Homework 1

Let us assume the right-handed global coordinate system as depicted in the Fig. 1. There is a camera located in the origin looking in the direction of the x axis. The camera will subsequently rotate around the y axis (see Fig. 1). Using the Catmull-Rom interpolation, we will rotate[1] the camera around the y axis as a function of the parameter $t \in [0, 1]$. For $t = 0$ the camera is looking in the direction of the x axis (Fig. 1 a)), for $t = 1$ the camera is looking in the direction of the z axis (Fig. 1 b)). Rotations in the 3D space will be represented by the quaternions q_0, q_1, q_2 and q_3. The quaternion $q0 = q1$ will represents the camera rotation from its initial orientation into the orientation[2] where the camera view direction is in the direction of the x axis (Fig. 1 a)). Quaternion q_2 will represent an clockwise rotation from the initial orientation into the orientation where the camera view direction is in the direction of the z axis (Fig. 1 b)). Quaternion q_3 will represent an clockwise rotation from the initial orientation into the orientation where the camera view direction is in the negative direction of the x axis (Fig. 1 c)).

Using the Catmull-Rom method interpolate the quaternions[3] q_0, q_1, q_2 and q_3 according to the particular parameter t and then calculate the normalized camera view direction vector rotated into the resulting orientation. Define the parameter t as $t = \frac{1}{d+m}$, where m is the number of the month in your birthday date, while d is the day number.

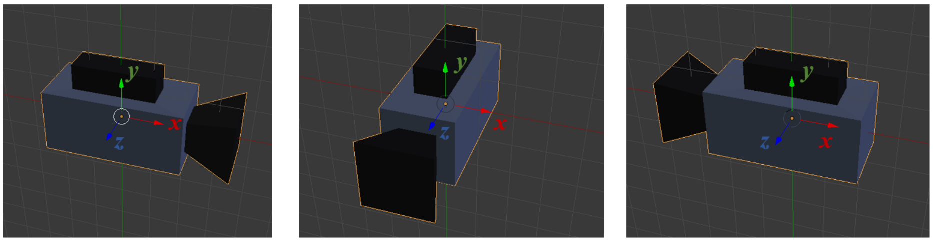
a) Express defined camera rotations (Fig. 1) around the y axis in the form of the unit quaternions q_0, q_1, q_2 and q_3.
b) Compute the quaternion q_t using Catmull-Rom interpolation for your parameter $t = \frac{1}{d+m}$. In each step of the computation verify, if the resulting quaternion is unit.
c) Compute the inverse of the quaternion q_t to perform the rotation of the camera view vector. Express camera view direction in initial orientation as an unit vector v.

[1] Note that we are using the right-handed coordinate system while we are rotating clockwise.

[2] Note that at the beginning we have the camera already rotated in the orientation where the camera is looking in the direction of the x axis.

[3] We can imagine normalized quaternions as a "points" on the 4D unit sphere. Catmull-Rom interpolation will compute new "points" between q_1 and q_2 on the sphere surface depending on the parameter t.

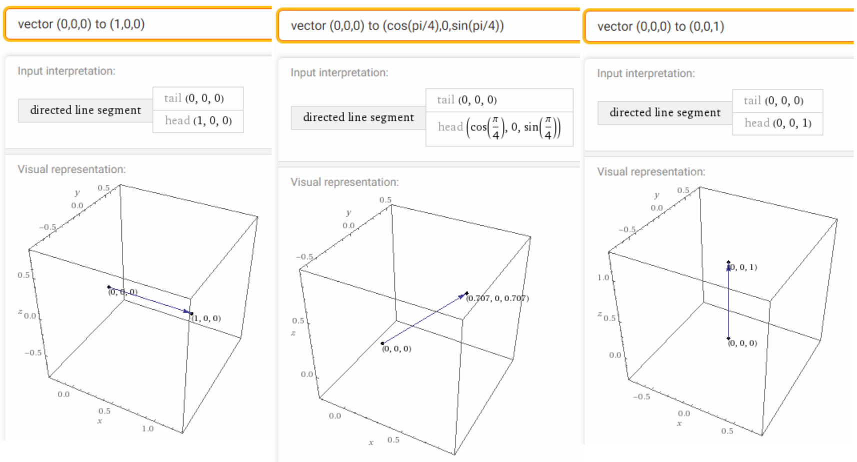
Fig. 1: The camera orientations.



a) b) c)

d) Rotate the vector v in order to obtain the view direction in the camera orientation satisfying your parameter $t = \frac{1}{d+m}$. Express rotated view direction as a unit vector v_t.
e) Plot three camera view directions for $t = 0$, $t = \frac{1}{d+m}$ and $t = 1$ (see Fig. 2).

Fig. 2: Example of three subsequent view directions satisfying camera orientations for the particular parameter t. Examples are plotted using WolframAlpha.



Explain in detail each calculation step.