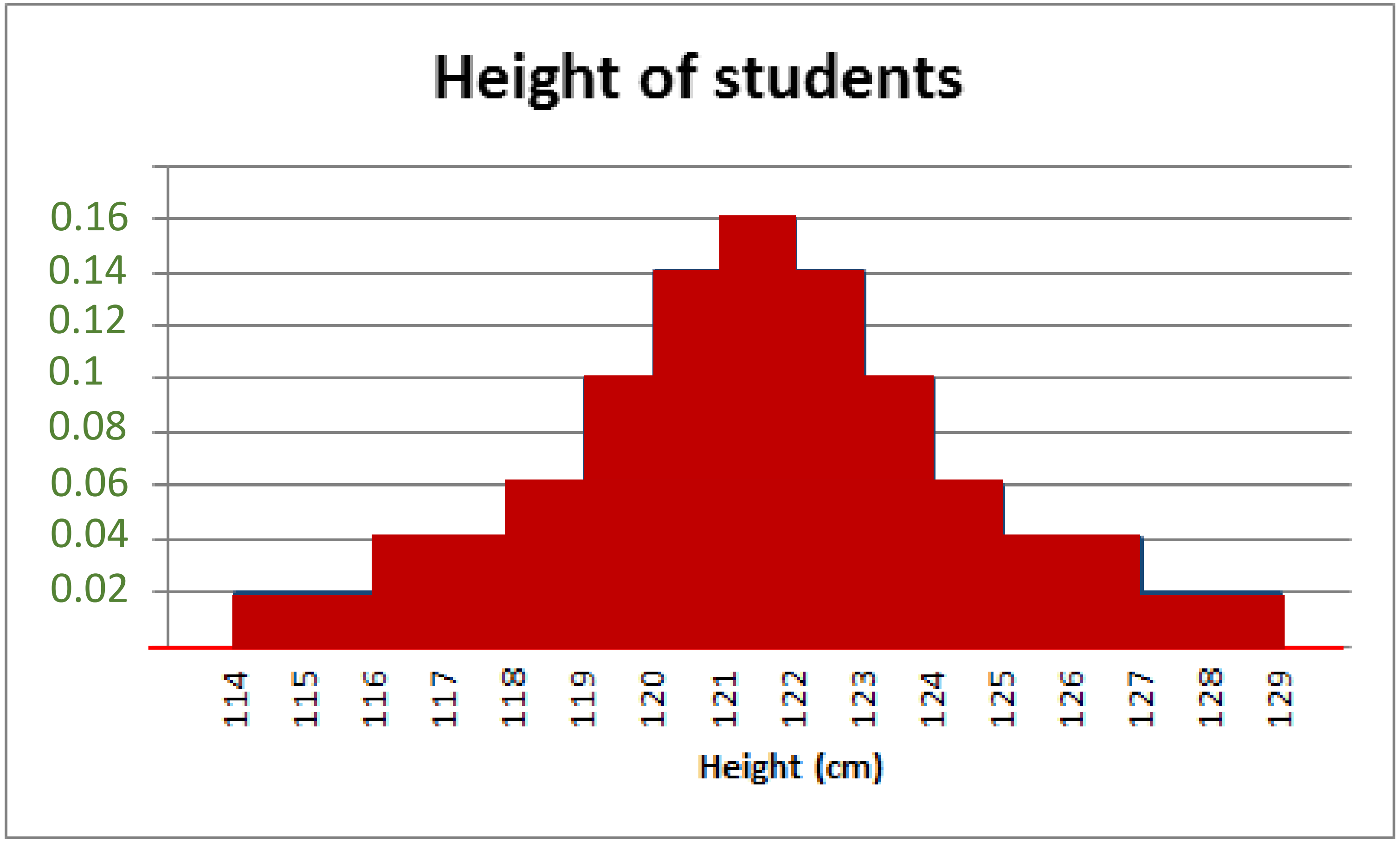


$$\int_{-\infty}^{\infty} x f(x)\, dx$$

$$0.02 \int_{128}^{129} x\, dx = \\ = 0.02 \times 128.5$$

$0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5 + 0.04 \times 126.5 + 0.02 \times 127.5 + 0.02 \times 128.5$

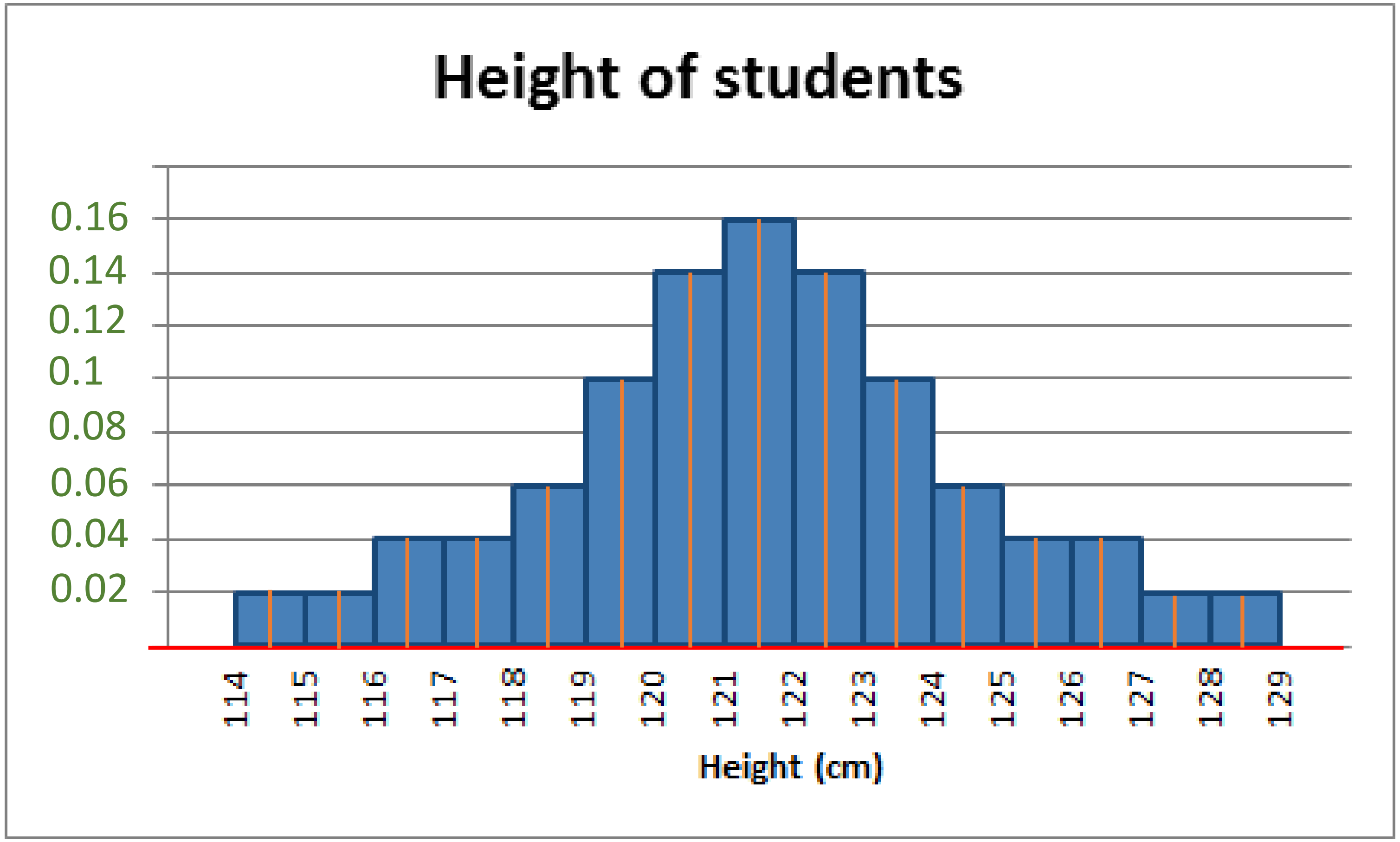
expected value of a random variable



$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$E[X] = 0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 + \\ + 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5 + 0.04 \times 126.5 + 0.02 \times 127.5 + 0.02 \times 128.5 = 121.5$$

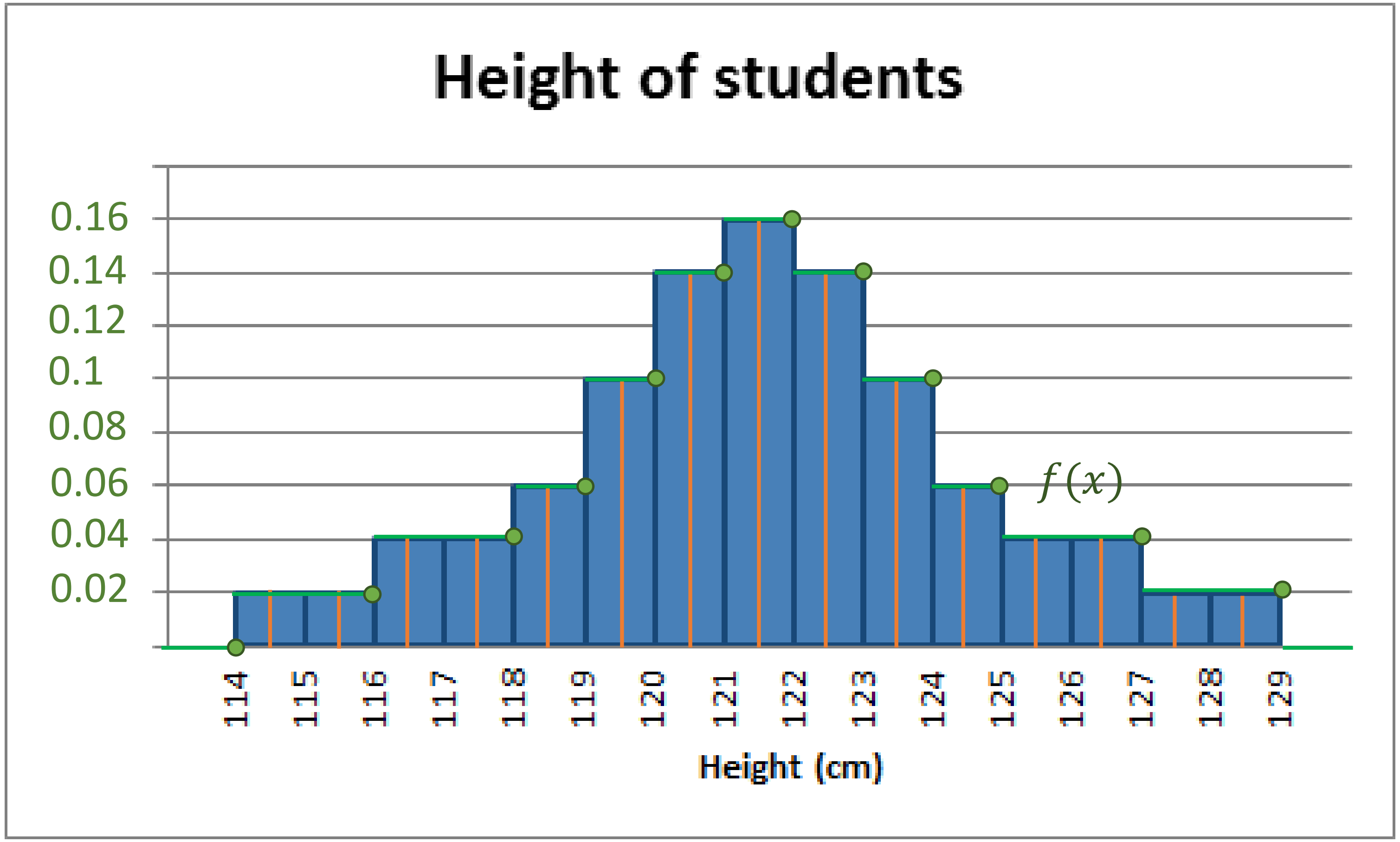
expected value of a random variable



$$E[g(X)] = \sum_x f(x)g(x)$$

$E[X] = 0.02 \times 114.5 + 0.02 \times 115.5 + 0.04 \times 116.5 + 0.04 \times 117.5 + 0.06 \times 118.5 + 0.1 \times 119.5 + 0.14 \times 120.5 + 0.16 \times 121.5 +$
$+ 0.14 \times 122.5 + 0.1 \times 123.5 + 0.06 \times 124.5 + 0.04 \times 125.5 + 0.04 \times 126.5 + 0.02 \times 127.5 + 0.02 \times 128.5 = 121.5$

expected value of a random variable

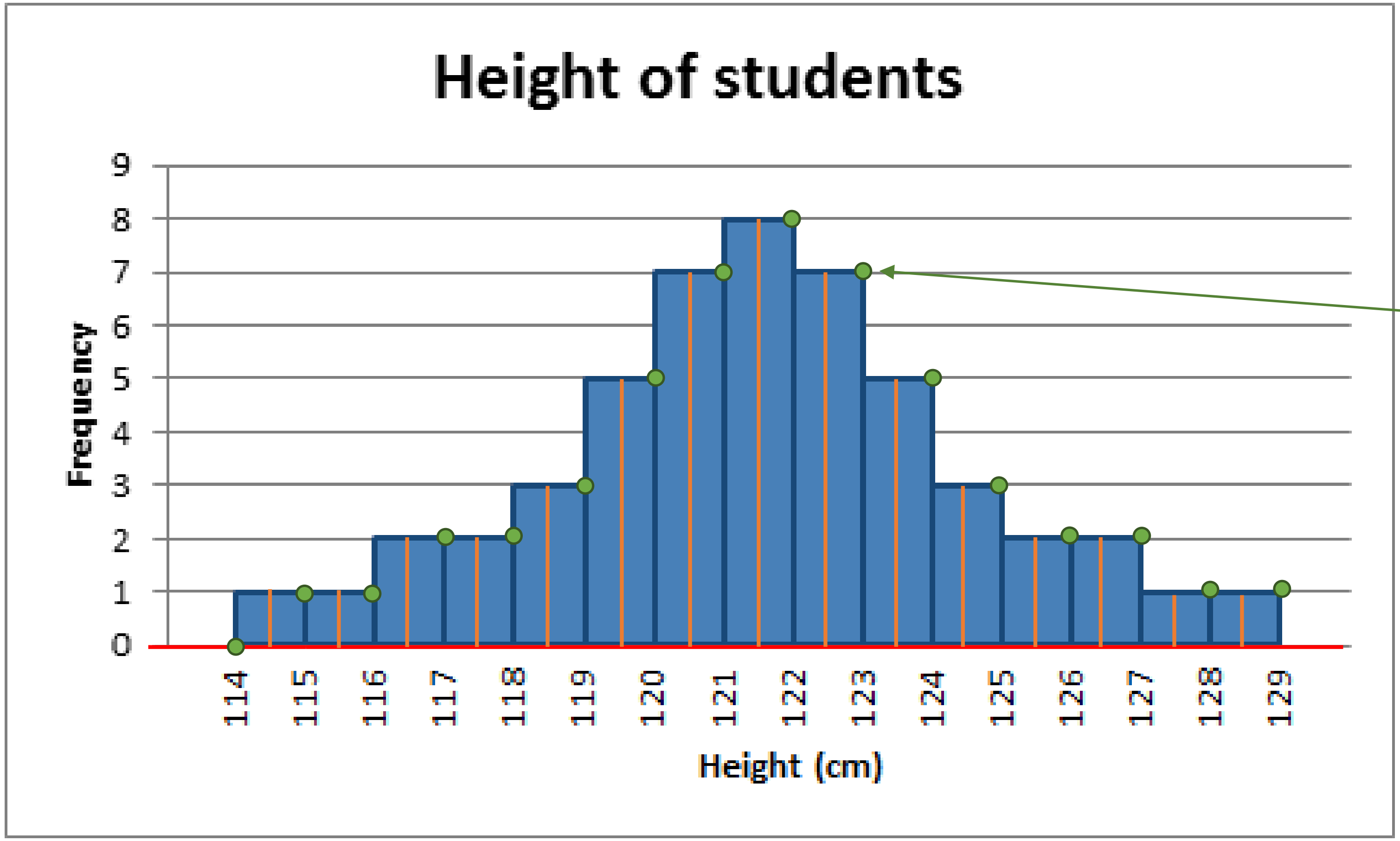


$$E[g(X)] = \sum_x f(x)g(x)$$

$$g(x) = x - \frac{1}{2}$$

$$E[X] = \frac{1}{50} \times 114.5 + \frac{1}{50} \times 115.5 + \frac{2}{50} \times 116.5 + \frac{2}{50} \times 117.5 + \frac{3}{50} \times 118.5 + \frac{5}{50} \times 119.5 + \frac{7}{50} \times 120.5 + \frac{8}{50} \times 121.5 +$$
$$+ \frac{7}{50} \times 122.5 + \frac{5}{50} \times 123.5 + \frac{3}{50} \times 124.5 + \frac{2}{50} \times 125.5 + \frac{2}{50} \times 126.5 + \frac{1}{50} \times 127.5 + \frac{1}{50} \times 128.5 = 121.5$$

expected value of a random variable



$$E[g(X)] = \sum_x f(x)g(x)$$

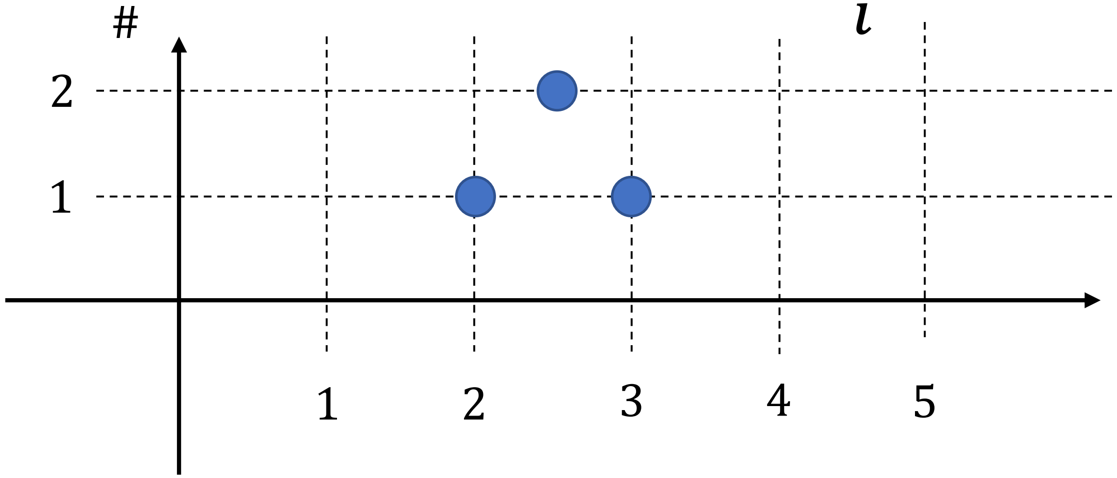
$$g(x) = x - \frac{1}{2}$$

$$f(x) = \frac{n_x}{\sum_j n_j}$$

$E[X] = \frac{1}{50}(1 \times 114.5 + 1 \times 115.5 + 2 \times 116.5 + 2 \times 117.5 + 3 \times 118.5 + 5 \times 119.5 + 7 \times 120.5 + 8 \times 121.5 +$
$+7 \times 122.5 + 5 \times 123.5 + 3 \times 124.5 + 2 \times 125.5 + 2 \times 126.5 + 1 \times 127.5 + 1 \times 128.5) = 121.5$

variance of a random variable

$$E[X] = \frac{1}{N}\sum_i n_i x_i$$

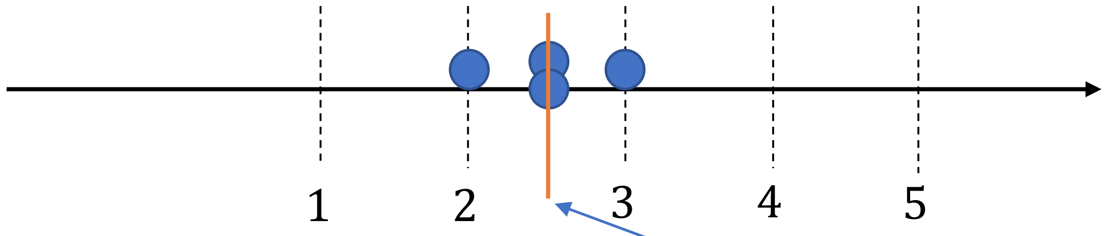


i	n_i	x_i
1	1	2
2	2	2.5
3	1	3

$$E[X] = \frac{1}{4}(2 + 2 \times 2.5 + 3)$$

variance of a random variable

$$E[X] = \frac{1}{N}\sum_i n_i x_i$$

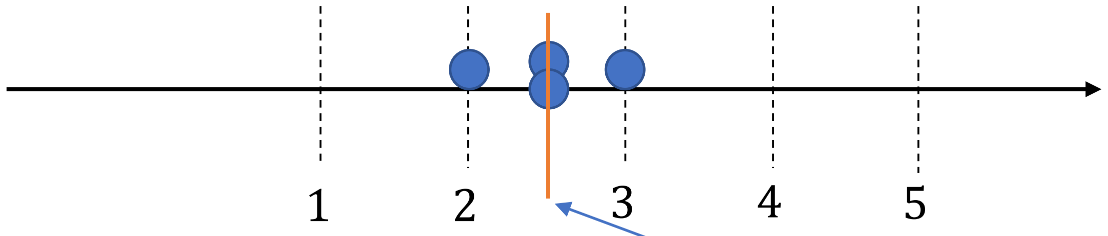


$$E[X] = \frac{1}{4}(2 + 2 \times 2.5 + 3) = 2.5$$

$$E[(X - E[X])] = \frac{1}{4}(2 - 2.5 + 2 \times (2.5 - 2.5) + 3 - 2.5)$$

variance of a random variable

$$E[X] = \frac{1}{N}\sum_i n_i x_i$$

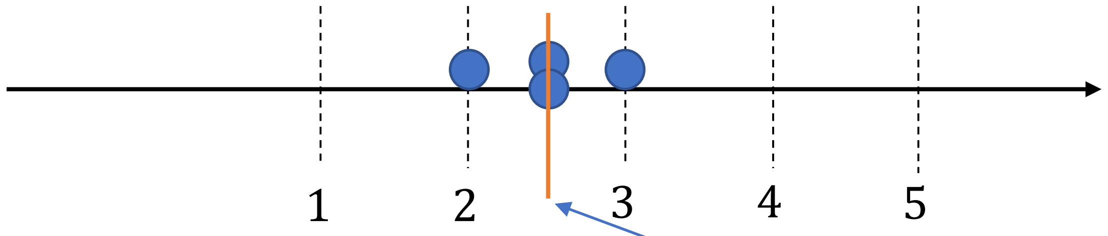


$$E[X] = \frac{1}{4}(2 + 2 \times 2.5 + 3) = 2.5$$

$$E[(X - E[X])^2] = \frac{1}{4}(2 - 2.5 + 2 \times (2.5 - 2.5) + 3 - 2.5)$$

variance of a random variable

$$E[X] = \frac{1}{N}\sum_i n_i x_i$$



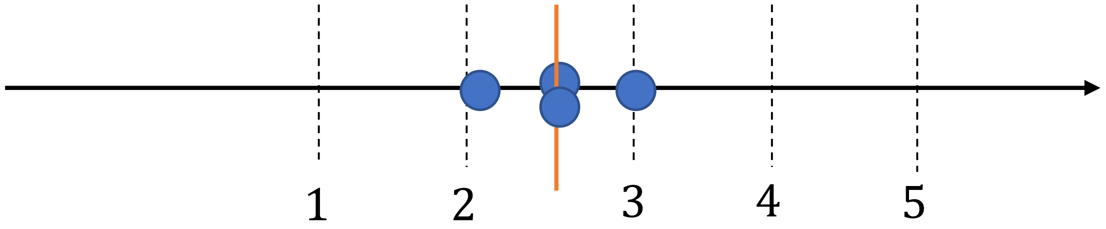
$$E[X] = \frac{1}{4}(2 + 2 \times 2.5 + 3) = 2.5$$

$$E[(X - E[X])^2] = \frac{1}{4}((2 - 2.5)^2 + 2 \times (2.5 - 2.5)^2 + (3 - 2.5)^2) = 0.125$$

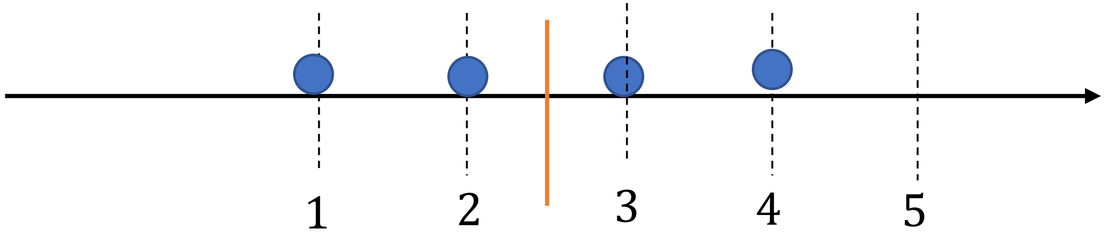
variance of a random variable

$$Var(X) = E[(X - E[X])^2]$$

$$Var(X) = E[X^2] - E[X]^2$$



$$Var(X) = \frac{1}{4}((2-2.5)^2 + 2 \times (2.5-2.5)^2 + (3-2.5)^2) = 0.125$$

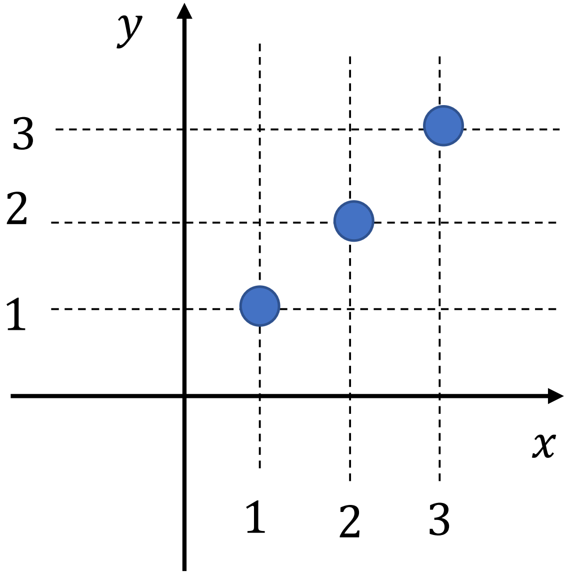


$$Var(X) = \frac{1}{4}((1-2.5)^2 + (2-2.5)^2 + (3-2.5)^2 + (4-2.5)^2) = 1.25$$

covariance of two random variables

$$Var(X) = Cov(X, X)$$

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$$



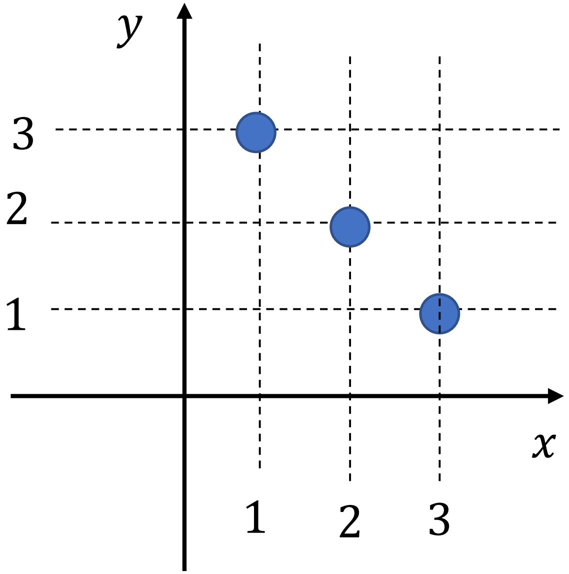
$$Var(X) = Cov(X, X) =$$

$$= \frac{1}{3}((1-2)^2 + (2-2)^2 + (3-2)^2) = +\frac{2}{3}$$

covariance of two random variables

$$Var(X) = Cov(X,X)$$

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$$



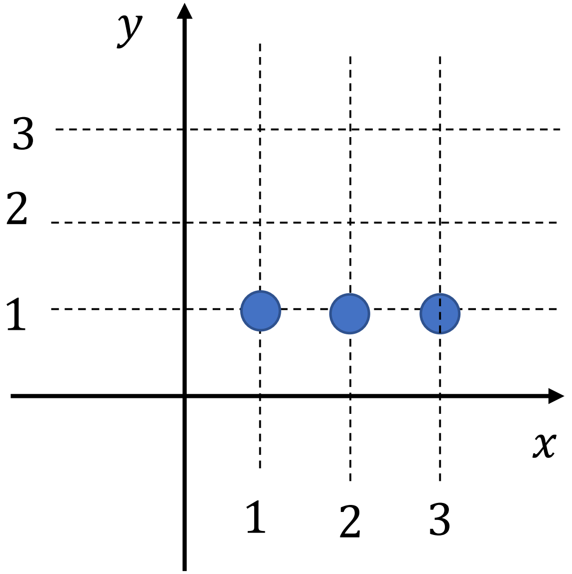
$$Cov(X,Y) =$$

$$= \frac{1}{3}\left(\underset{-}{(1-2)}\underset{+}{(3-2)} + (2-2)^2 + \underset{+}{(3-2)}\underset{-}{(1-2)}\right) = -\frac{2}{3}$$

covariance of two random variables

$$Var(X) = Cov(X, X)$$

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$$



$$Cov(X,Y) =$$

$$= \frac{1}{3}((1-2) + (2-2) + (3-2))(1-1)) = 0$$

covariance matrix

$$\mathbf{X} = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}$$

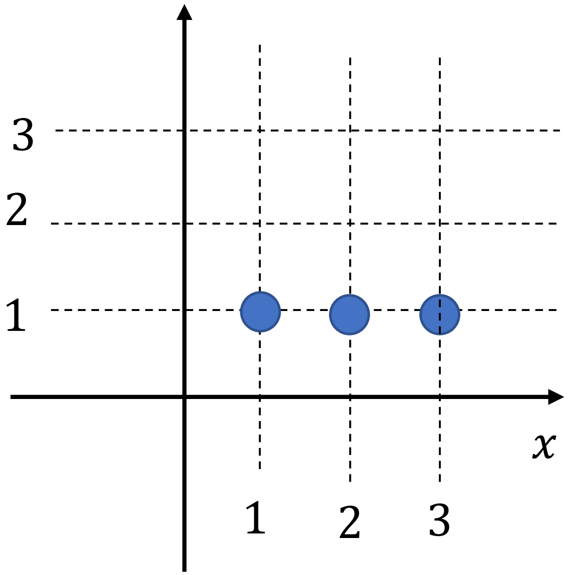
$$\begin{bmatrix} E[(X_1 - E[X_1])(X_1 - E[X_1])] & E[(X_1 - E[X_1])(X_2 - E[X_2])] & \cdots & E[(X_1 - E[X_1])(X_n - E[X_n])] \\ E[(X_2 - E[X_2])(X_1 - E[X_1])] & E[(X_2 - E[X_2])(X_2 - E[X_2])] & \cdots & E[(X_2 - E[X_2])(X_n - E[X_n])] \\ \vdots & \vdots & \ddots & \vdots \\ E[(X_n - E[X_n])(X_1 - E[X_1])] & E[(X_n - E[X_n])(X_2 - E[X_2])] & \cdots & E[(X_n - E[X_n])(X_n - E[X_n])] \end{bmatrix}$$

covariance matrix

$$\mathbf{x} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\begin{bmatrix} Cov(X,X) & Cov(X,Y) \\ Cov(Y,X) & Cov(Y,Y) \end{bmatrix}$$

covariance matrix



$$Cov(X,Y) = Cov(Y,X) =$$

$$= \frac{1}{3}((1-2)+(2-2)+(3-2))(1-1)) = 0$$

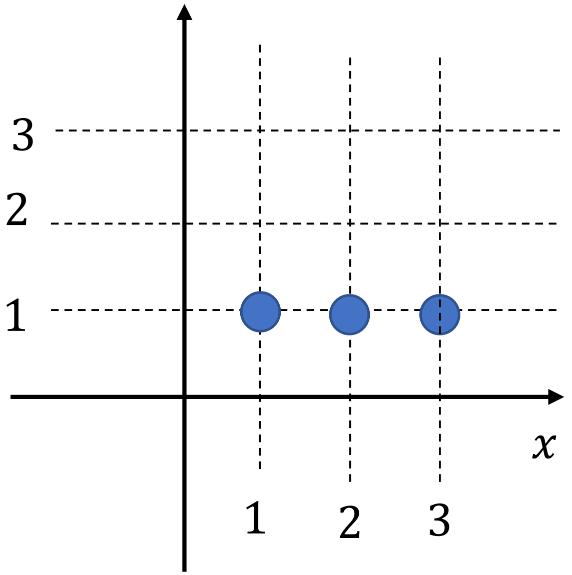
$$Cov(Y,Y) =$$

$$= \frac{1}{3}((1-1)+(1-1)+(1-1))(1-1)) = 0$$

$$Cov(X,X) =$$

$$= \frac{1}{3}((1-2)^2+(2-2)^2+(3-2)^2)) = \frac{2}{3}$$

covariance matrix



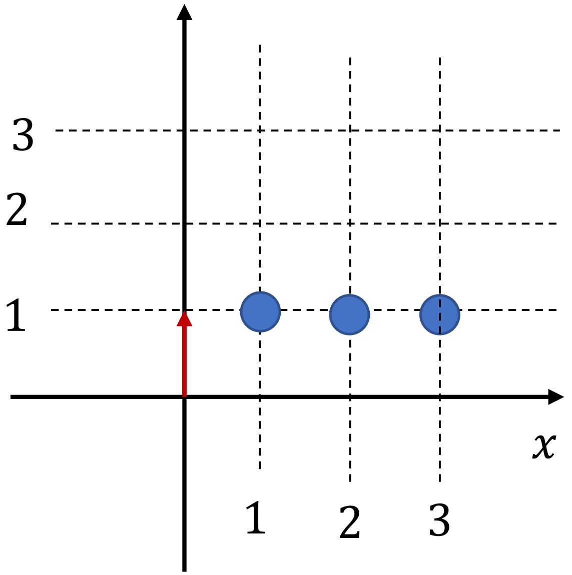
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix}$$

covariance matrix



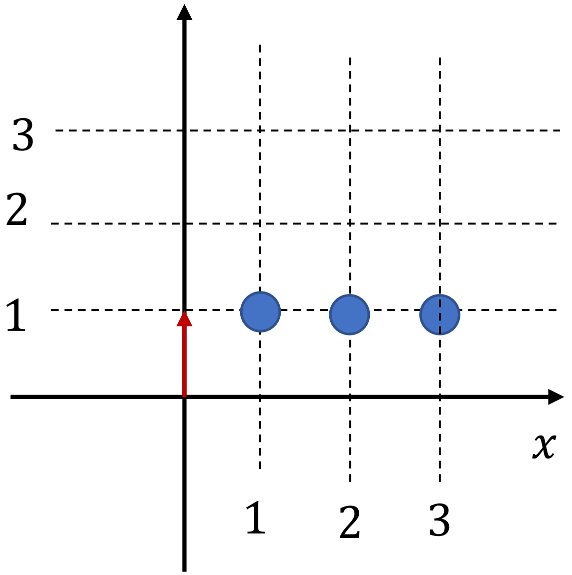
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

covariance matrix



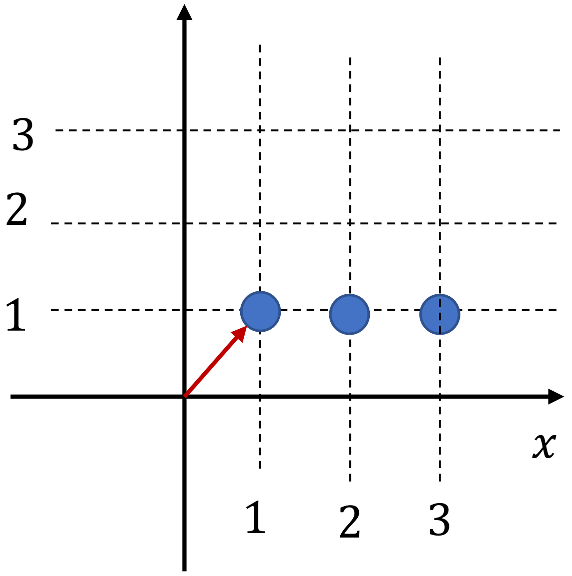
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

covariance matrix



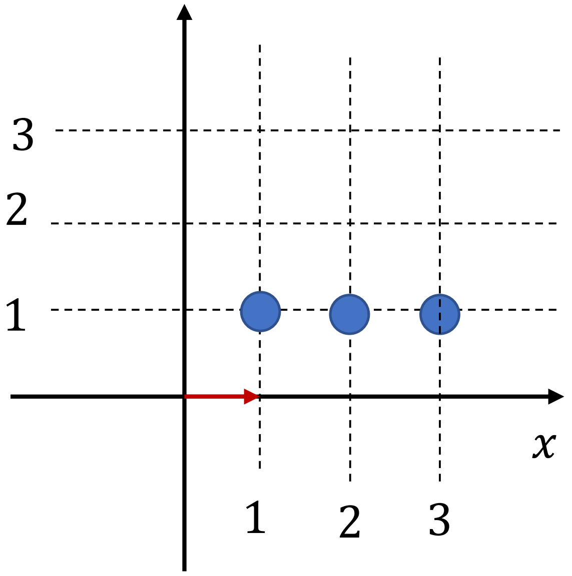
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ 0 \end{bmatrix}$$

covariance matrix



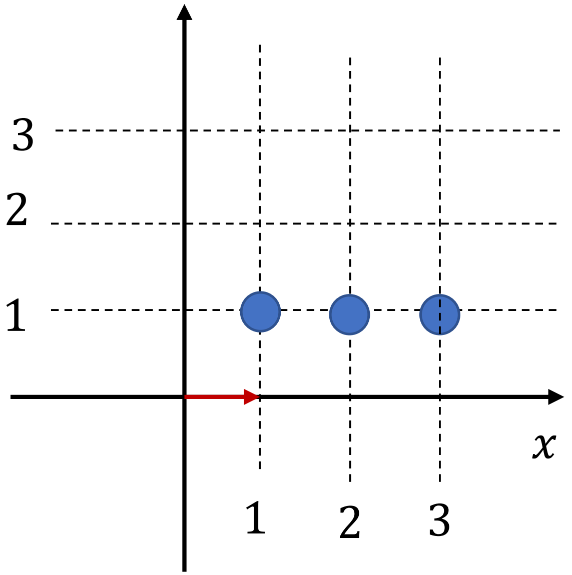
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ 0 \end{bmatrix}$$

covariance matrix



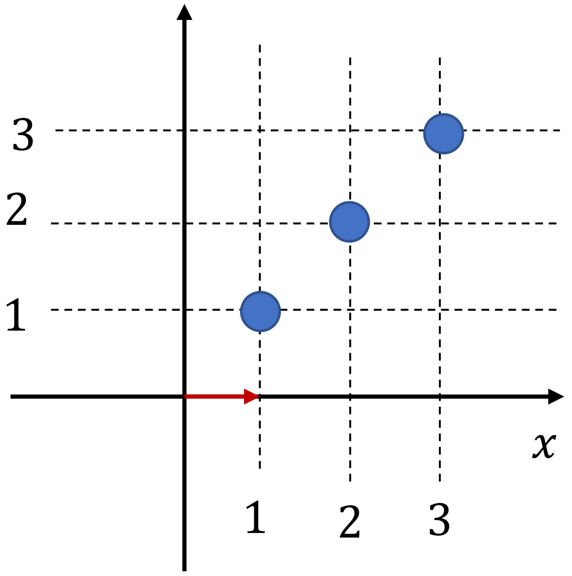
$$Cov(X,Y) = Cov(Y,X) = 0$$

$$Cov(Y,Y) = 0$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

covariance matrix



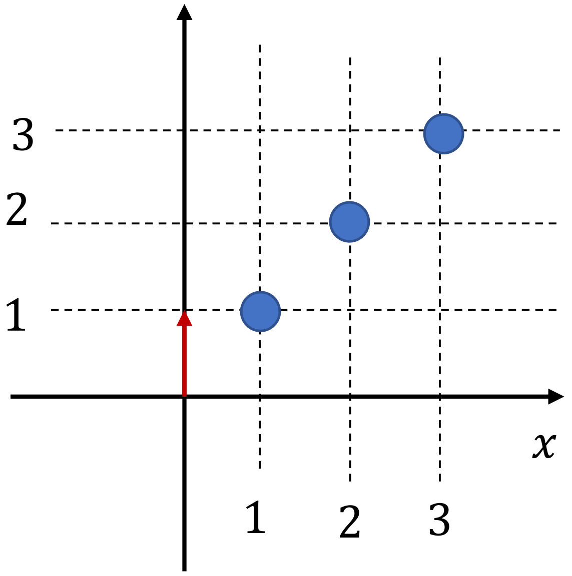
$$Cov(X,Y) = Cov(Y,X) = \frac{2}{3}$$

$$Cov(Y,Y) = \frac{2}{3}$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$$

covariance matrix



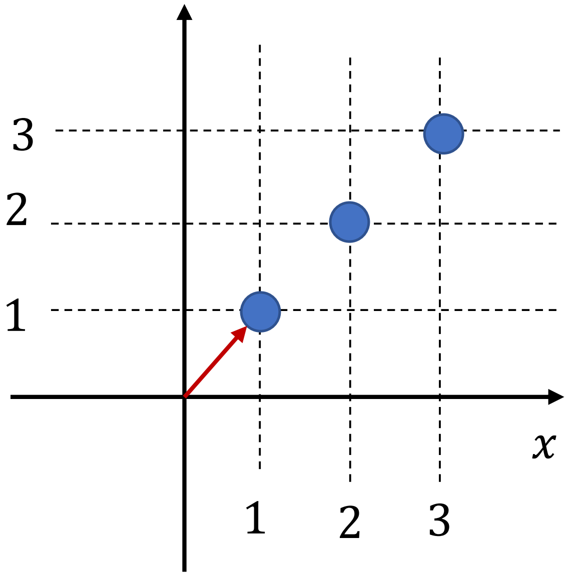
$$Cov(X,Y) = Cov(Y,X) = \frac{2}{3}$$

$$Cov(Y,Y) = \frac{2}{3}$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$$

covariance matrix



$$Cov(X,Y) = Cov(Y,X) = \frac{2}{3}$$

$$Cov(Y,Y) = \frac{2}{3}$$

$$Cov(X,X) = \frac{2}{3}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigenvector vector of a linear transformation

$$K\boldsymbol{v} = \lambda\boldsymbol{v}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

eigenvector vector of a linear transformation

$$K\boldsymbol{v} = \lambda\boldsymbol{v}$$

eigenvector eigenvalue

$$K - \lambda\boldsymbol{I} = \boldsymbol{M}$$

$$M\boldsymbol{v} = \boldsymbol{0}$$

eigenvector vector of a linear transformation

$$K\boldsymbol{v} = \lambda\boldsymbol{v}$$

eigenvector eigenvalue

$$K - \lambda\boldsymbol{I} = \boldsymbol{M}$$

$$M\boldsymbol{v} = \boldsymbol{0} \quad \Leftrightarrow \quad |M| = \boldsymbol{0}$$

determinant

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$K - \lambda \boldsymbol{I} = \boldsymbol{M}$$

$$|M| = \left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right|$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$\left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix}$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0 \qquad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$(\lambda - 1)(\lambda - 3) = 0$$

$$\lambda = 1; \lambda = 3$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$\lambda = 1: \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$x \in \mathbb{R}; x \neq 0$

$\lambda = 1:$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \quad v_{\lambda=1} = \begin{bmatrix} x \\ -x \end{bmatrix}$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$$\lambda = 3: \quad \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

eigenvector vector of a linear transformation

$$K = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|K - \lambda \boldsymbol{I}| = 0$$

$x \in \mathbb{R}; x \neq 0$

$$\lambda = 3: \quad \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \quad v_{\lambda=3} = \begin{bmatrix} x \\ x \end{bmatrix}$$