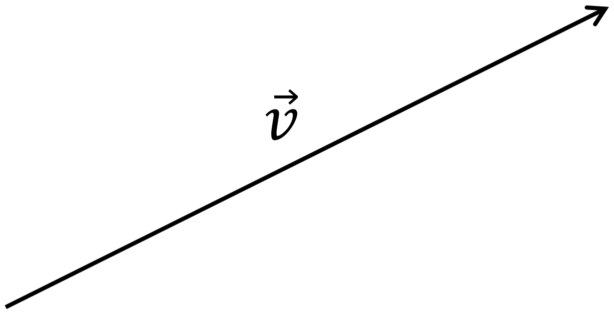
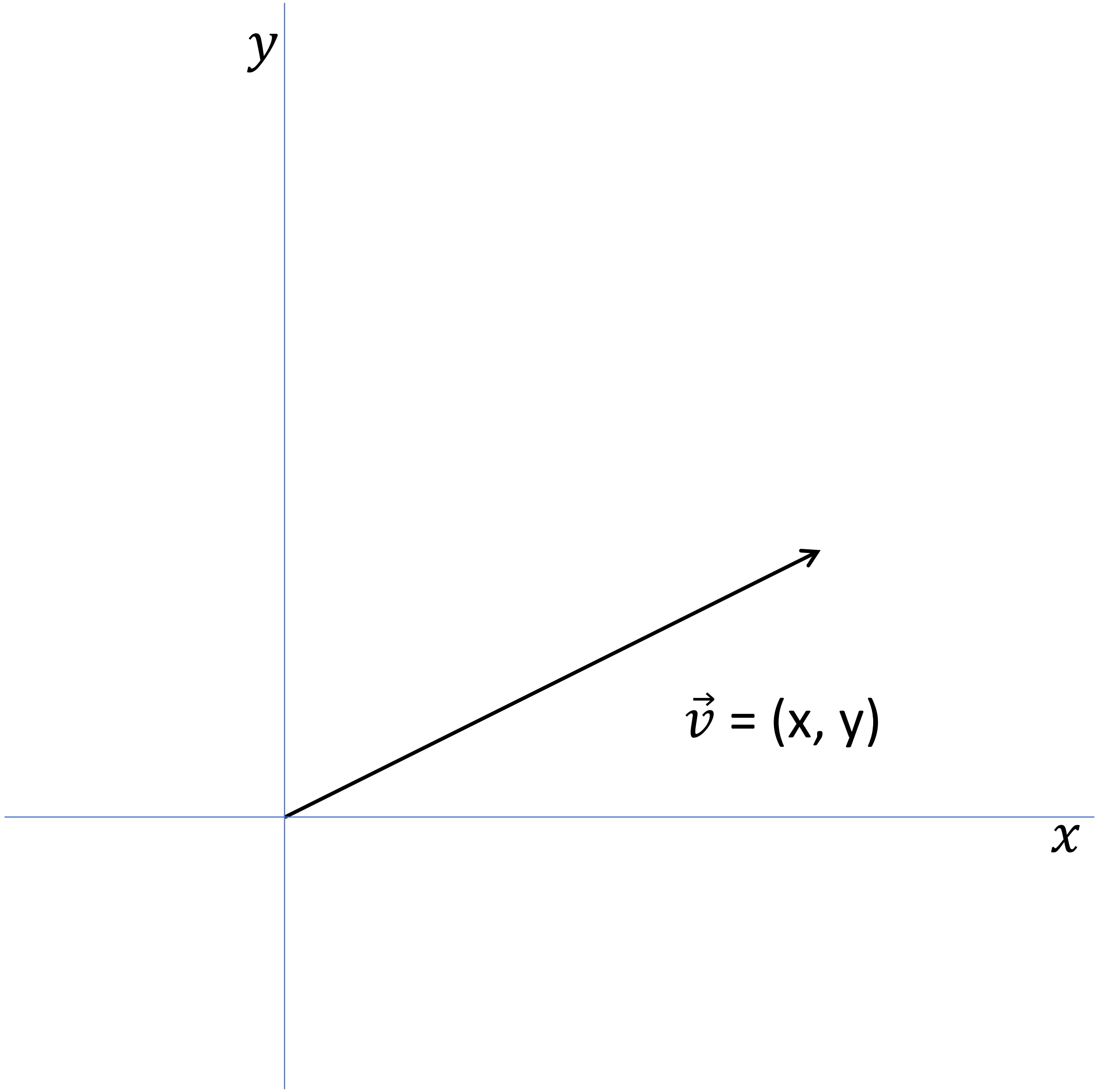
Euclidean vector

- direction
- magnitude / length



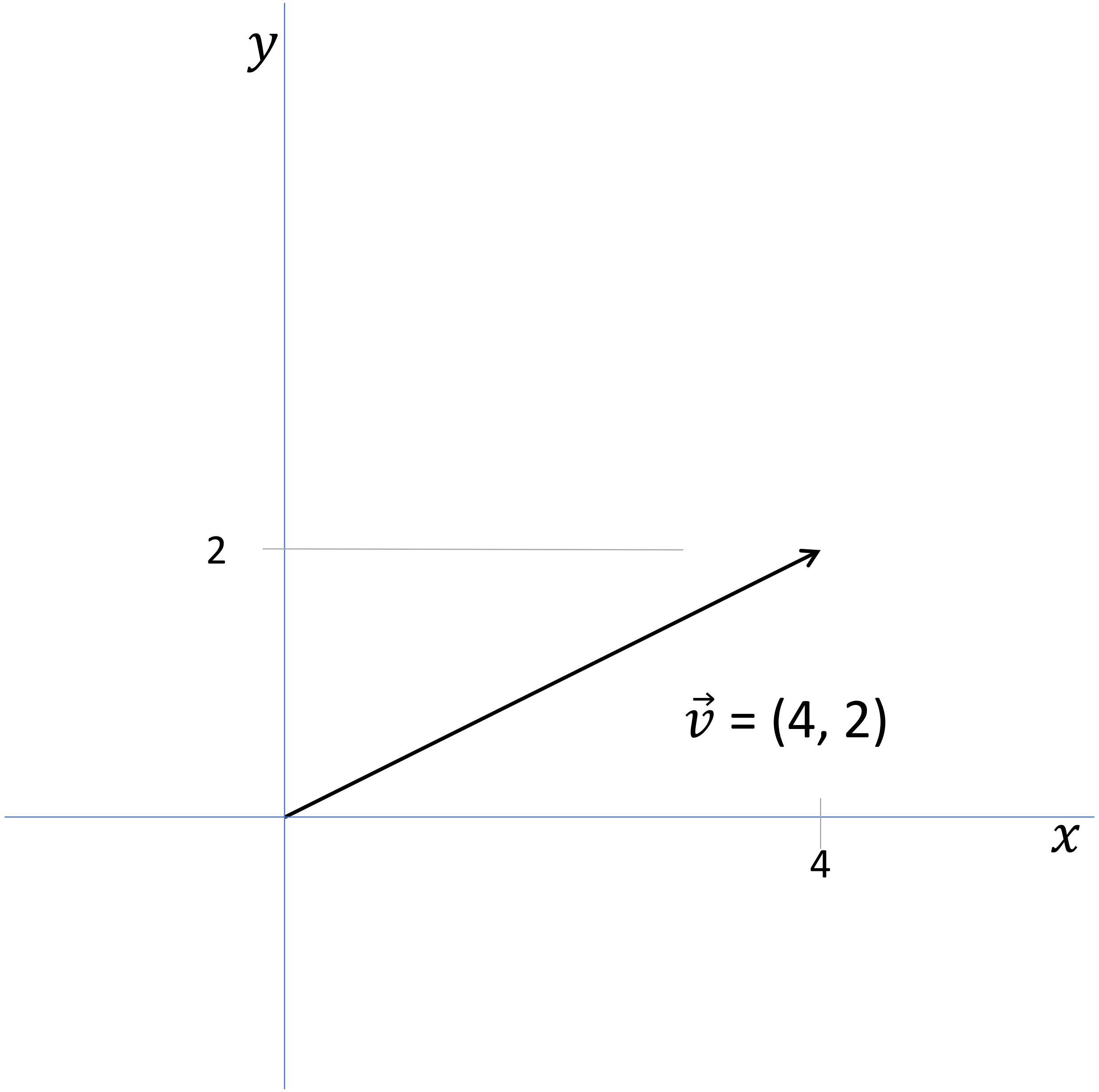
Euclidean vector

- direction
- magnitude / length



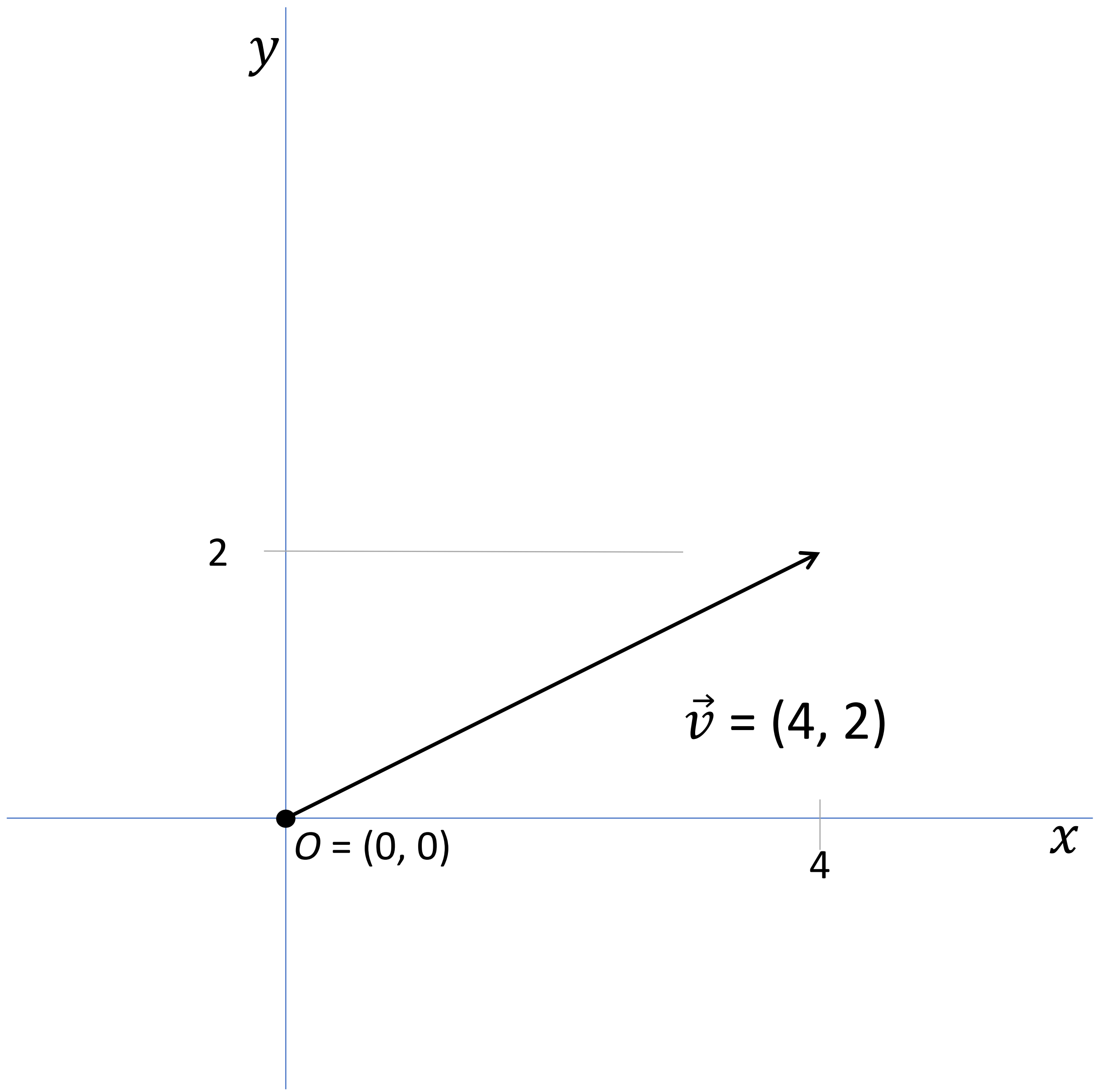
Euclidean vector

- direction
- magnitude / length



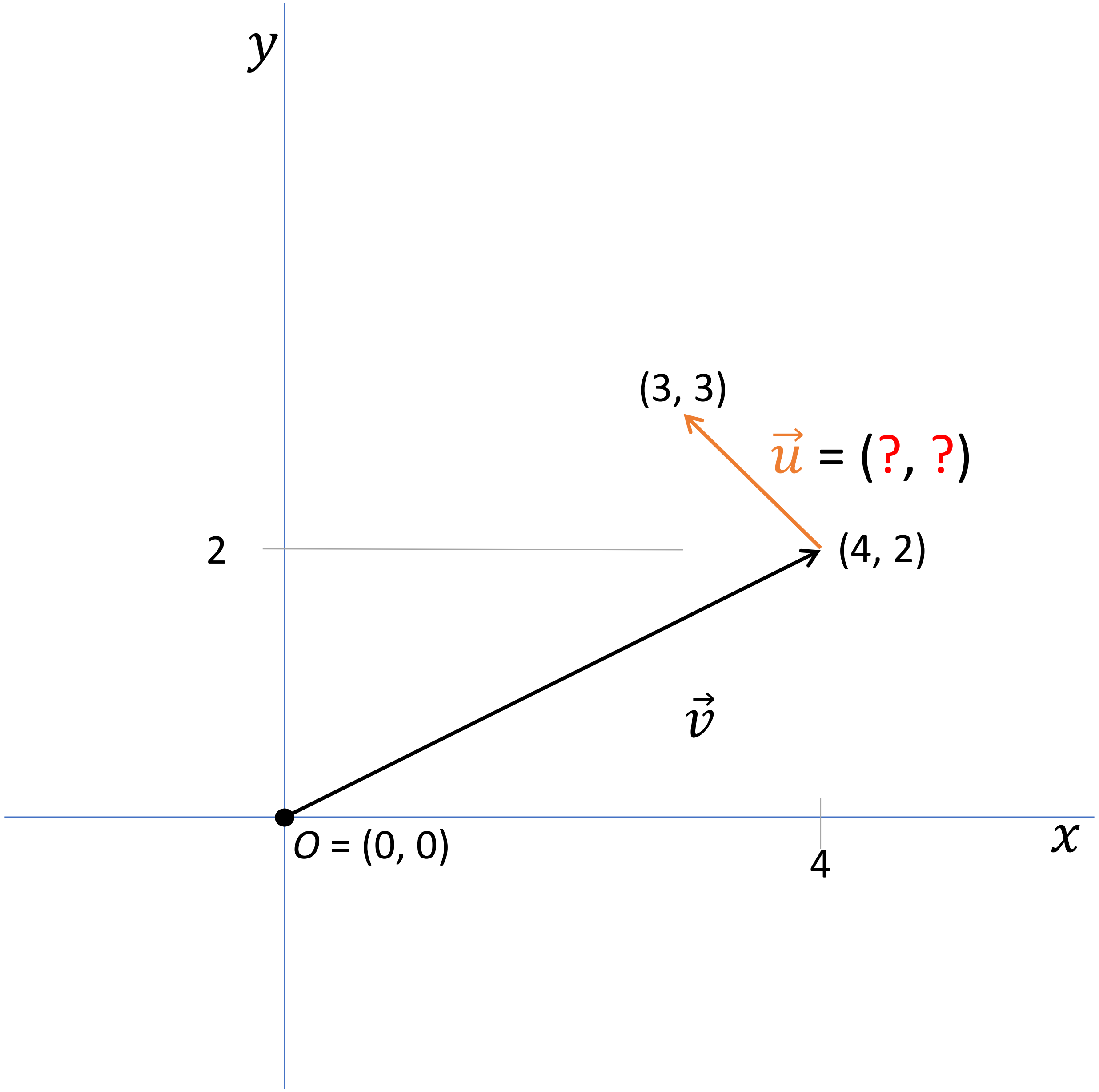
Euclidean vector

- direction
- magnitude / length



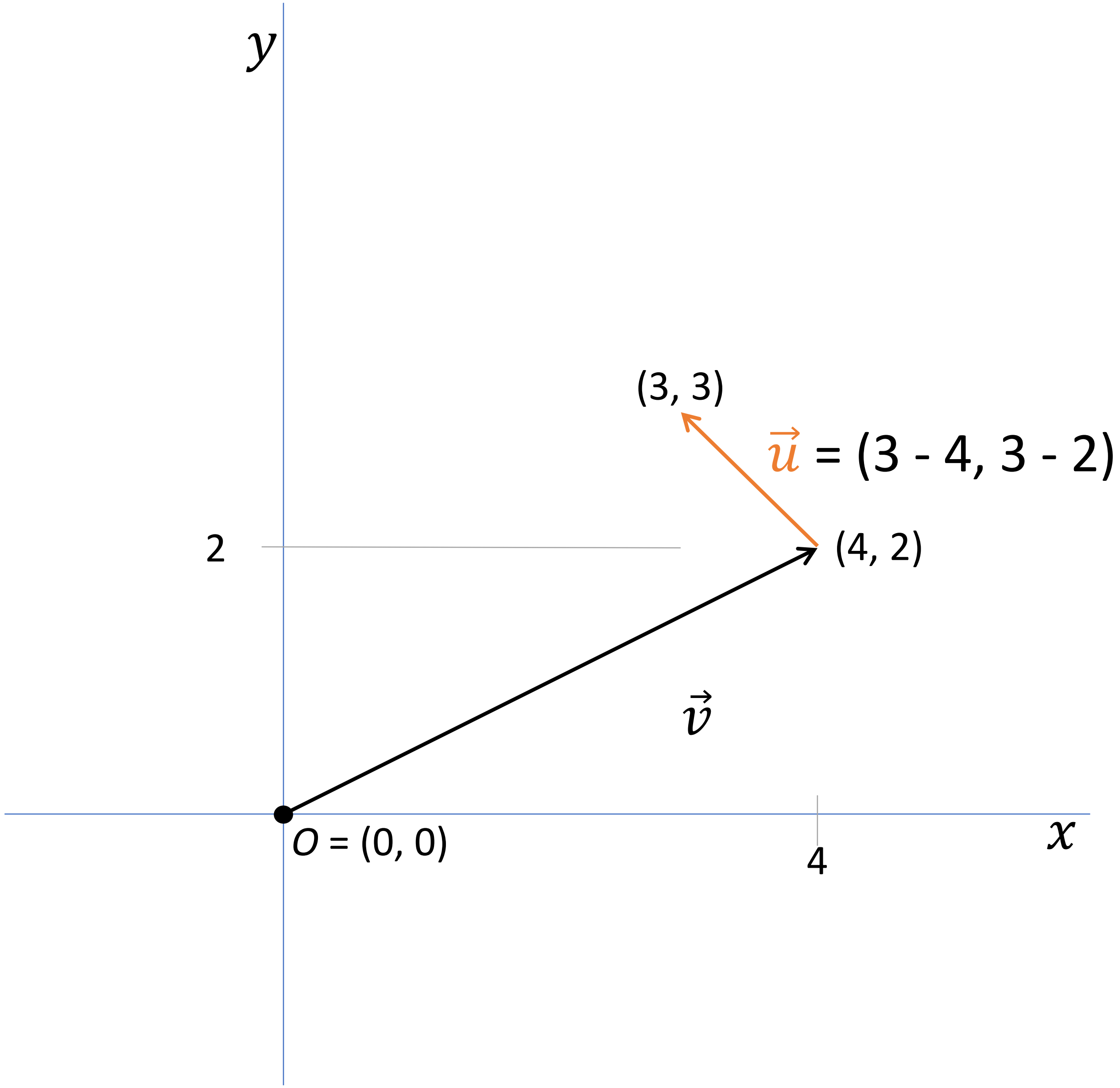
Euclidean vector

- direction
- magnitude / length



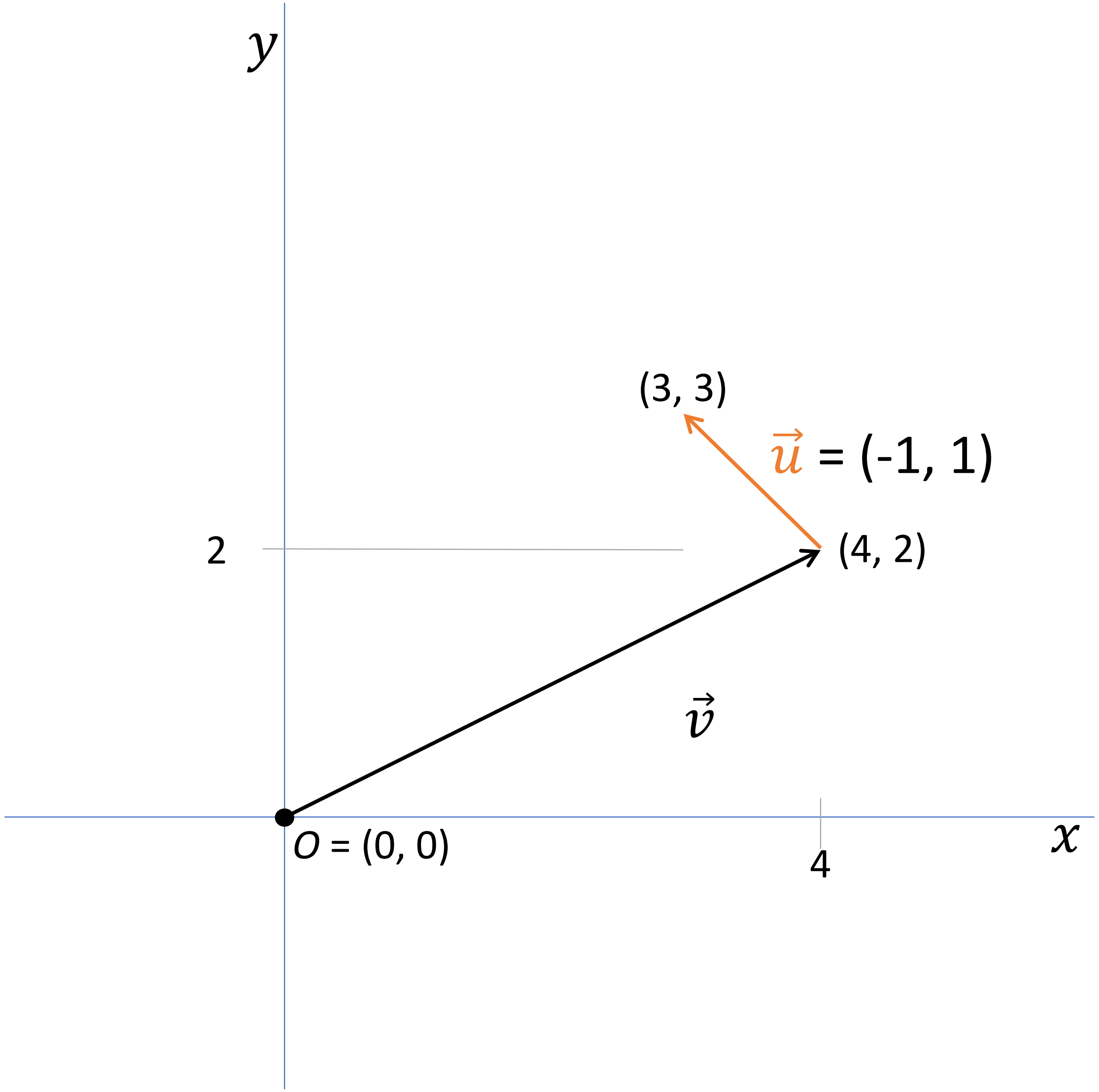
Euclidean vector

- direction
- magnitude / length



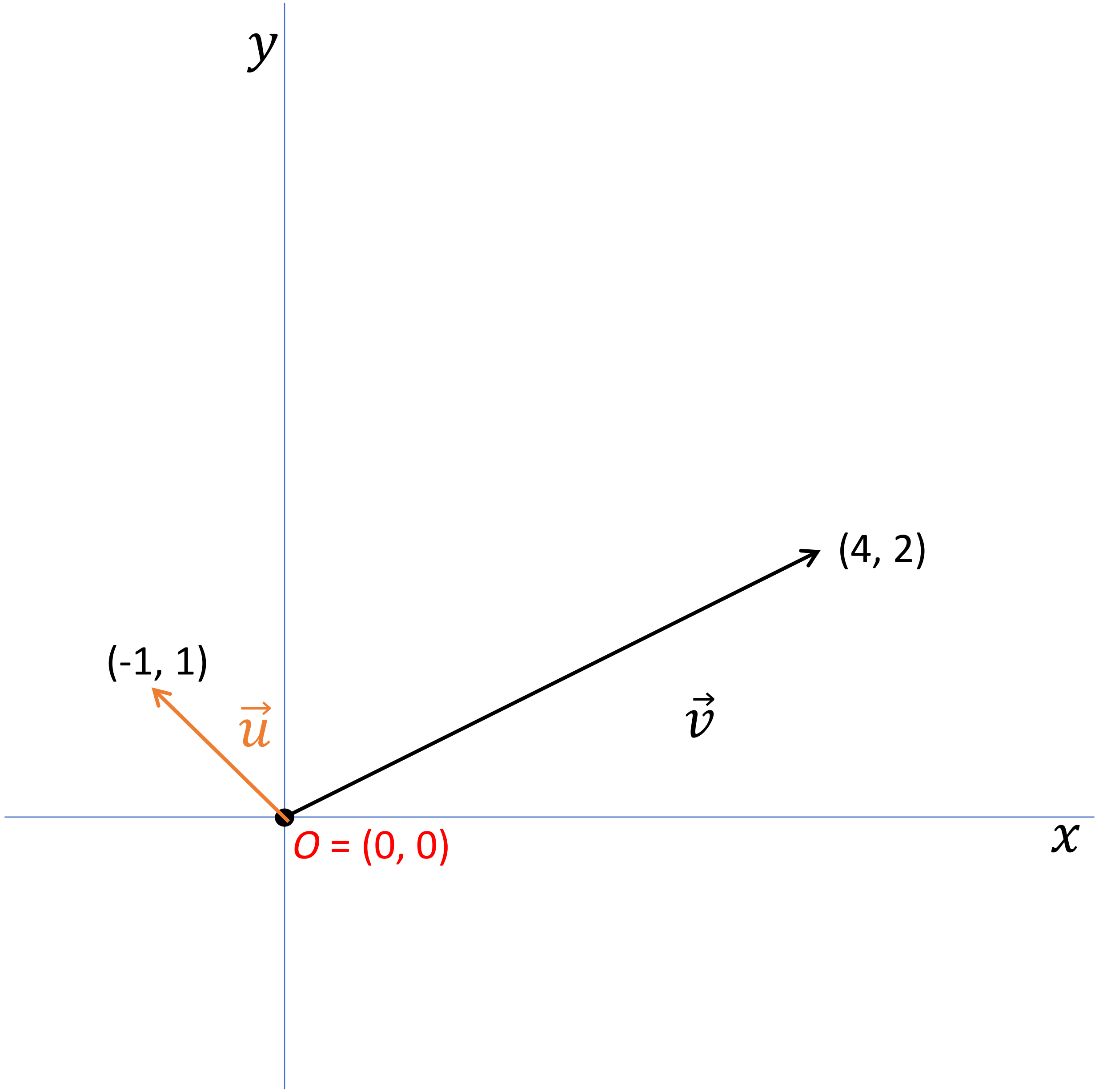
Euclidean vector

- direction
- magnitude / length



Euclidean vector

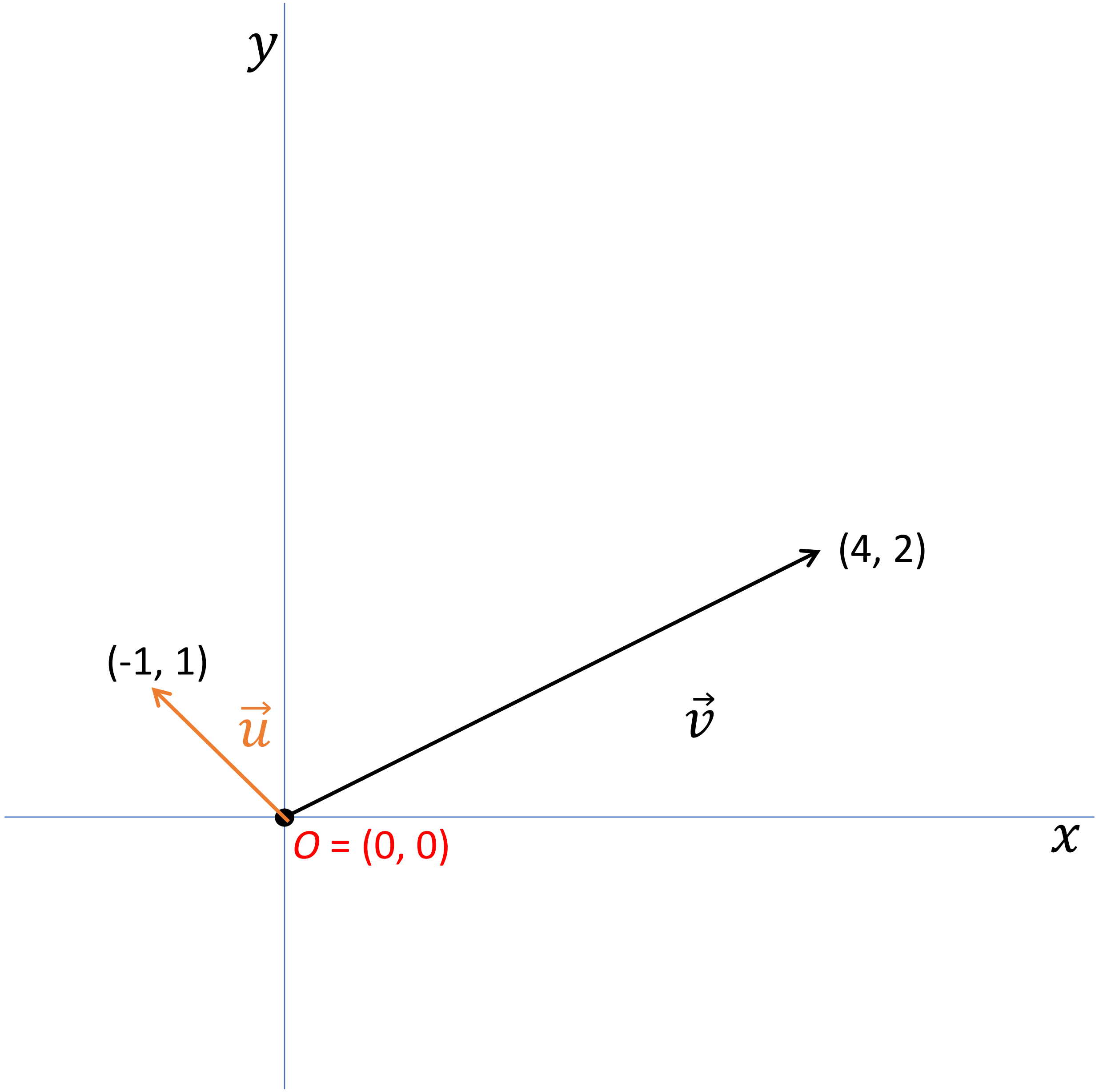
- direction
- magnitude / length



Euclidean vector

- direction
- magnitude / length

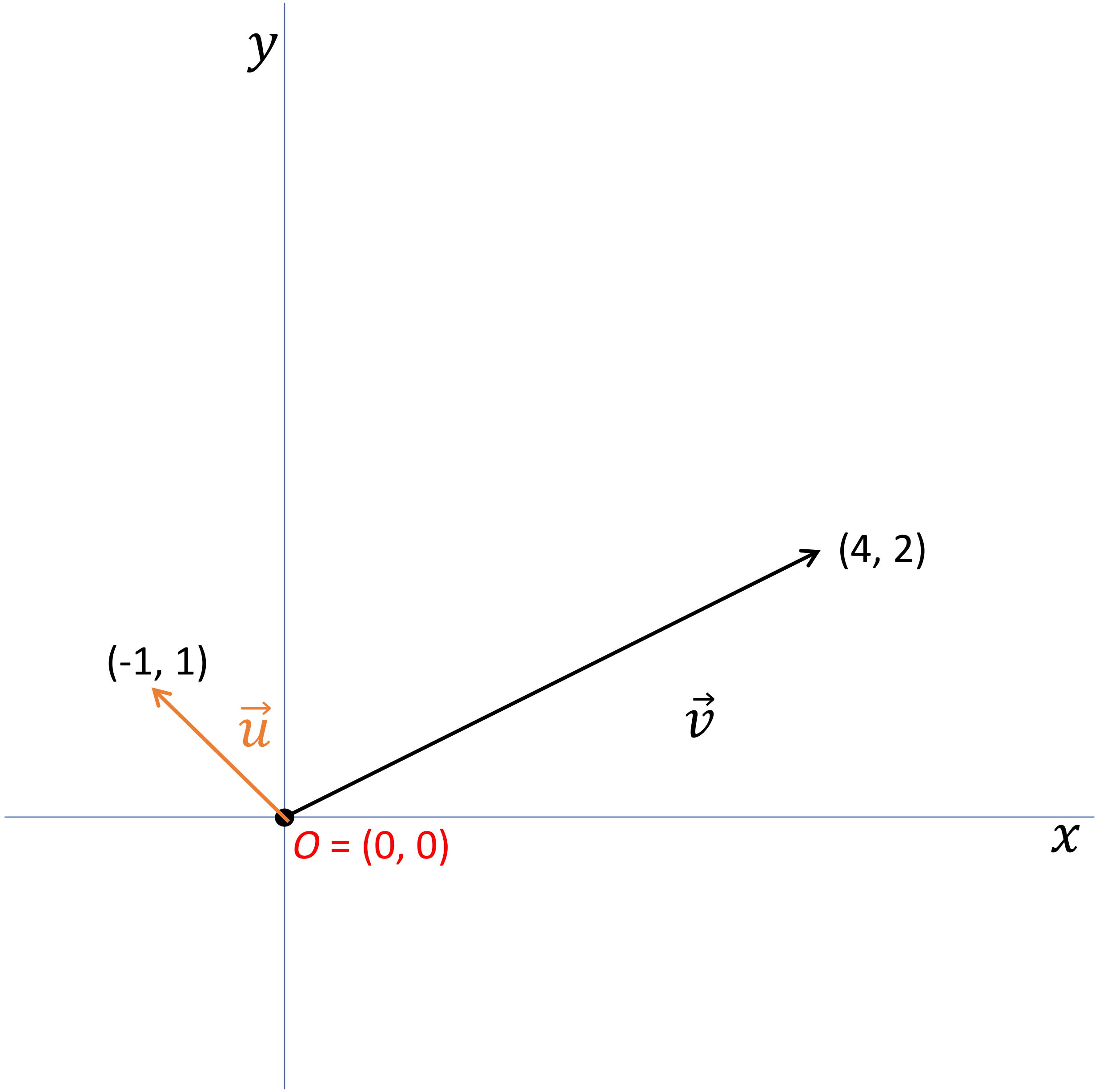
$$\vec{w} \overset{?}{=} \vec{v} + \vec{u}$$



Euclidean vector

- direction
- magnitude / length

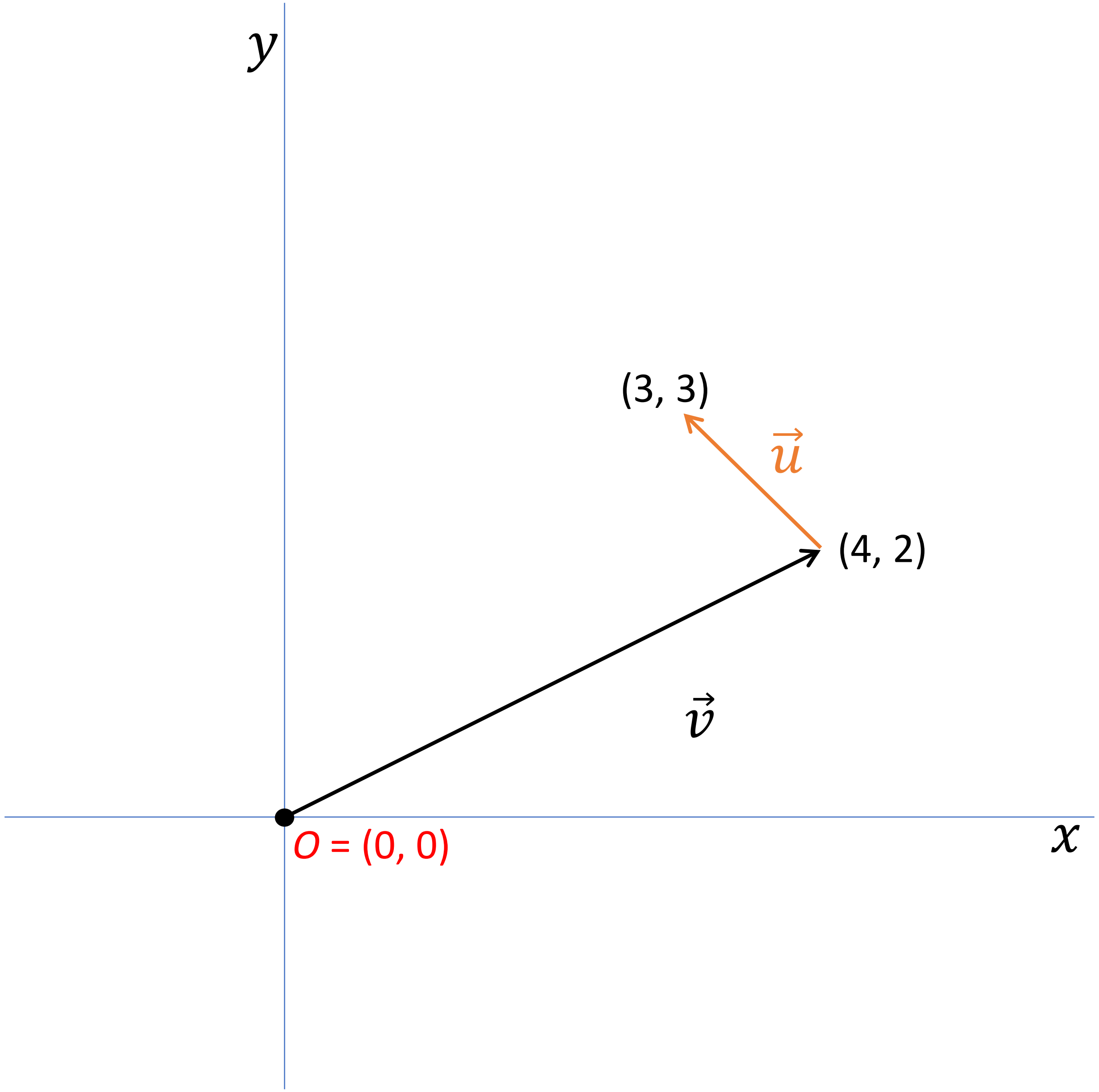
$$\vec{w} = (4, 2) + (-1, 1)$$



Euclidean vector

- direction
- magnitude / length

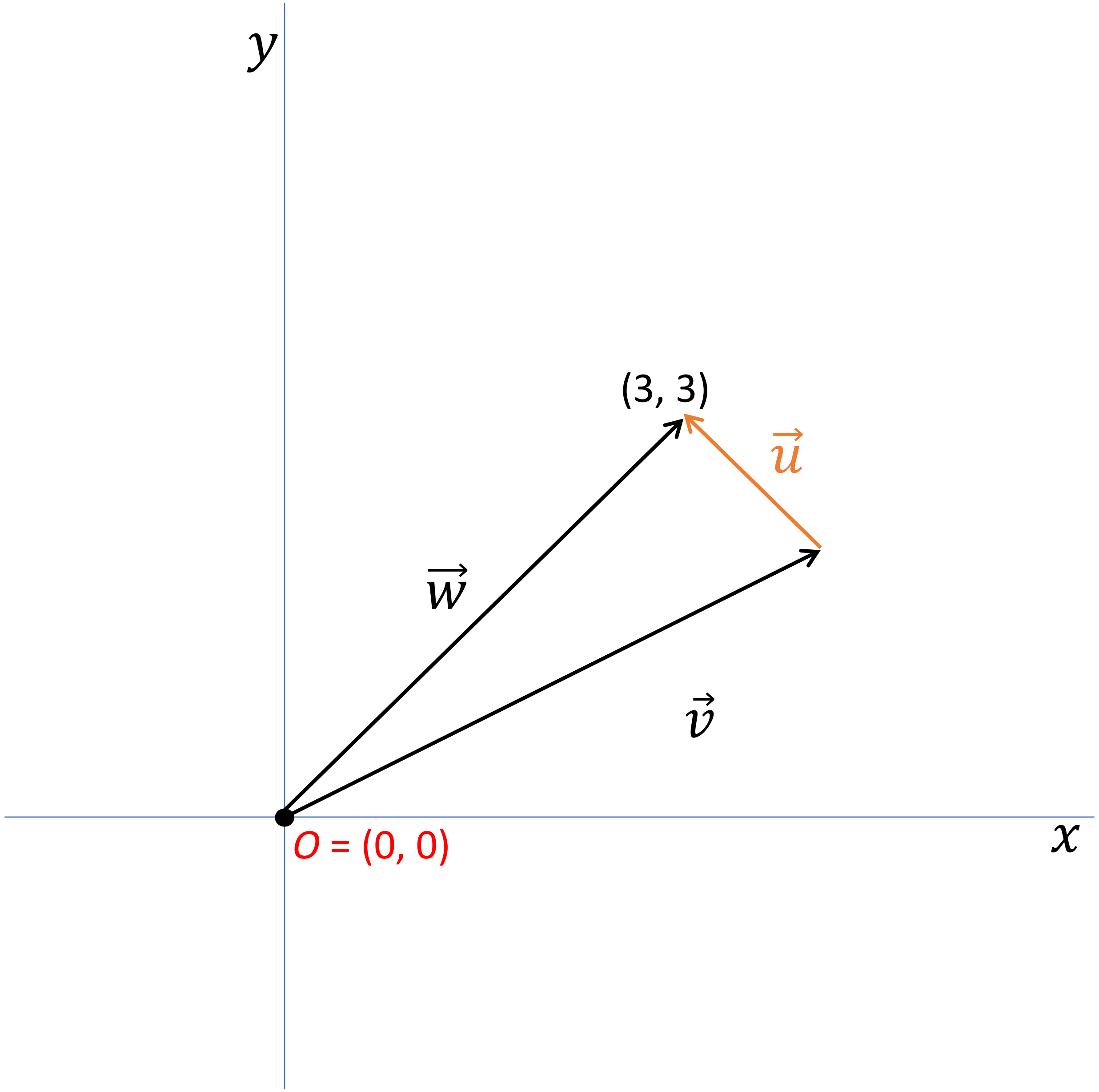
$$\vec{w} = (4 - 1, 2 + 1)$$



Euclidean vector

- direction
- magnitude / length

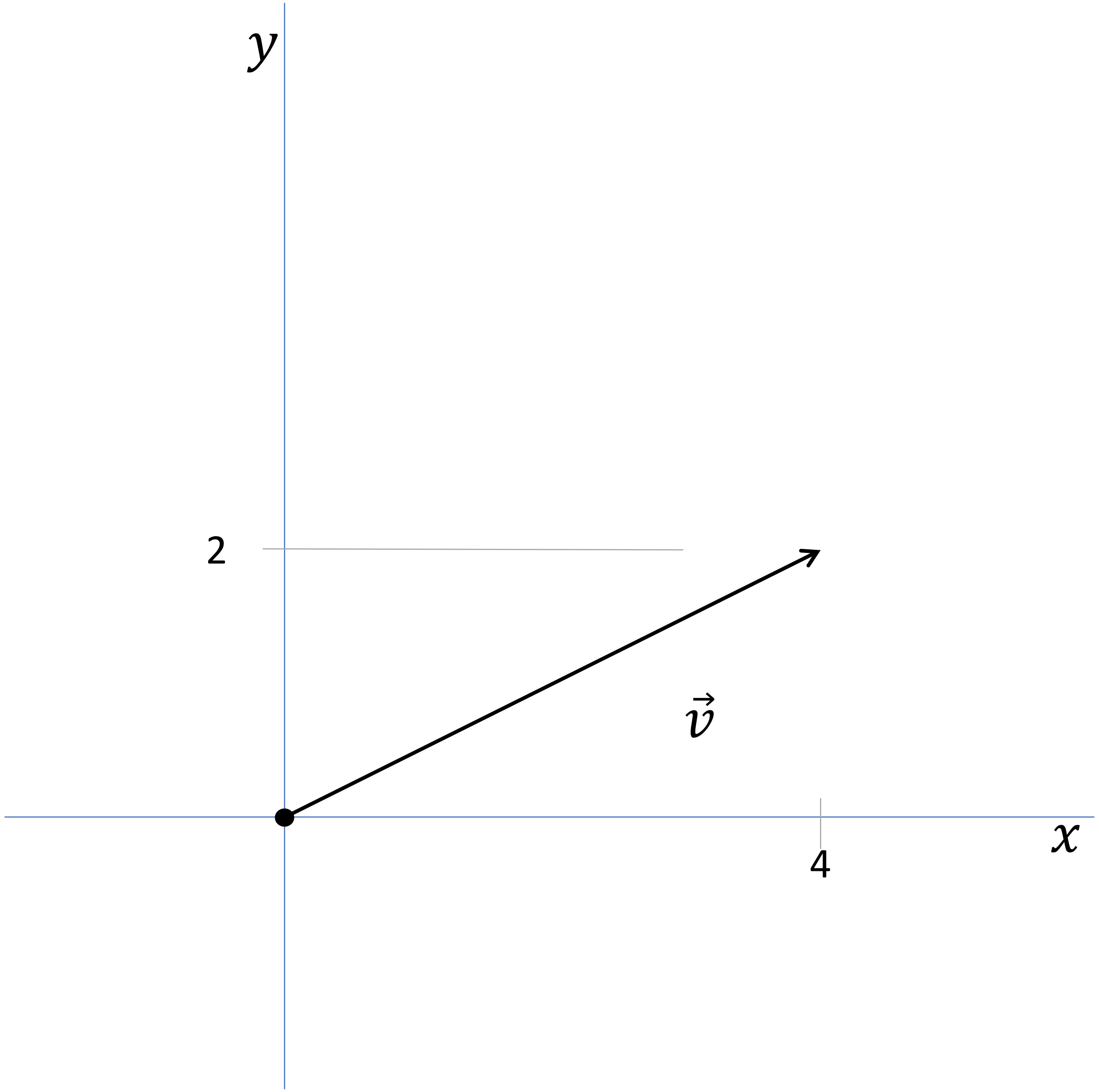
$\vec{w} = (3, 3)$



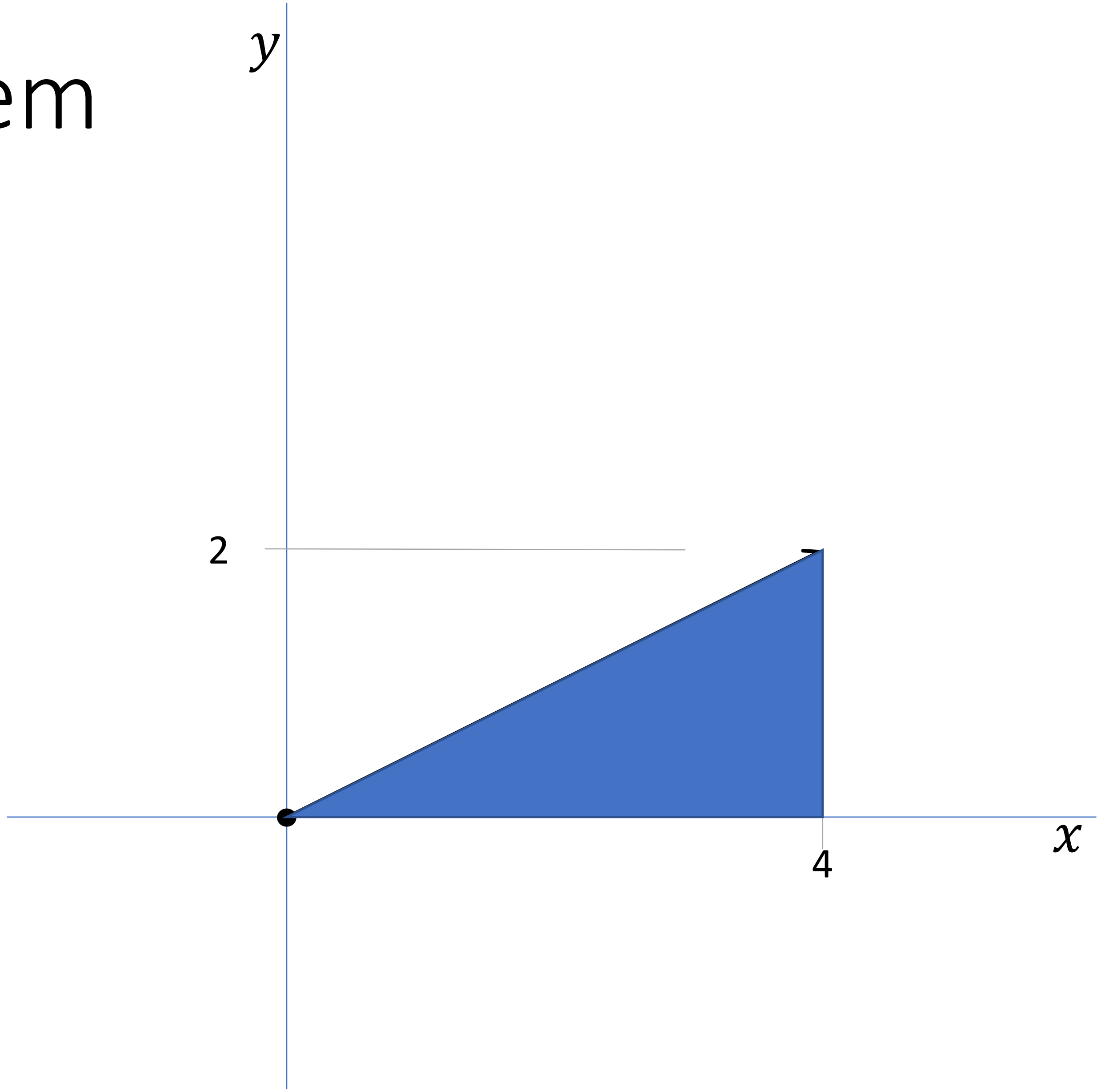
Euclidean vector

- direction
- magnitude / length

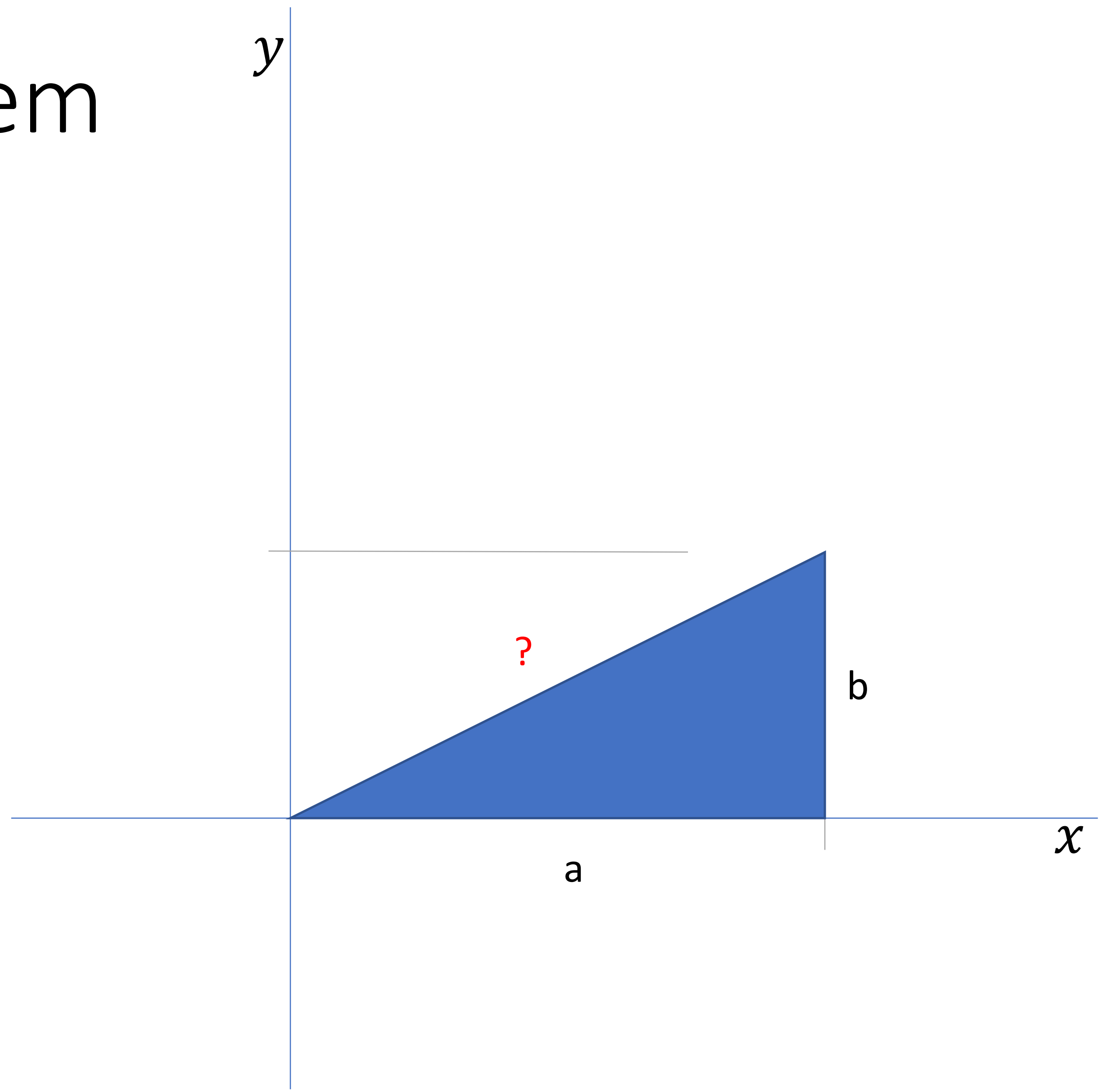
$$|\vec{v}| = ?$$



Pythagorean theorem



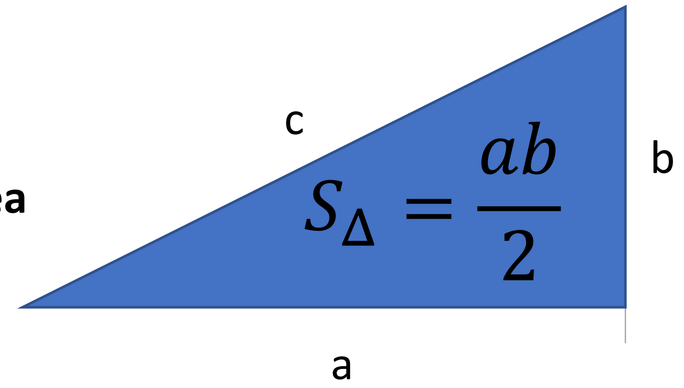
Pythagorean theorem



Pythagorean theorem

C = ?

triangle area

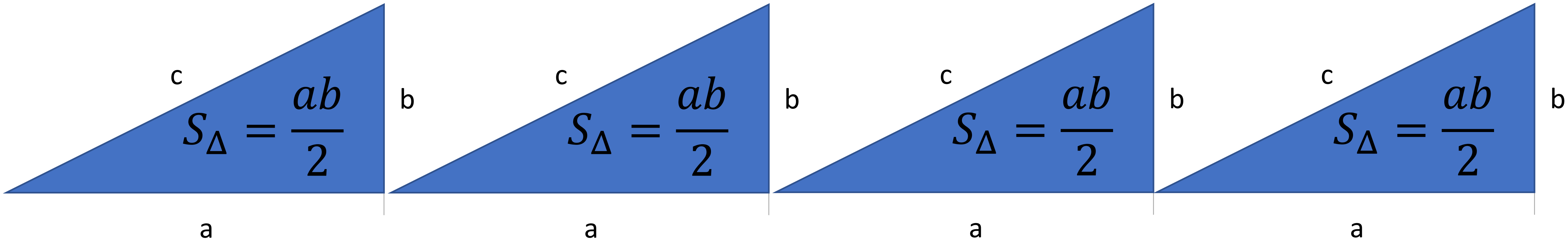


$$S_\Delta = \frac{ab}{2}$$

Pythagorean theorem

c = ?

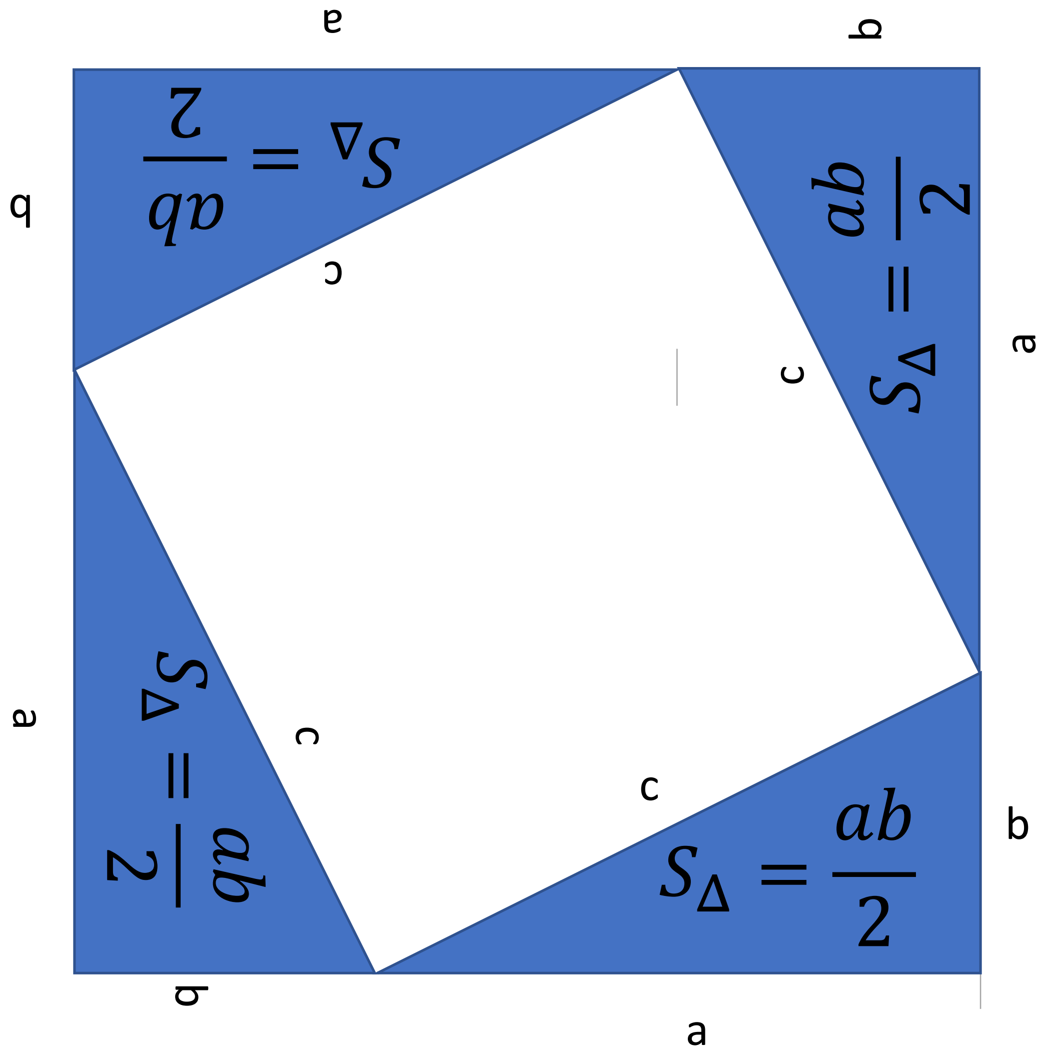
$$S = 4S_{\Delta} = 2ab$$



Pythagorean theorem

c = ?

$$S = 4S_{\Delta} = 2ab$$

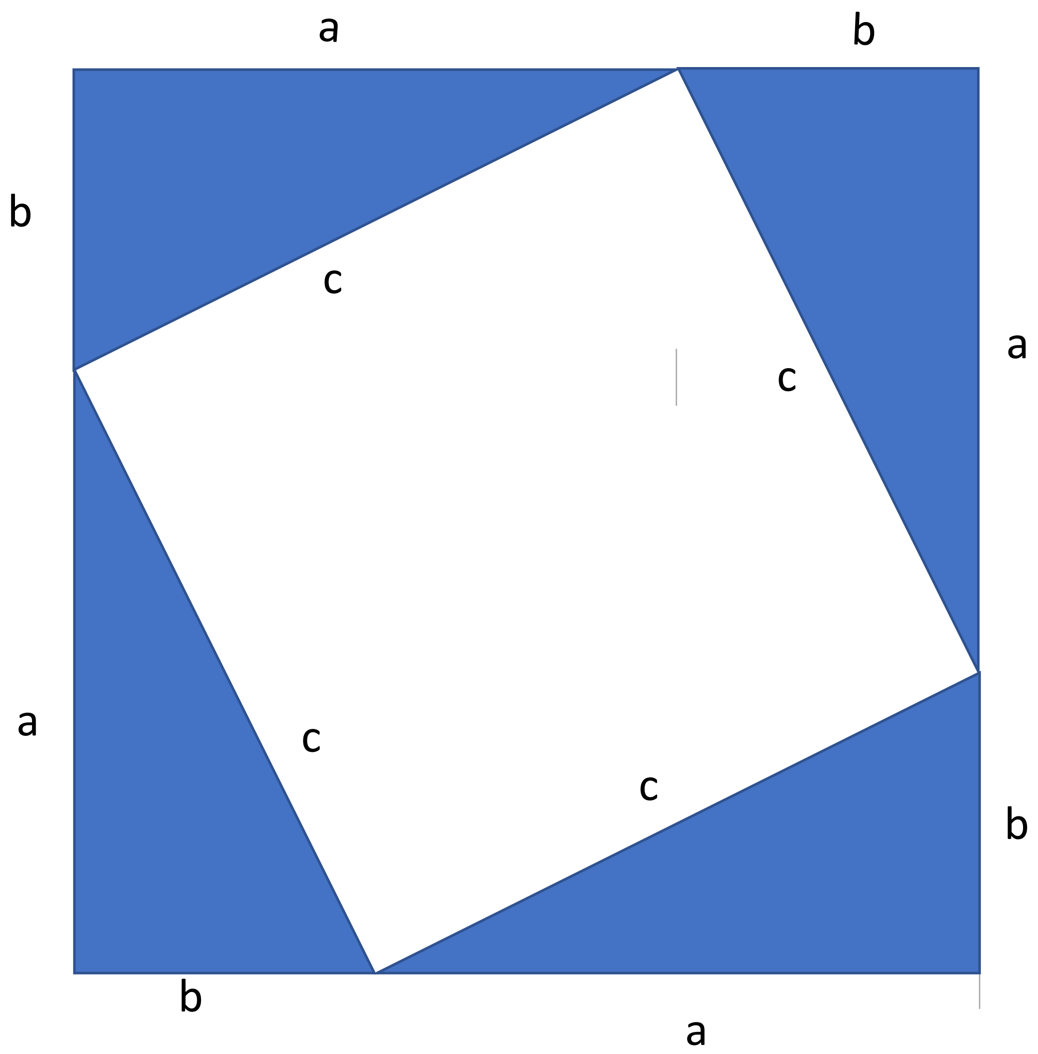


Pythagorean theorem

$$S_{\blacksquare} = (a+b)^2$$

c = ?

$$S = 4S_{\Delta} = 2ab$$

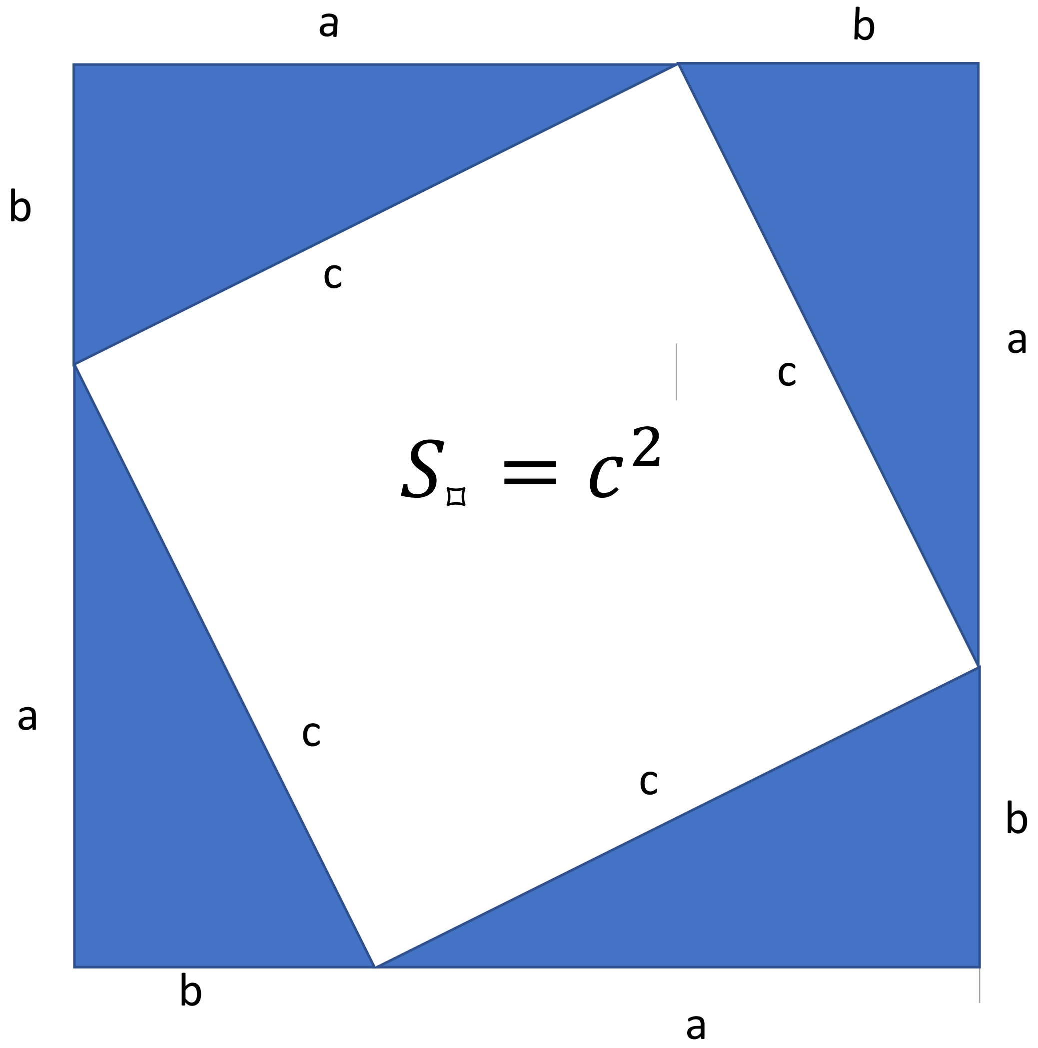


Pythagorean theorem

$$S_{■} = (a+b)^2$$

c = ?

$$S = 4S_{\Delta} = 2ab$$



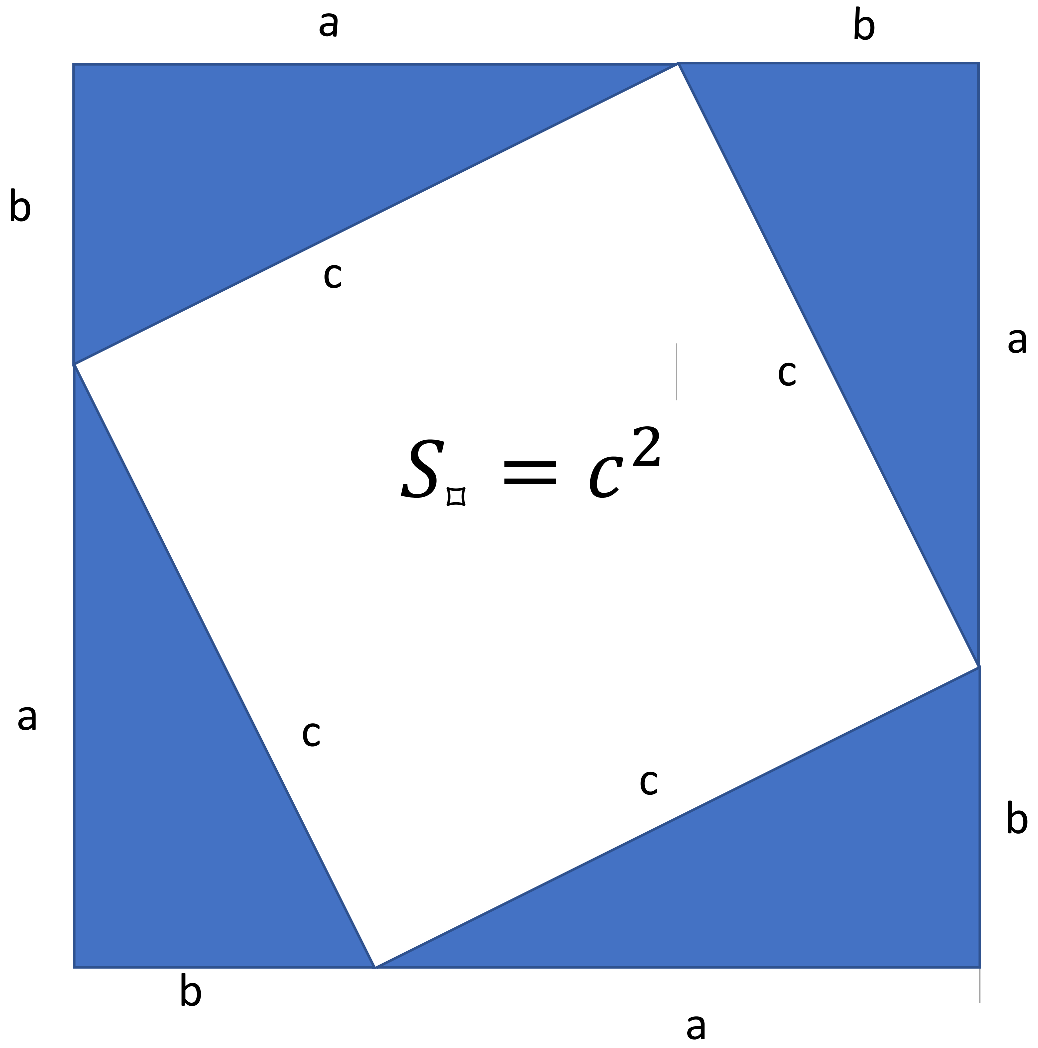
Pythagorean theorem

$S_{\blacksquare} = (a + b)^2$

c = ?

$S = 4S_{\Delta} = 2ab$

$S = S_{\blacksquare} - S_{¤}$



Pythagorean theorem

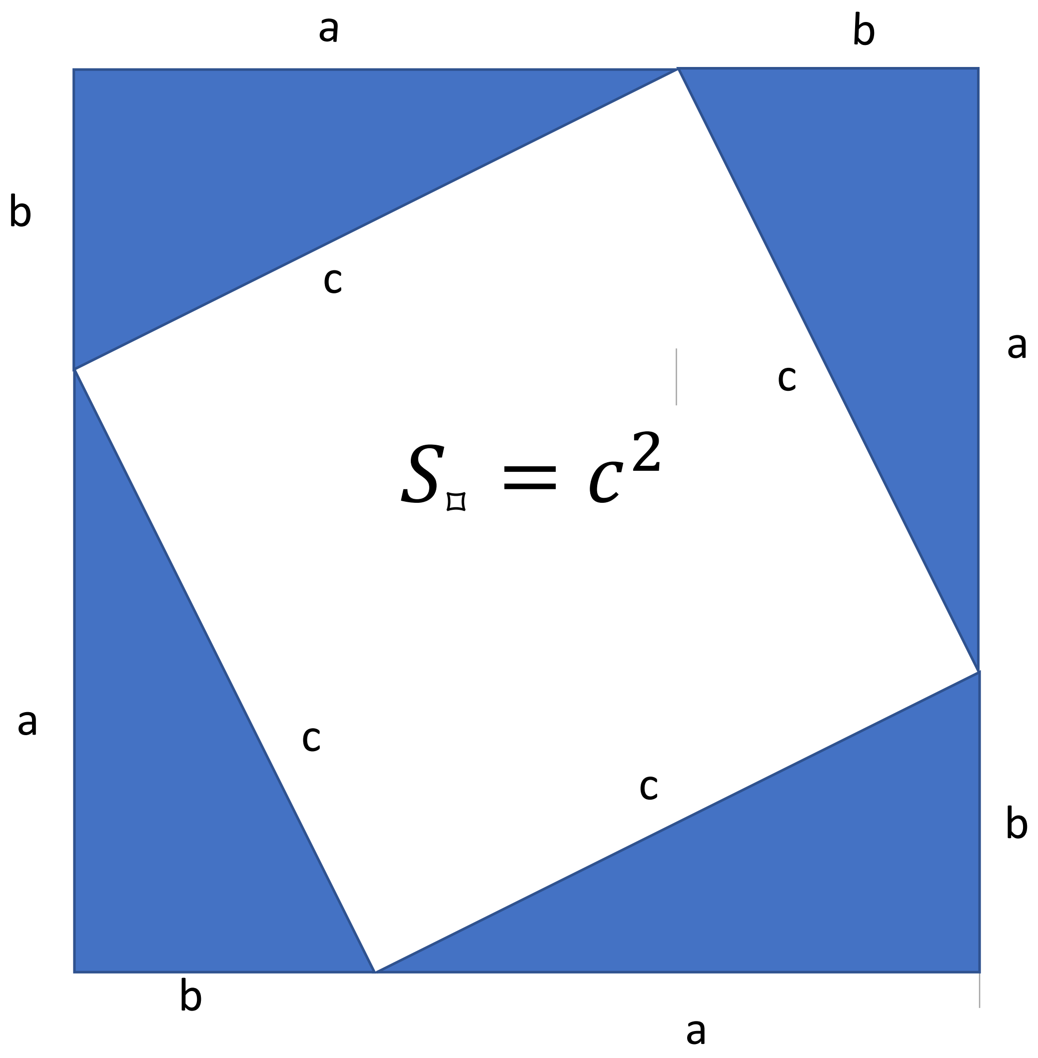
$$S_{\blacksquare} = (a + b)^2$$

$$S = 4S_{\Delta} = 2ab$$

$$S = S_{\blacksquare} - S_{\square}$$

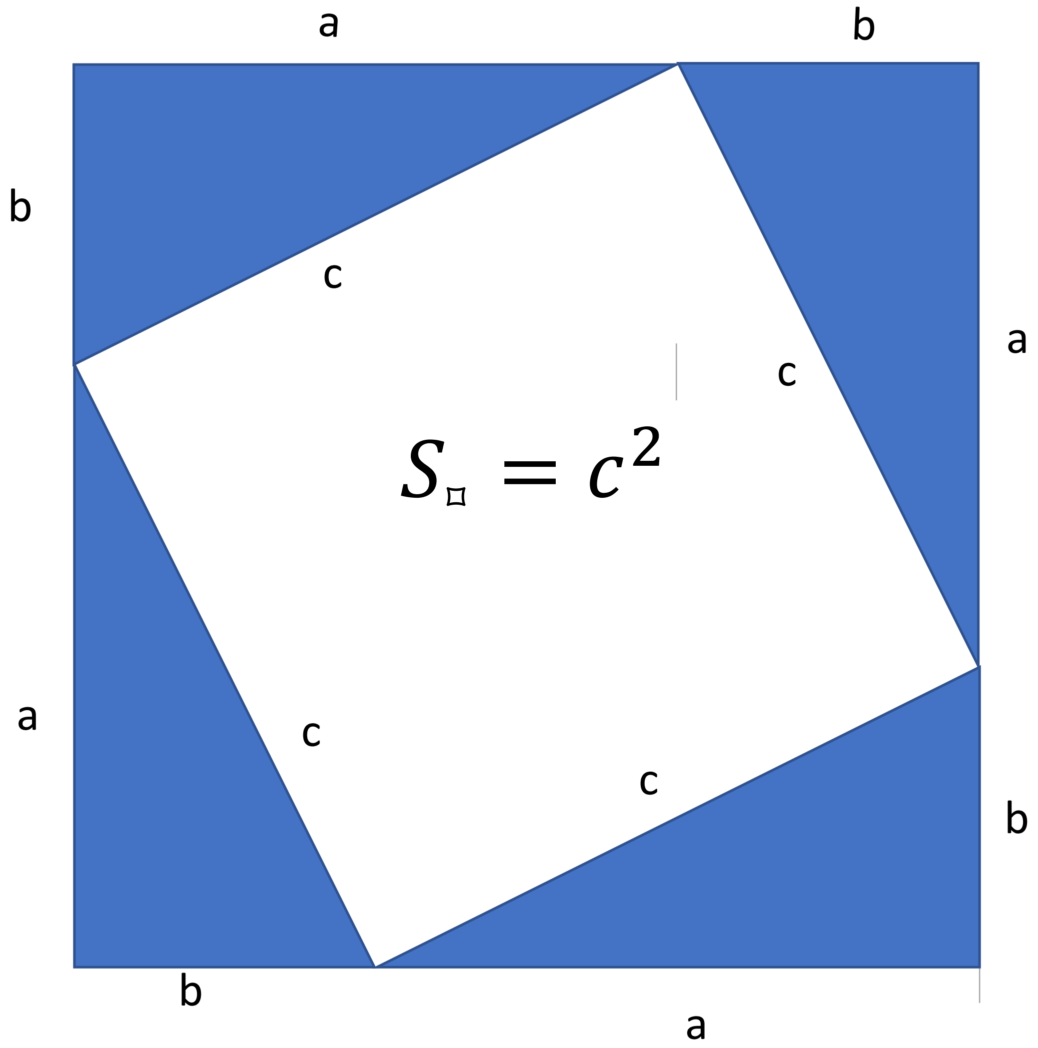
c = ?

$$2ab = (a + b)^2 - c^2$$



Pythagorean theorem

$$S_{\blacksquare} = (a+b)^2$$



$$S = 4S_{\Delta} = 2ab$$

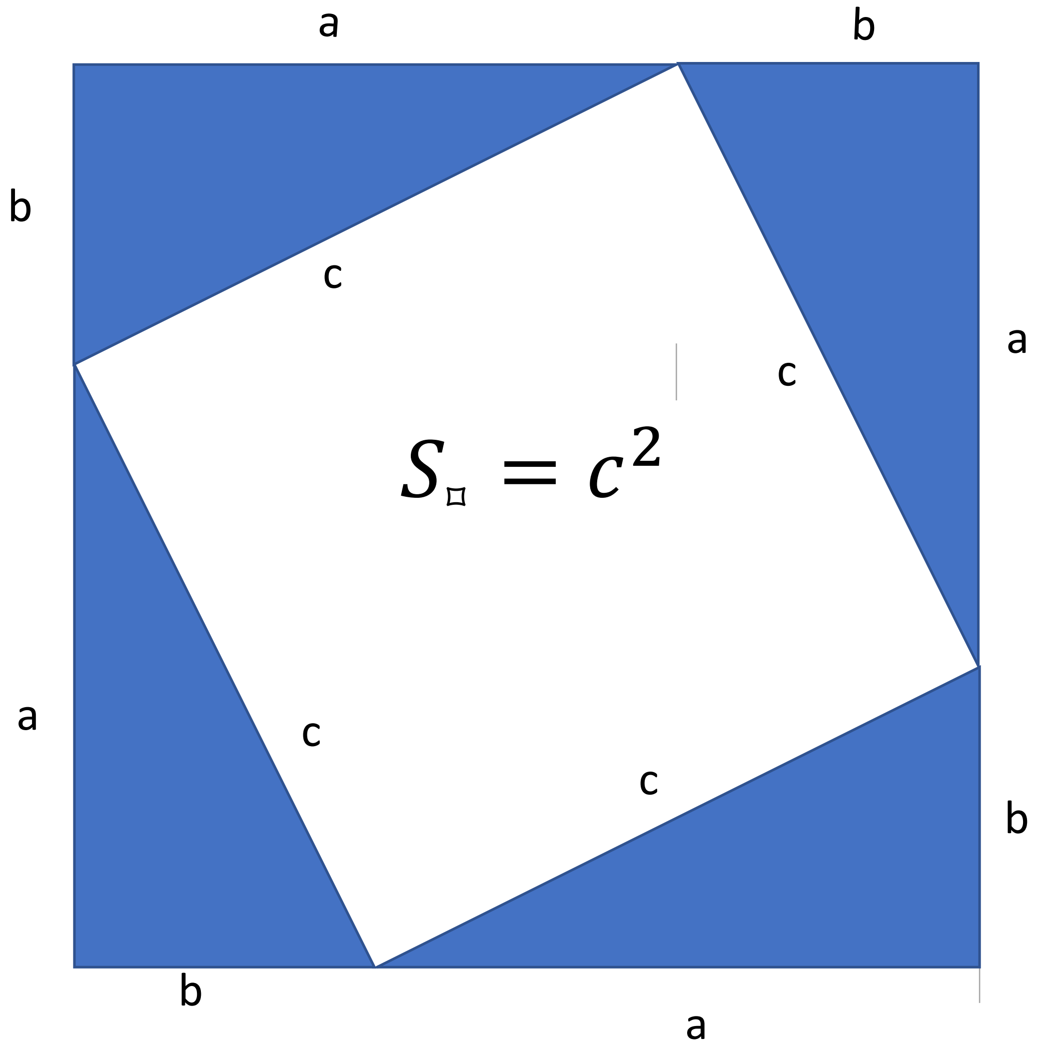
$$S = S_{\blacksquare} - S_{\square}$$

c = ?

$$2ab = a^2 + 2ab + b^2 - c^2$$

Pythagorean theorem

$$S_{\blacksquare} = (a+b)^2$$



$$S = 4S_{\Delta} = 2ab$$

$$S = S_{\blacksquare} - S_{\square}$$

c = ?

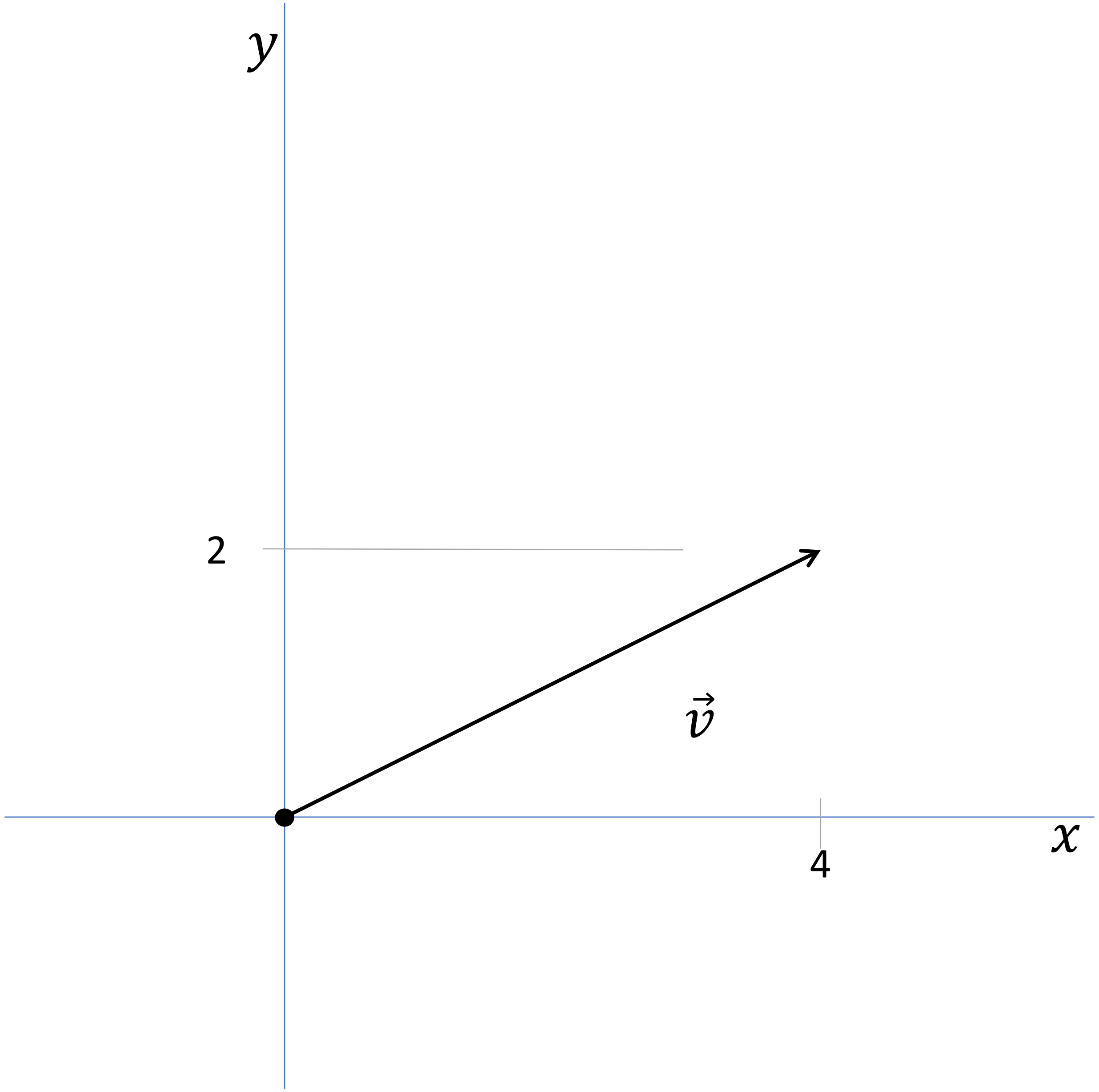
$$2ab = a^2 + 2ab + b^2 - c^2$$

$$c^2 = a^2 + b^2$$

Euclidean vector

- direction
- magnitude / length

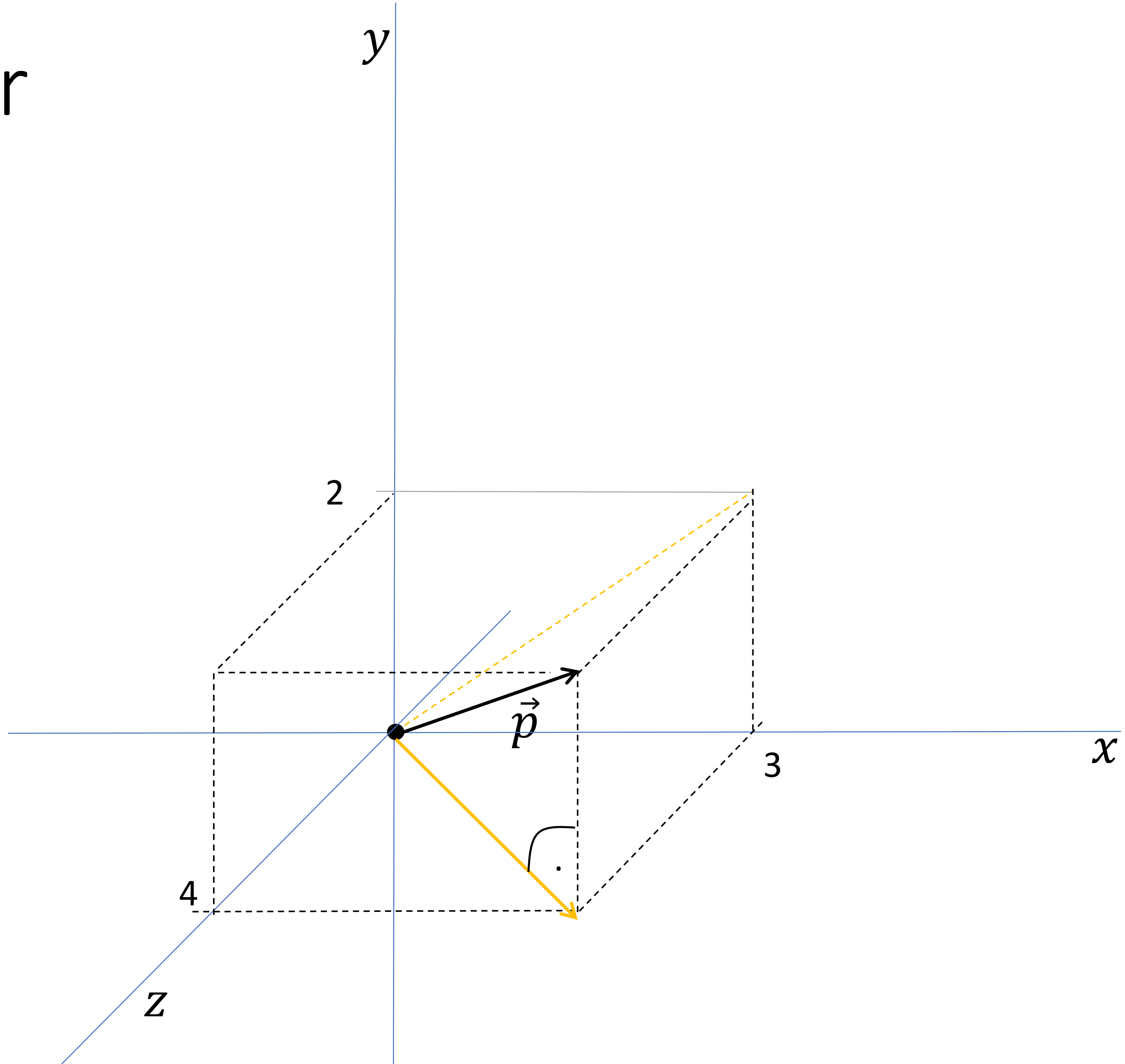
$$|\vec{v}| = \sqrt{4^2 + 2^2}$$



Euclidean vector

- direction
- magnitude / length

$|\vec{p}| = ?$

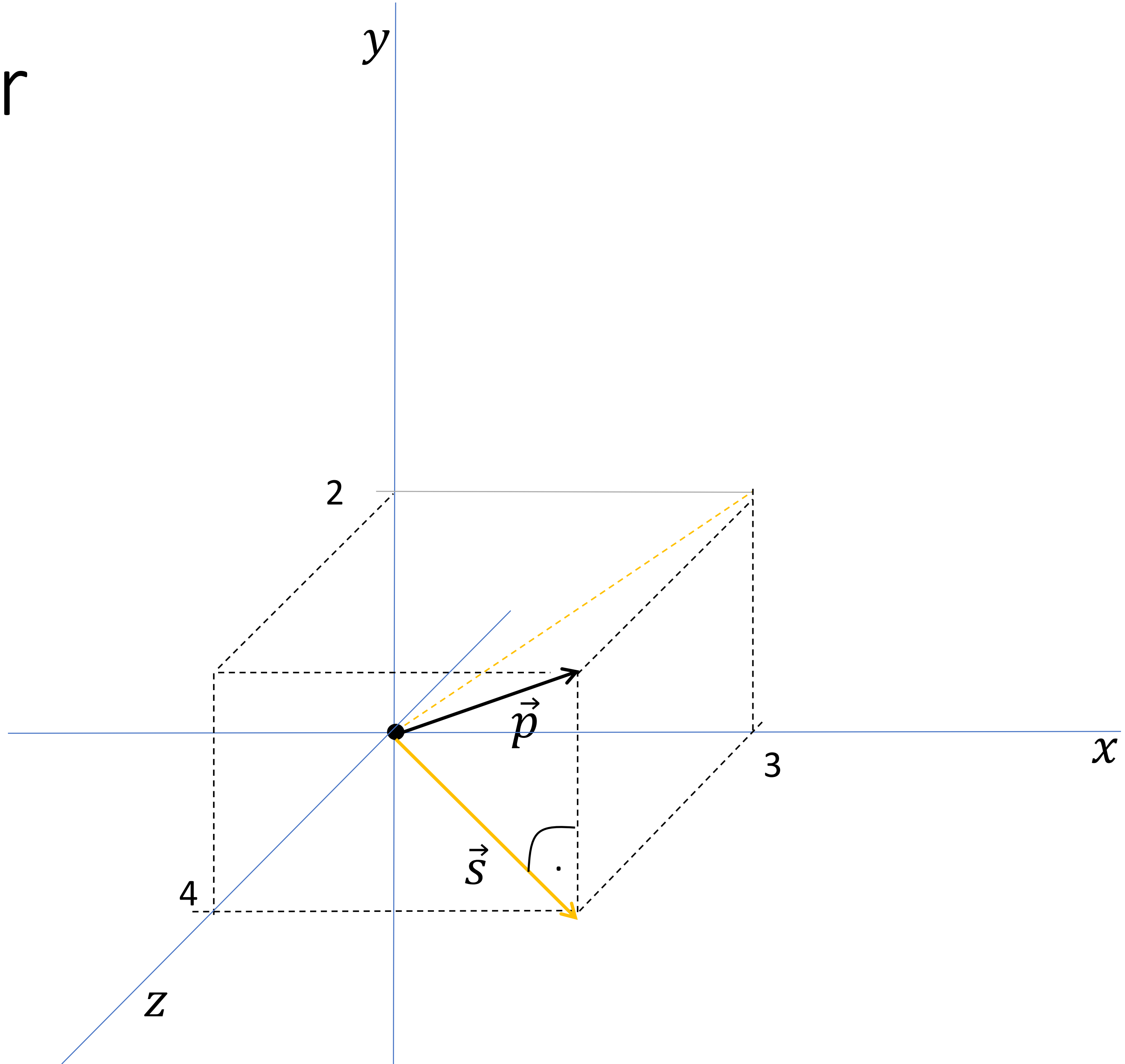


Euclidean vector

- direction
- magnitude / length

$|\vec{p}| = ?$

$|\vec{s}| = ?$

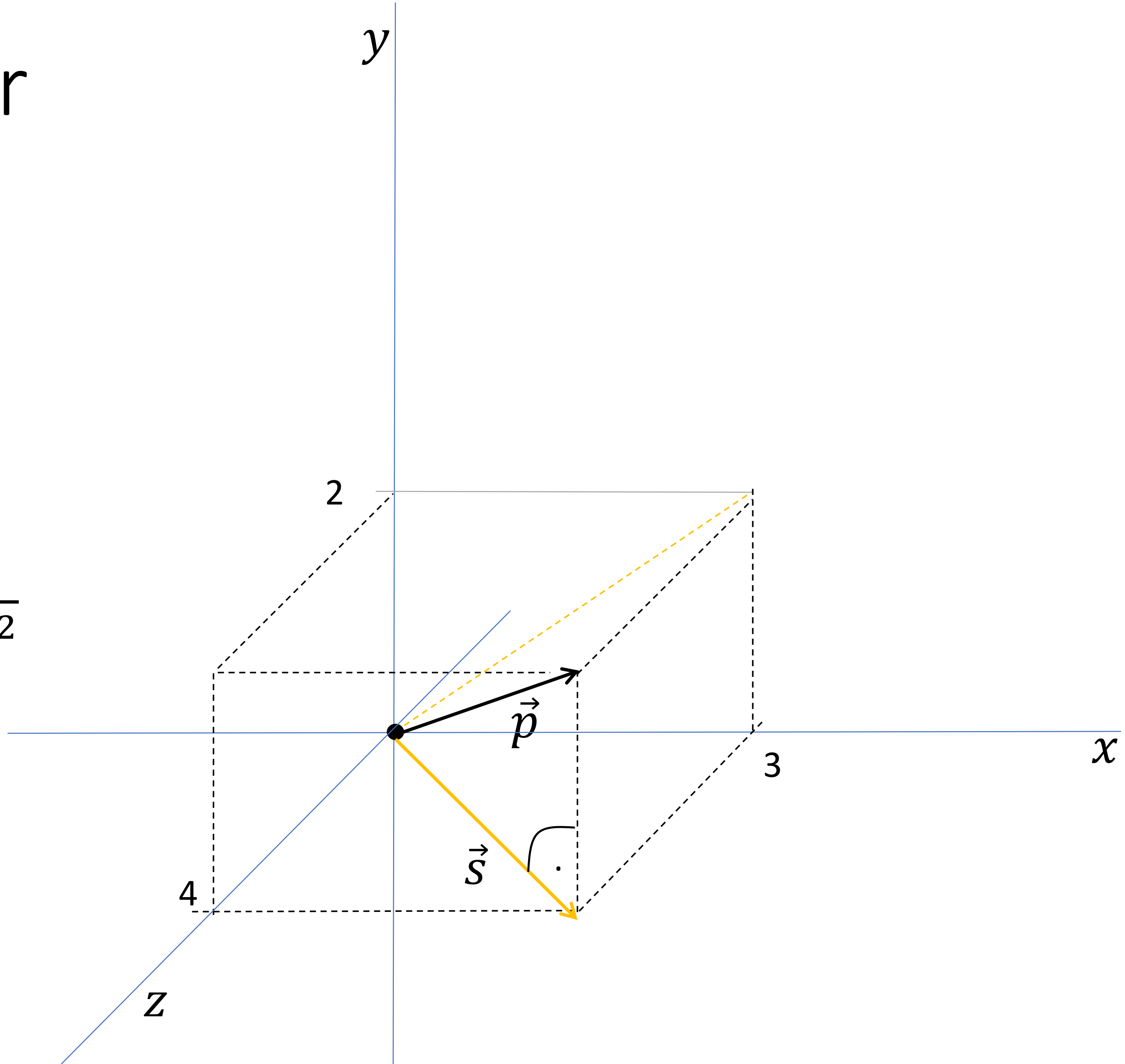


Euclidean vector

- direction
- magnitude / length

$|\vec{p}| = ?$

$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$

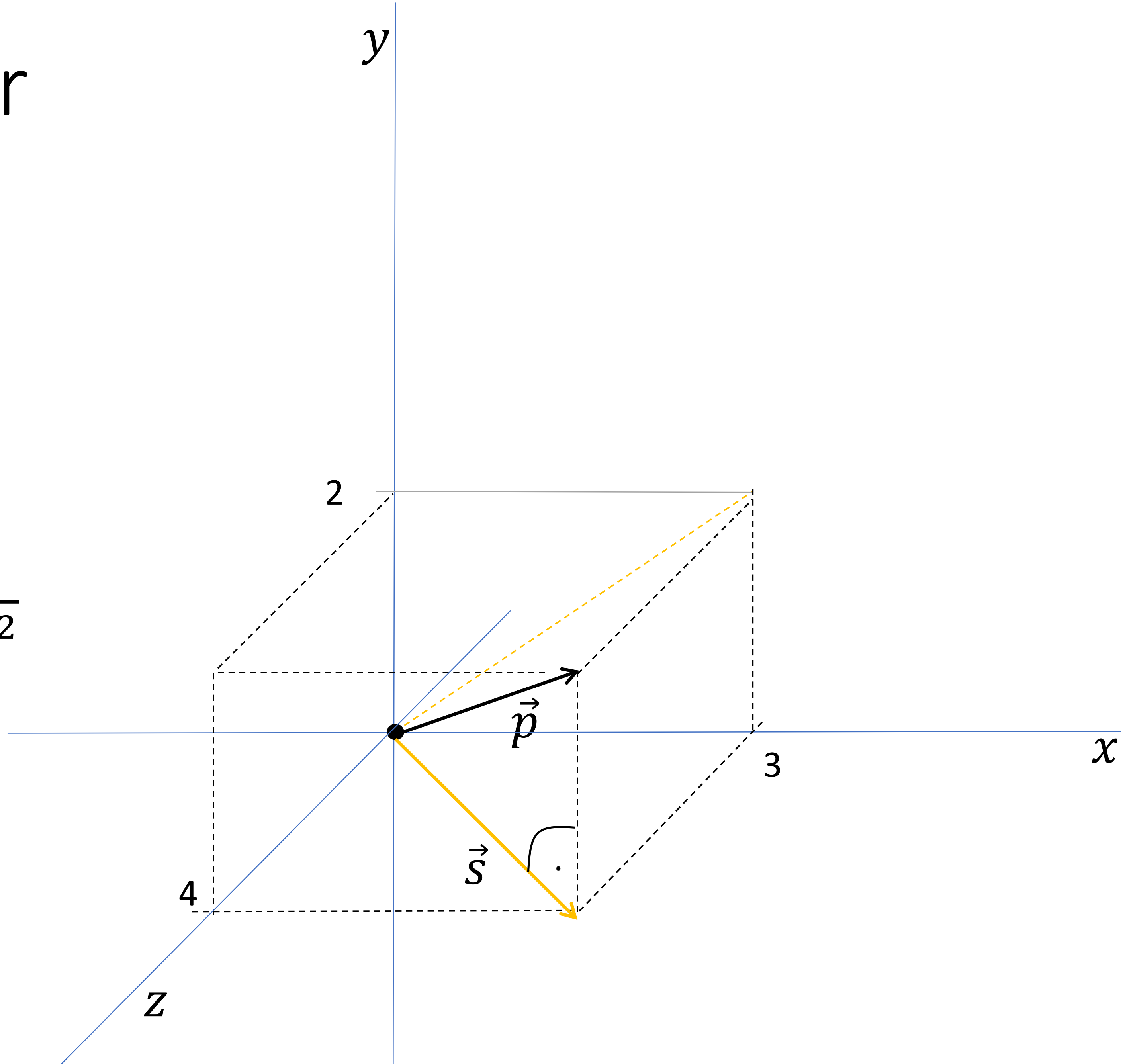


Euclidean vector

- direction
- magnitude / length

$$|\vec{p}| = \sqrt{|\vec{s}|^2 + y^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$

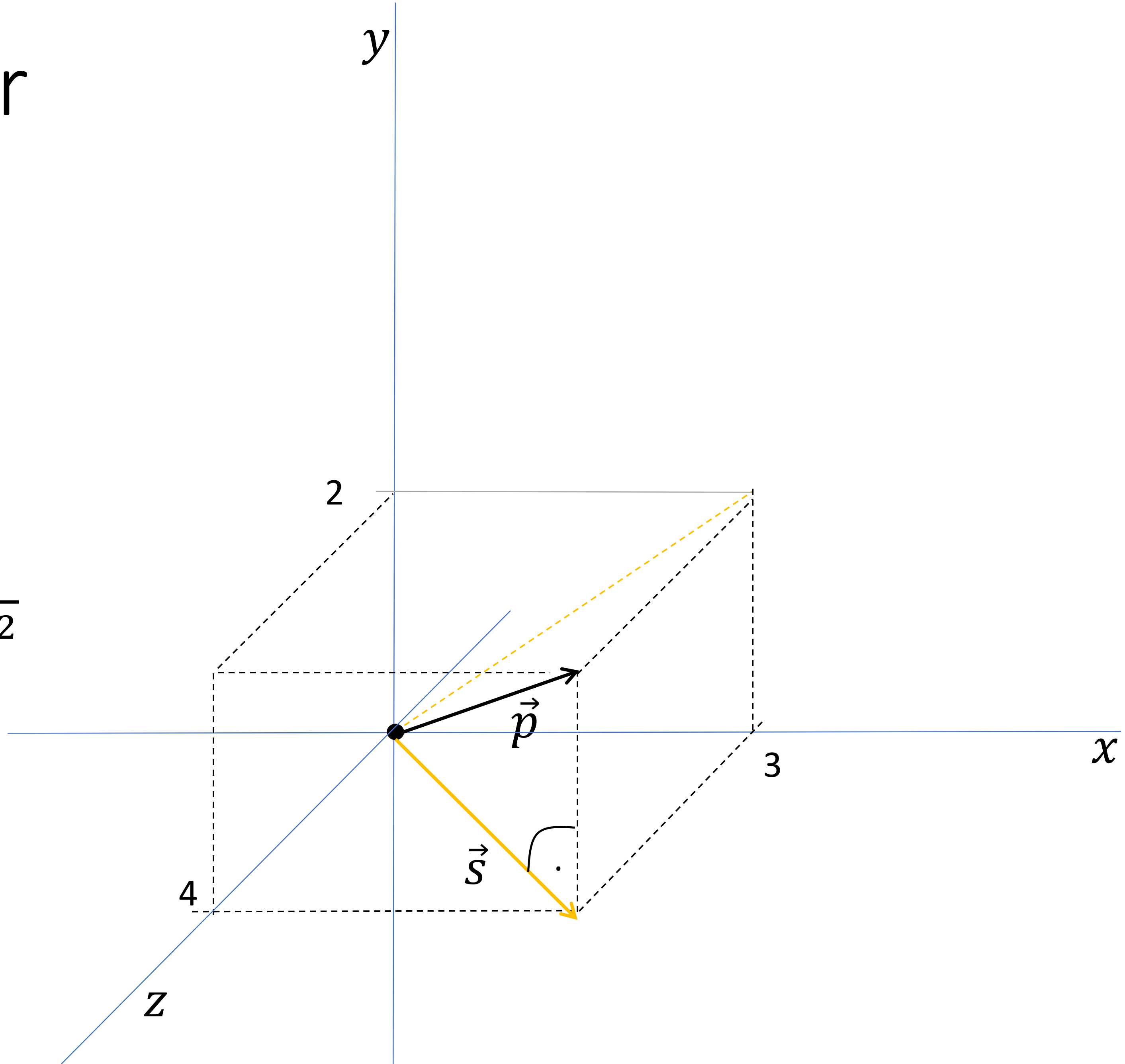


Euclidean vector

- direction
- magnitude / length

$$|\vec{p}| = \sqrt{\sqrt{x^2 + z^2}^2 + y^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$

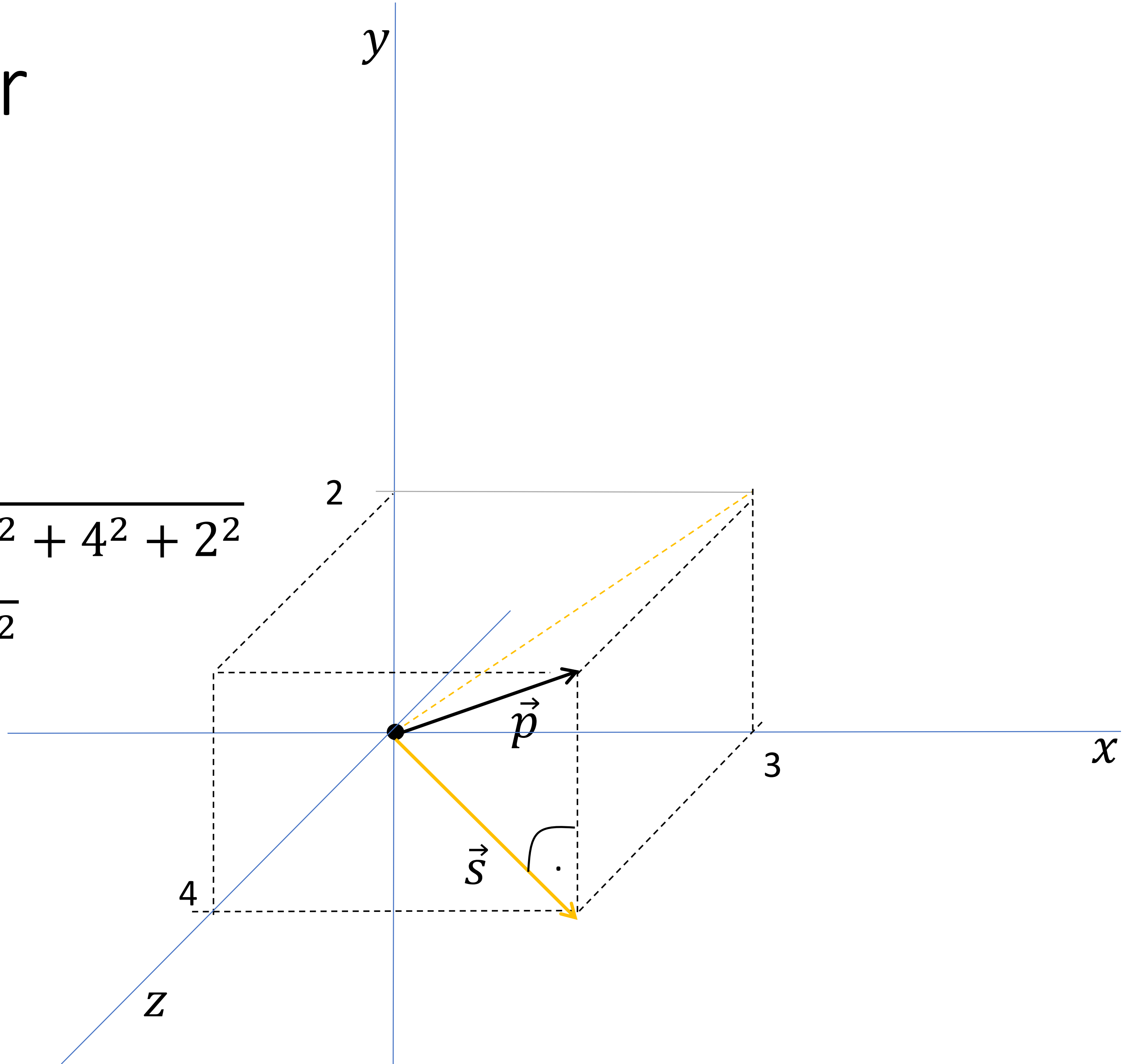


Euclidean vector

- direction
- magnitude / length

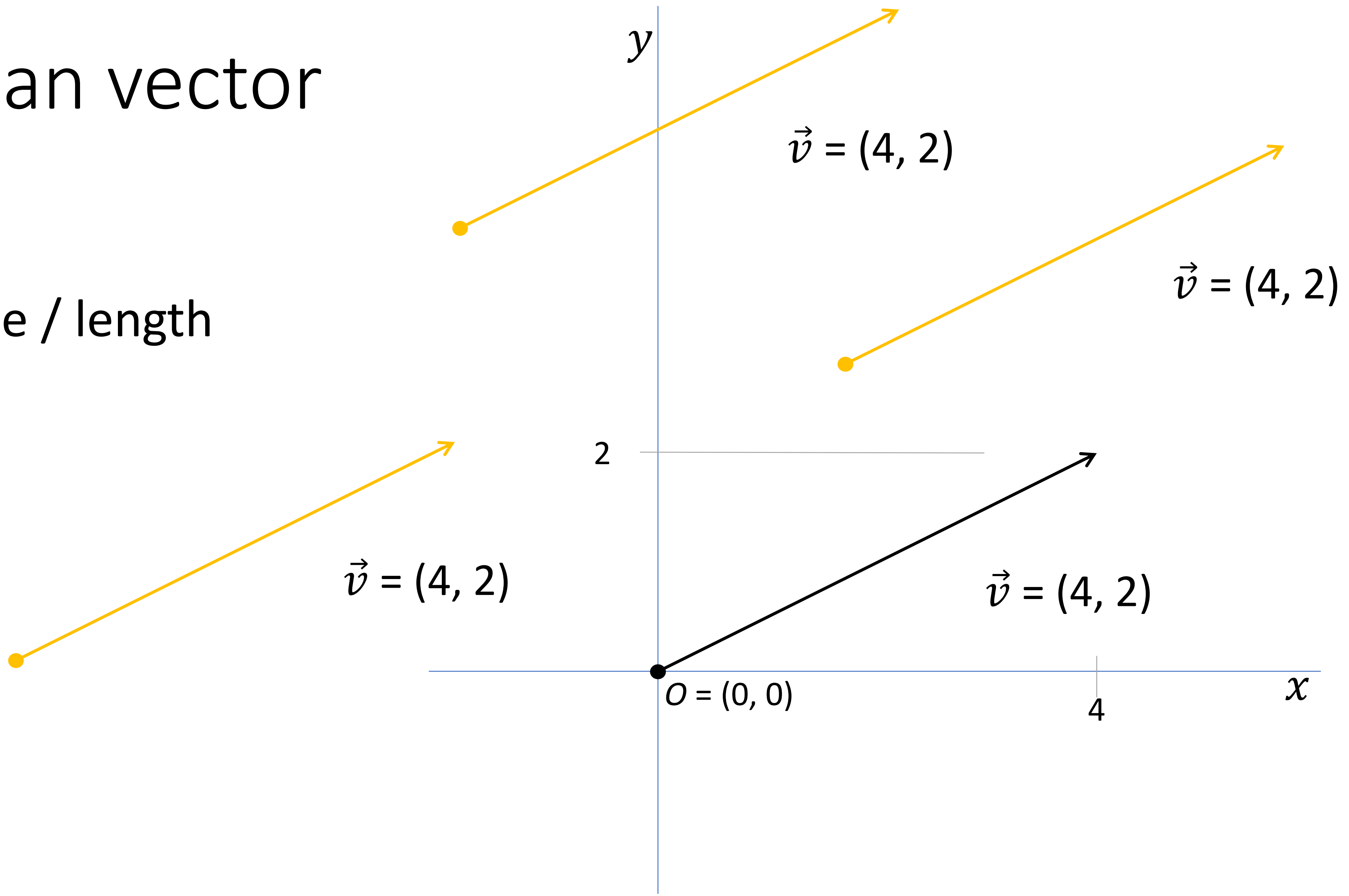
$$|\vec{p}| = \sqrt{x^2 + z^2 + y^2} = \sqrt{3^2 + 4^2 + 2^2}$$

$$|\vec{s}| = \sqrt{x^2 + z^2} = \sqrt{3^2 + 4^2}$$



Euclidean vector

- direction
- magnitude / length

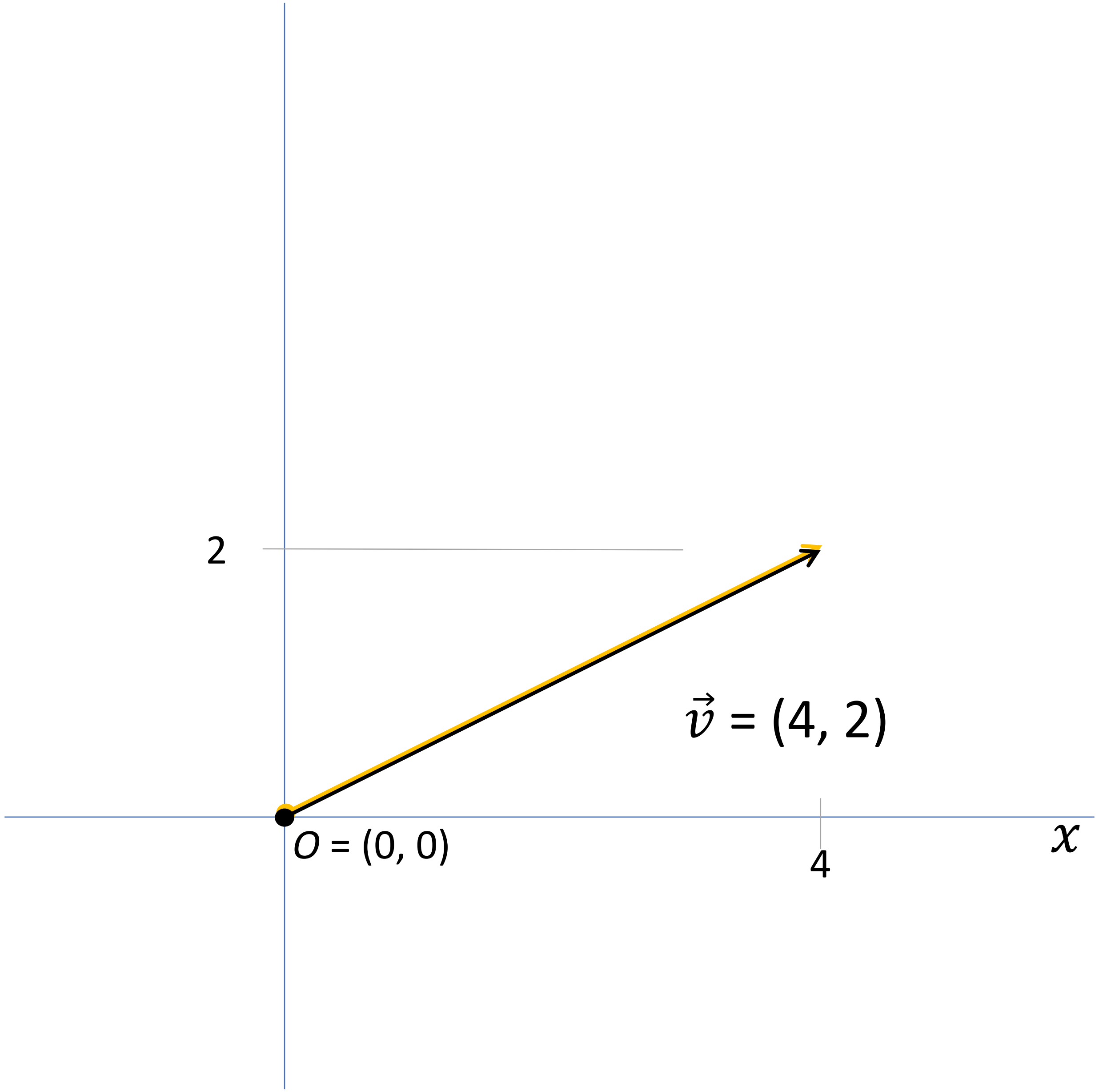


```cpp
#include <stdio.h>
#include <math.h>

class Triple {
  public:
    float x, y, z;

    Triple() : x(0), y(0), z(0) {}

    Triple(const float x, const float y, const float z)
      : x(x), y(y), z(z) {}
};

class Vector3D: public Triple {
  public:
    Vector3D(const float x, const float y, const float z)
      : Triple(x, y, z) {}

    float magnitude() const {
      return sqrt(x*x + y*y + z*z);
    }
};

class Point3D: public Triple {
  public:
    Point3D(const float x, const float y, const float z)
      : Triple(x, y, z) {}

    Point3D translate(const Vector3D &vector) {
      x += vector.x;
      y += vector.y;
      z += vector.z;
    }

    Vector3D operator-(const Point3D& p) const {
      return Vector3D(x - p.x, y - p.y, z - p.z);
    }
};
```

```cpp
int main(void) {

    Point3D p0( 0.0f, 0.0f, 1.0f);
    Point3D p1( 1.0f, 2.0f, 3.0f);

    Vector3D v = p1 - p0;
    printf ("v=(%f, %f, %f)\n", v.x, v.y, v.z);//v=(1.000000, 2.000000, 2.000000)

    p0.translate(v);
    printf ("p0=(%f, %f, %f)\n", p0.x, p0.y, p0.z);//p0=(1.000000, 2.000000, 3.000000)

    v.translate(Vector3D( 1.0f, 2.0f, 3.0f));//error
    float len = p0.magnitude();//error

    return 0;
}
```

Normalized vector

- direction
- magnitude / length

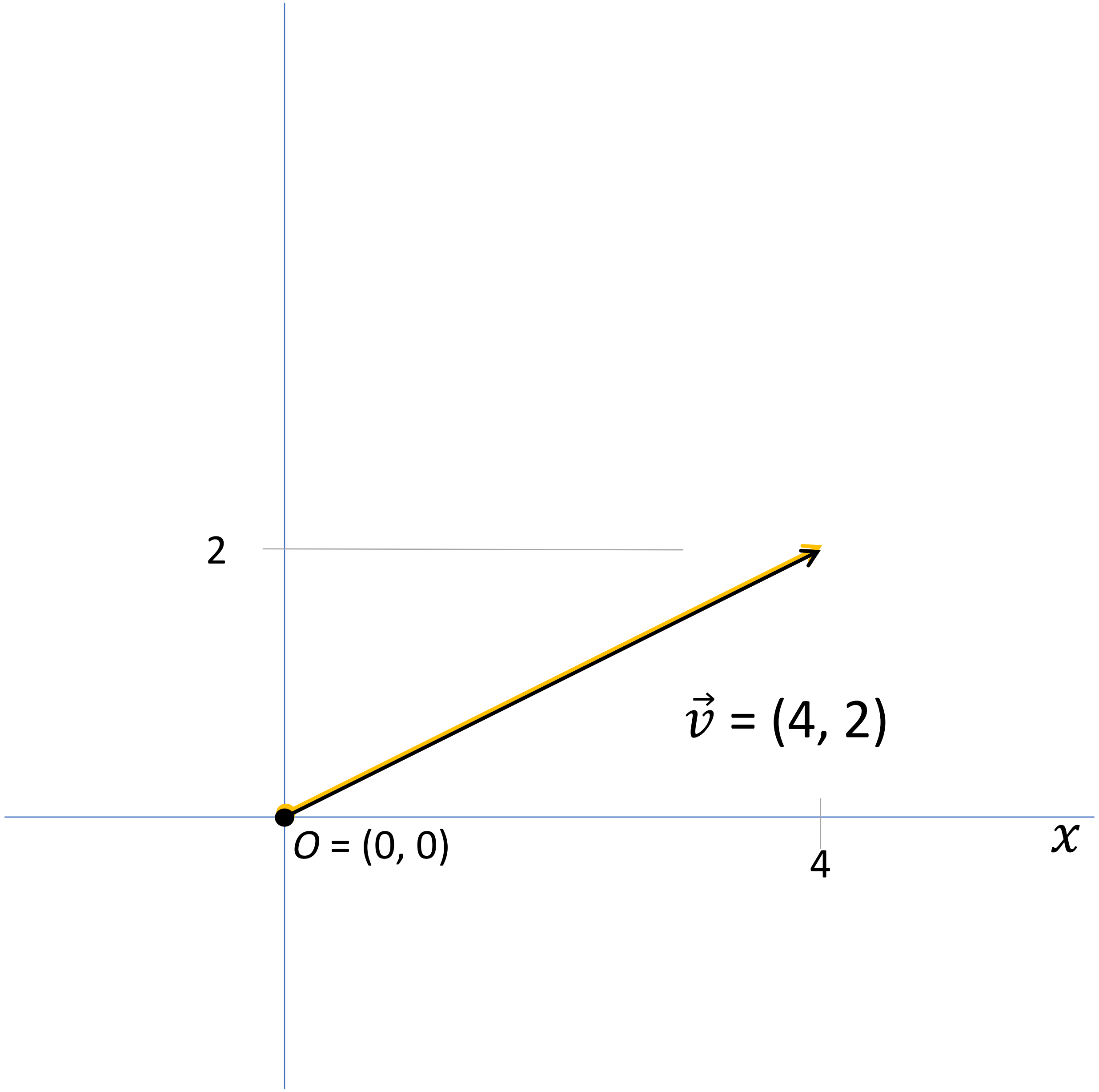


Normalized vector

- direction

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = \sqrt{20}$$



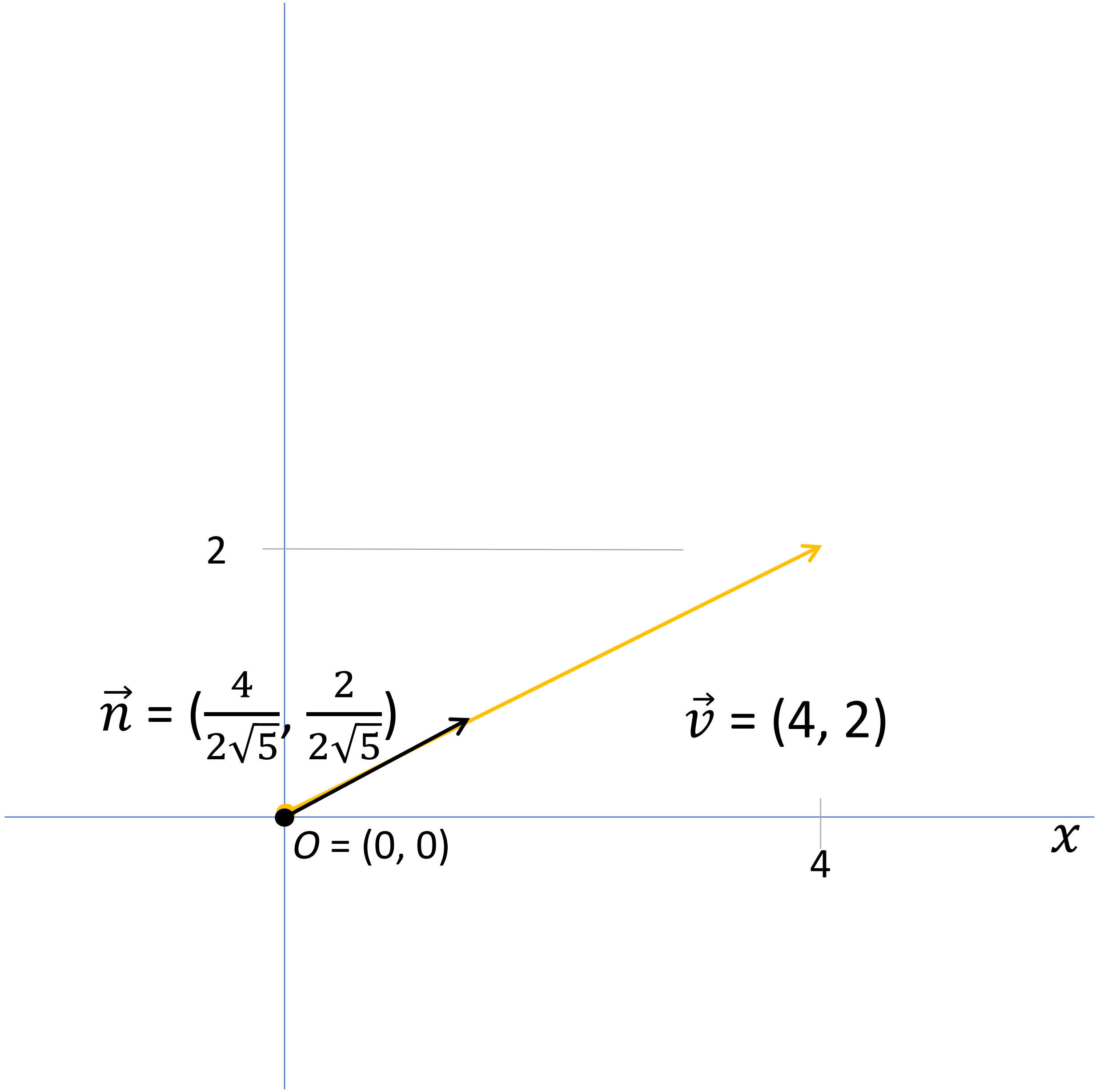
Normalized vector

- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$



Normalized vector

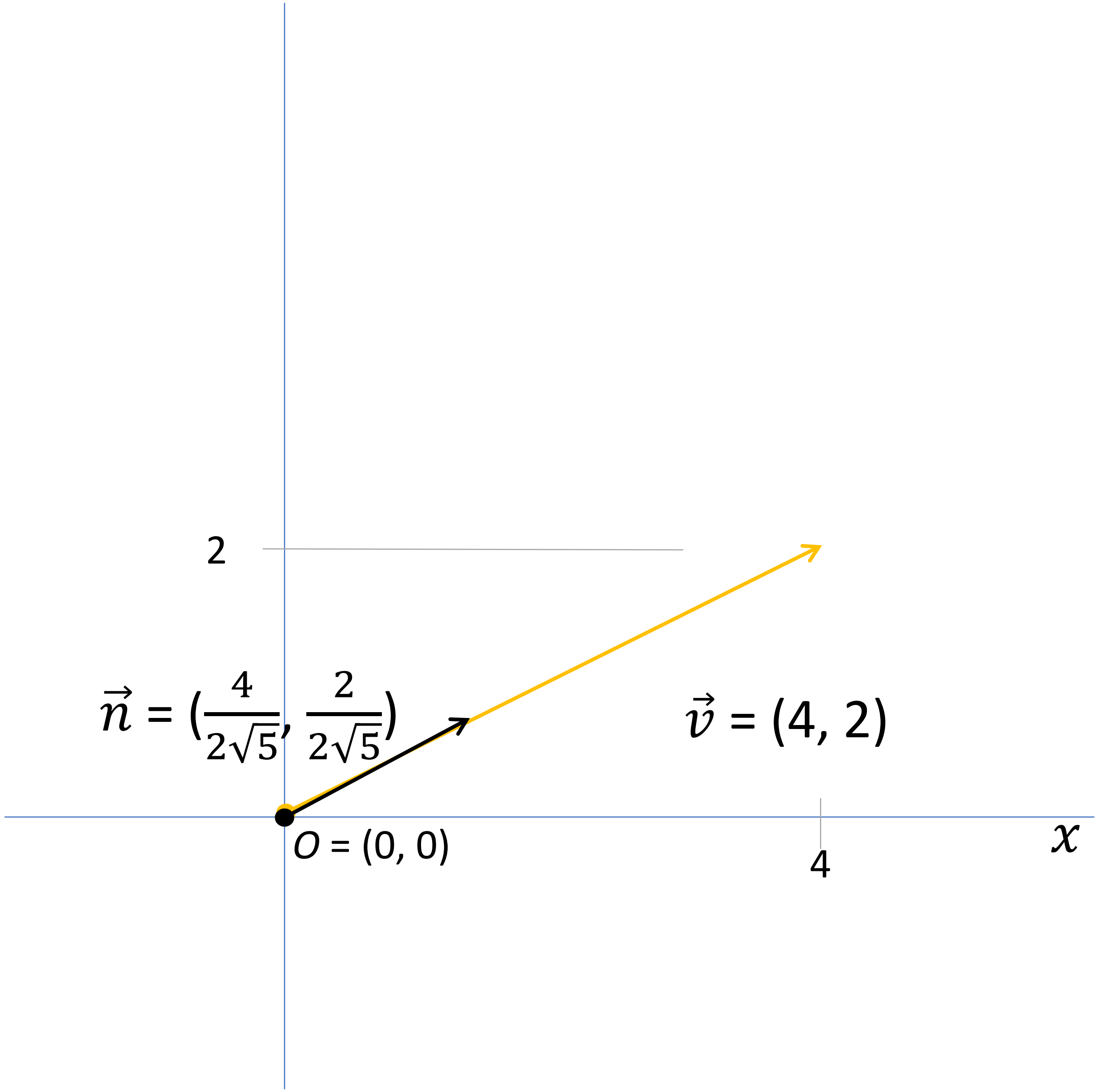
- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

$$|\vec{n}| = ?$$



Normalized vector

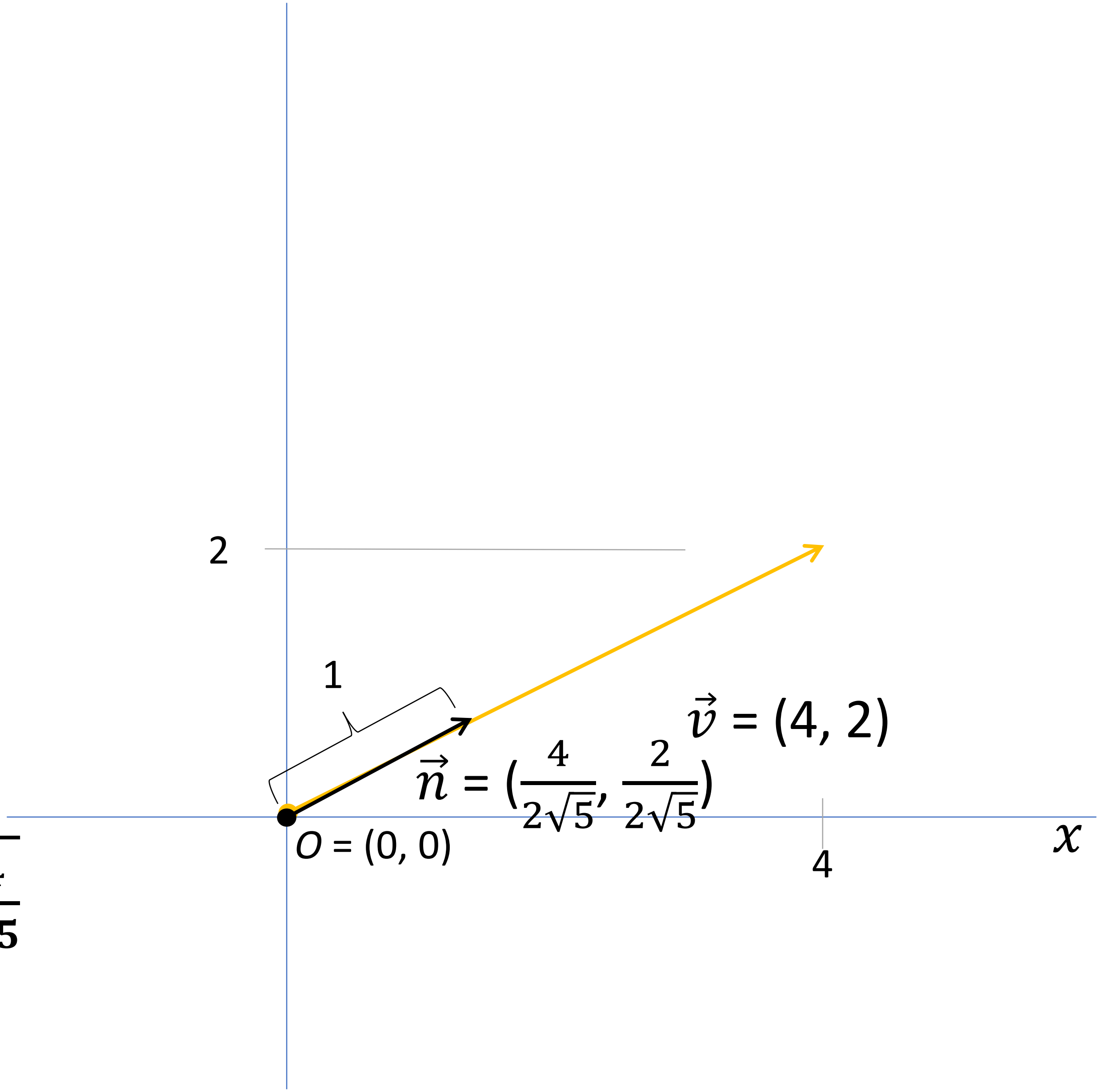
- direction

$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$|\vec{v}| = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

$$|\vec{n}| = \sqrt{\left(\frac{4}{2\sqrt{5}}\right)^2 + \left(\frac{2}{2\sqrt{5}}\right)^2} = \sqrt{\frac{\mathbf{16+4}}{\mathbf{2\times2\times5}}}$$



Normalized vector

- direction

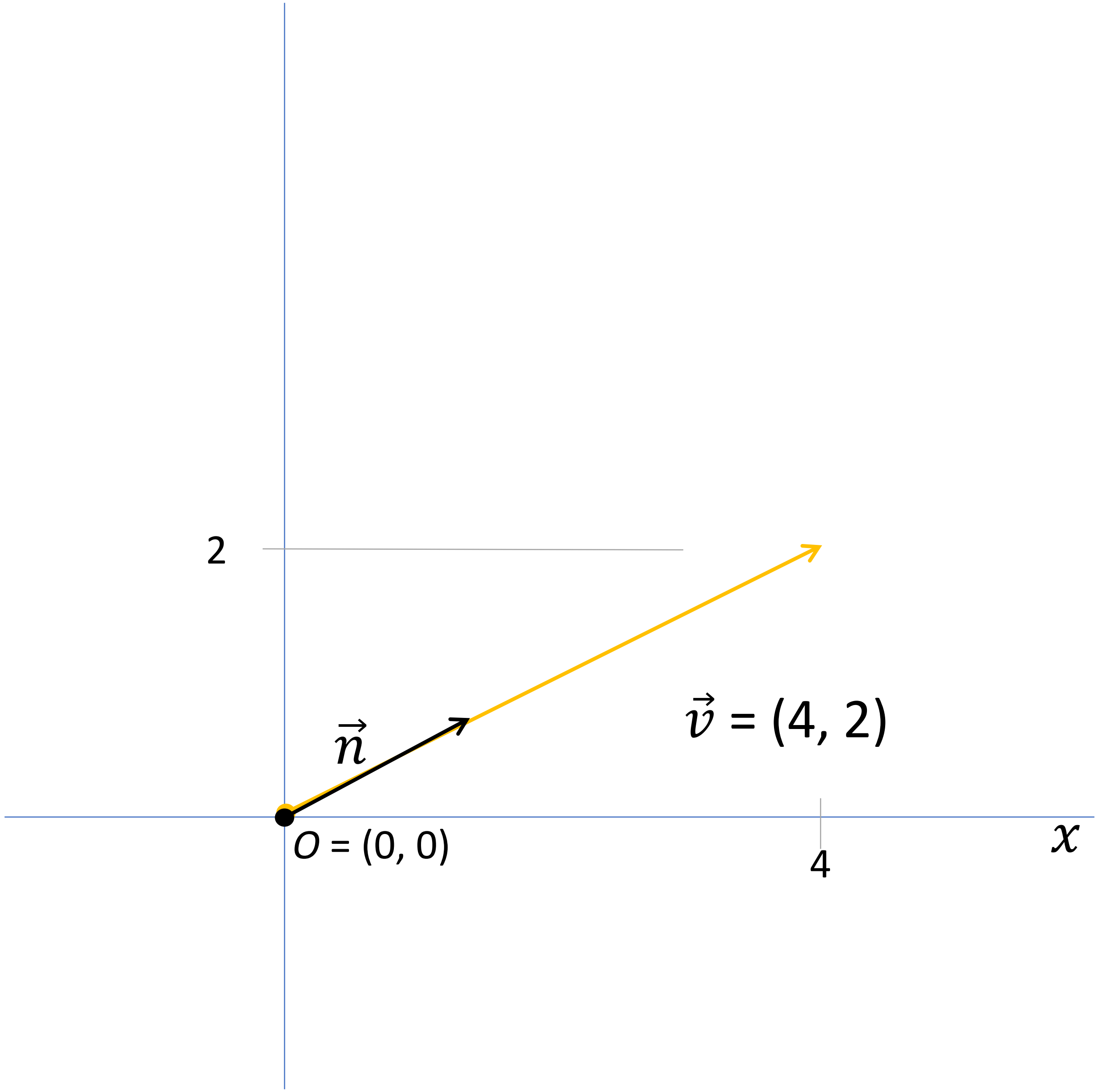
$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4,\ 2)}{2\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

- scalar vector multiplication

$$c\,\vec{n} = ?$$



Normalized vector

- direction

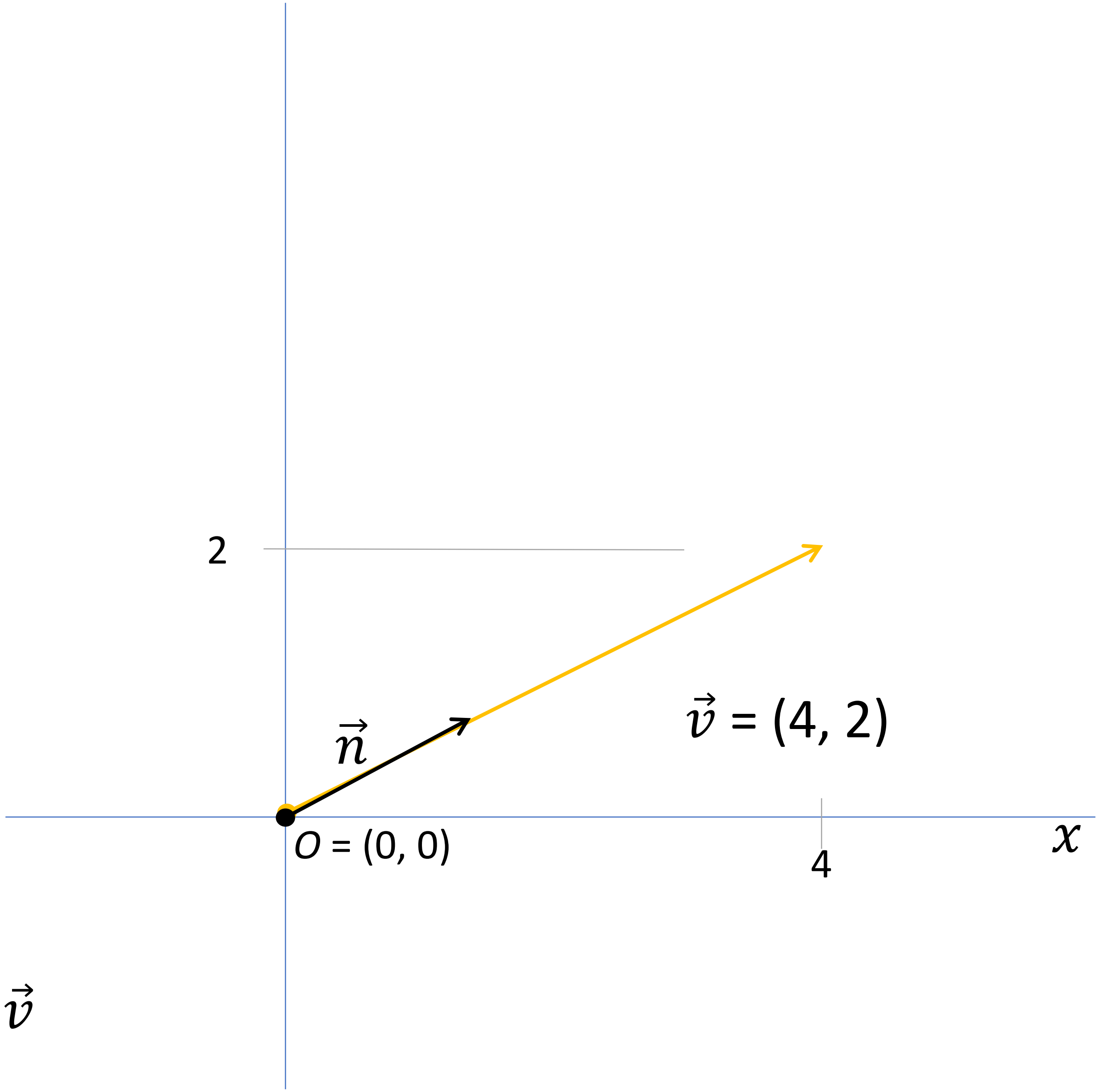
$$\vec{n} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4, 2)}{2\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{4^2 + 2^2} = 2\sqrt{5}$$

- scalar vector multiplication

$$c\,\vec{n} = 2\sqrt{5}\,\left(\frac{4}{2\sqrt{5}}, \frac{2}{2\sqrt{5}}\right) = (4, 2) = \vec{v}$$



Normalized vector

- direction

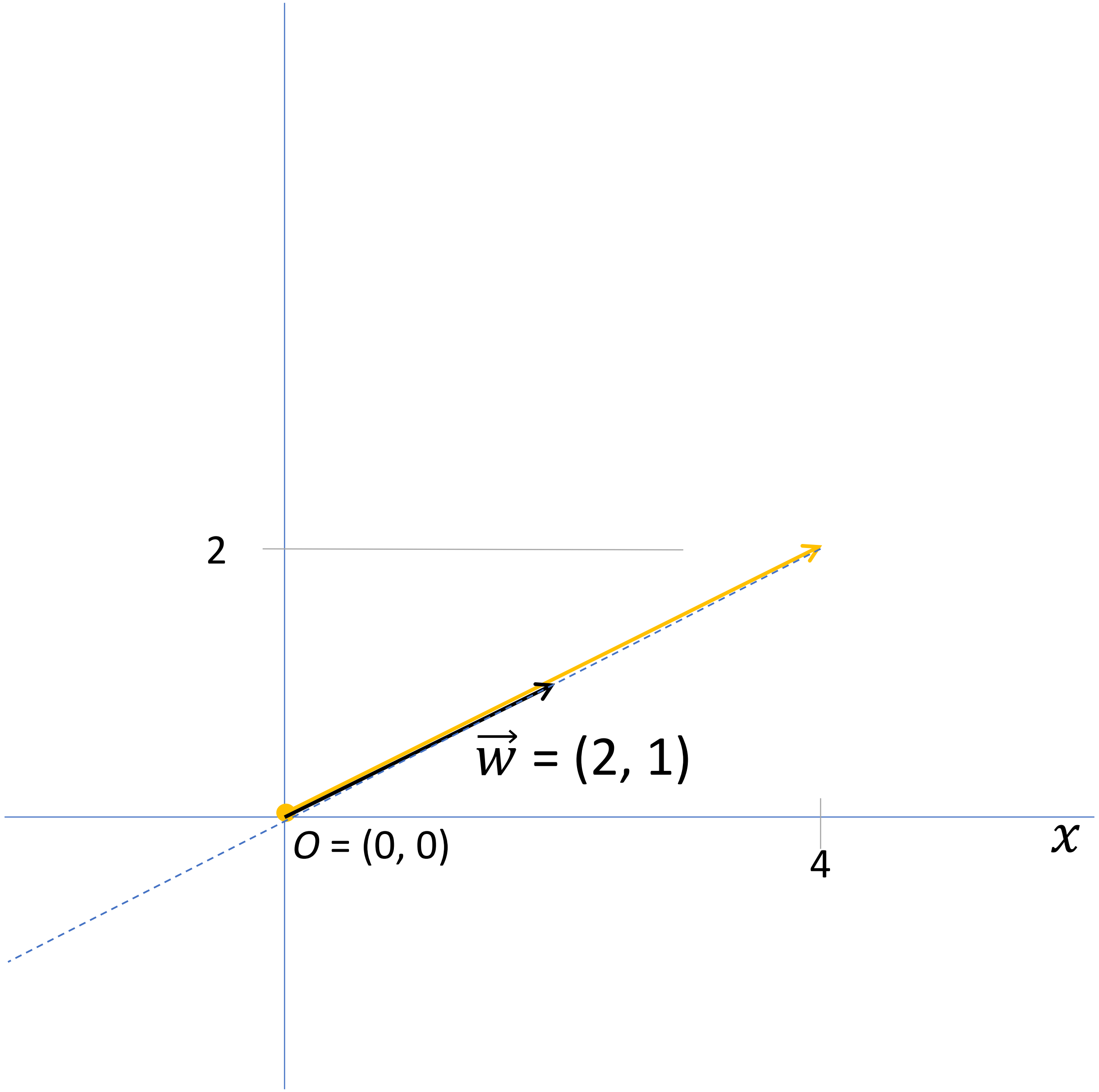
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2, 1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$2\,\vec{w} = 2\,(2, 1) = (4, 2)$$



Normalized vector

- direction

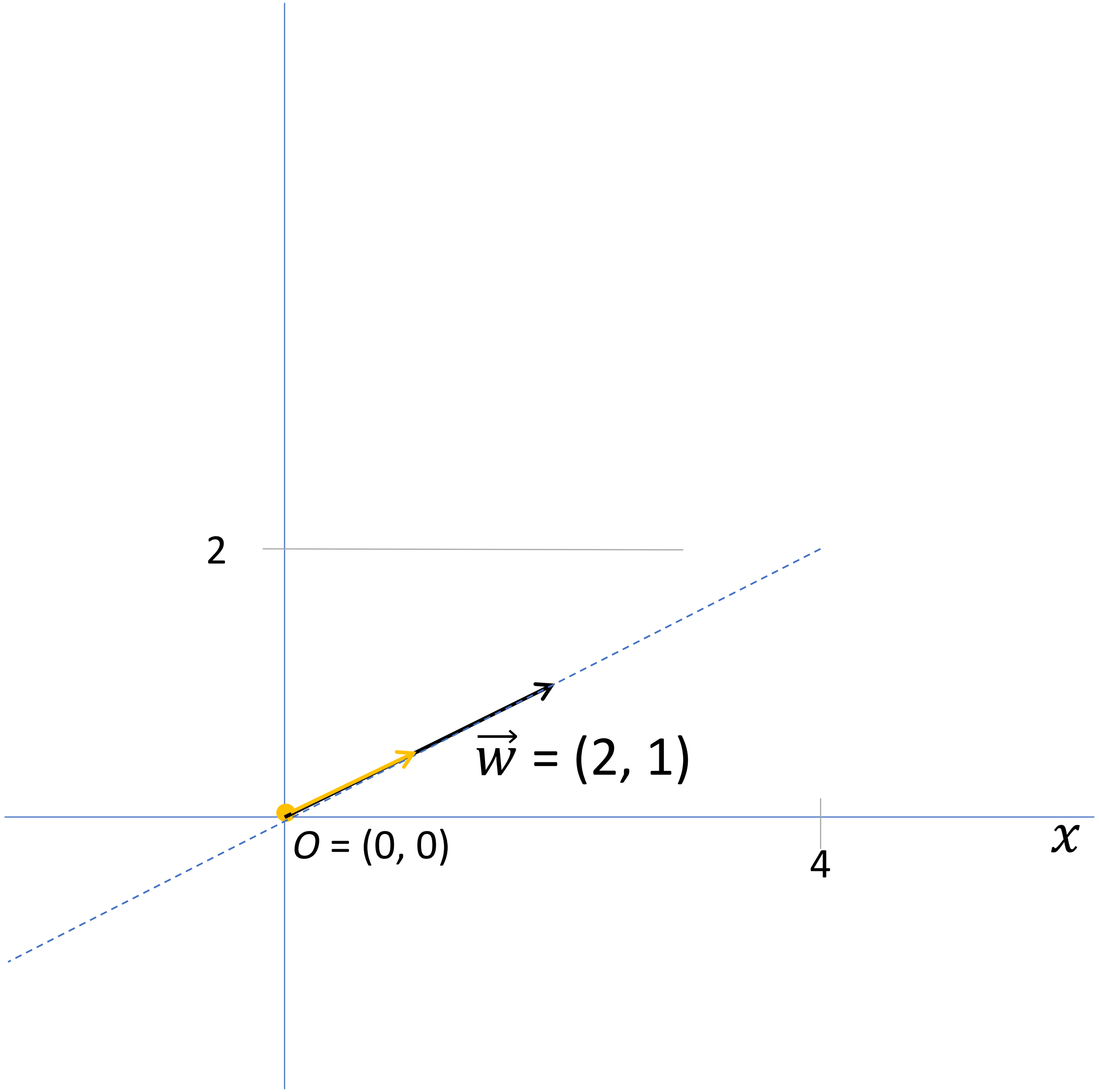
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2,1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$\frac{1}{2}\vec{w} = \frac{1}{2}(2,1) = (1, \frac{1}{2})$$



Normalized vector

- direction

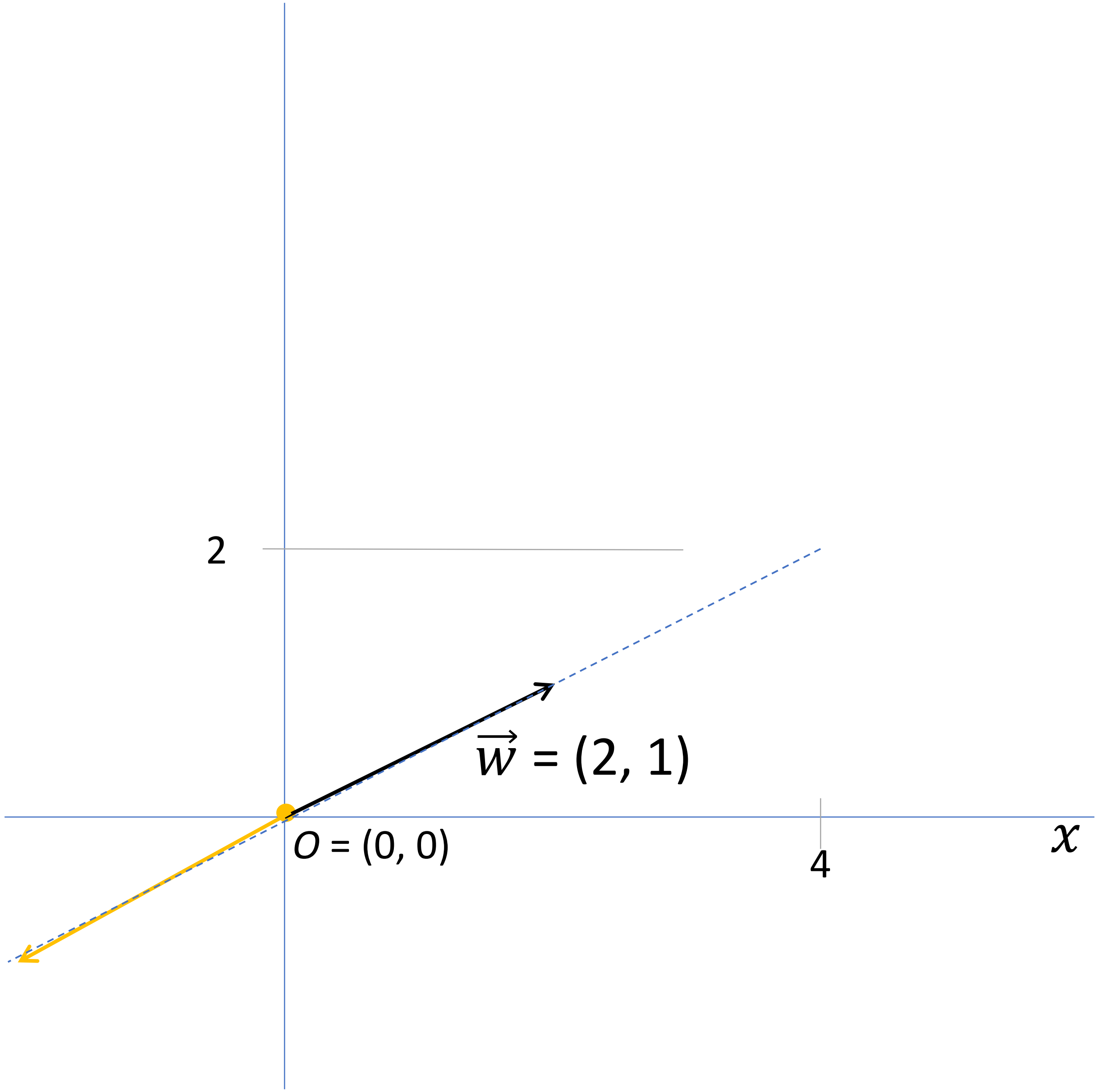
$$\vec{n} = \frac{\vec{w}}{|\vec{w}|} = \frac{(2, 1)}{\sqrt{5}}$$

- magnitude / length

$$c = \sqrt{2^2 + 1^2} = \sqrt{5}$$

- scalar vector multiplication

$$-1\,\vec{w} = -1\,(2, 1) = (-2, -1) = -\vec{w}$$



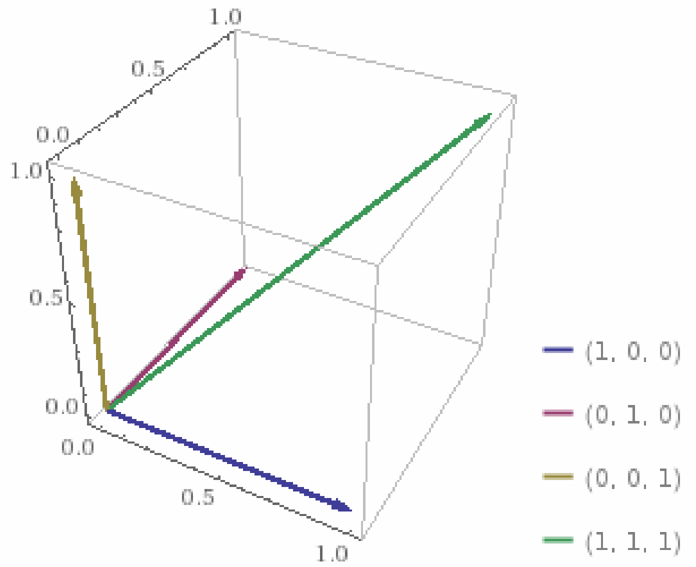
vector (1,0,0) (0,1,0) (0,0,1) (1,1,1)

http://m.wolframalpha.com/

Input interpretation:

{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)}

Vector plot:



Vector lengths:

Approximate forms

(1, 0, 0)	1
(0, 1, 0)	1
(0, 0, 1)	1
(1, 1, 1)	$\sqrt{3}$

- Linear combination

$\overrightarrow{e_1} = (1, 0,0)$
$\overrightarrow{e_2} = (0, 1,0)$
$\overrightarrow{e_3} = (0, 0,1)$

- $x\overrightarrow{e_1} + y\overrightarrow{e_2} + z\overrightarrow{e_3} = (x, y, z)$
- $\overrightarrow{e_1} + \overrightarrow{e_2} + \overrightarrow{e_3} = (1,1,1)$

Matrix

- rectangular array arranged in rows and columns

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

- # rows = 2
- # columns = 3

Matrix

- rectangular array arranged in rows and columns

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

- # rows = 2
- # columns = 3

$$2 \times 3$$

Matrix

- Transpose

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$3 \times 2$$

Matrix

- Transpose

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$3 \times 2$$

- vector as a matrix

$$(1 \;\; 2 \;\; 3)^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$3 \times 1$$

Matrix

- Matrix = 2D array

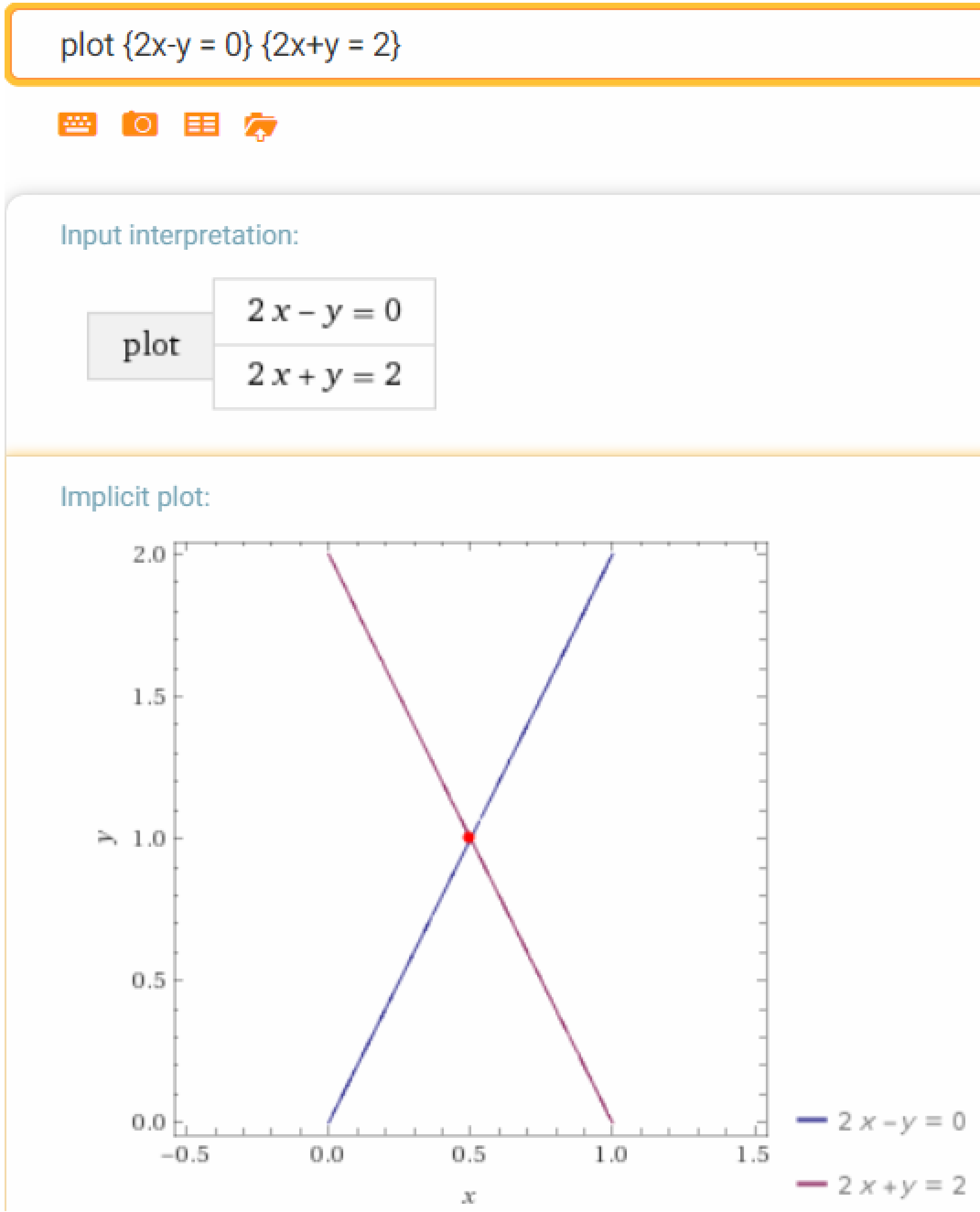
$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

- Vector = a row or column

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Matrix

$$2x - y = 0$$
$$2x + y = 2$$

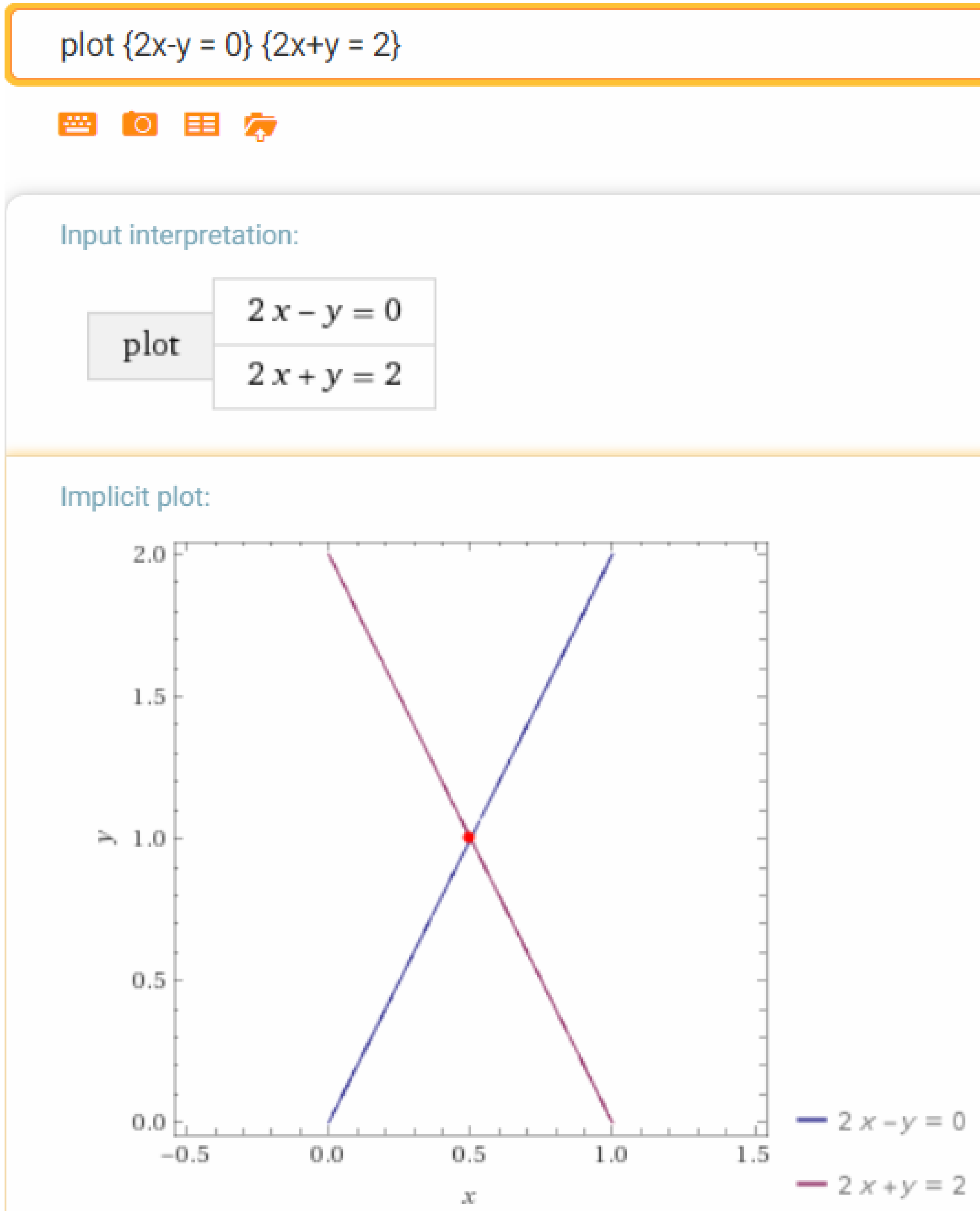


Matrix

$$2x - y = 0$$
$$2x + y = 2$$

multiplication

$$\begin{pmatrix} 2x - y \\ 2x + y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



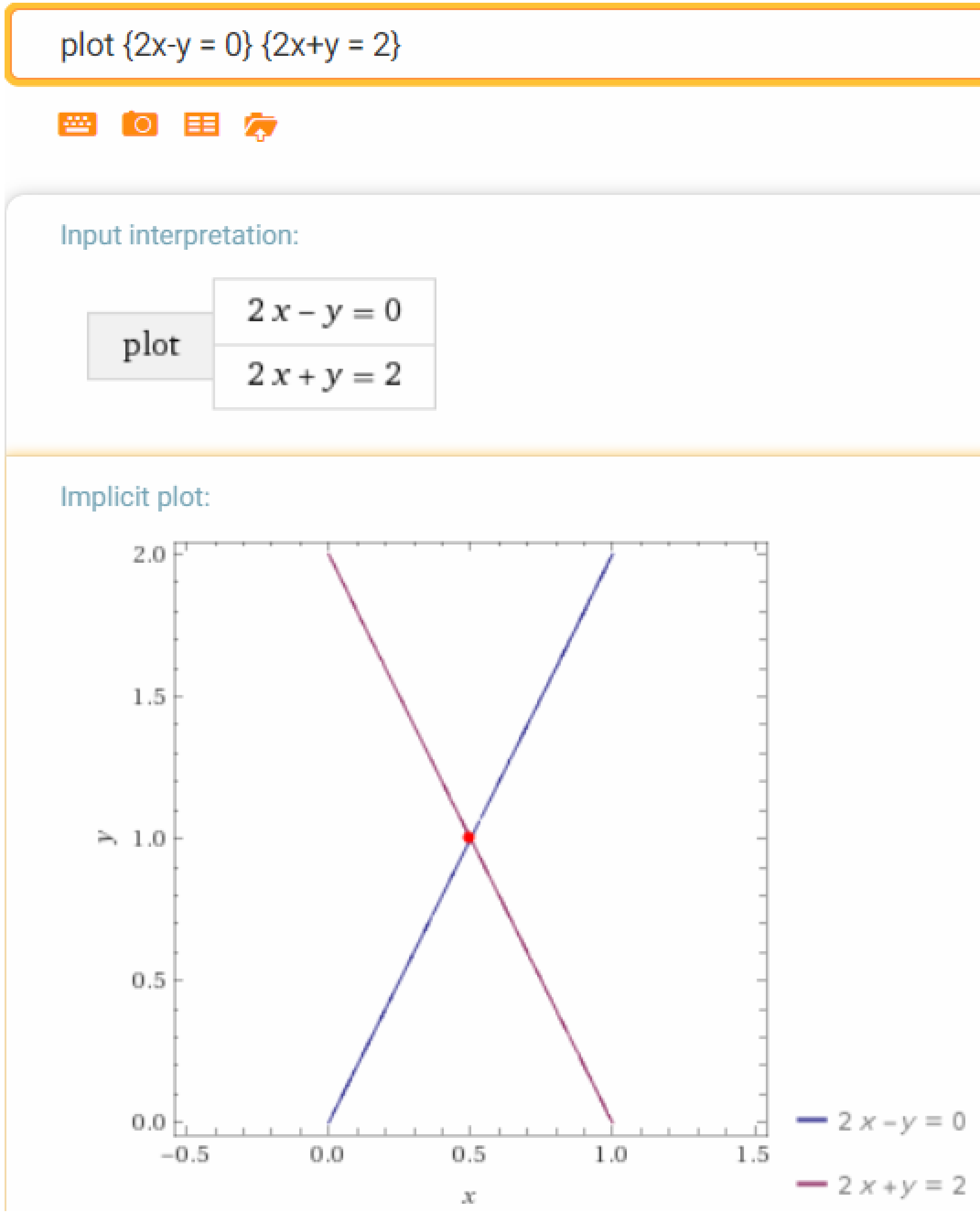
Matrix

$$2x - y = 0$$
$$2x + y = 2$$

multiplication

$$\begin{pmatrix} 2x - y \\ 2x + y \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$



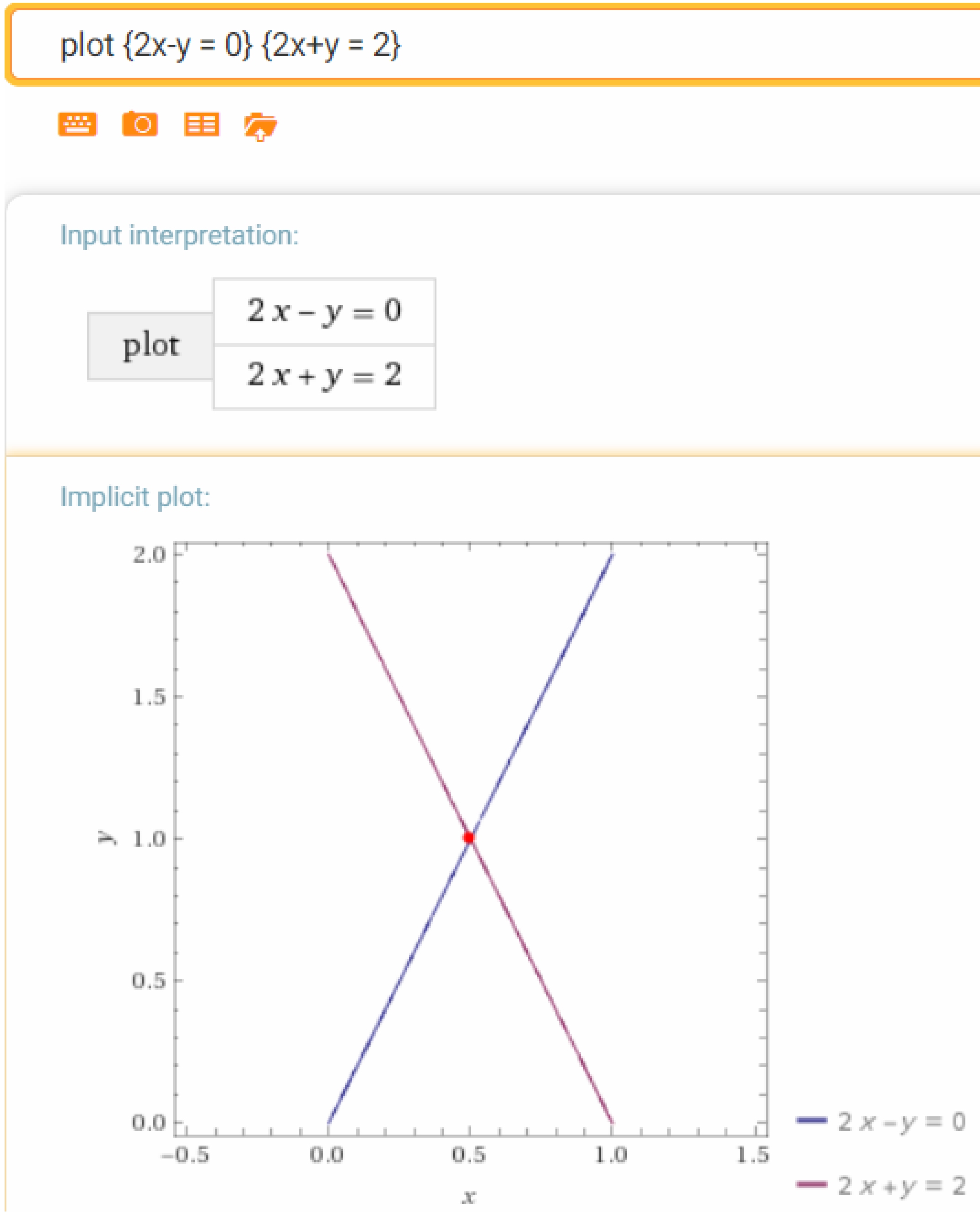
Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & 1 & 2 \end{array}\right) \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 2 & 1 & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 \end{pmatrix} \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 1 \end{array}\right)$$



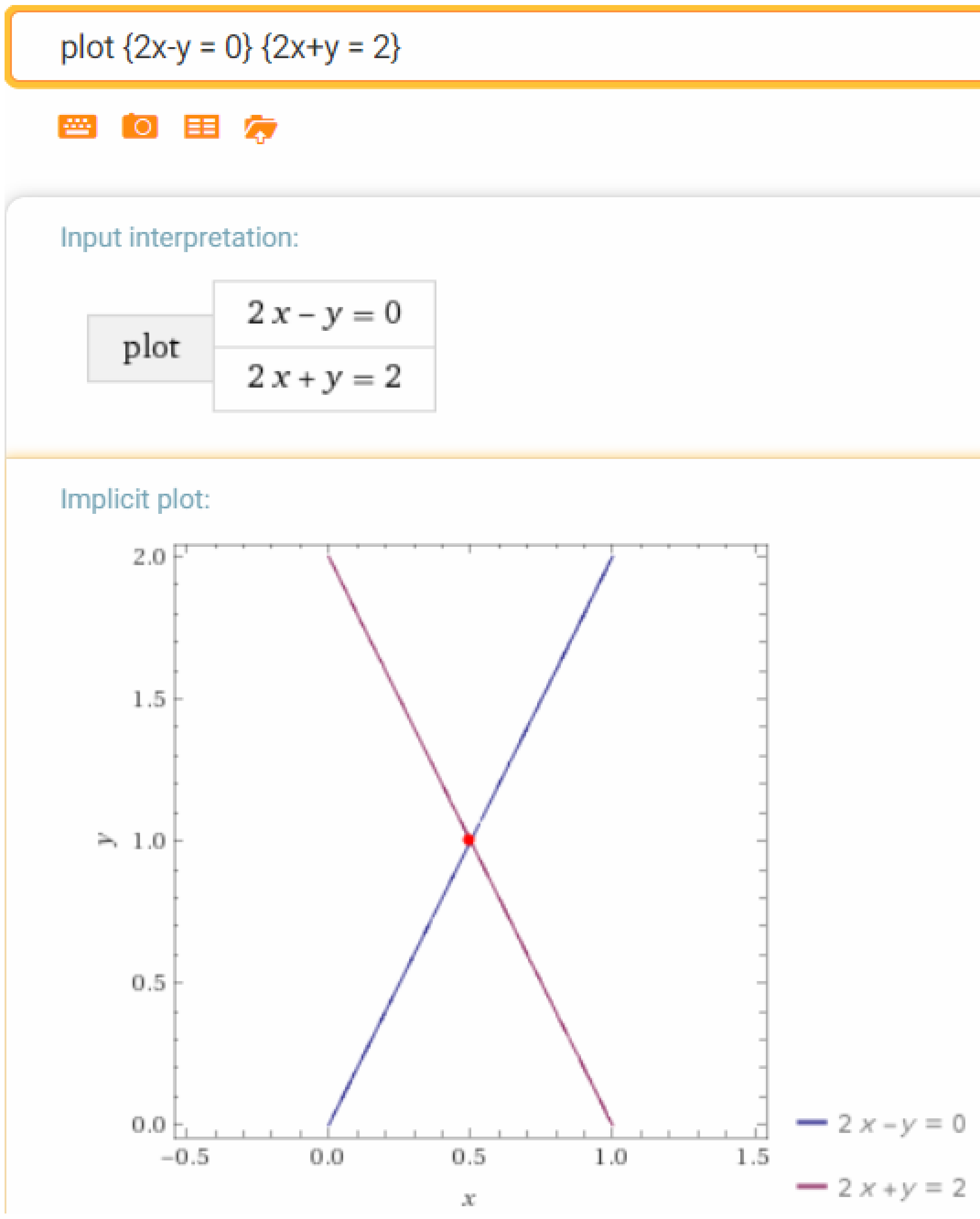
Matrix

$$\left(\begin{array}{cc|c} 2 & -1 & 0 \\ 2 & 1 & 2 \end{array}\right) \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 2 & 1 & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 1 \end{pmatrix} \sim$$

$$\sim \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 1 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

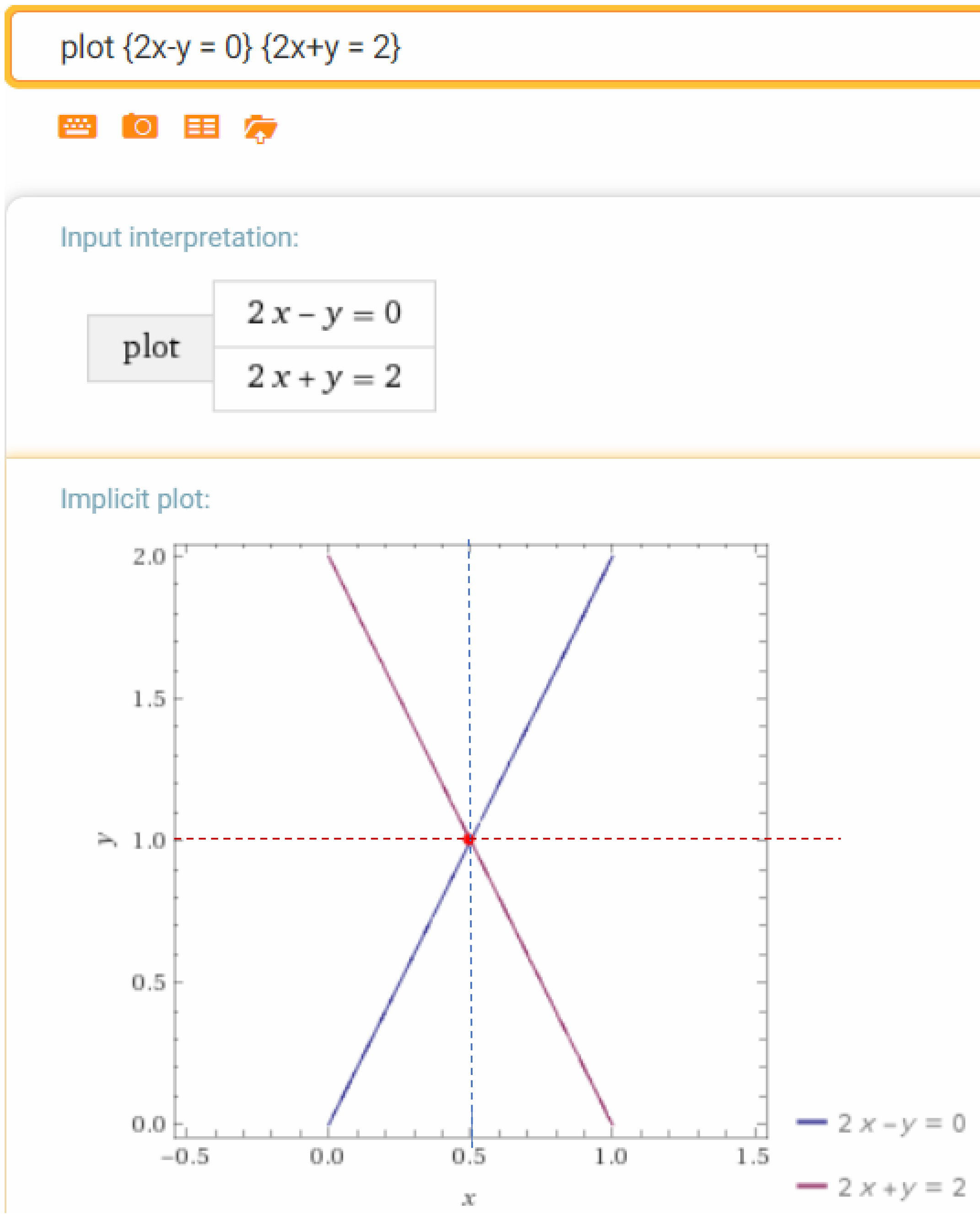


Matrix

$$\left(\begin{array}{cc|c}2 & -1 & 0\\ 2 & 1 & 2\end{array}\right)\sim\left(\begin{array}{cc|c}1 & 0 & \frac{1}{2}\\ 0 & 1 & 1\end{array}\right)$$

$$\begin{pmatrix}2 & -1\\ 2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 2\end{pmatrix}$$

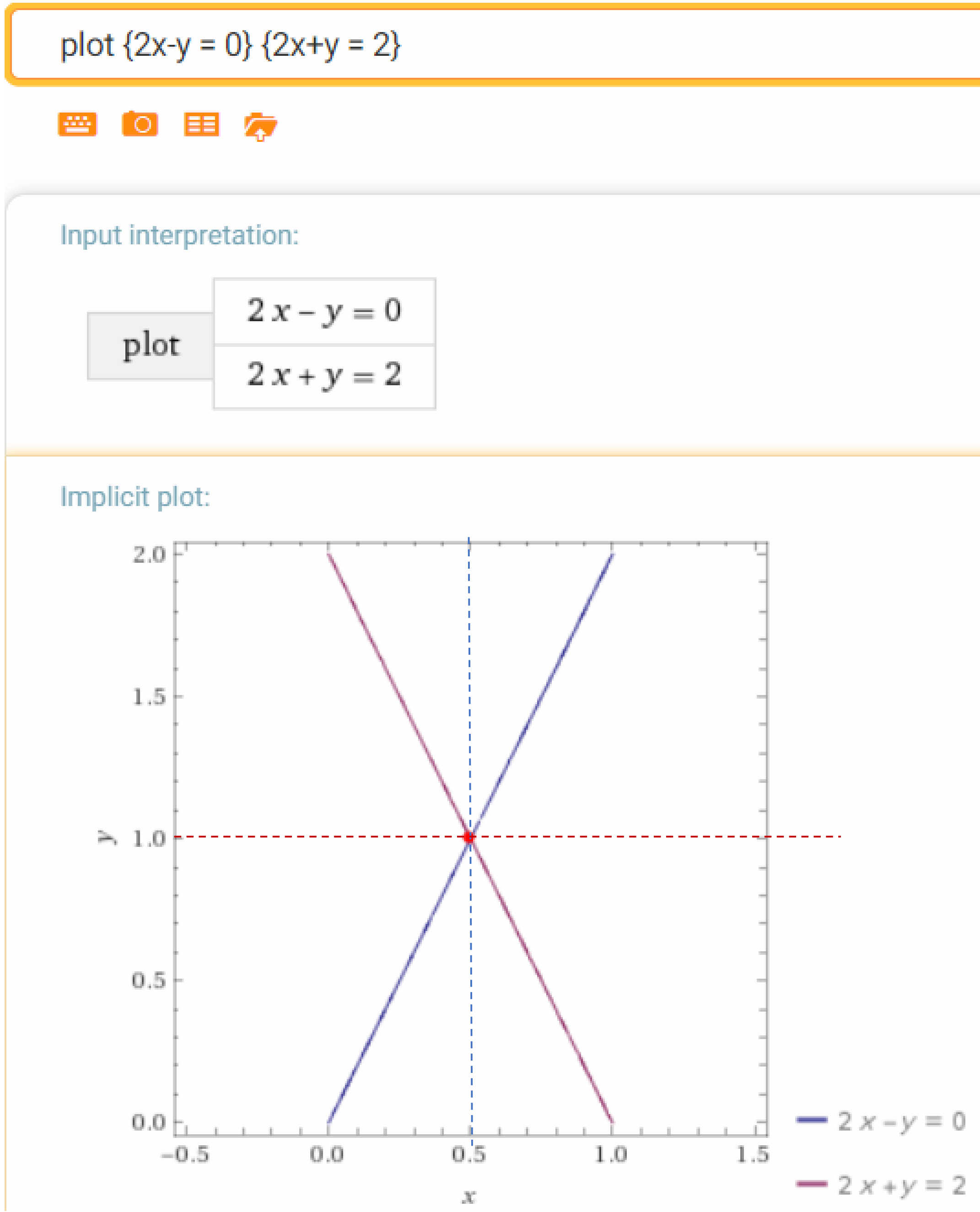
$$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\ 1\end{pmatrix}$$



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

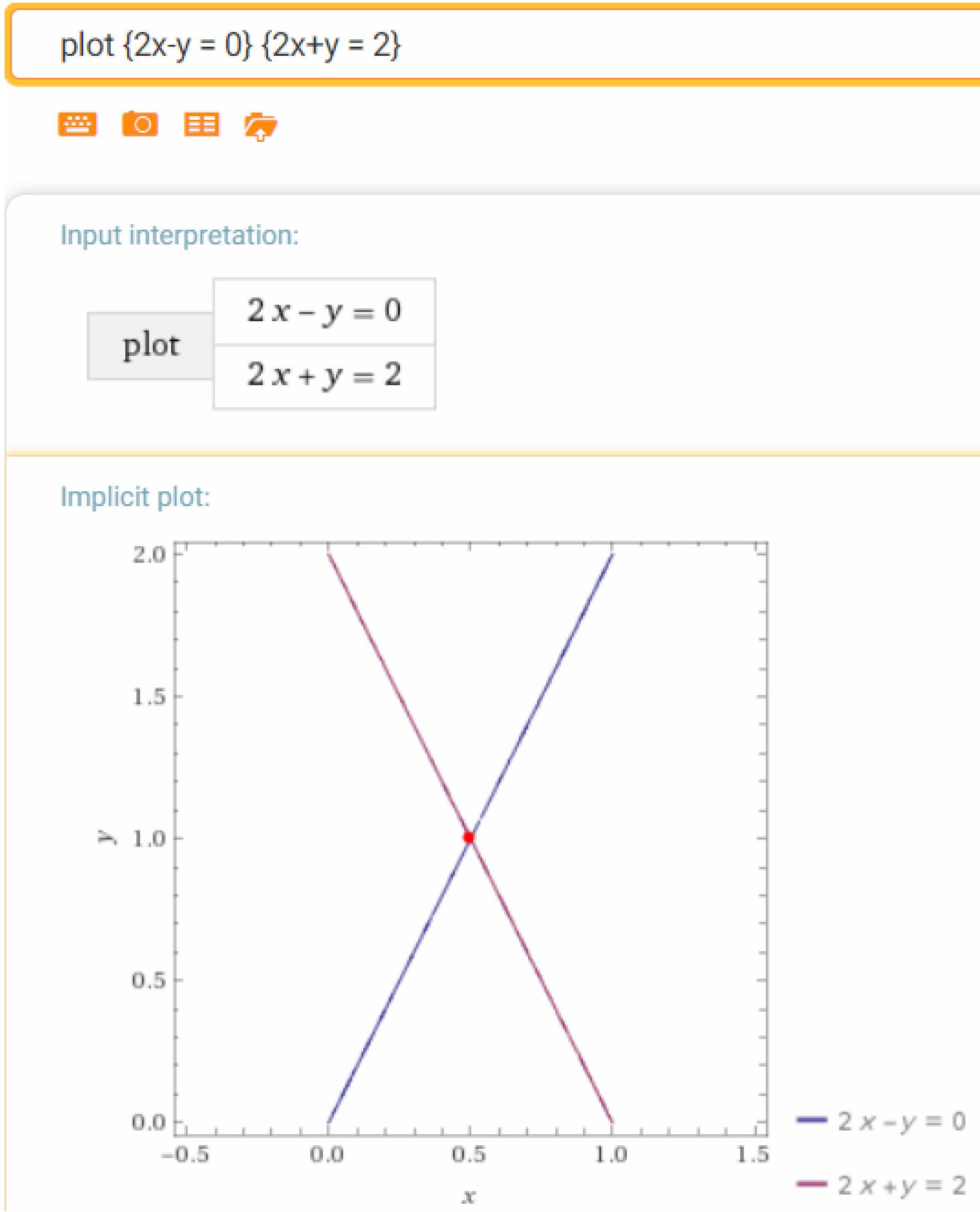


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x\begin{pmatrix} 2 \\ 2 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$



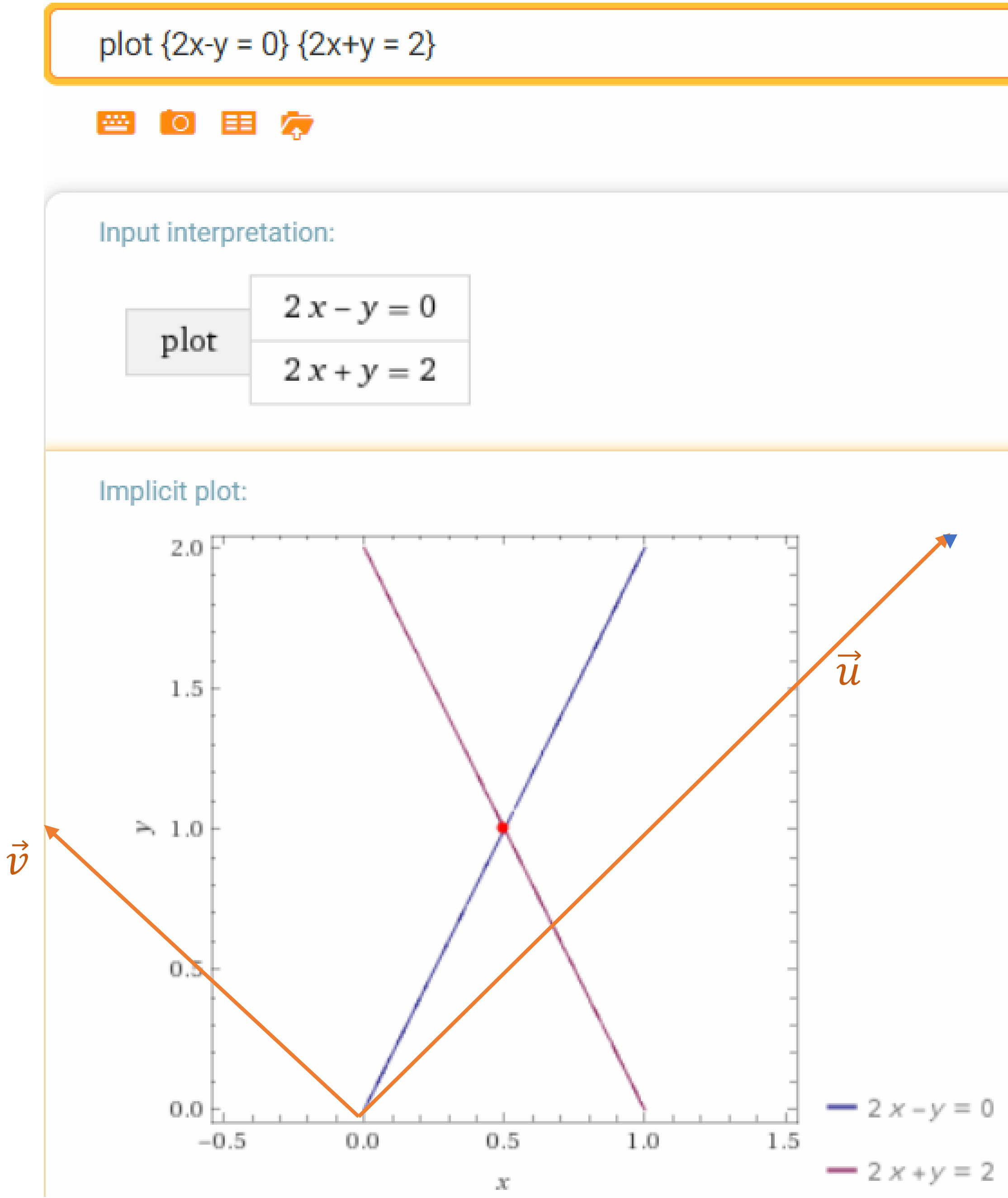
Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



Matrix

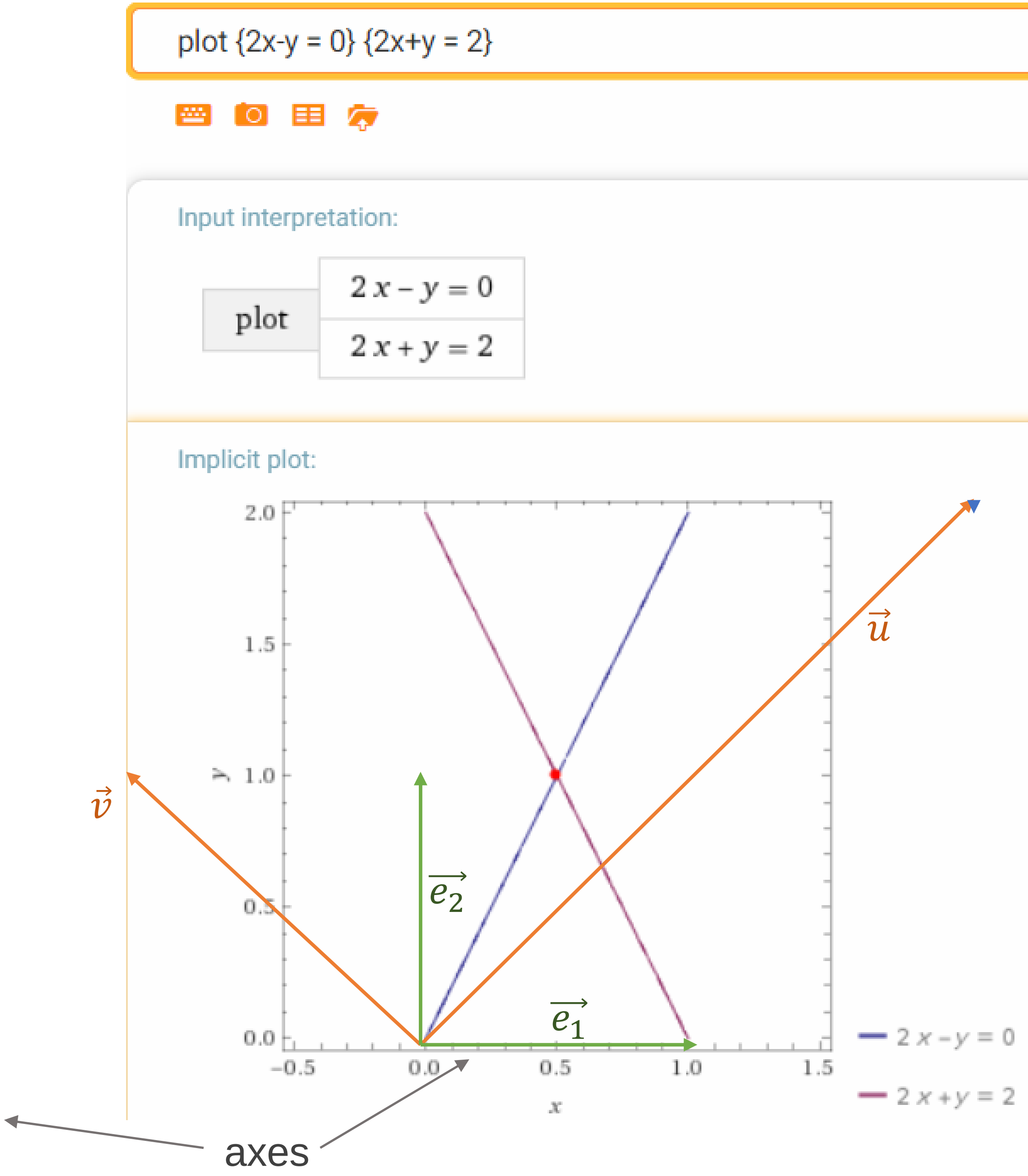
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

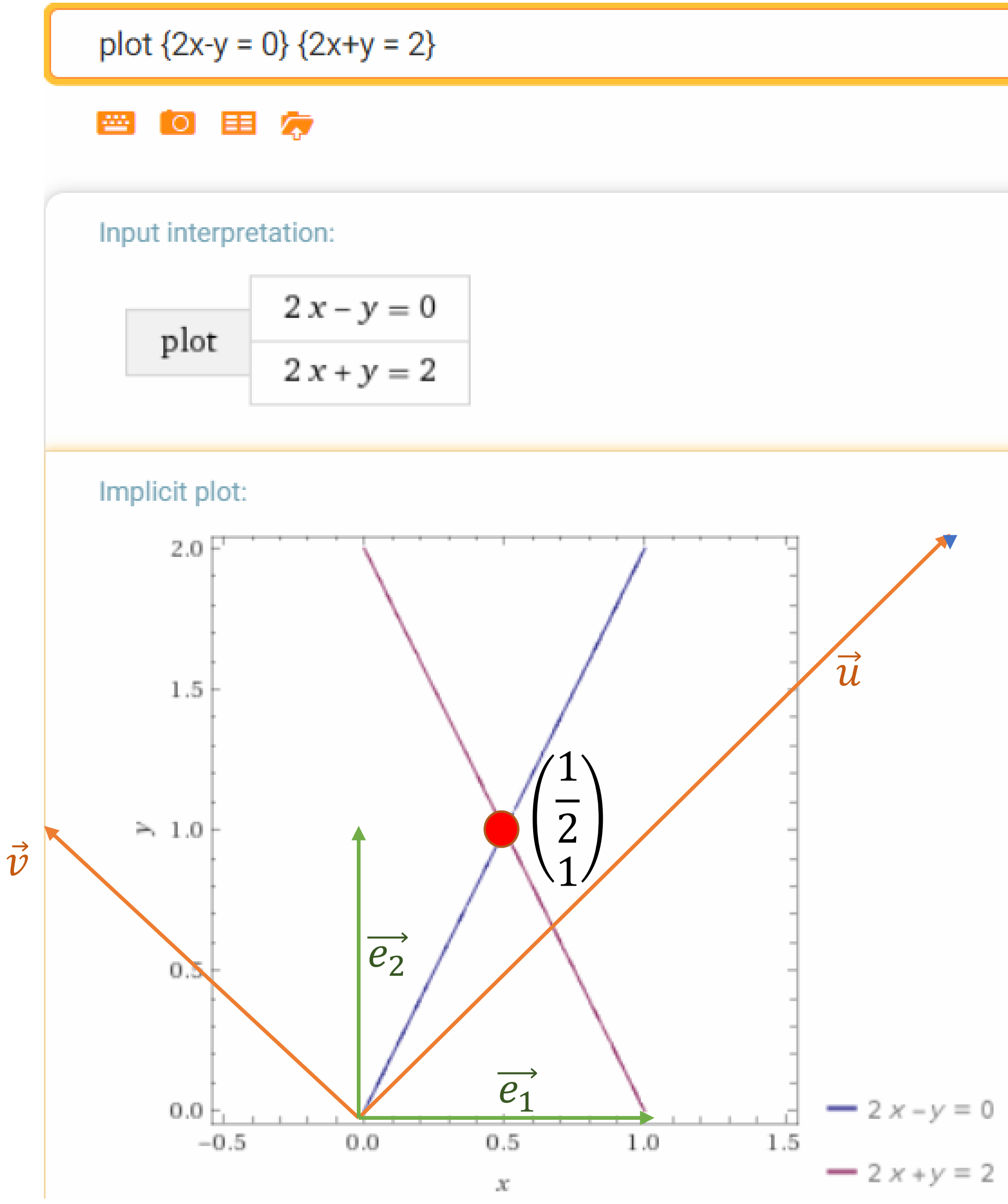
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$$

$$x \begin{pmatrix} 2 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \overrightarrow{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

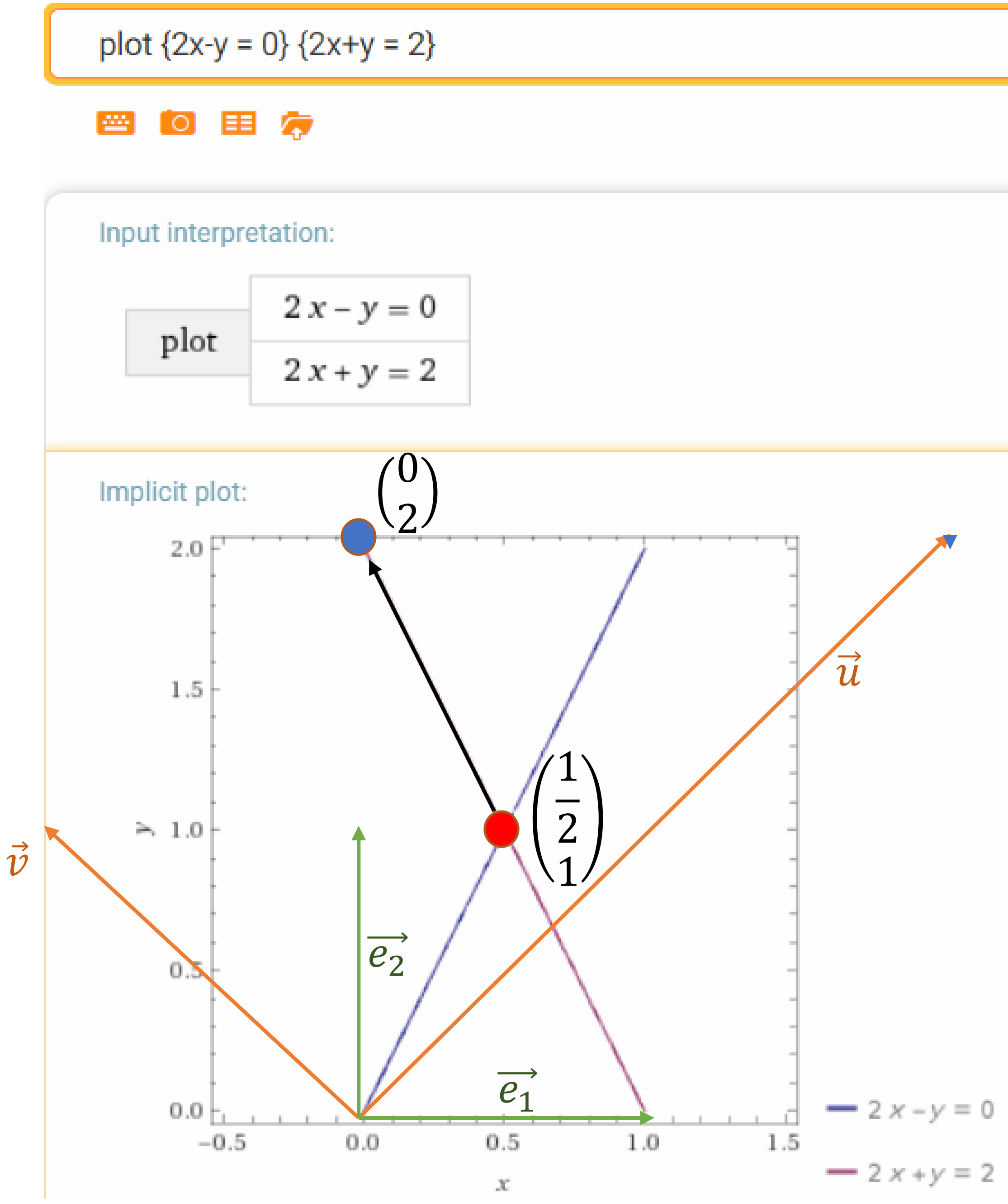
$$\begin{pmatrix}2 & -1\\ 2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}0\\ 2\end{pmatrix}$$

$$\begin{pmatrix}2 & -1\\ 2 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ 1\end{pmatrix} = \begin{pmatrix}0\\ 2\end{pmatrix}$$

$$x\begin{pmatrix}2\\ 2\end{pmatrix} + y\begin{pmatrix}-1\\ 1\end{pmatrix} = \begin{pmatrix}0\\ 2\end{pmatrix}$$

$$\vec{u} = \begin{pmatrix}2\\ 2\end{pmatrix};\ \vec{v} = \begin{pmatrix}-1\\ 1\end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix}1\\ 0\end{pmatrix};\ \vec{e_2} = \begin{pmatrix}0\\ 1\end{pmatrix}$$

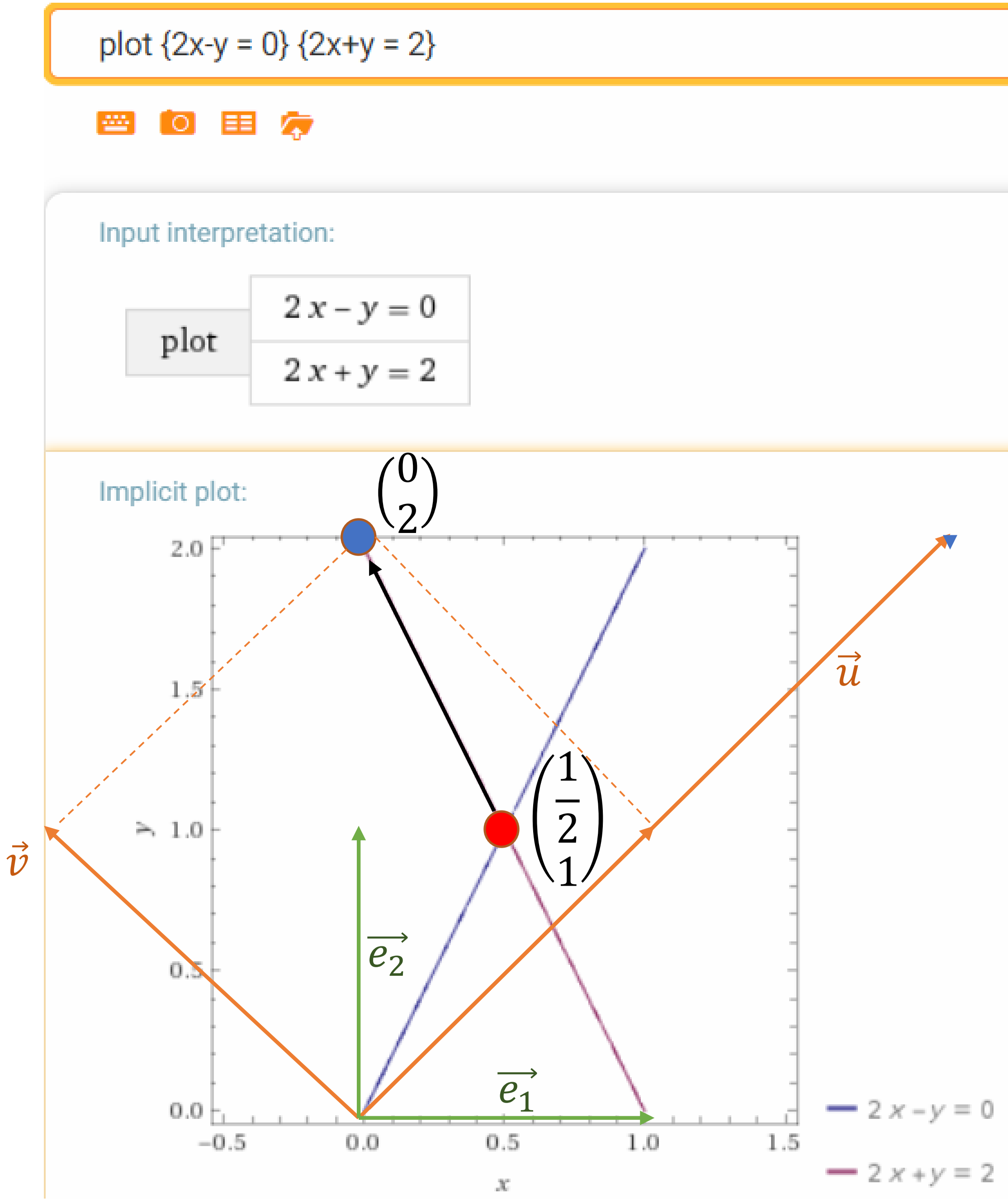


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$x\vec{u} + y\vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$



$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$; $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\overrightarrow{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\overrightarrow{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Matrix

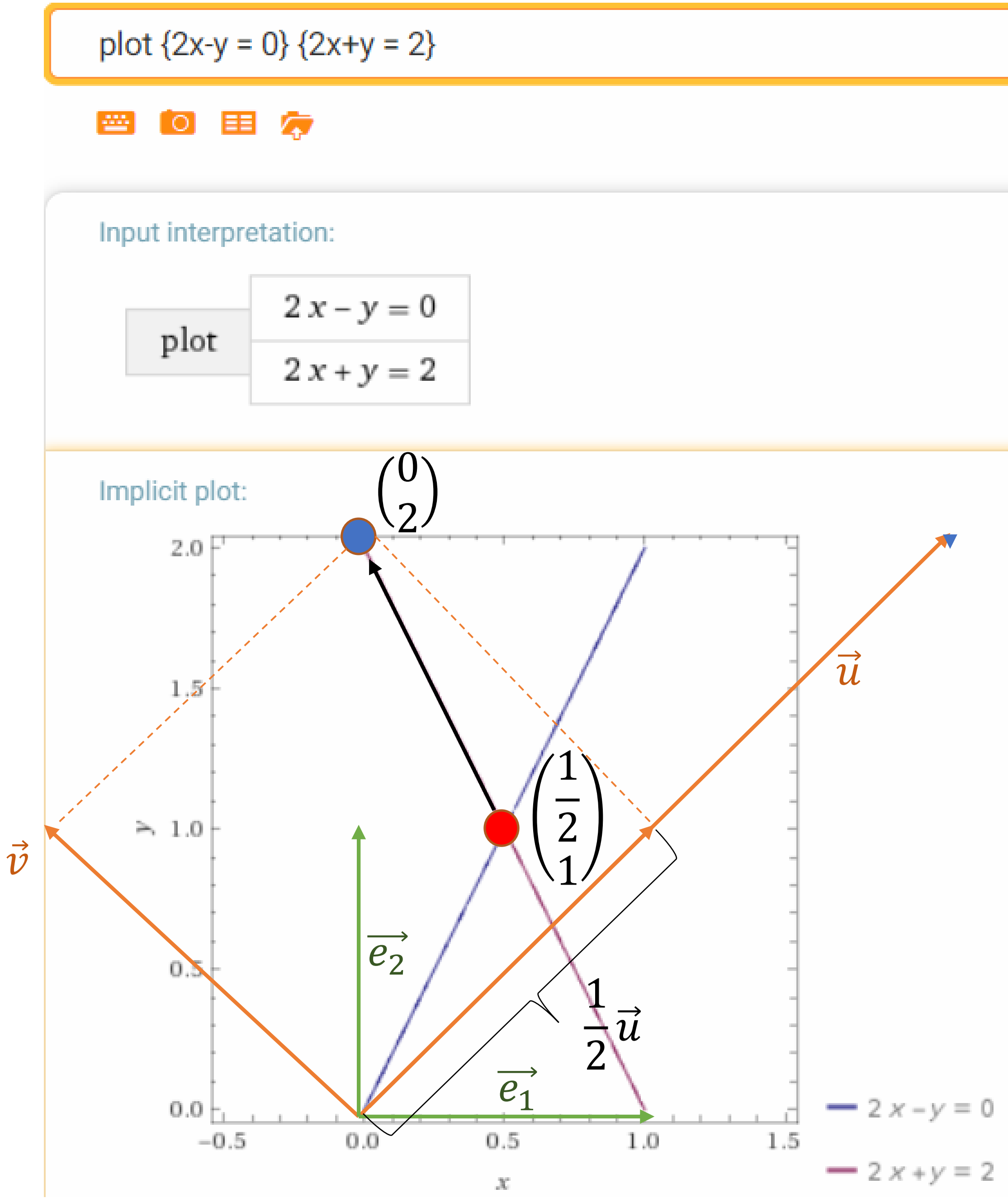
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\frac{1}{2}\vec{u} + y\vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

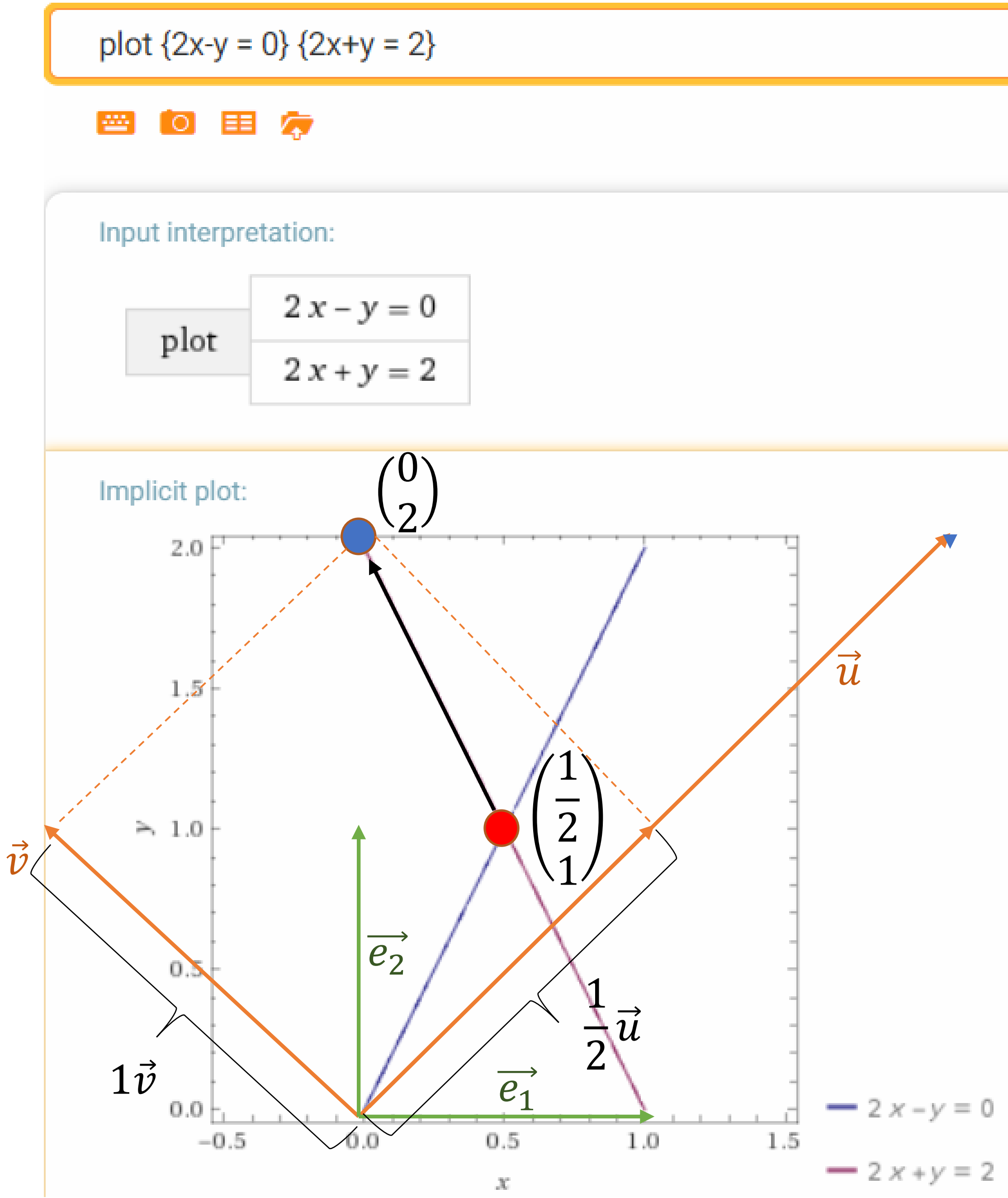
$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\frac{1}{2}\vec{u} + 1\vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

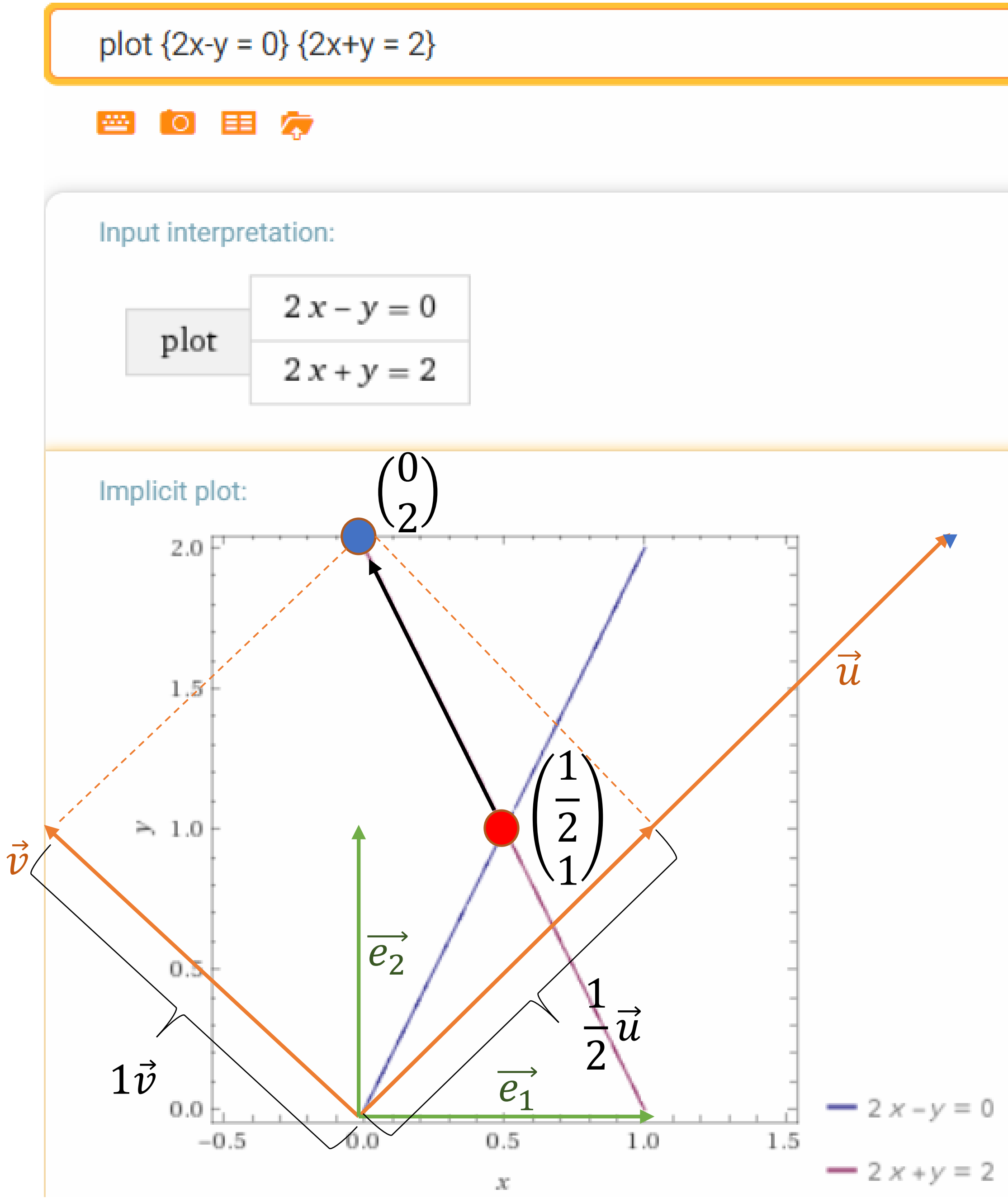


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\overrightarrow{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \overrightarrow{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

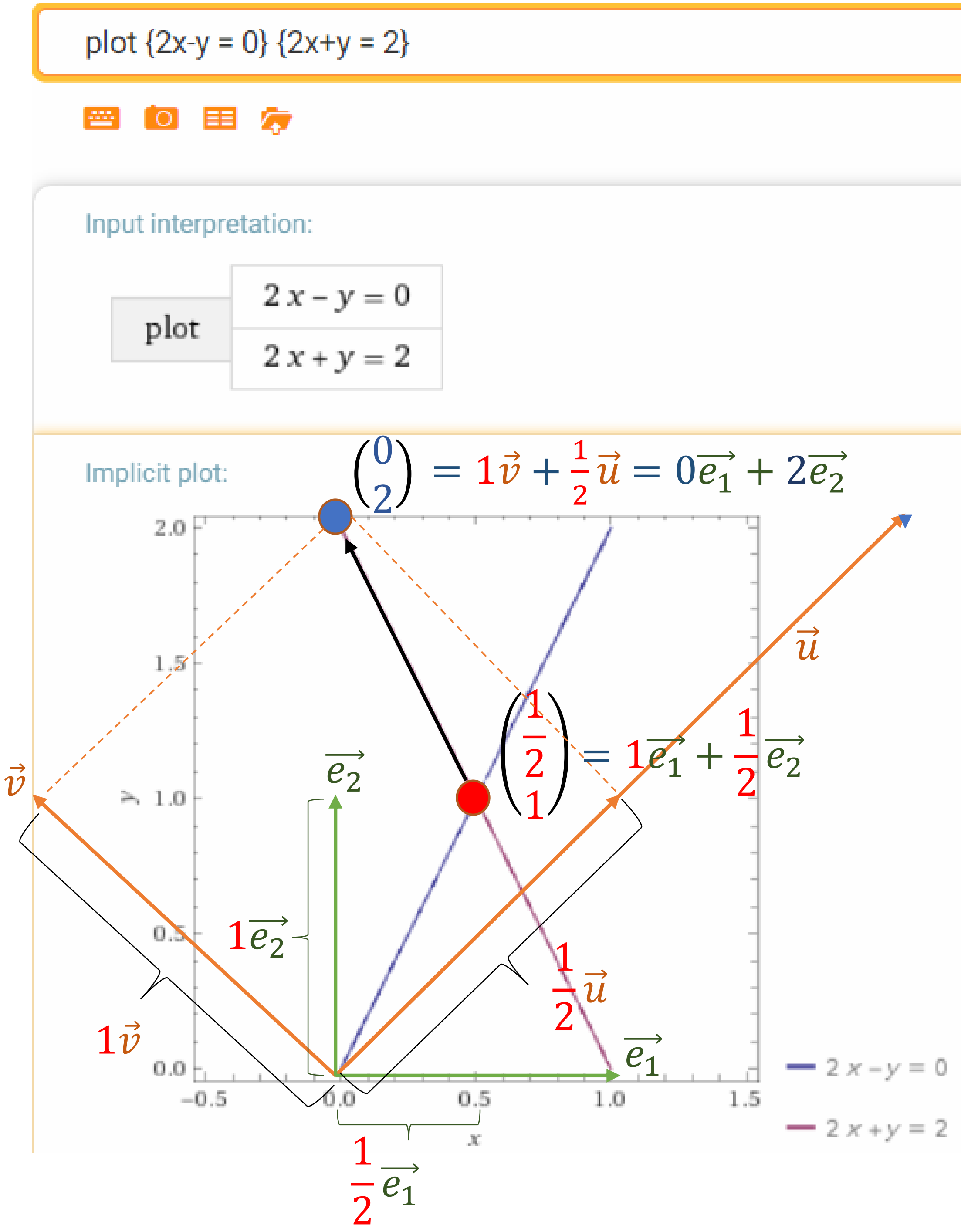


Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

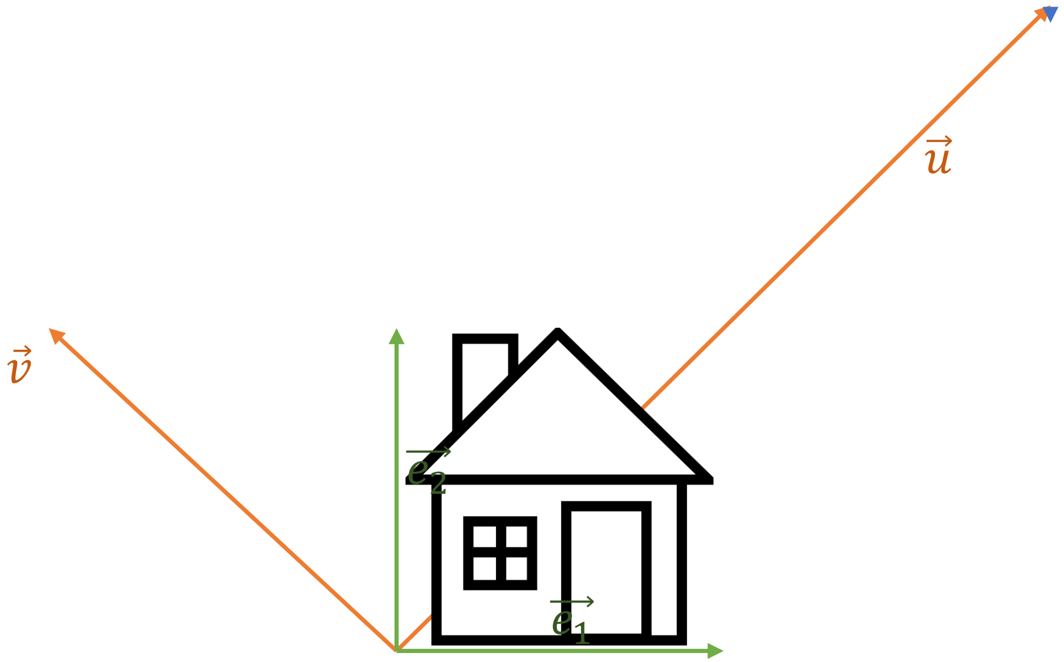
$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\ \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

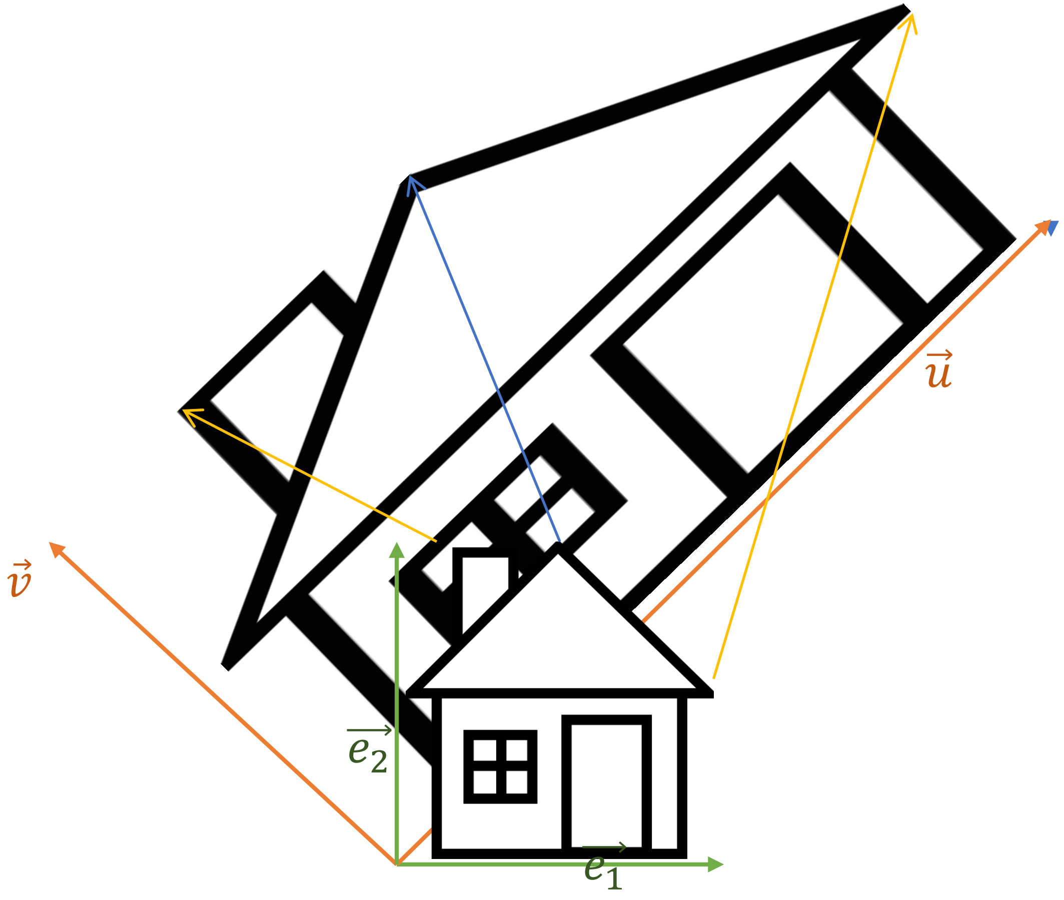


$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrix as a transformation

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

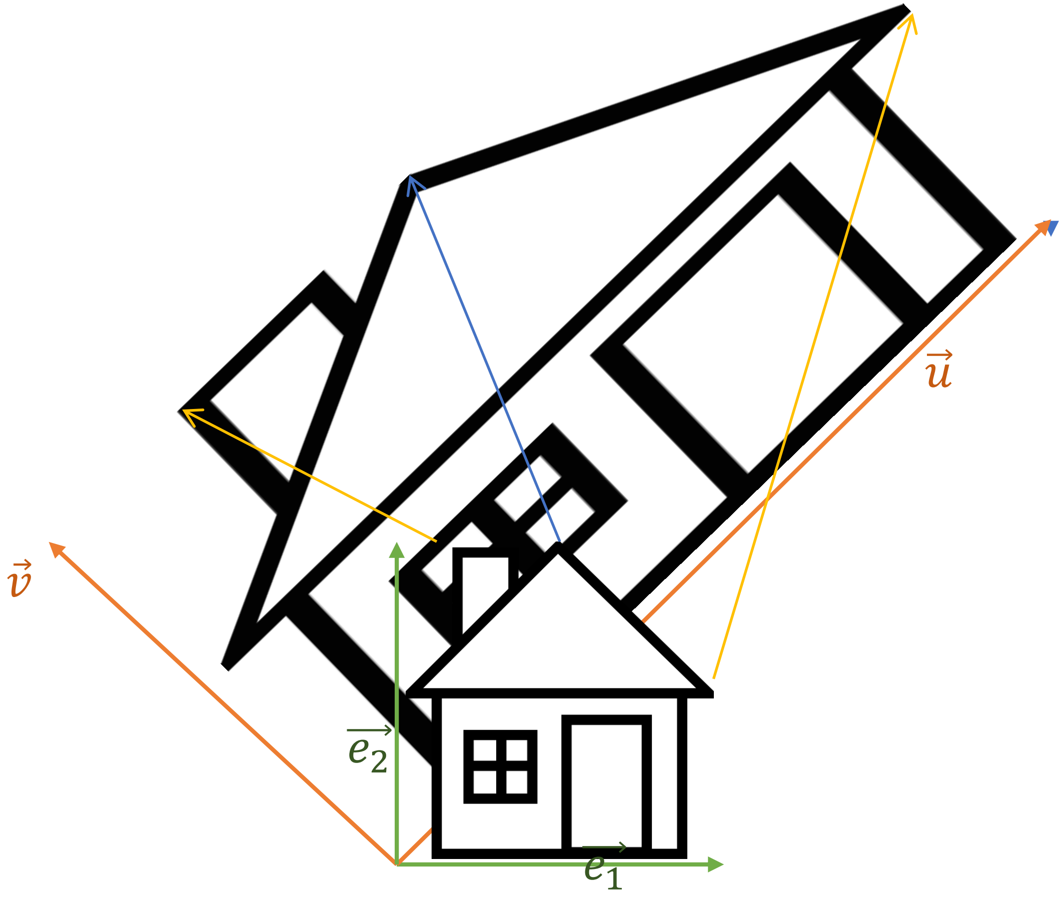


$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrix as a transformation

$$\begin{pmatrix} 2 & -1 \\ 2 & 1 \end{pmatrix}$$



$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$; $\vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\overrightarrow{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\overrightarrow{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

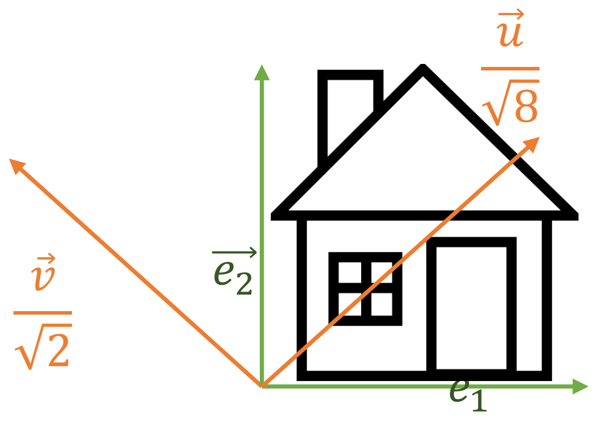
Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



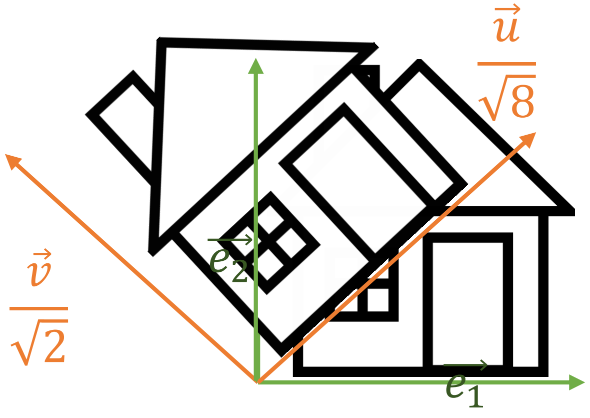
Matrix as a transformation

$$\begin{pmatrix} \cos 45^o & -\sin 45^o \\ \sin 45^o & \cos 45^o \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



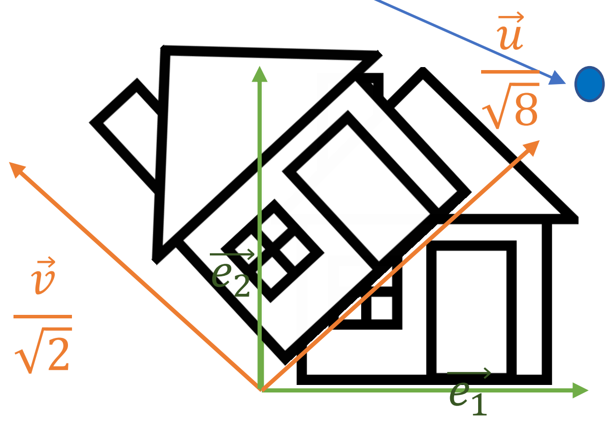
Matrix as a transformation

$$\begin{pmatrix} \cos 45^o & -\sin 45^o \\ \sin 45^o & \cos 45^o \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

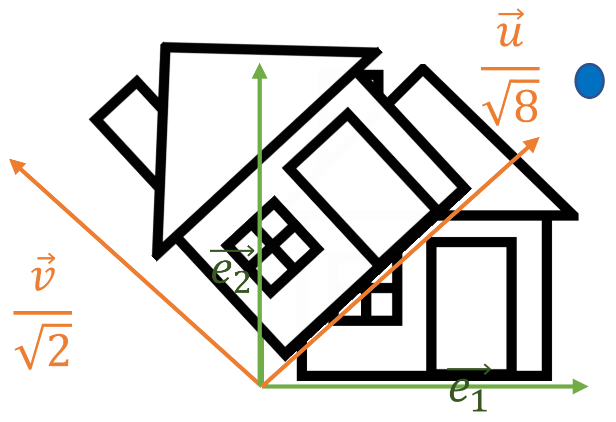
$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = ?$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$



$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}; \ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

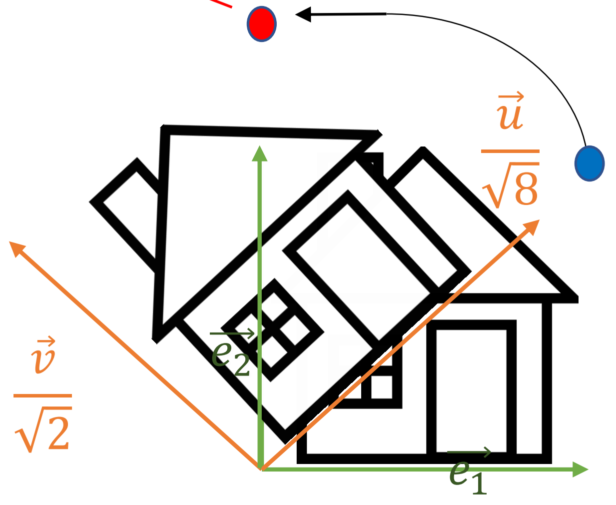
Matrix as a transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$$

$$\frac{\vec{u}}{|\vec{u}|} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

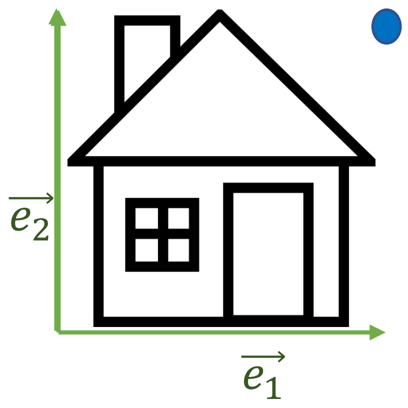
$$\vec{u} = \begin{pmatrix} 2 \\ 2 \end{pmatrix};\ \vec{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



Matrix as a transformation

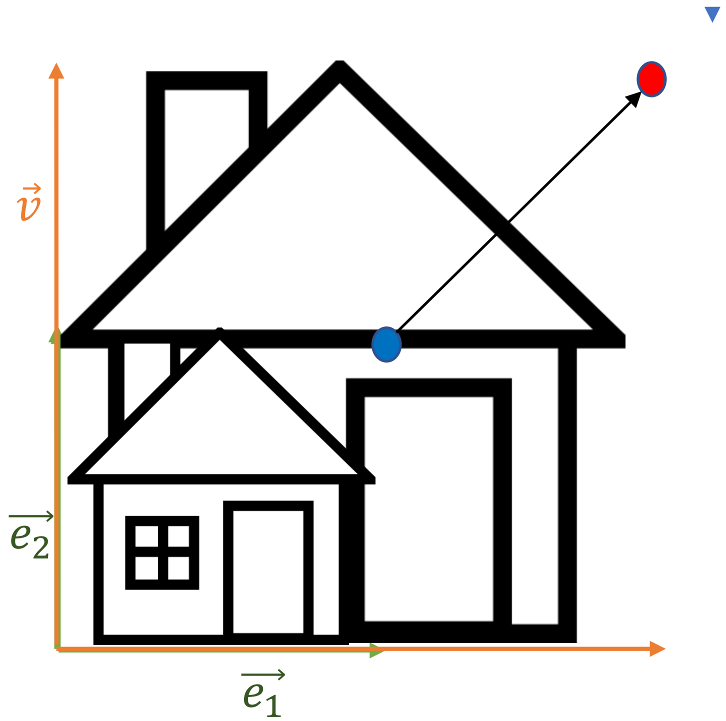
$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = ?$$



$$\vec{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \vec{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Matrix as a transformation

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$$



$$\vec{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix};\ \vec{v} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\overrightarrow{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix};\overrightarrow{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Dot product

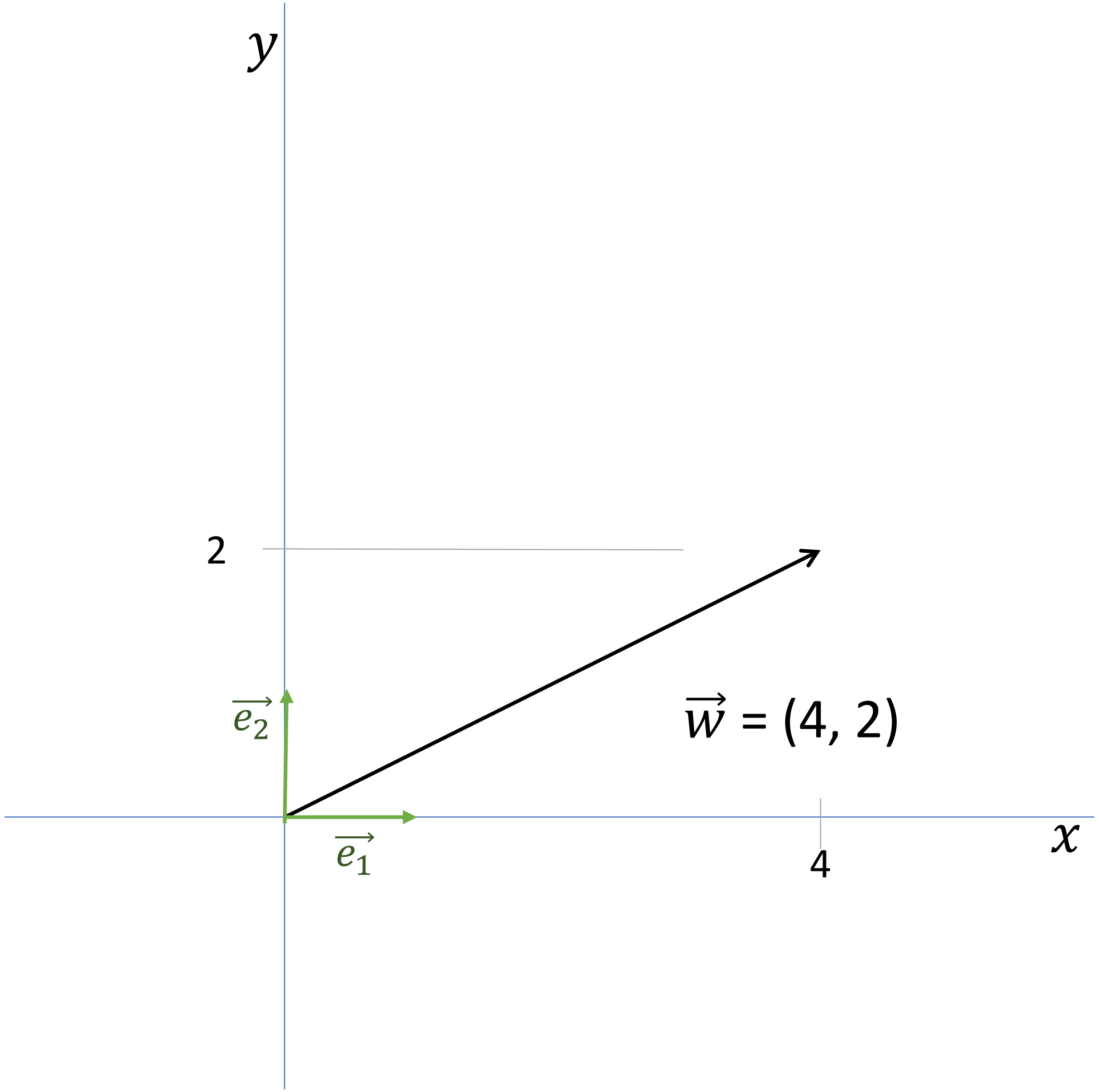
$$\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}; \vec{b} = \begin{pmatrix} b_x \\ b_2 \\ b_z \end{pmatrix}$$

$$\vec{a} \cdot \vec{b} = \begin{pmatrix} a_x & a_y & a_z \end{pmatrix} \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = a_x b_x + a_y b_y + a_z b_z$$

Dot product

$\overrightarrow{e_1} \cdot \vec{w}$ = ?

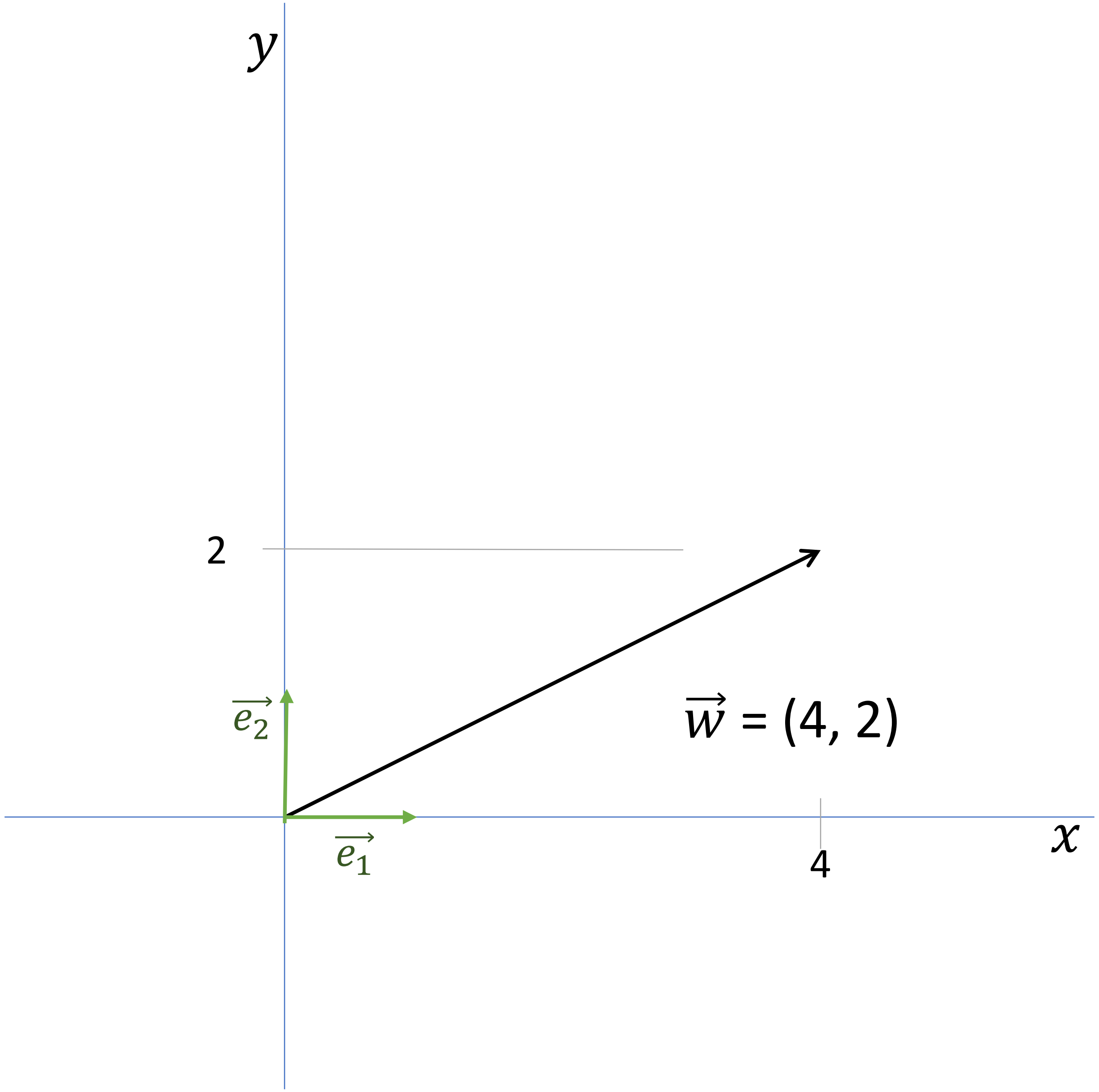
$\overrightarrow{e_2} \cdot \vec{w}$ = ?



Dot product

$$\vec{e_1} \cdot \vec{w} = (1 \;\; 0)\begin{pmatrix}4\\2\end{pmatrix} = 4$$

$$\vec{e_2} \cdot \vec{w} = (0 \;\; 1)\begin{pmatrix}4\\2\end{pmatrix} = 2$$

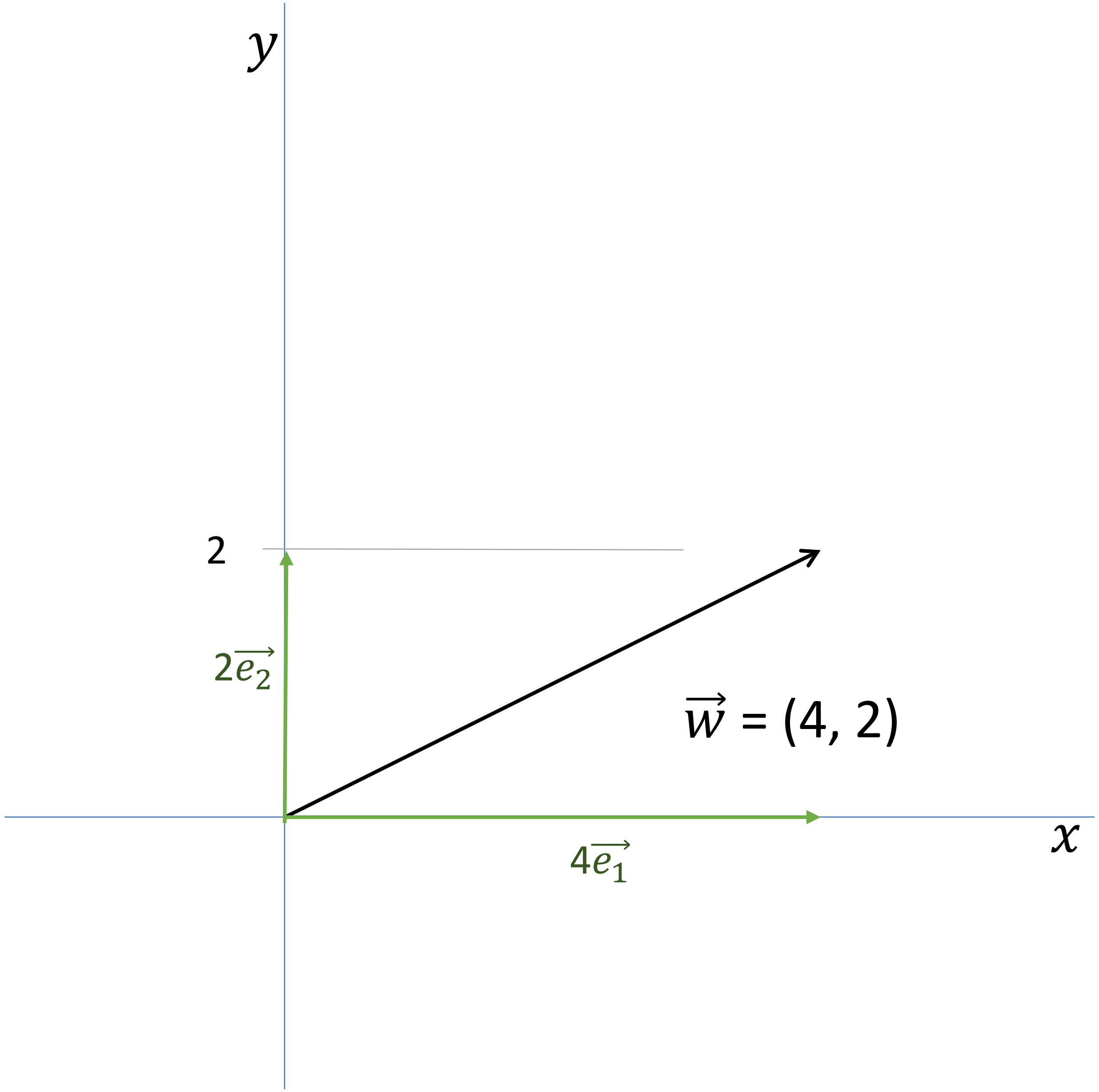


Dot product

$$\vec{e_1} \cdot \vec{w} = (1 \;\; 0)\begin{pmatrix}4\\2\end{pmatrix} = 4$$

$$\vec{e_2} \cdot \vec{w} = (0 \;\; 1)\begin{pmatrix}4\\2\end{pmatrix} = 2$$

$$\vec{w} = 4\vec{e_1} + 2\vec{e_2}$$



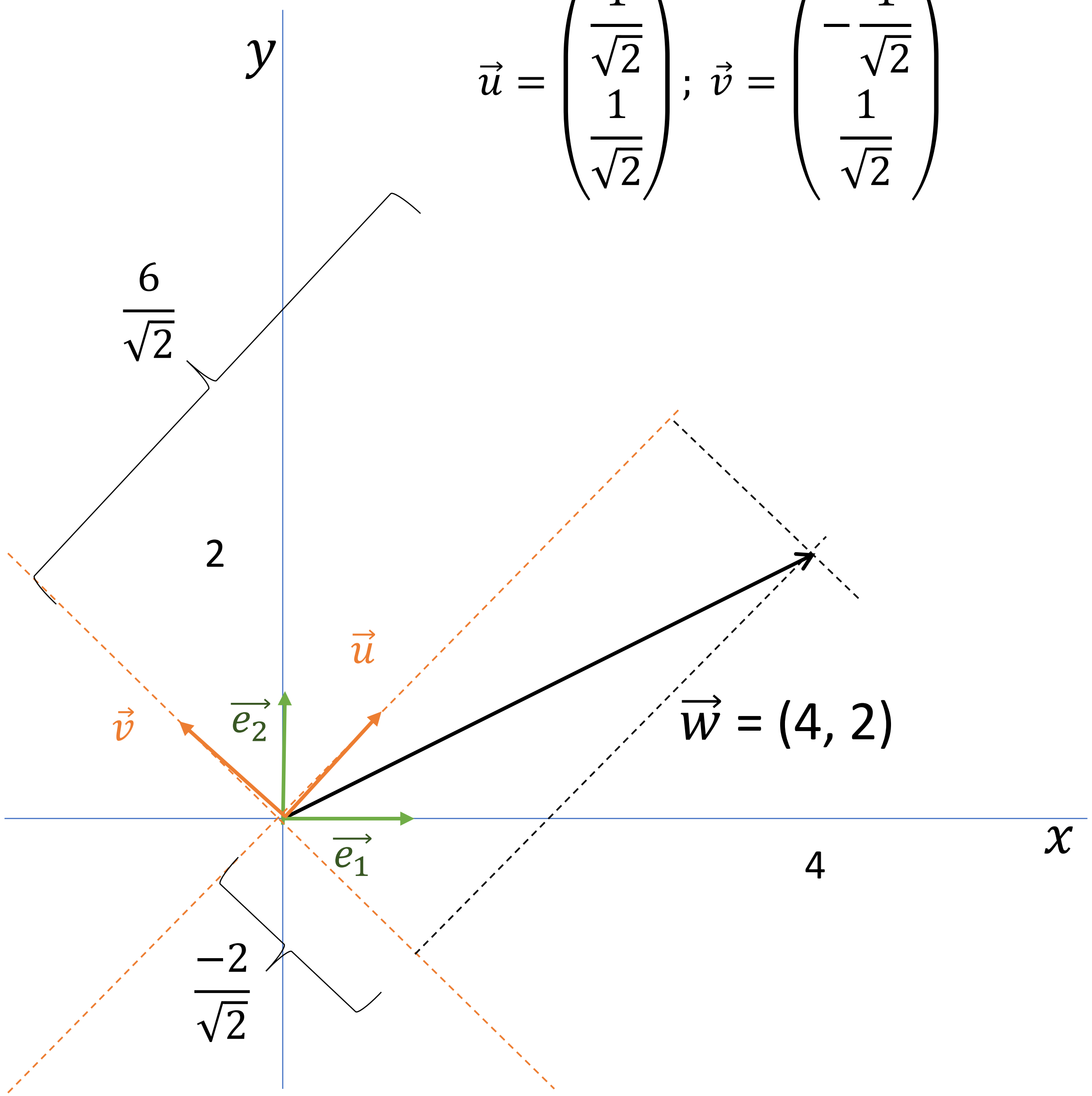
Dot product

$$\vec{u} \cdot \vec{w} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \frac{6}{\sqrt{2}}$$

$$\vec{v} \cdot \vec{w} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \frac{-2}{\sqrt{2}}$$

$$\vec{w} = \frac{6}{\sqrt{2}}\vec{u} + \frac{-2}{\sqrt{2}}\vec{v}$$

$$\vec{u} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}; \; \vec{v} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$



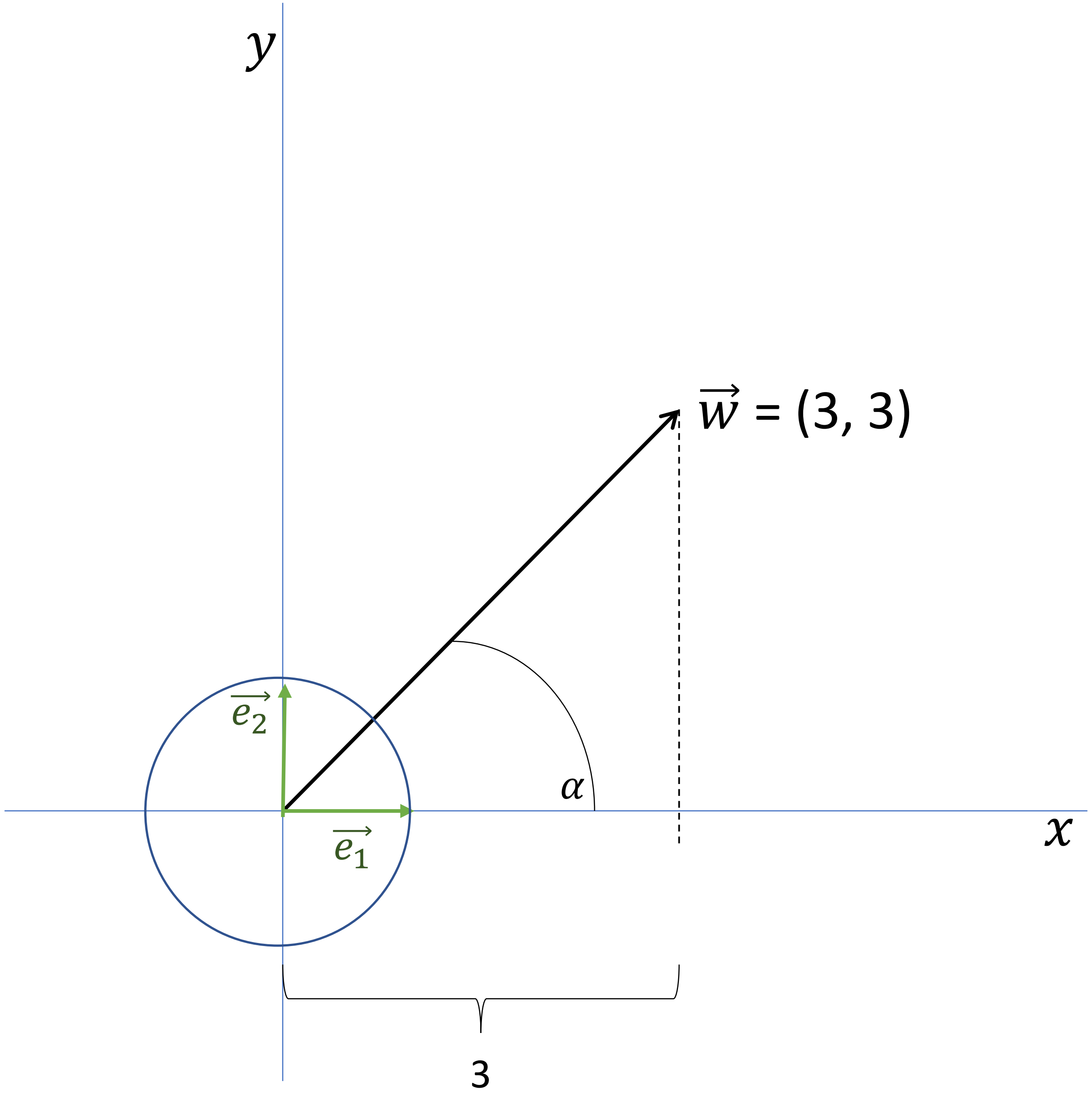
Dot product

$$\vec{e_1} \cdot \vec{w} = (1 \;\; 0)\begin{pmatrix}3\\3\end{pmatrix} = 3$$

$$|\vec{w}| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$|\vec{w}| \cos\alpha = \vec{e_1} \cdot \vec{w}$$

$$\frac{\vec{e_1} \cdot \vec{w}}{|\vec{w}|} = \frac{3}{3\sqrt{2}} = \cos 45^o$$



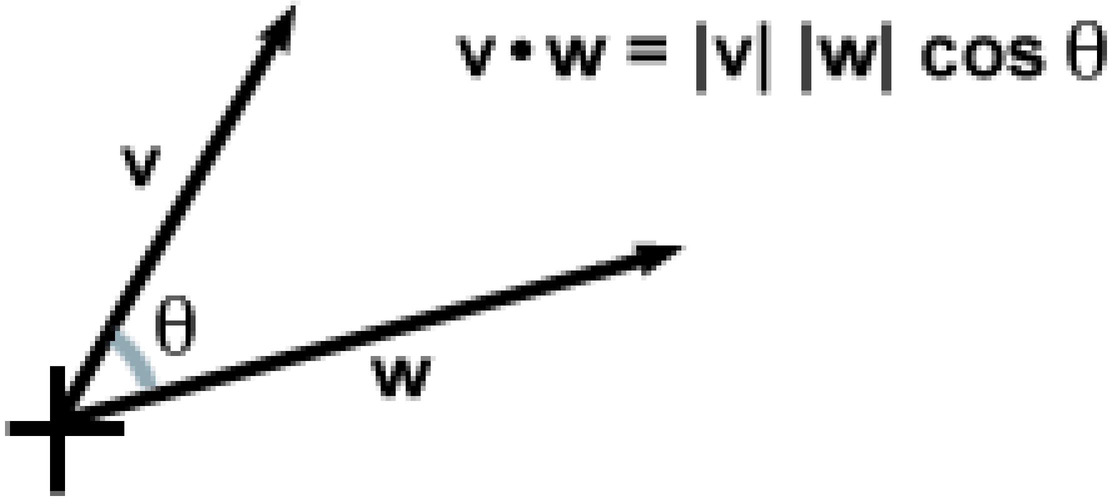
Dot product

$\vec{v} \cdot \vec{w}$ = $(1 \;\; 0)\begin{pmatrix} 3 \\ 3 \end{pmatrix}$ = 3

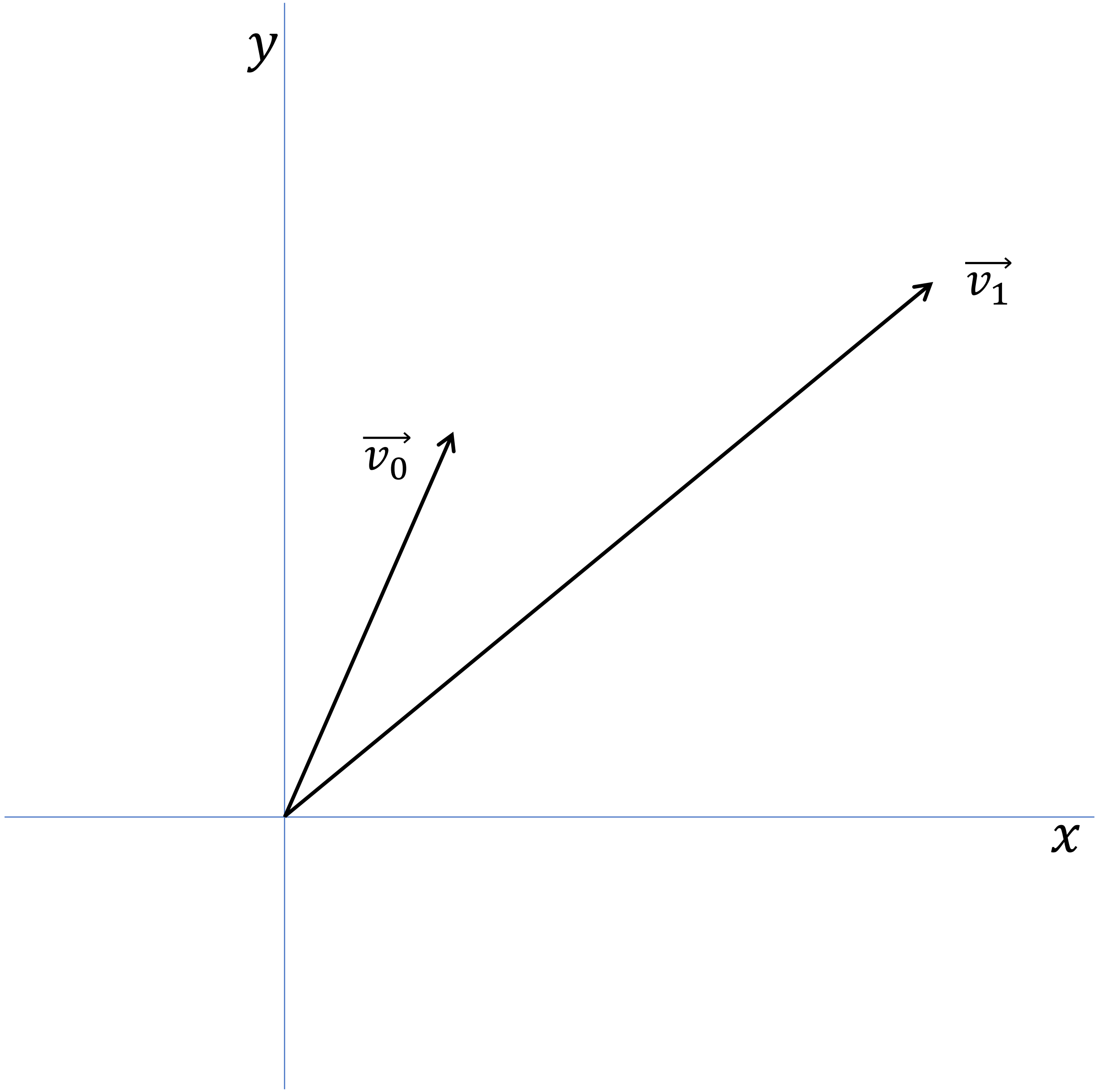
$$|\vec{w}| = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

$$|\vec{w}||\vec{v}| \cos\theta = \vec{v} \cdot \vec{w}$$

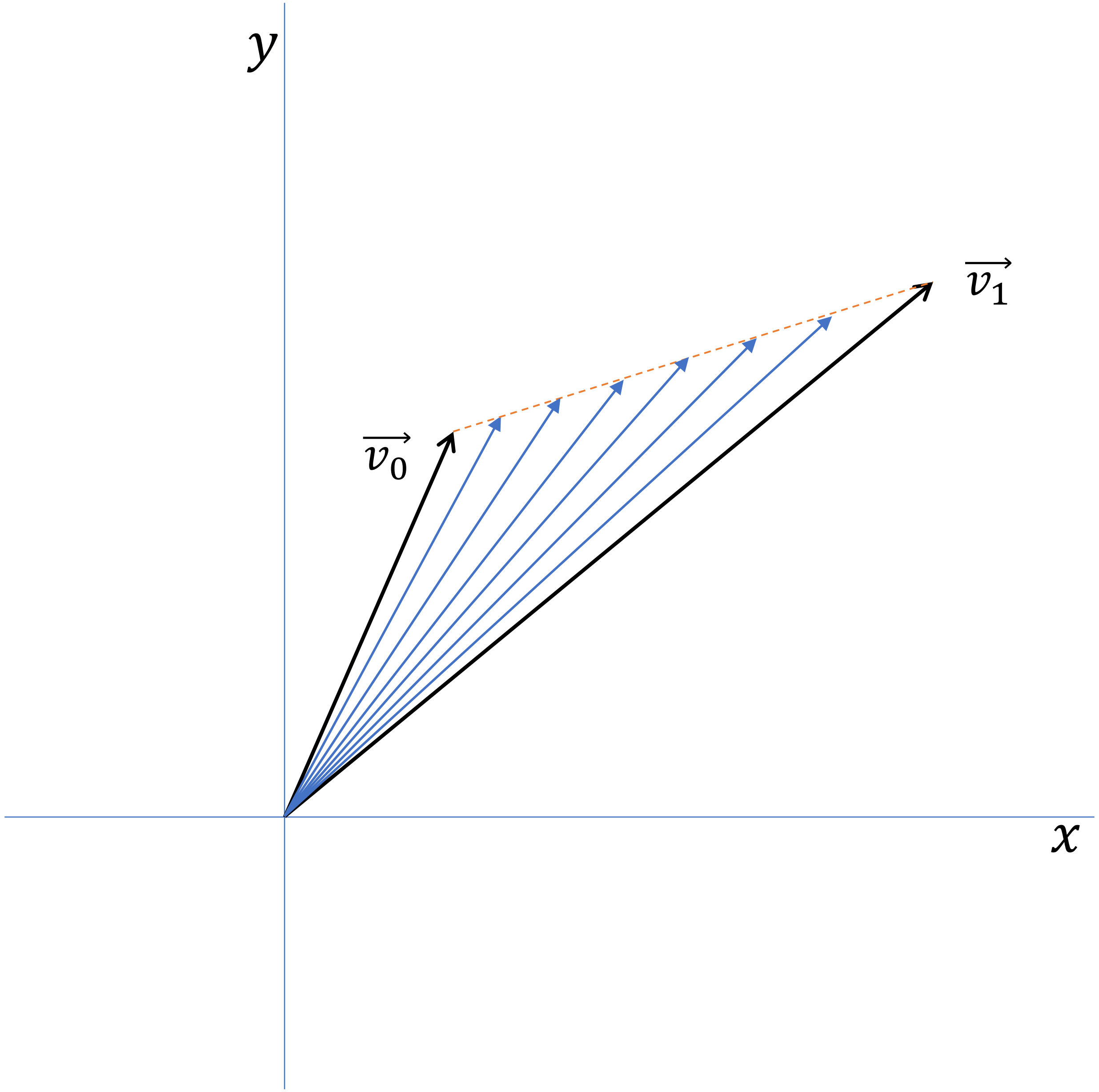
$$\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = \frac{3}{3\sqrt{2}} = \cos 45^o$$



Linear interpolation

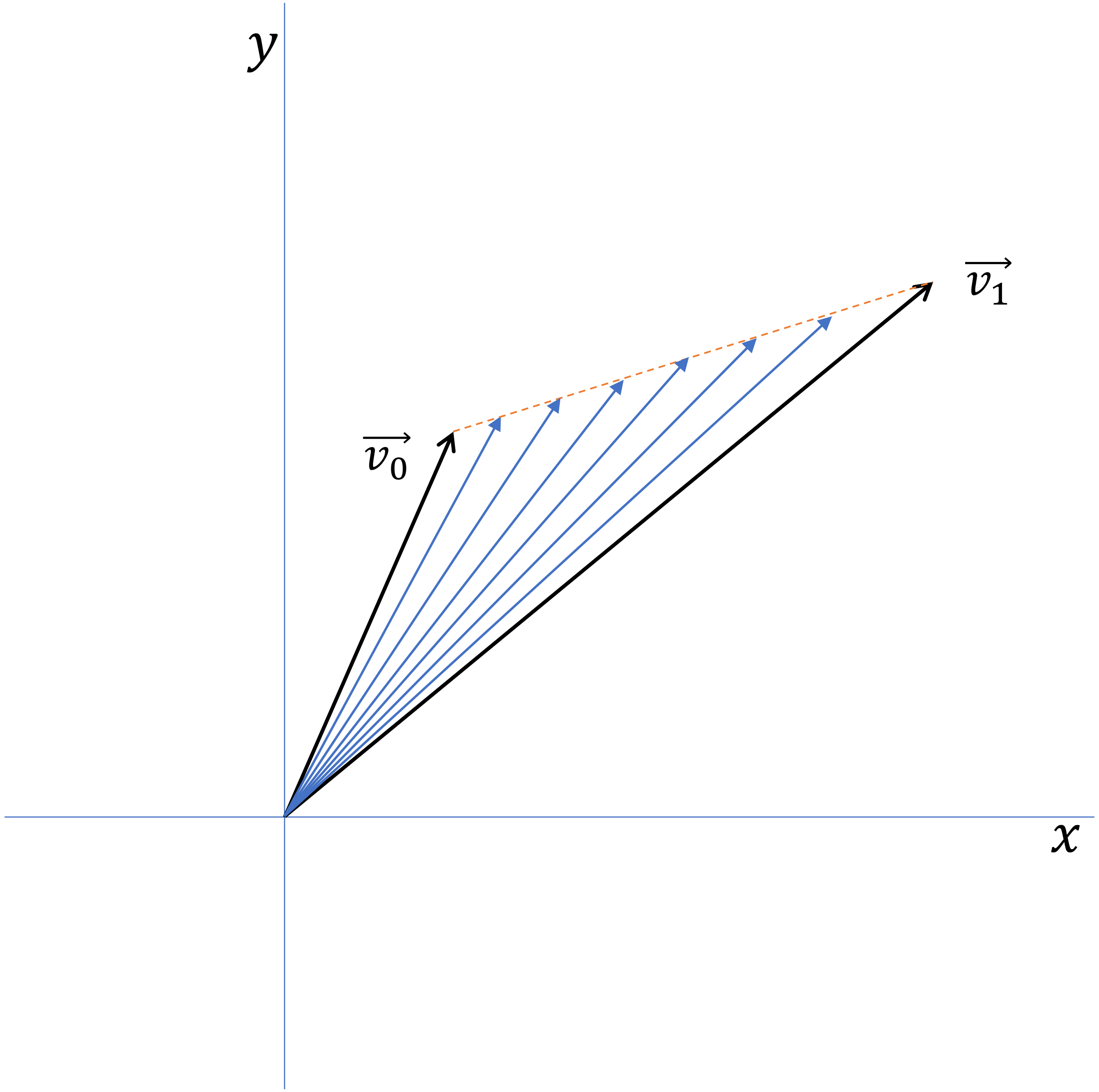


Linear interpolation



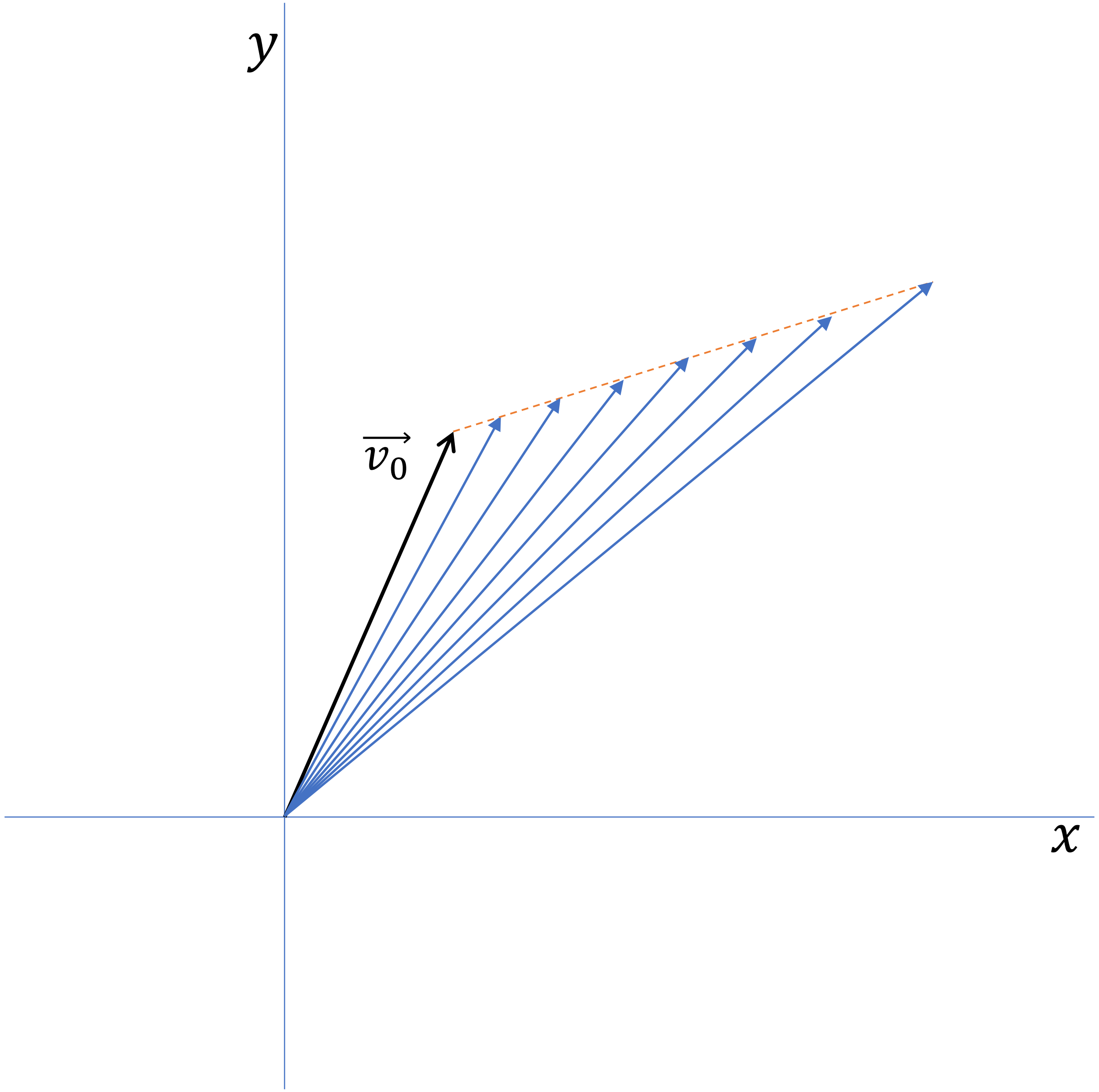
Linear interpolation

- For $t \in [0, 1]$



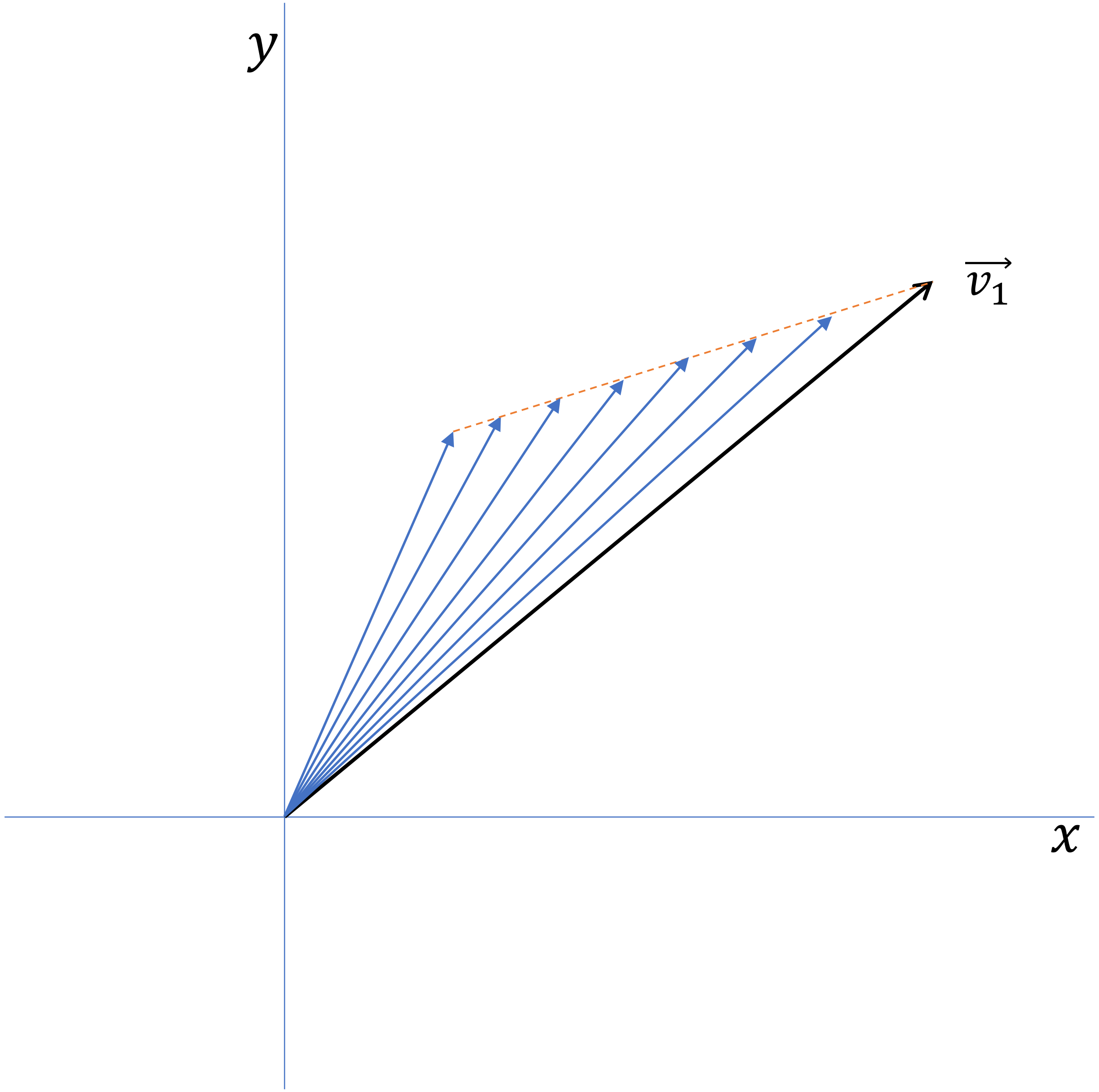
Linear interpolation

- For $t \in [0, 1]$
  - $t = 0$



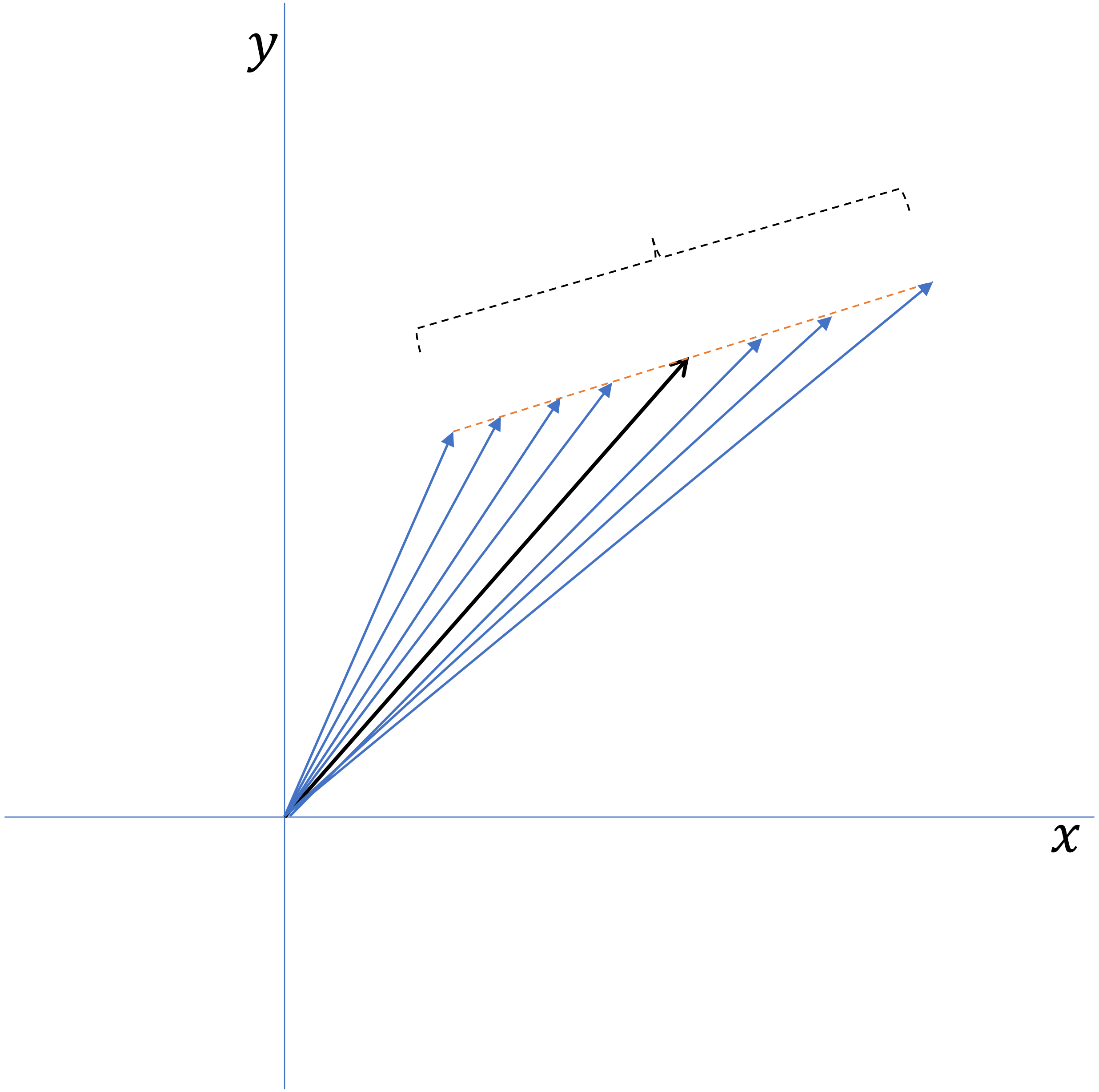
Linear interpolation

- For $t \in [0, 1]$
  - $t = 1$



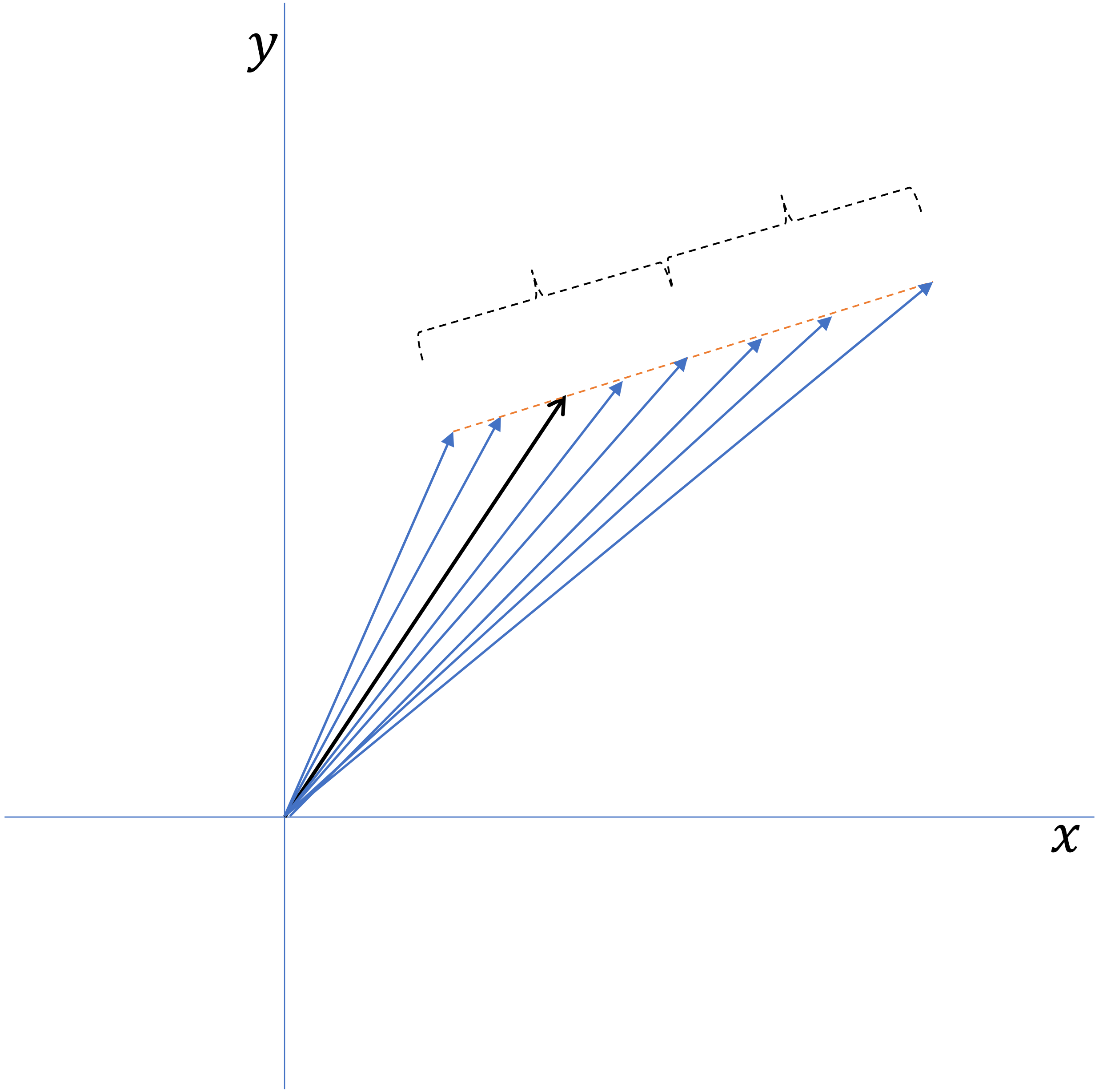
Linear interpolation

- For $t \in [0, 1]$
  - $t = \frac{1}{2}$



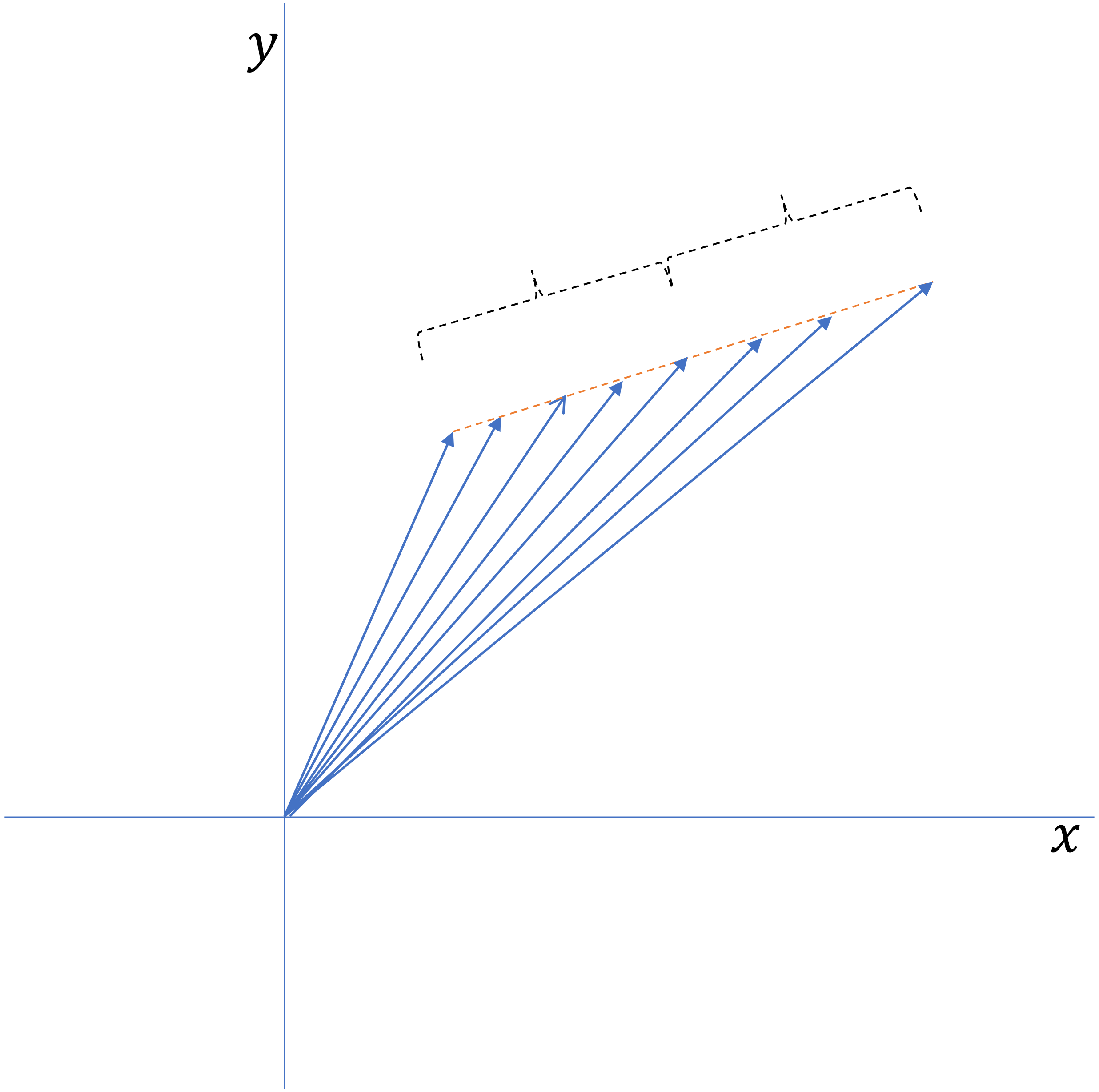
Linear interpolation

- For $t \in [0, 1]$
  - $t = \frac{1}{4}$



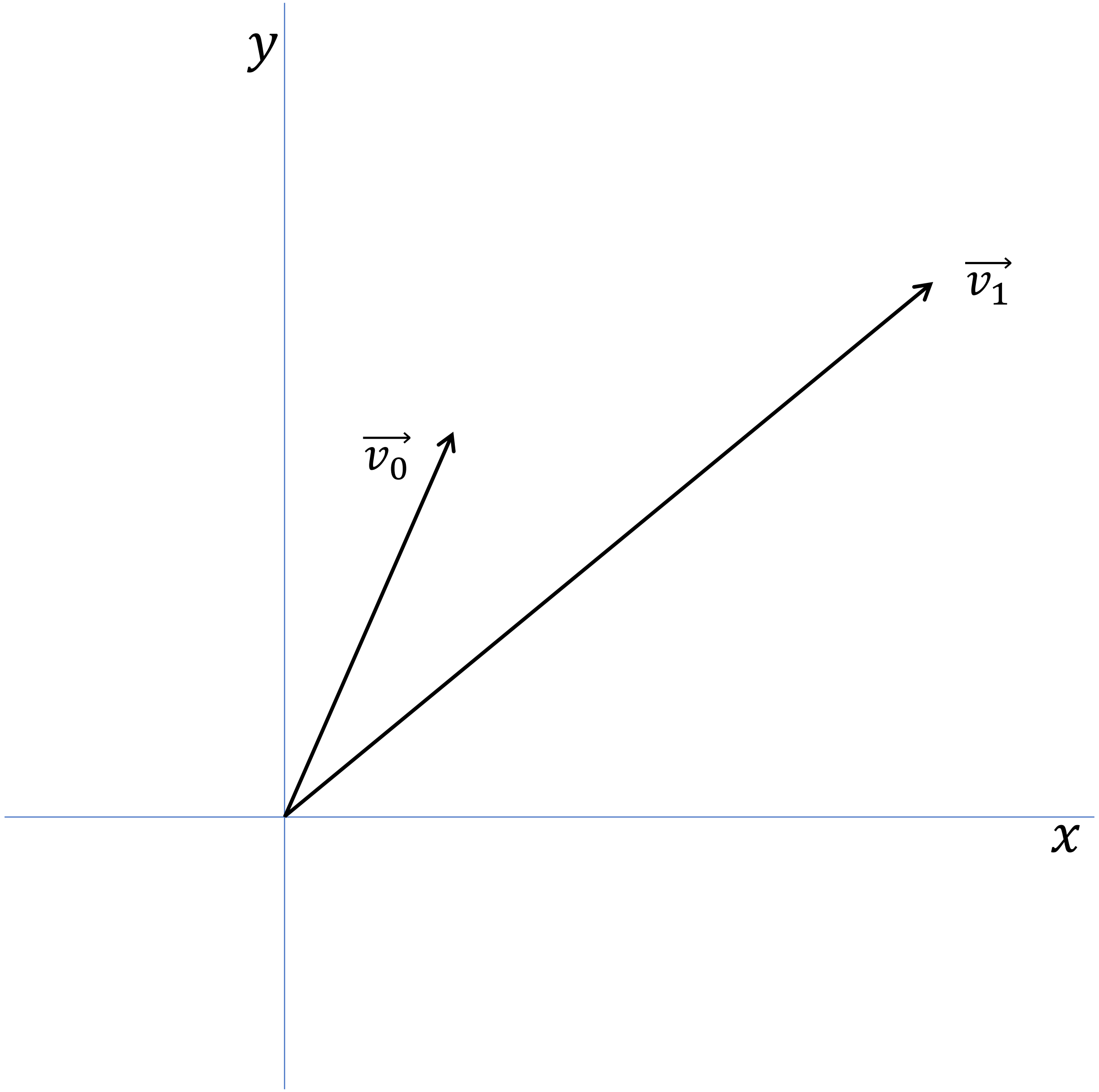
Linear interpolation

- For $t \in [0, 1]$
  - $t \stackrel{?}{=} \frac{3}{4}$



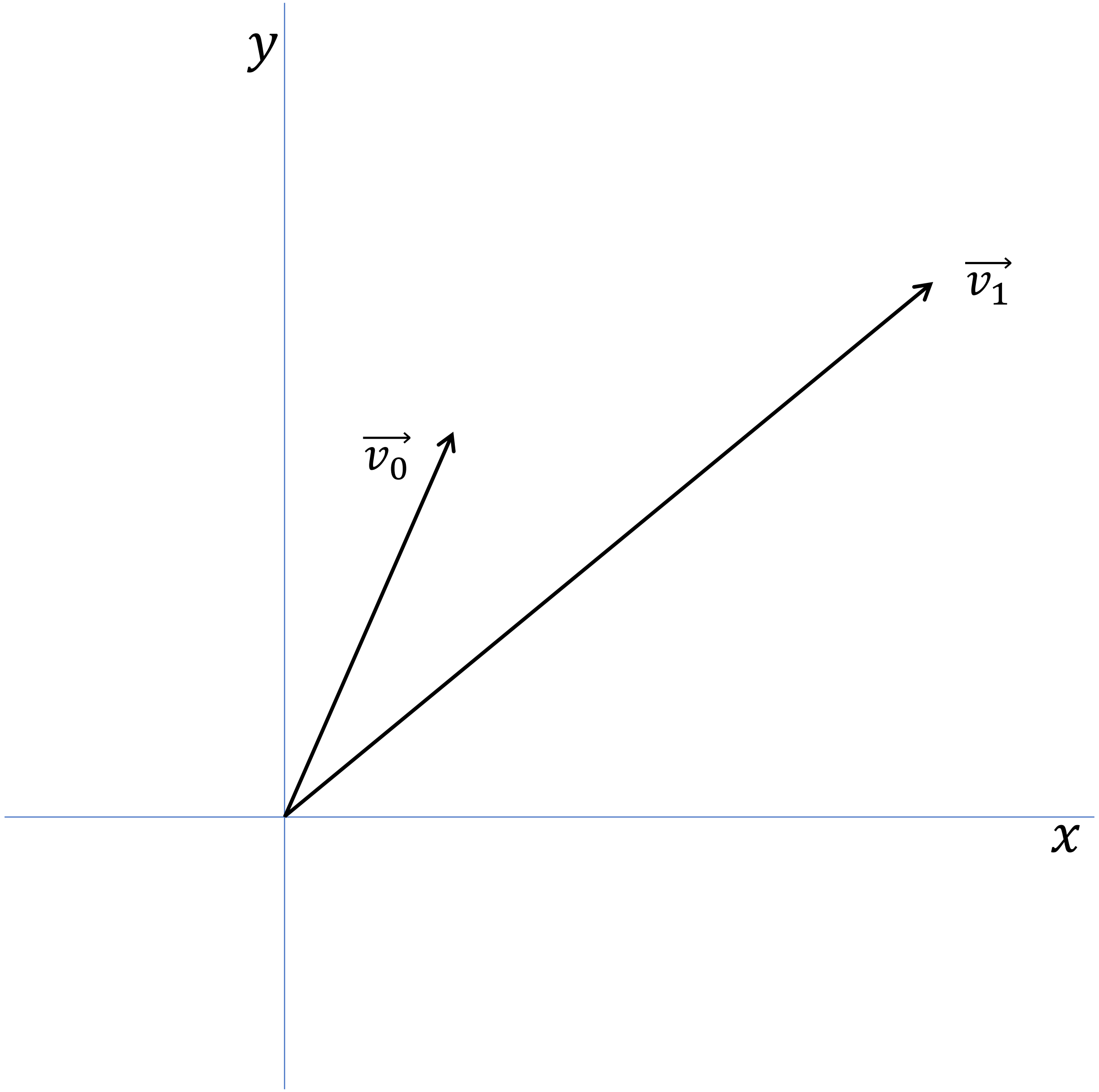
Linear interpolation

- For $t \in [0, 1]$
  - $t = 0 \Rightarrow 1\overrightarrow{v_0} + 0\overrightarrow{v_1}$



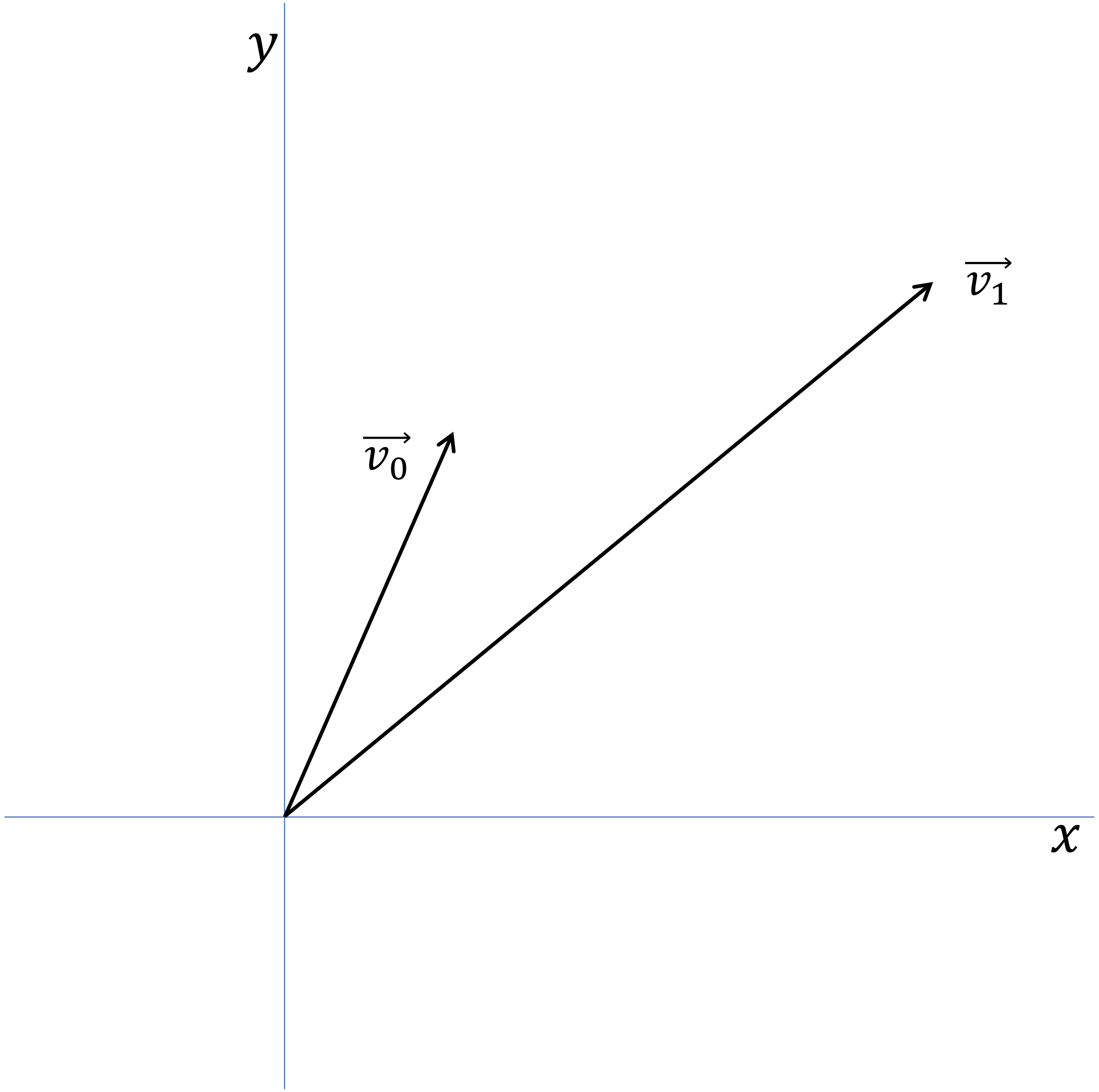
Linear interpolation

- For $t \in [0, 1]$
  - $t = 0 \Rightarrow 1\overrightarrow{v_0} + 0\overrightarrow{v_1}$
  - $t = 1 \Rightarrow 0\overrightarrow{v_0} + 1\overrightarrow{v_1}$



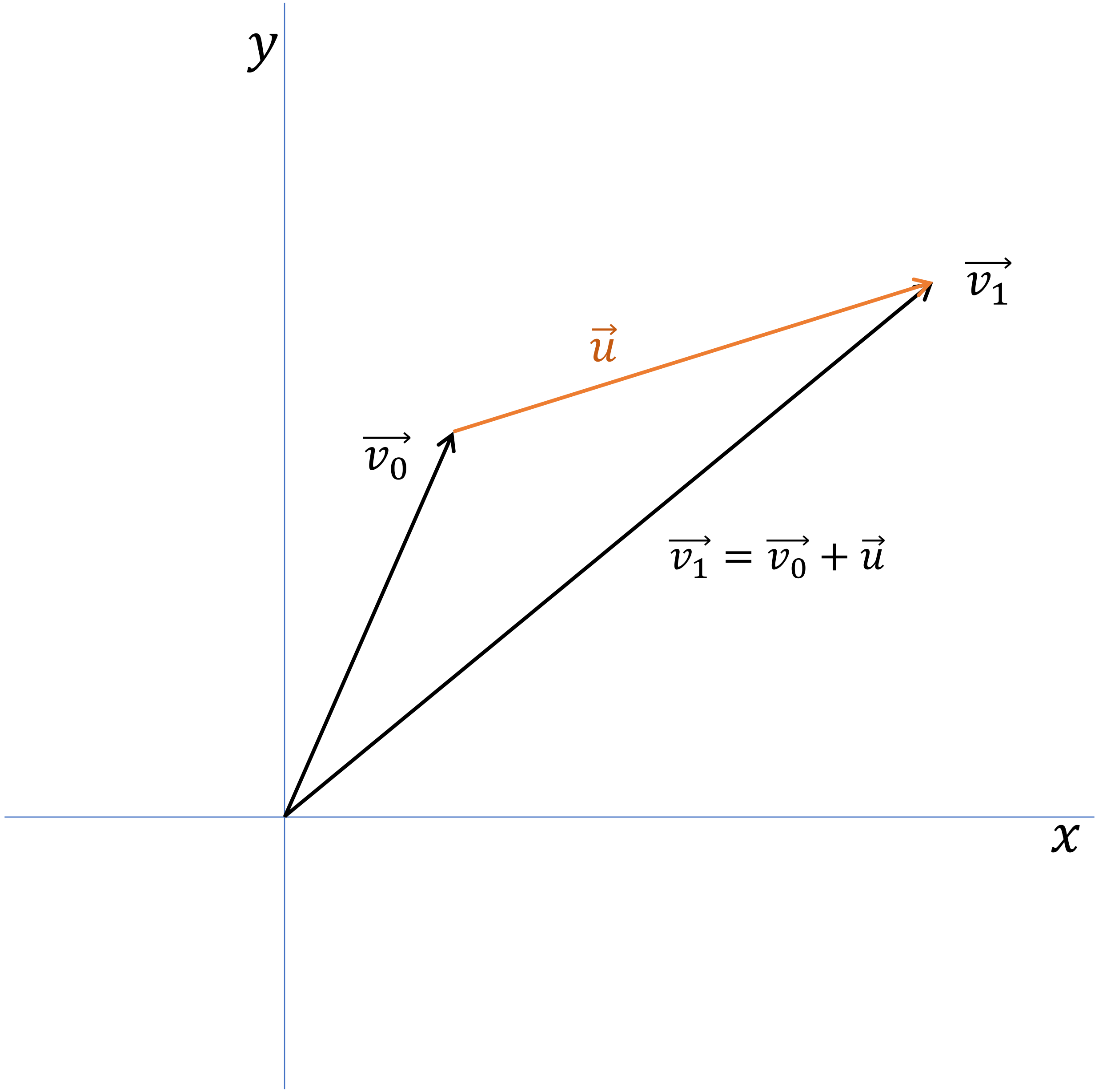
Linear interpolation

- For $t \in [0, 1]$
  - $t = 0 \Rightarrow 1\overrightarrow{v_0} + 0\overrightarrow{v_1}$
  - $t = 1 \Rightarrow 0\overrightarrow{v_0} + 1\overrightarrow{v_1}$
  - $a\overrightarrow{v_0} + b\overrightarrow{v_1}$



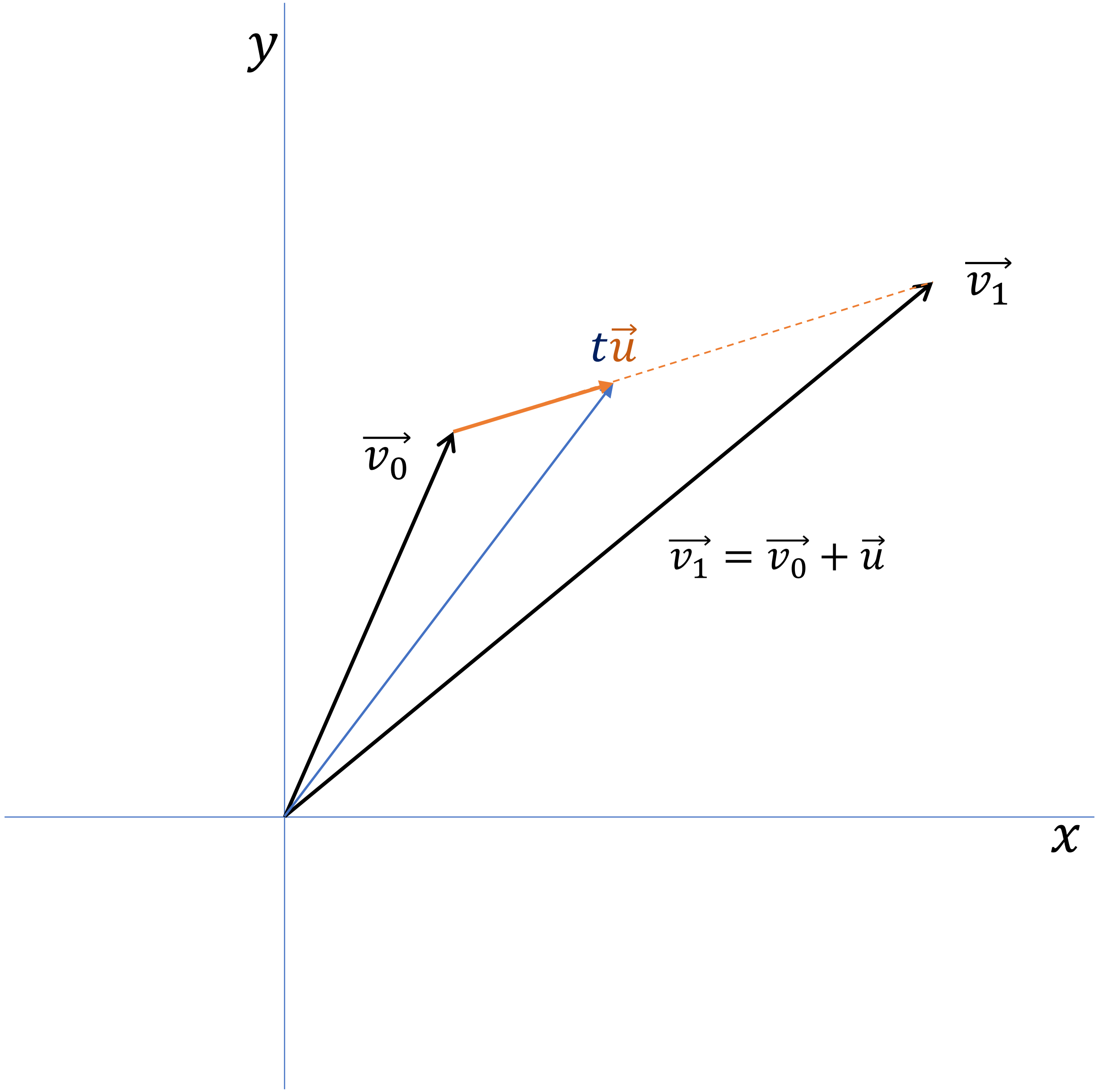
Linear interpolation

- For $t \in [0, 1]$
  - $a\overrightarrow{v_0} + b\overrightarrow{v_1}$



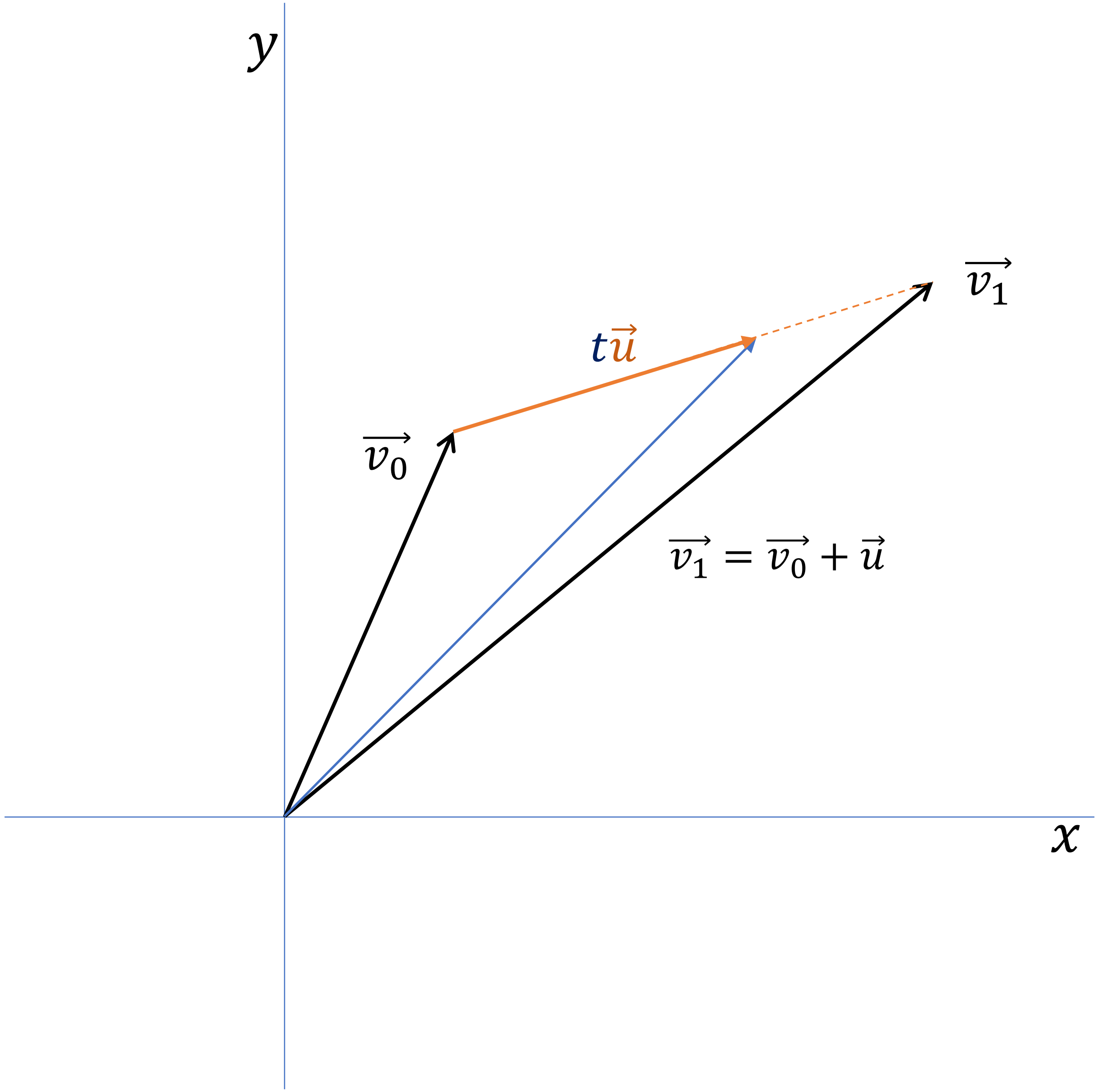
Linear interpolation

- For $t \in [0, 1]$
  - $a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_0} + t\vec{u}$



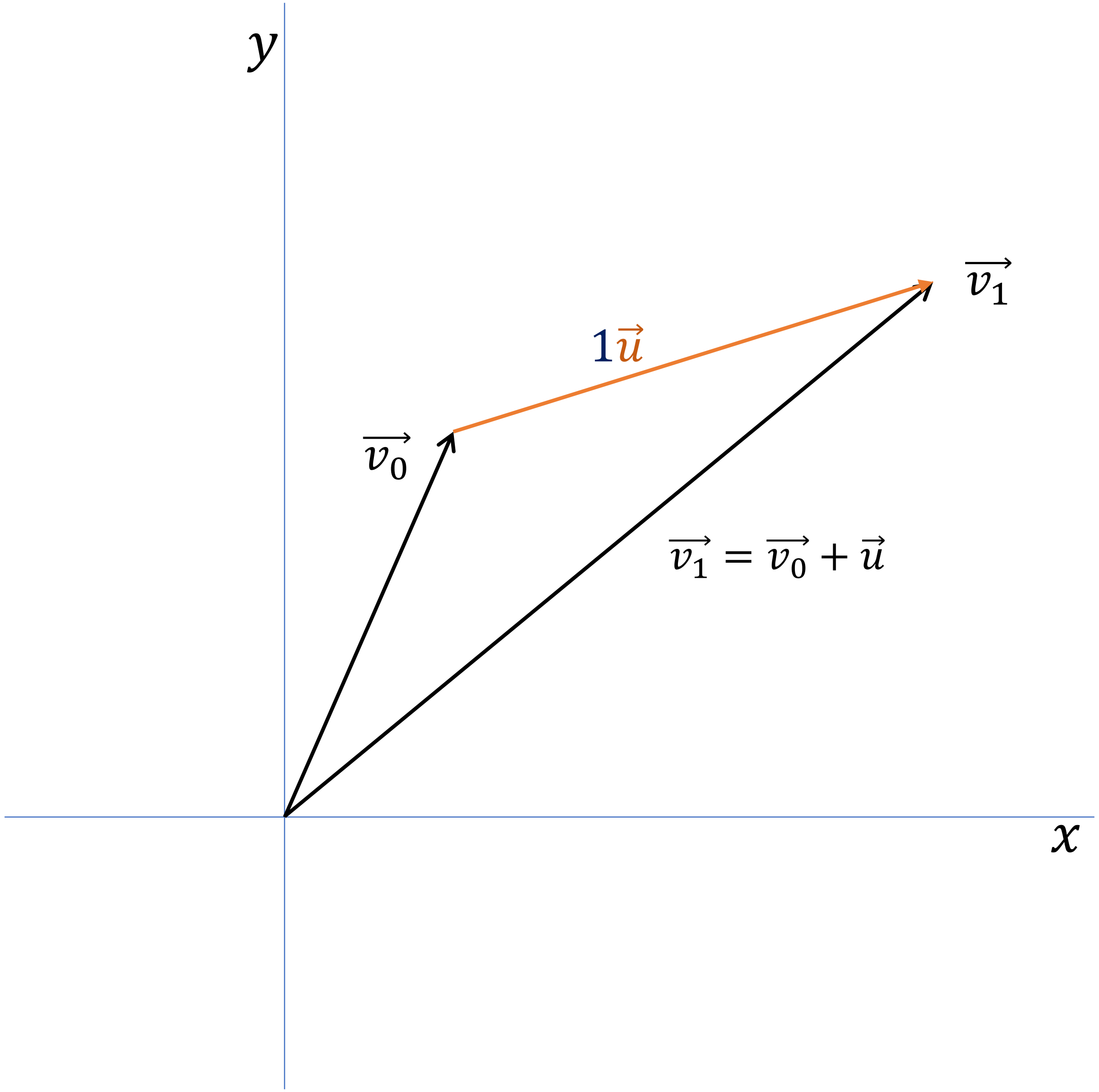
Linear interpolation

- For $t \in [0, 1]$
  - $a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_0} + t\vec{u}$



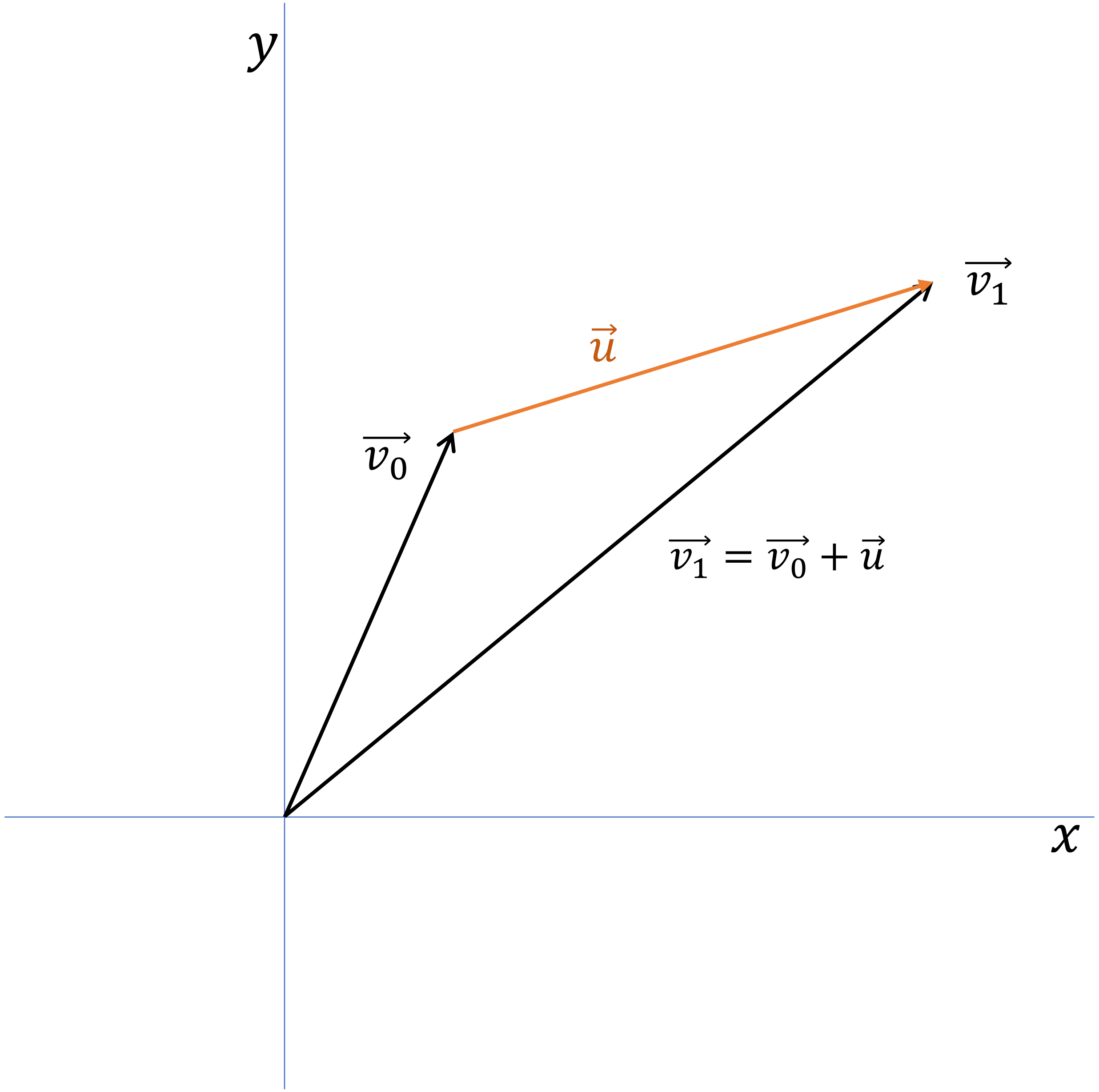
Linear interpolation

- For $t \in [0, 1]$
  - $a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_0} + 1\vec{u}$



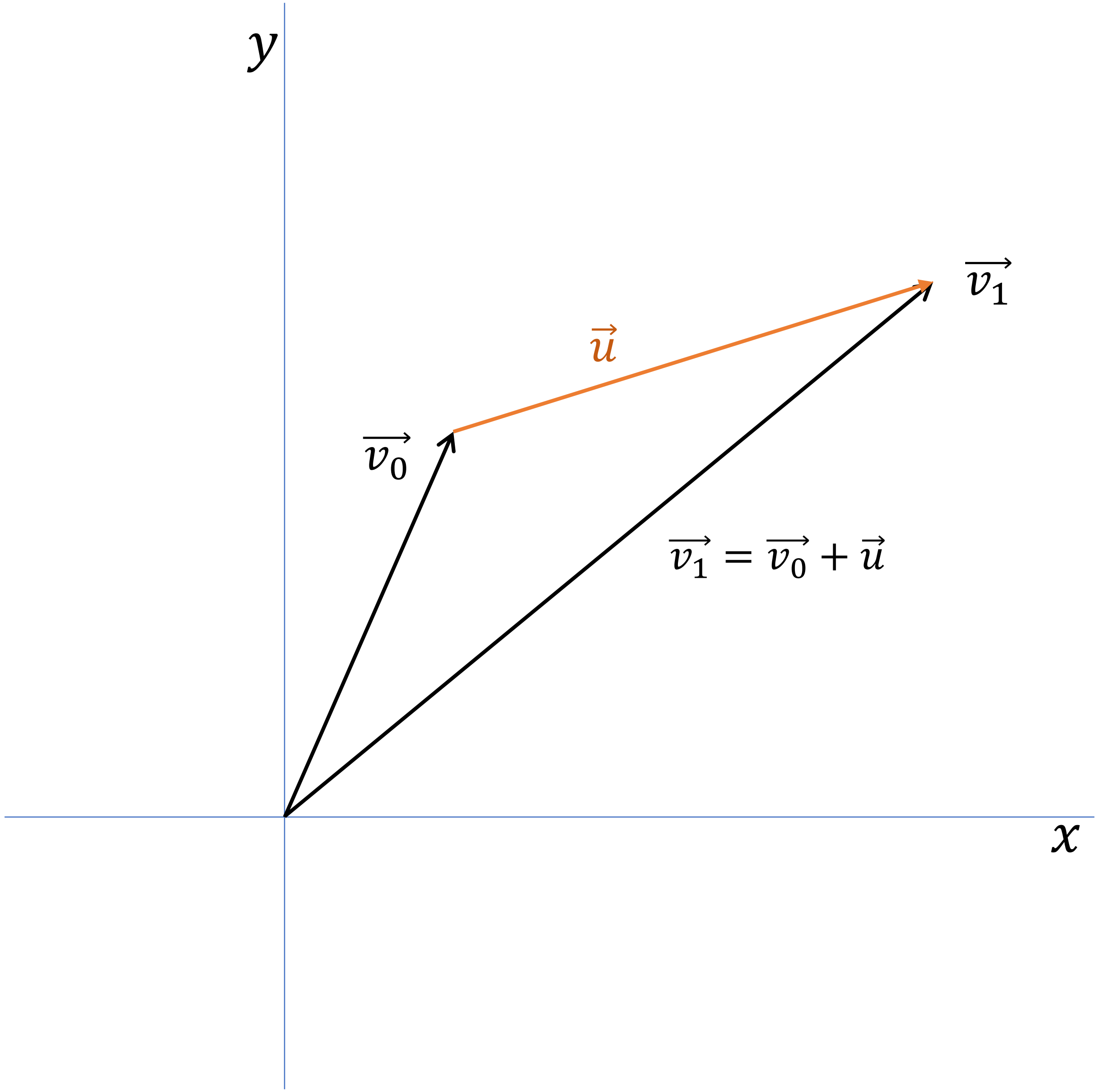
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$



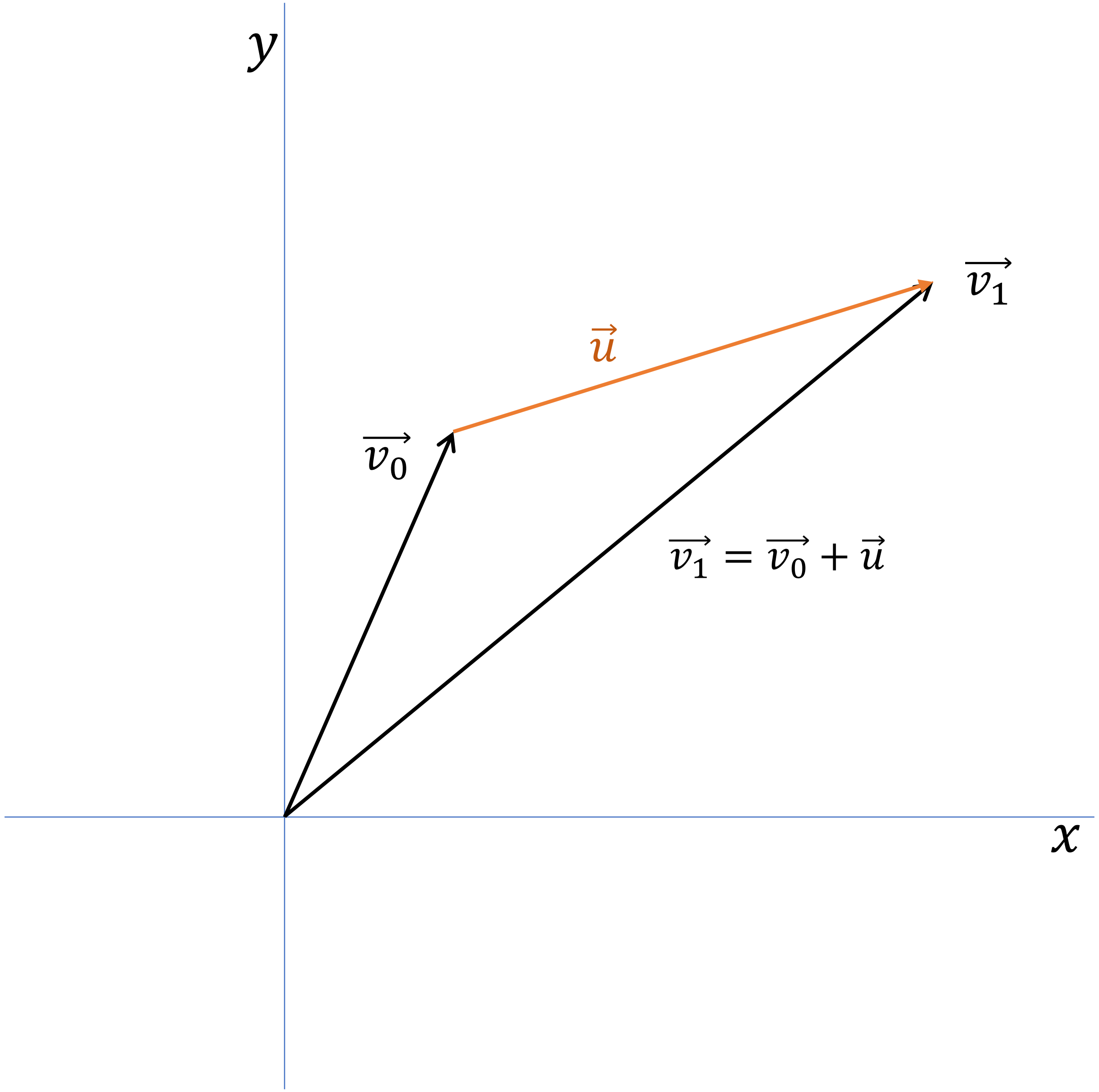
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$



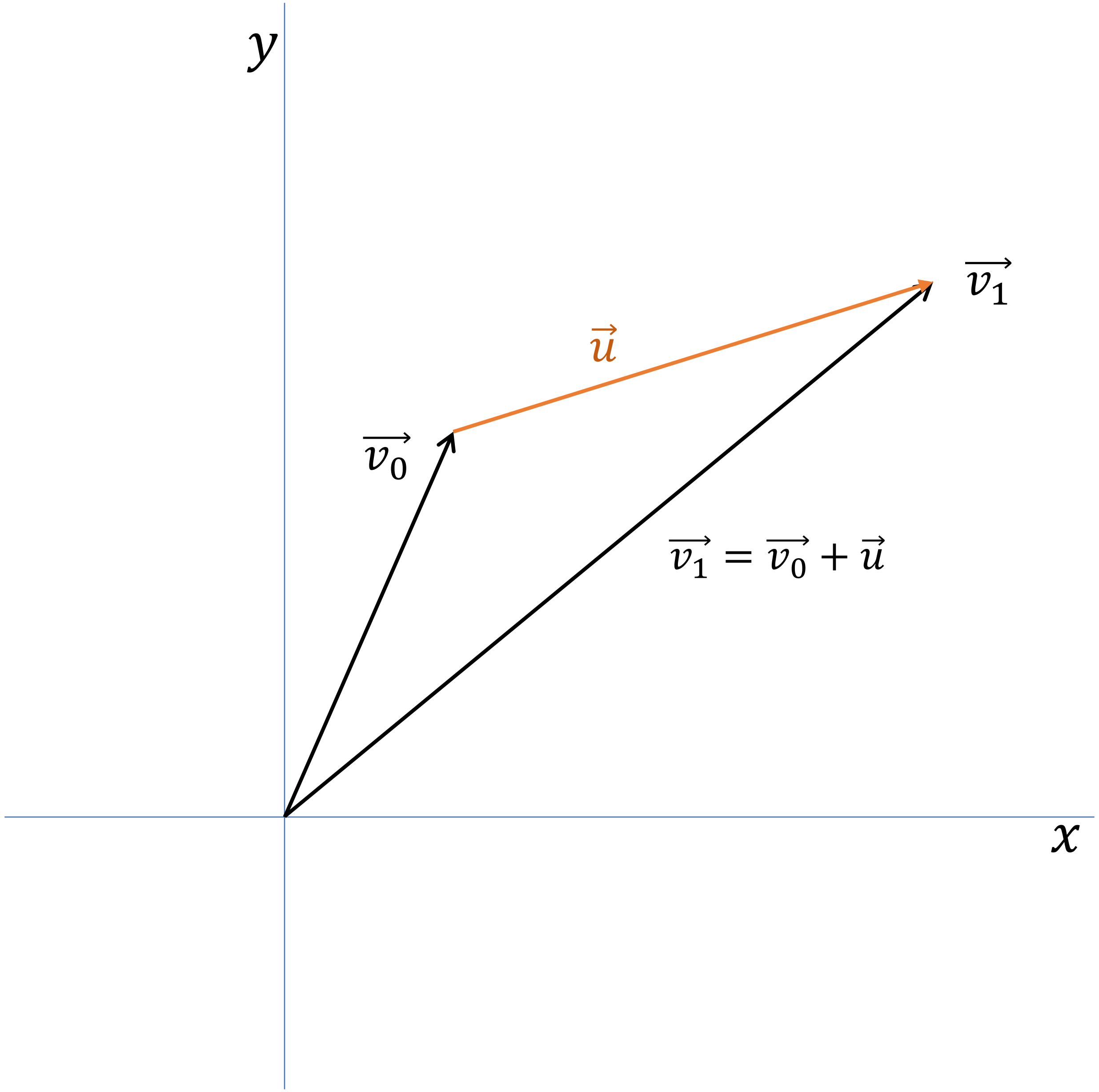
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t\vec{u}$



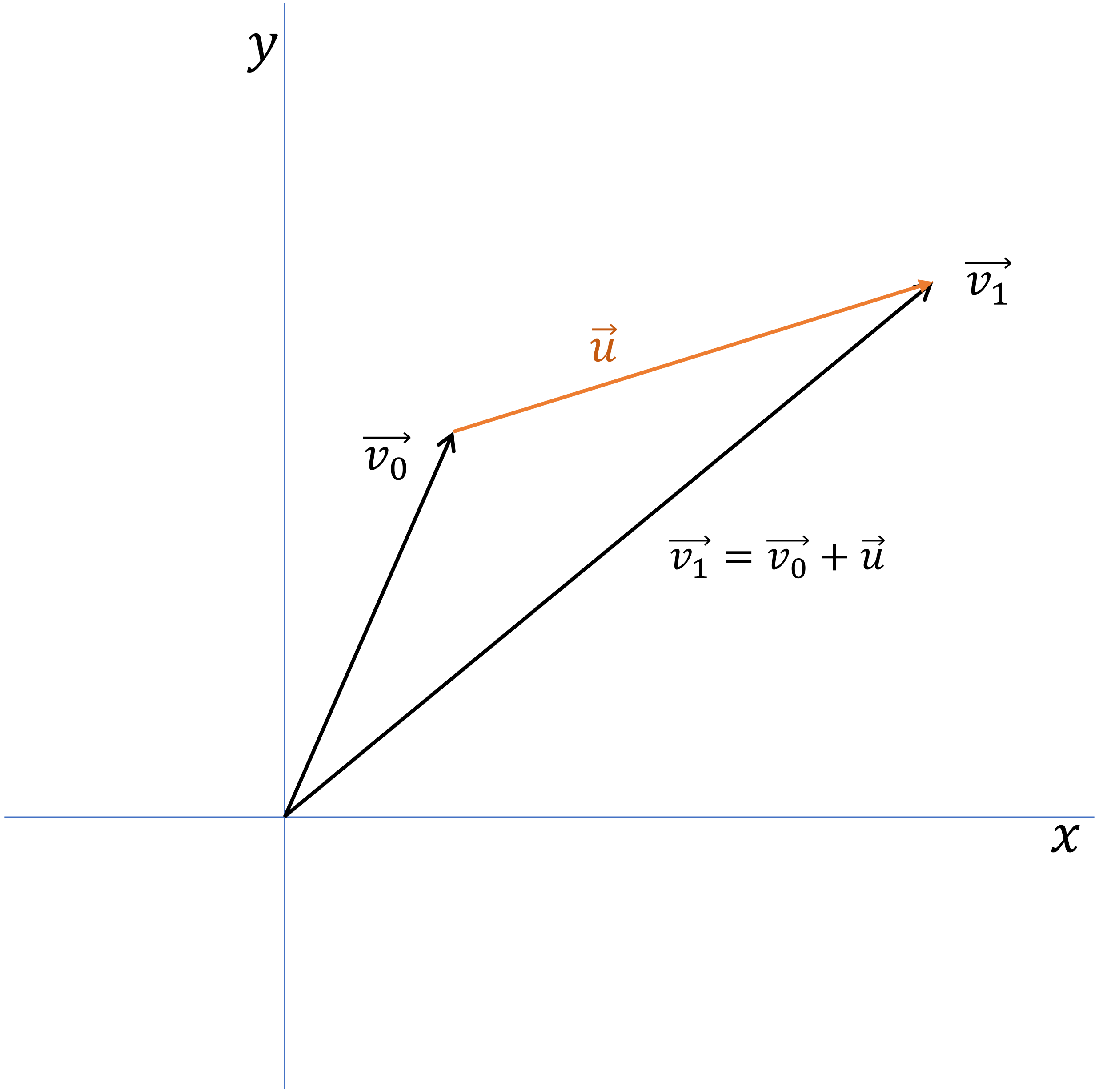
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t(\overrightarrow{v_1} - \overrightarrow{v_0})$



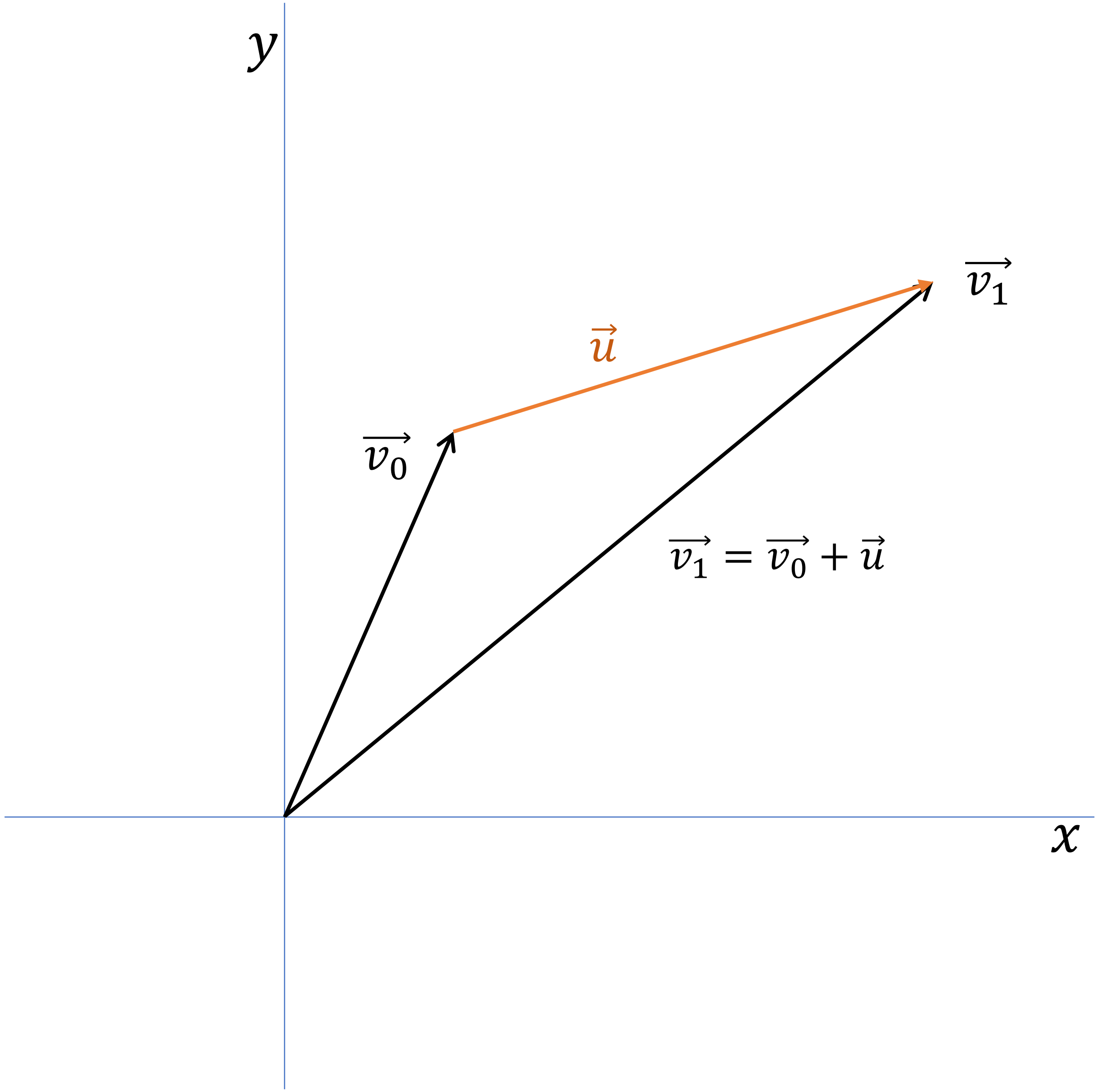
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t(\overrightarrow{v_1} - \overrightarrow{v_0})$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t\overrightarrow{v_1} - t\overrightarrow{v_0}$



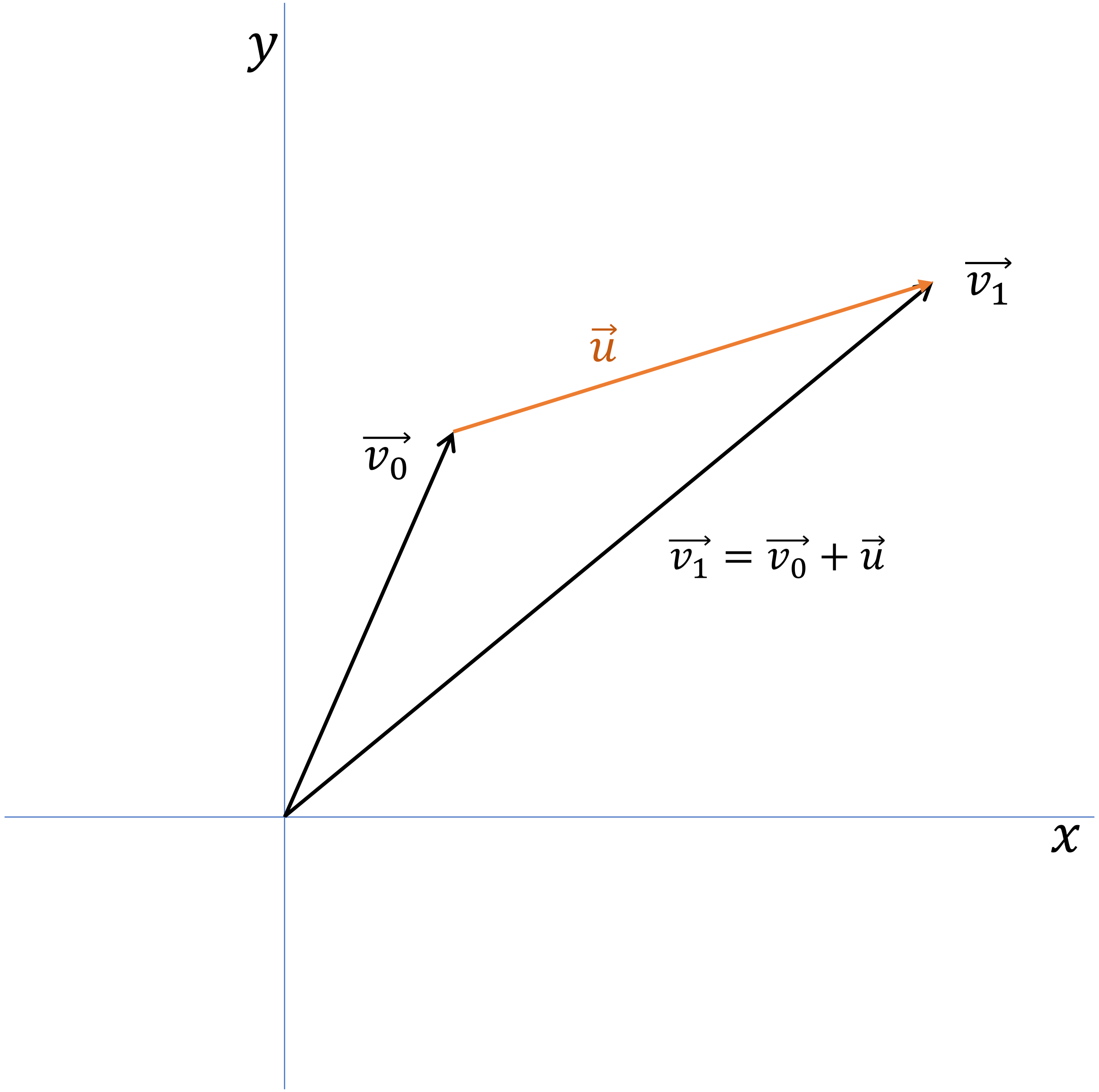
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t(\overrightarrow{v_1} - \overrightarrow{v_0})$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t\overrightarrow{v_1} - t\overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} - t\overrightarrow{v_0} + t\overrightarrow{v_1}$



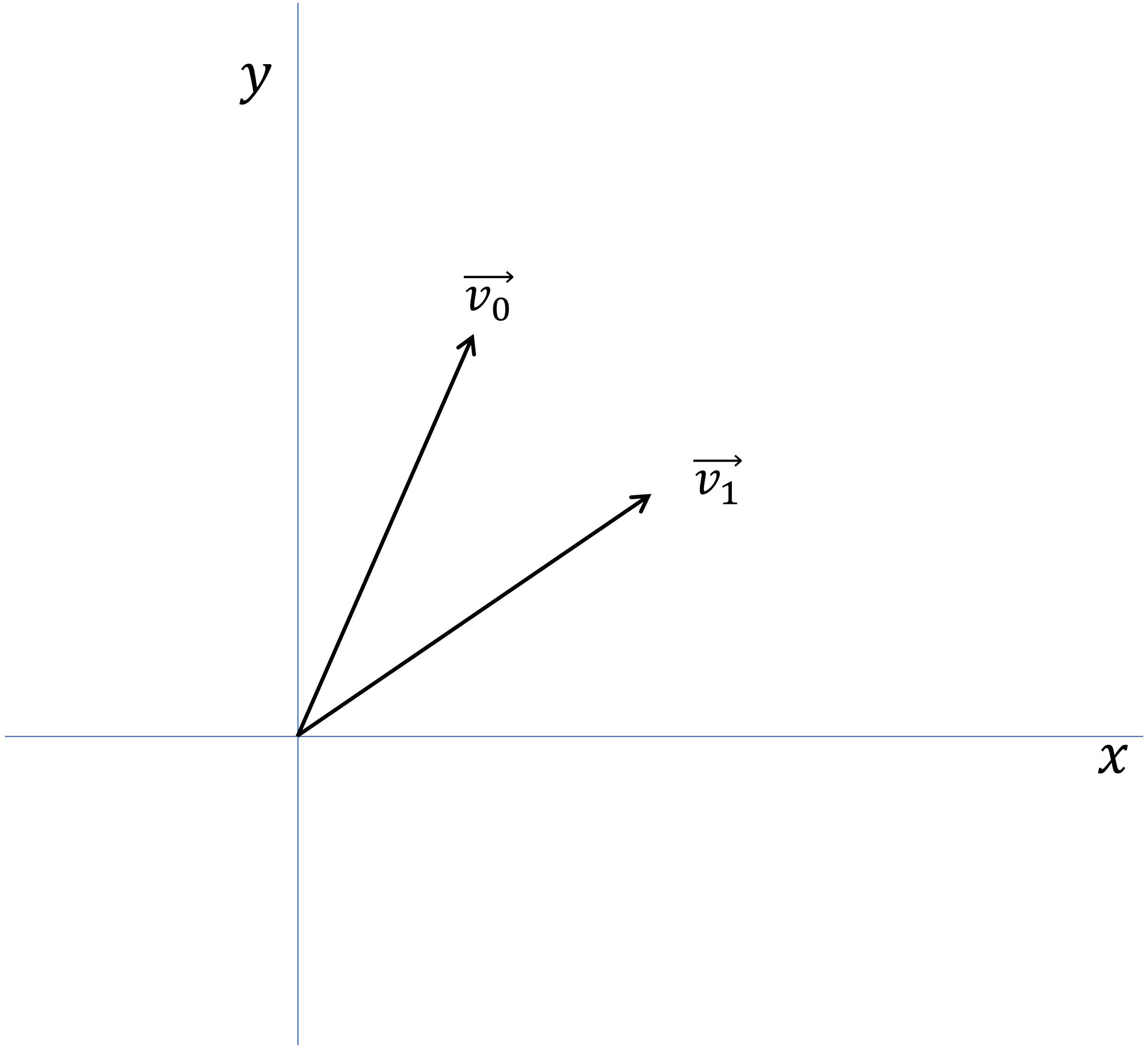
Linear interpolation

- For $t \in [0, 1]$
  - $\overrightarrow{v_t} = a\overrightarrow{v_0} + b\overrightarrow{v_1}$
  - $\overrightarrow{v_1} = \overrightarrow{v_0} + \vec{u}$
  - $\vec{u} = \overrightarrow{v_1} - \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t(\overrightarrow{v_1} - \overrightarrow{v_0})$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} + t\overrightarrow{v_1} - t\overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{v_0} - t\overrightarrow{v_0} + t\overrightarrow{v_1}$
  - $\overrightarrow{v_t} = (1 - t)\,\overrightarrow{v_0} + t\overrightarrow{v_1}$



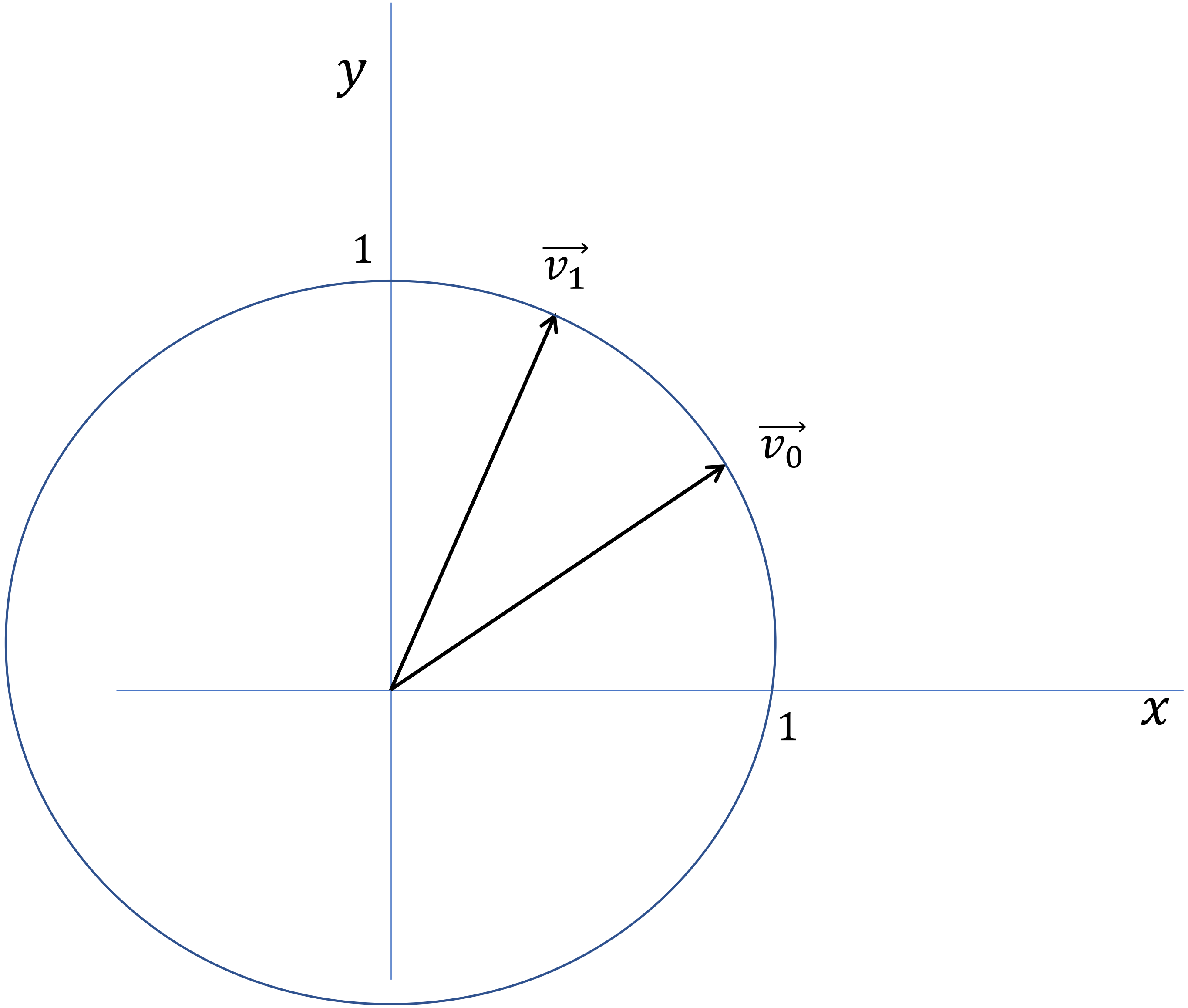
Spherical linear interpolation interpolation

- For $t \in [0, 1]$



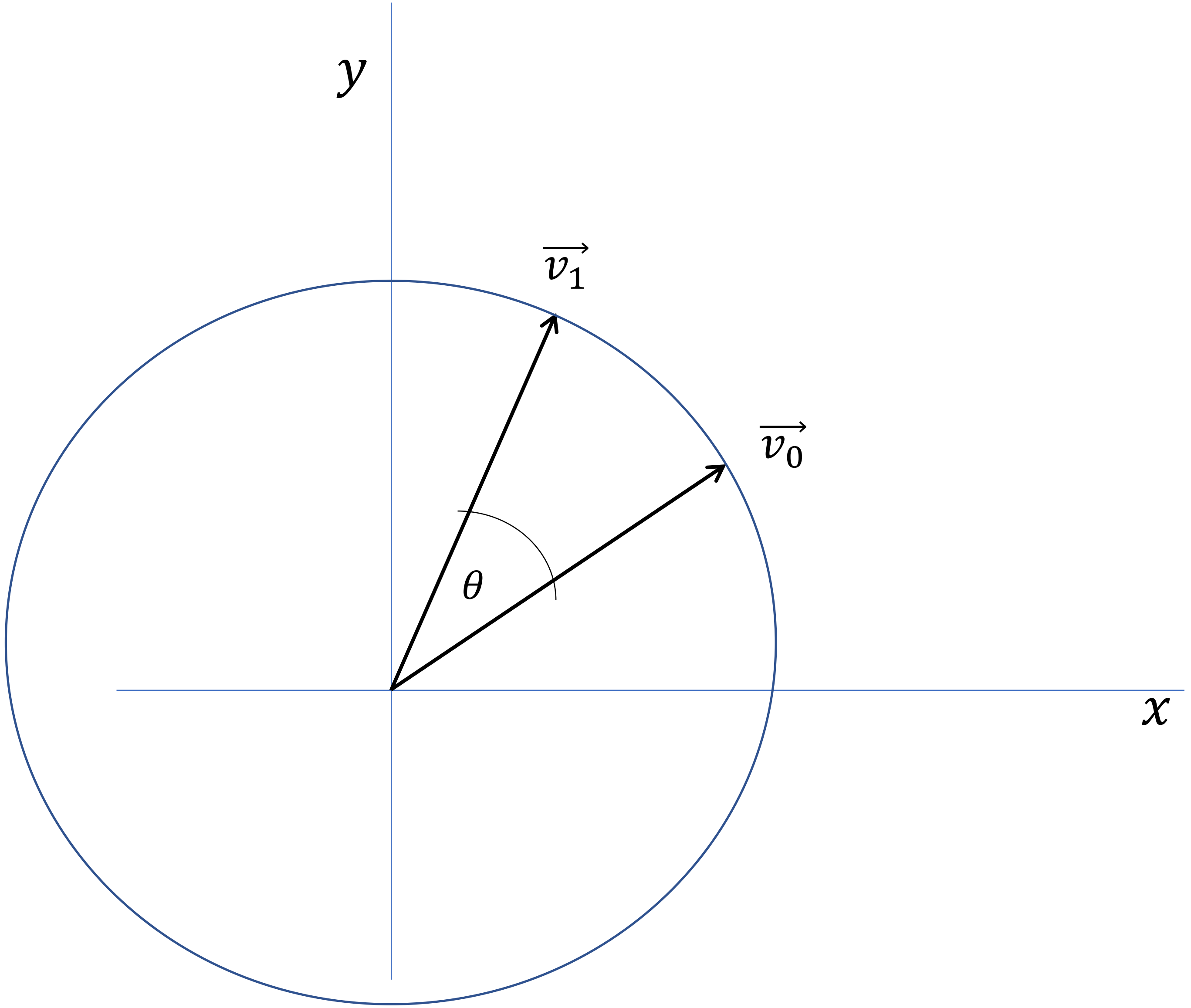
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$



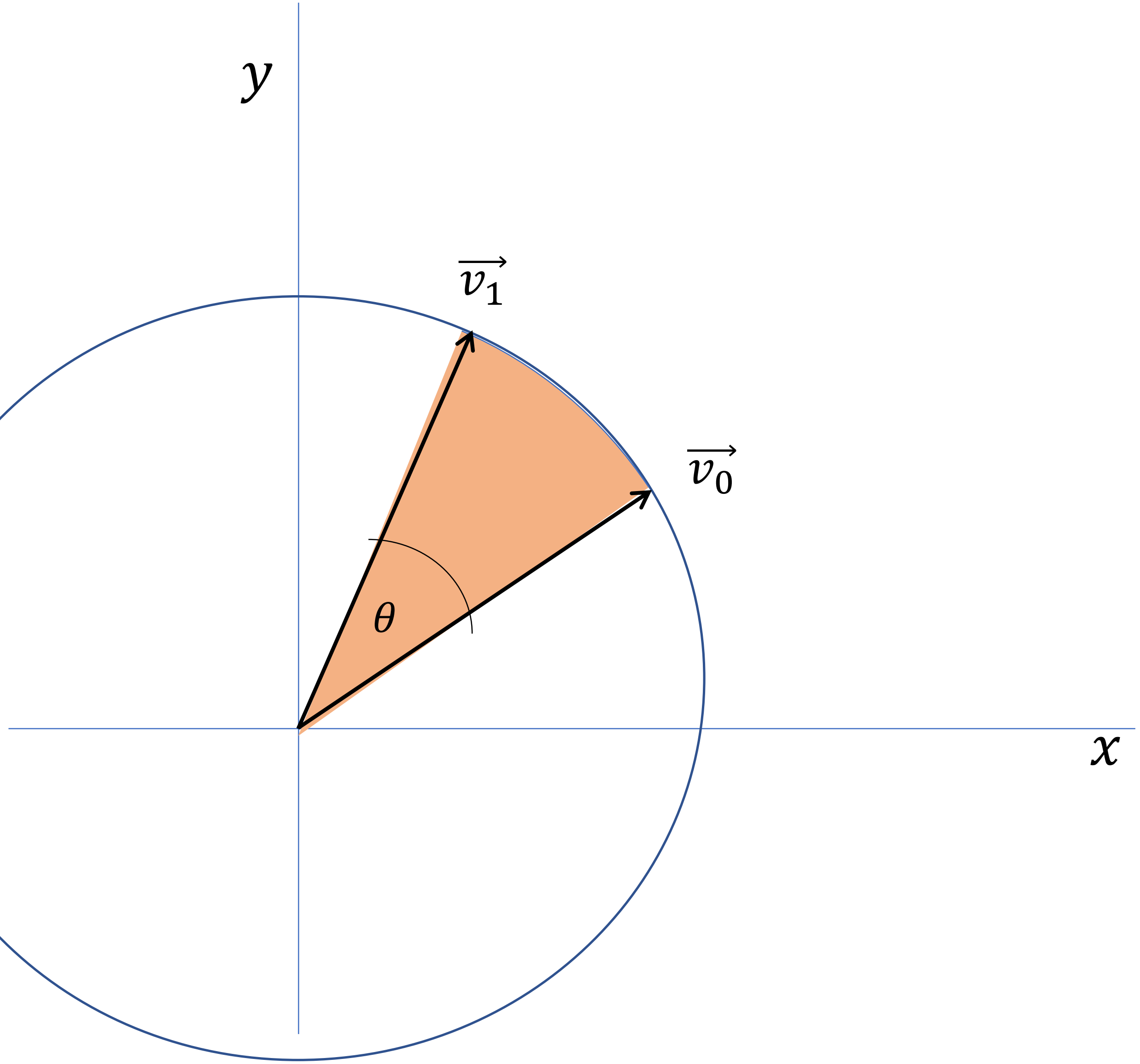
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$



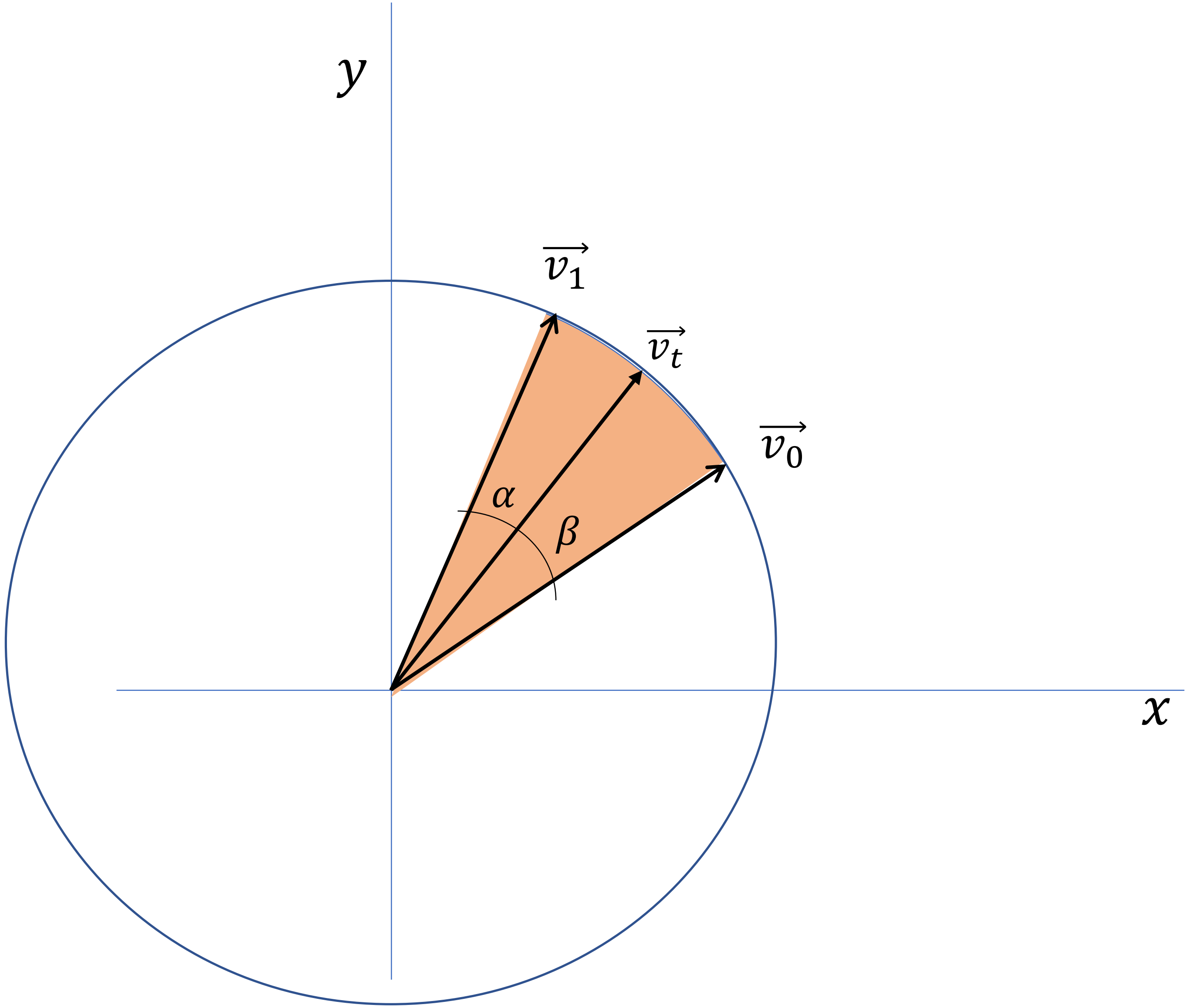
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$



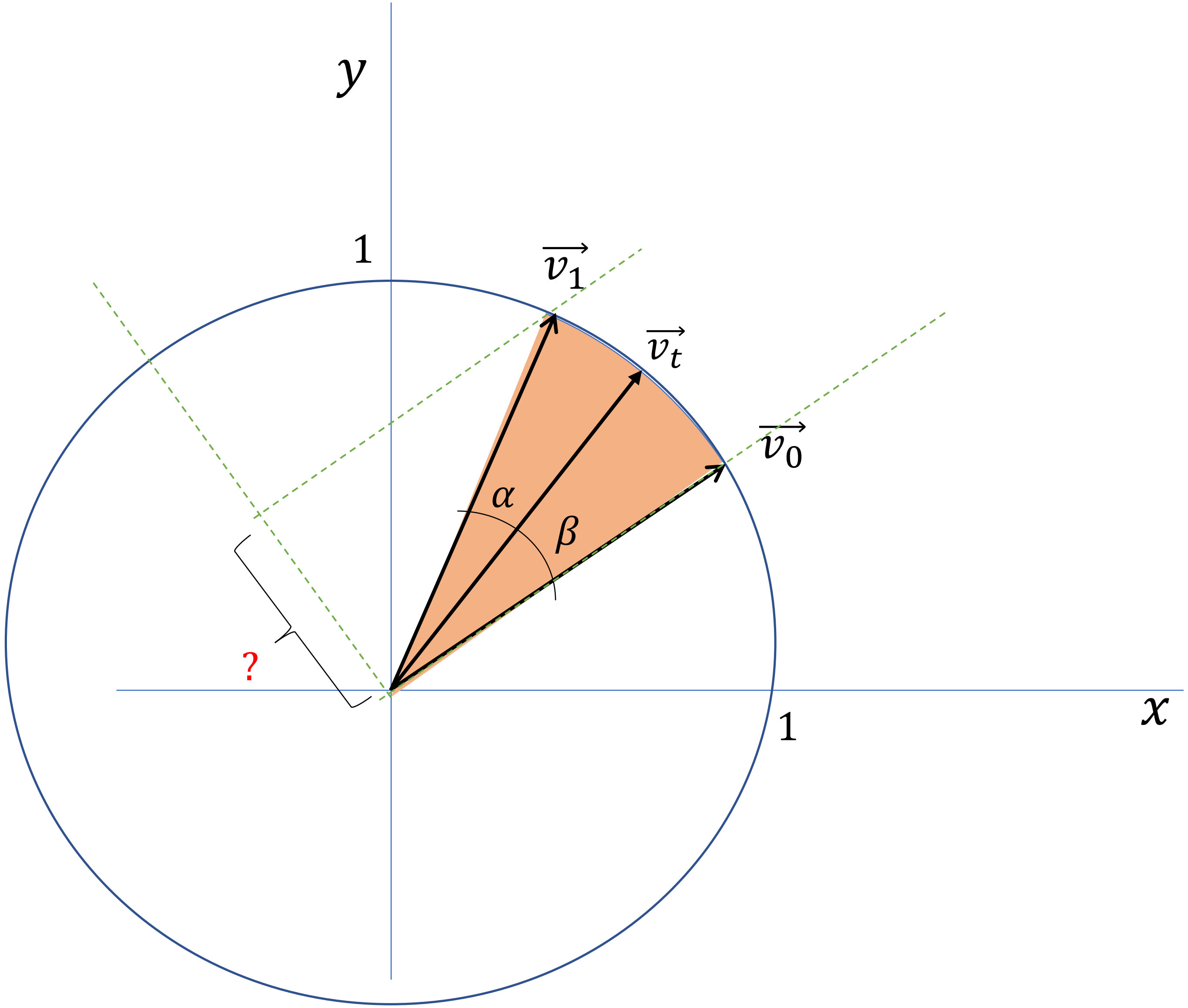
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$



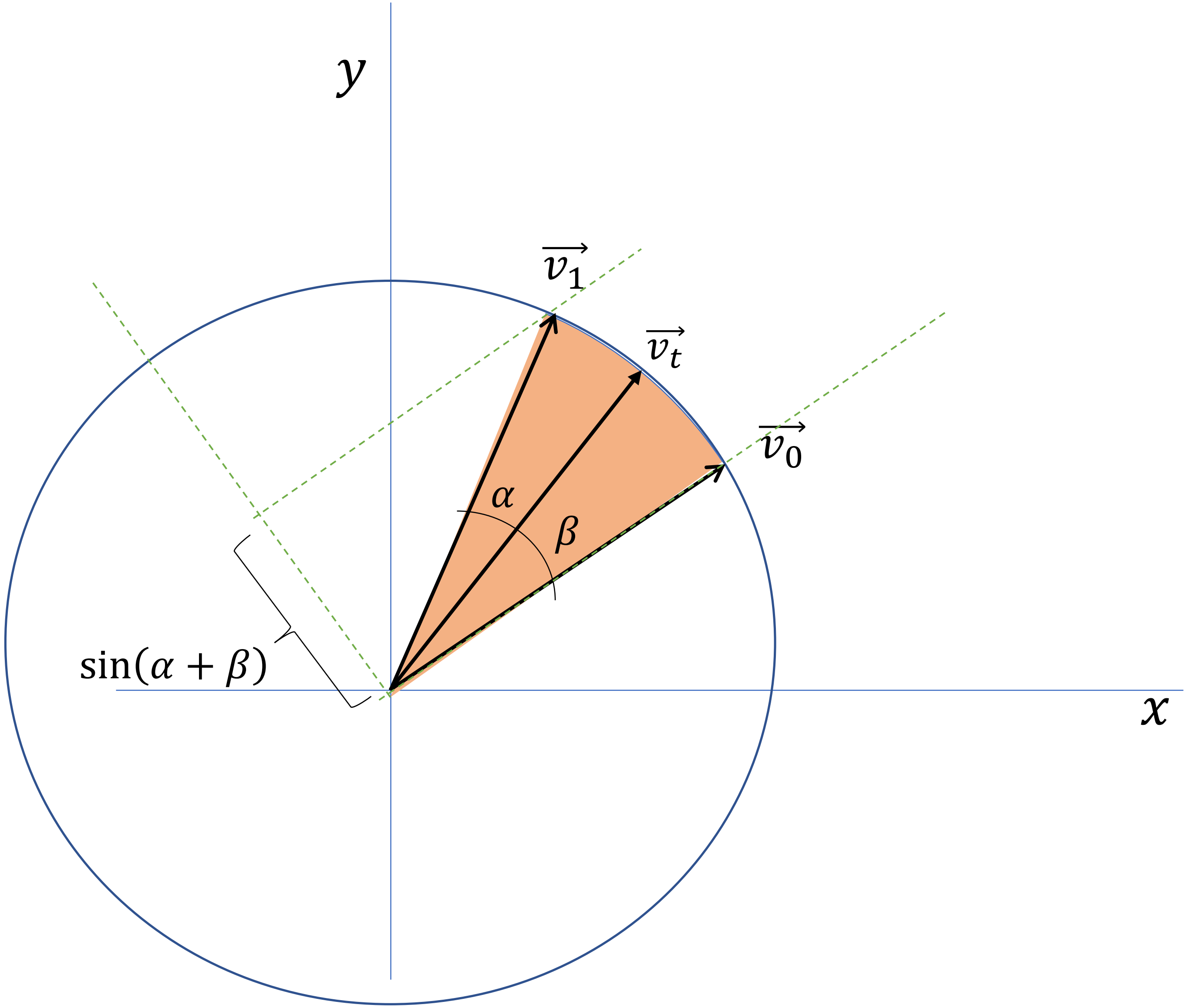
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$



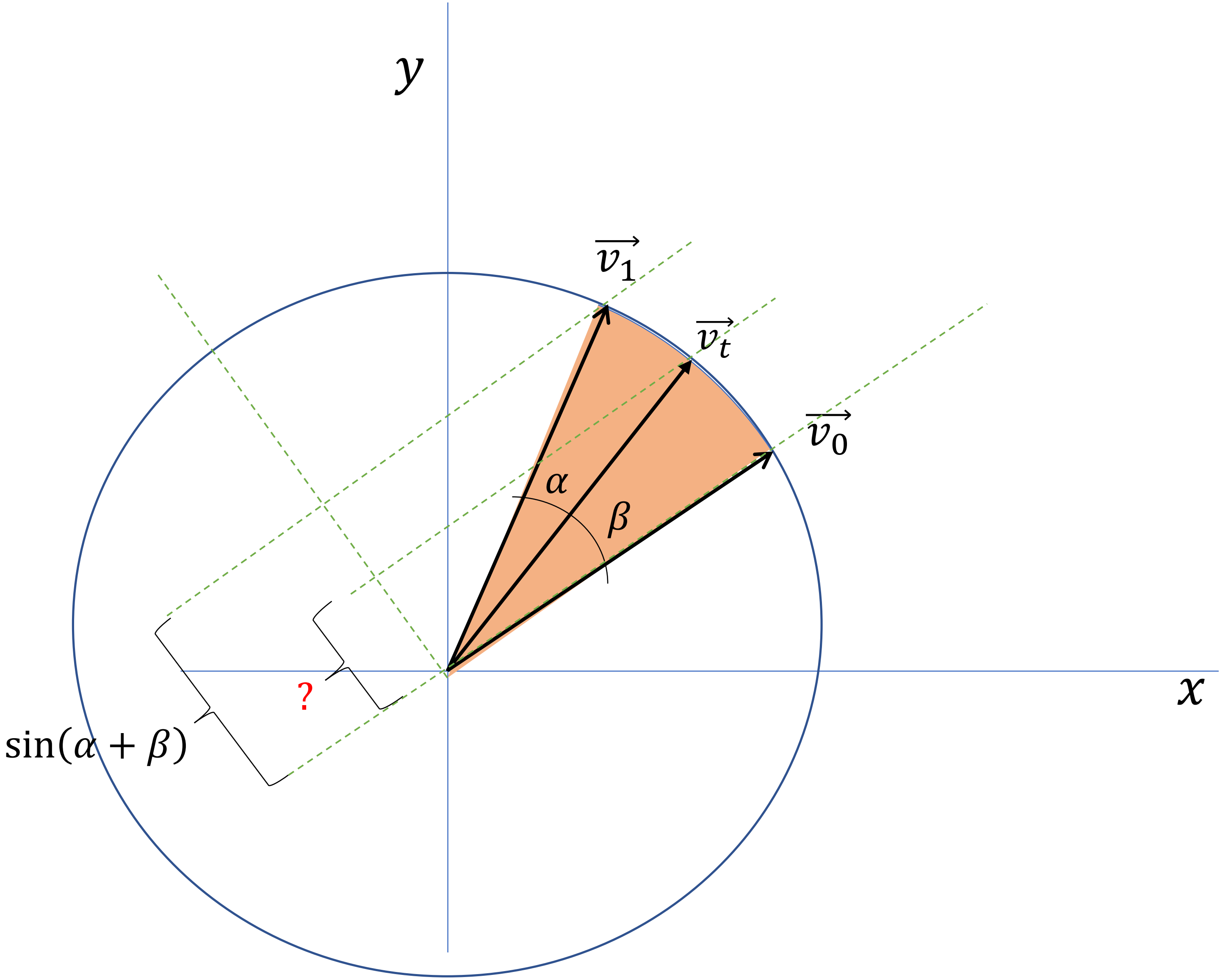
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$



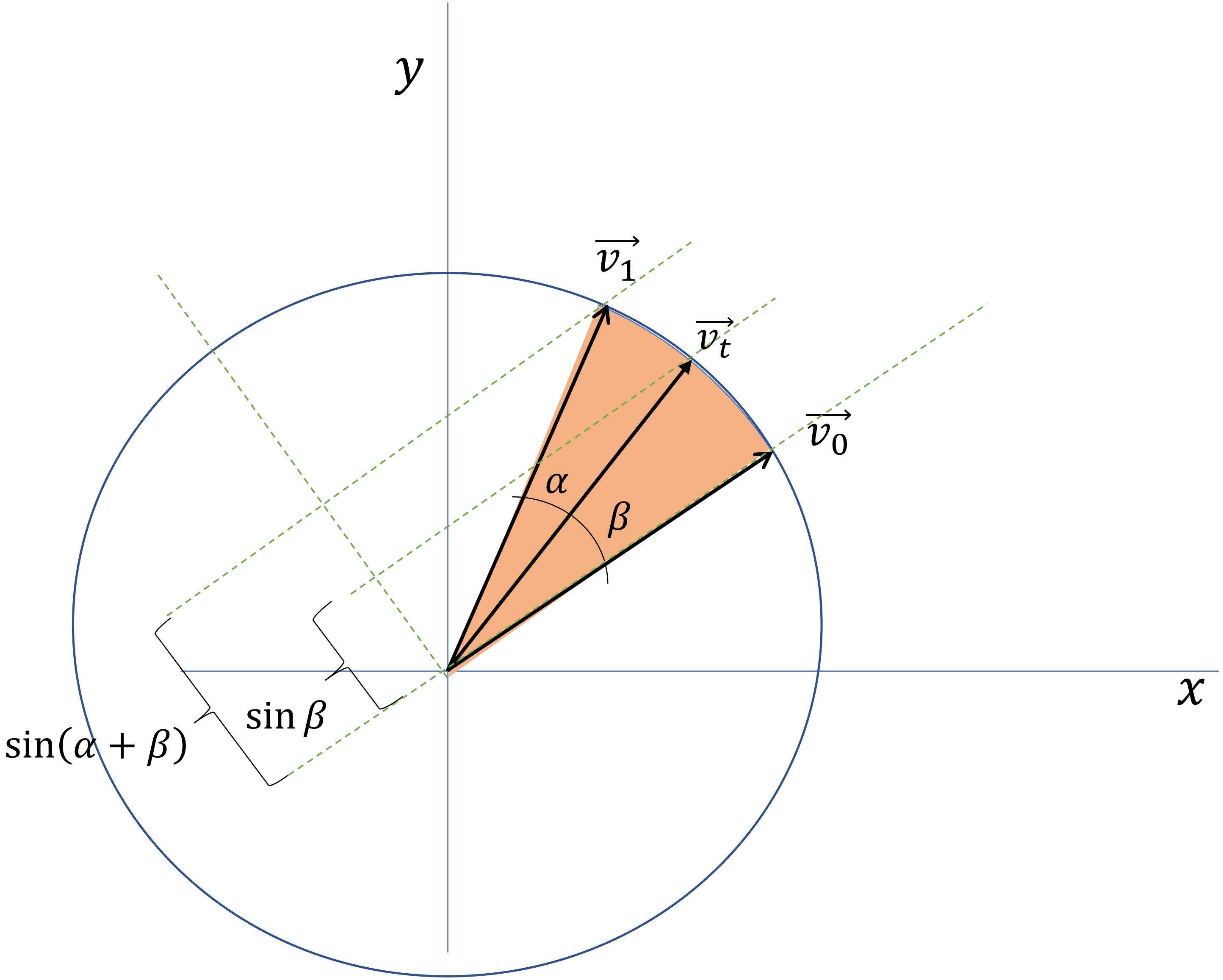
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$



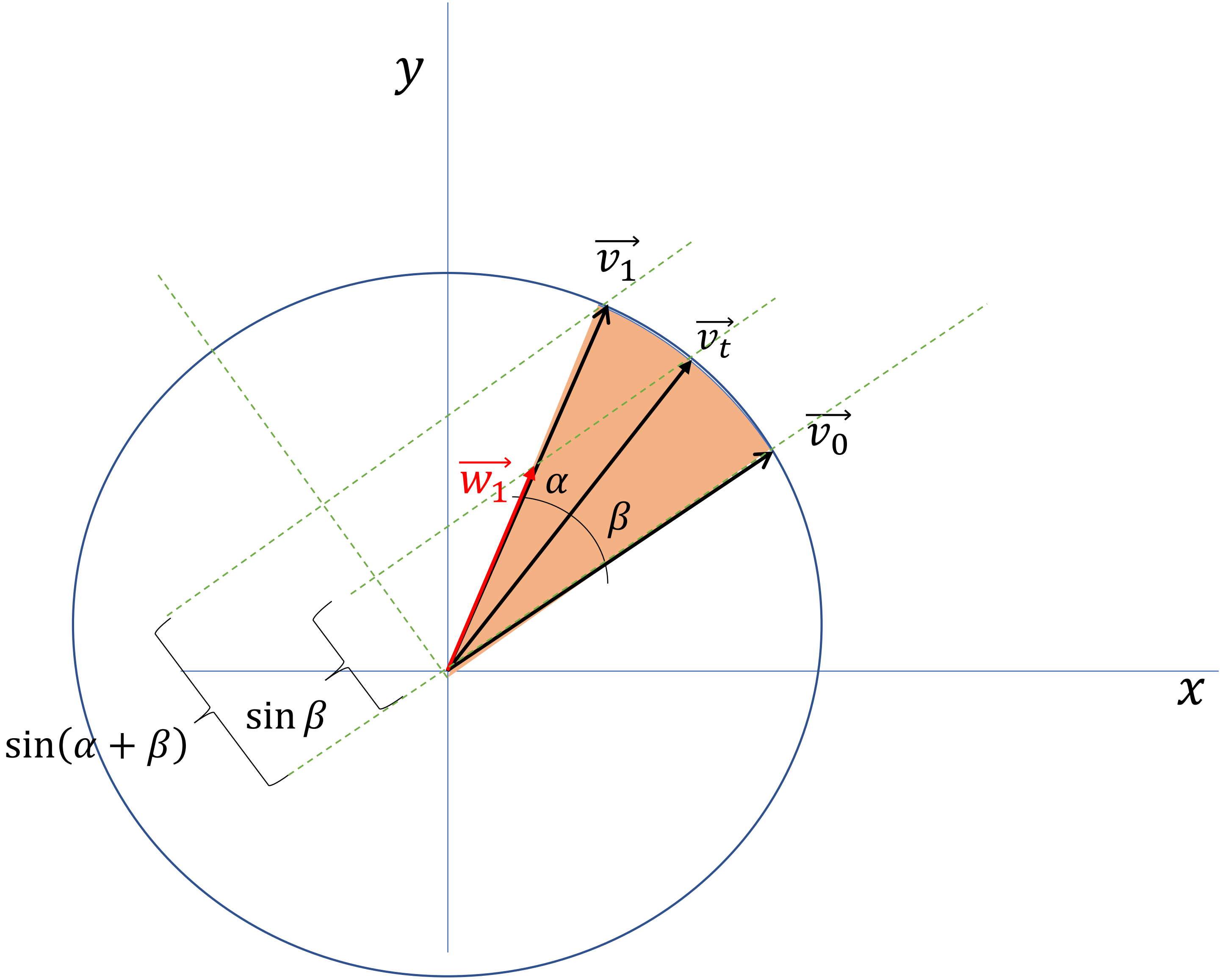
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$



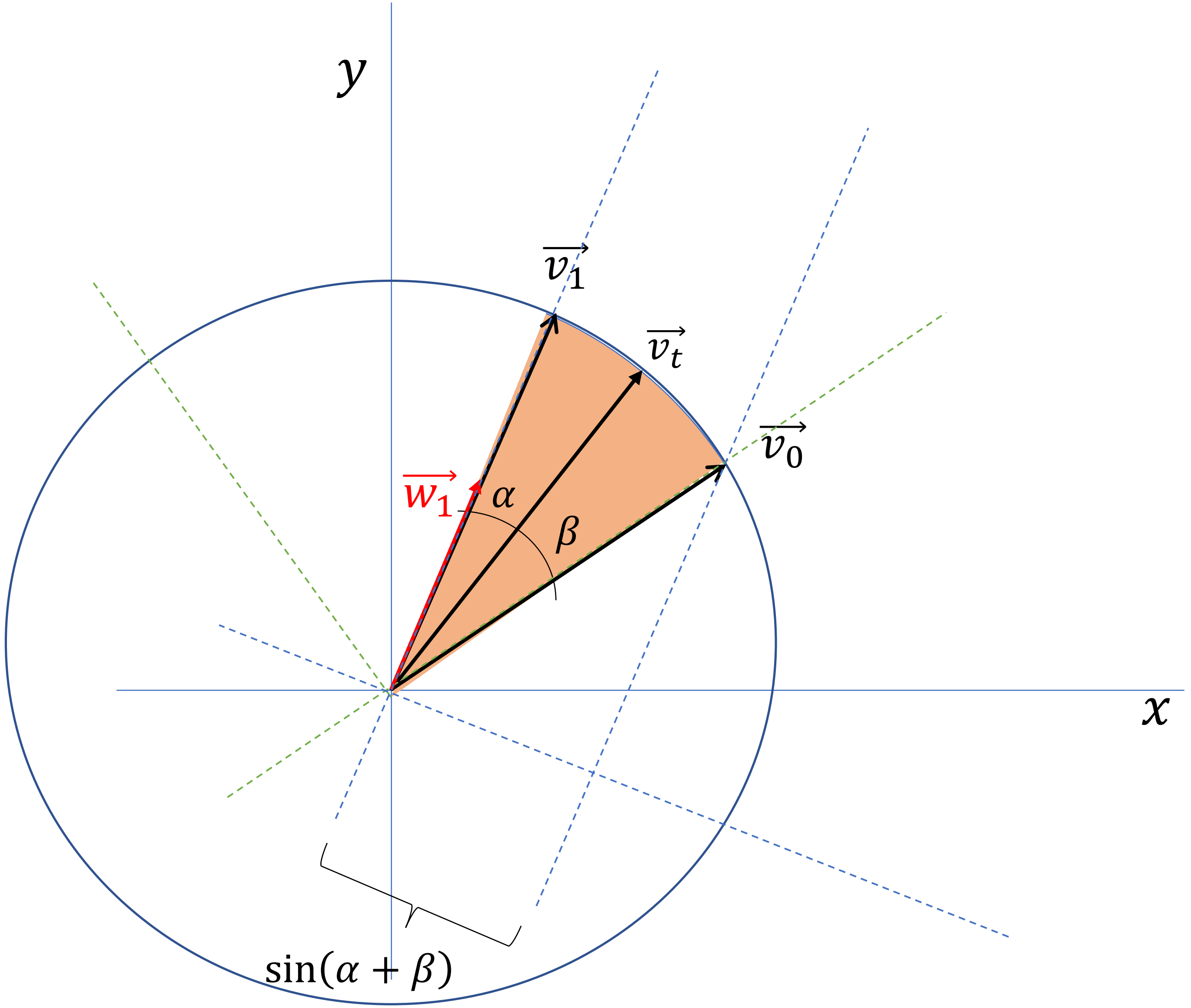
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin \beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$



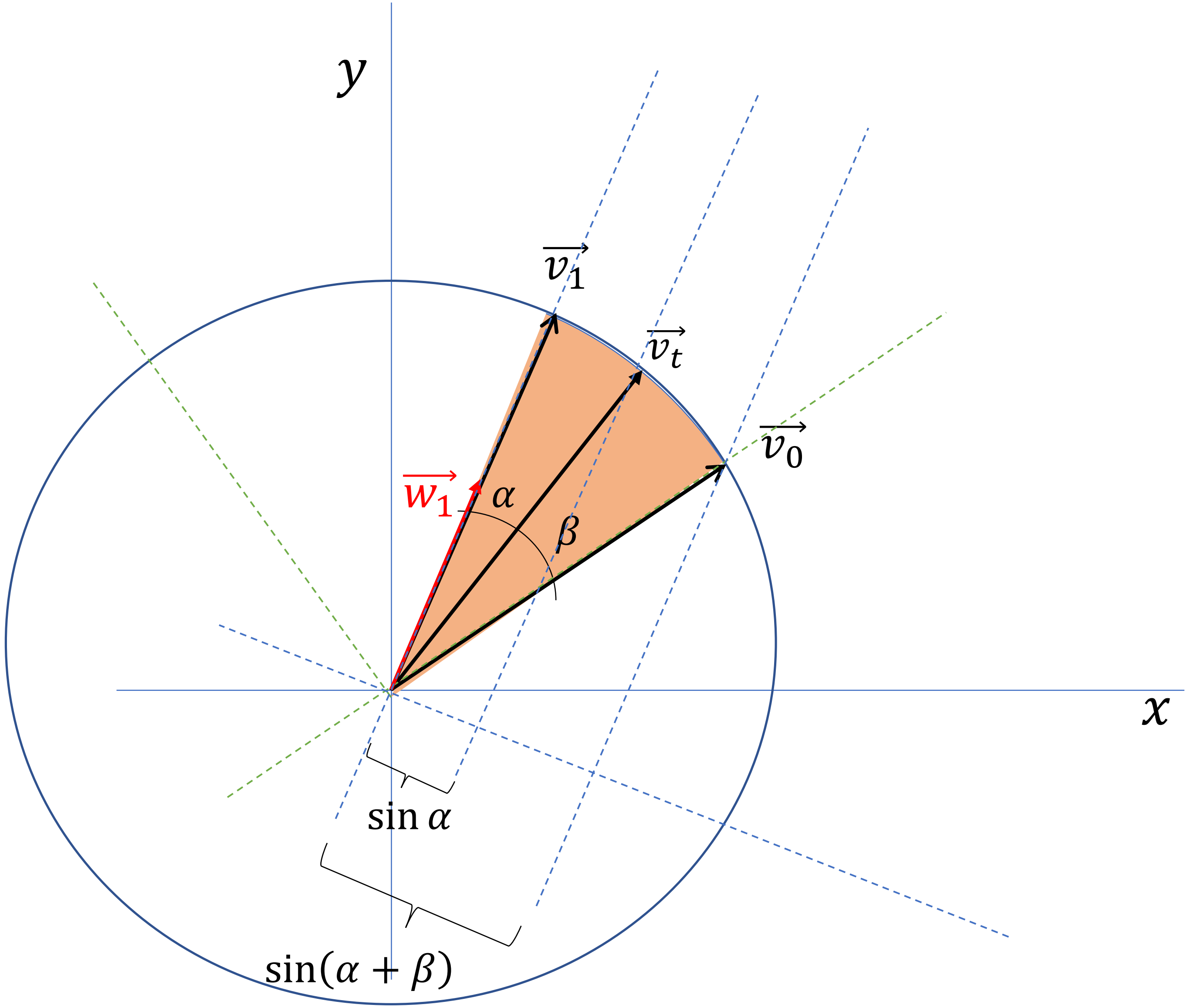
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin \beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$



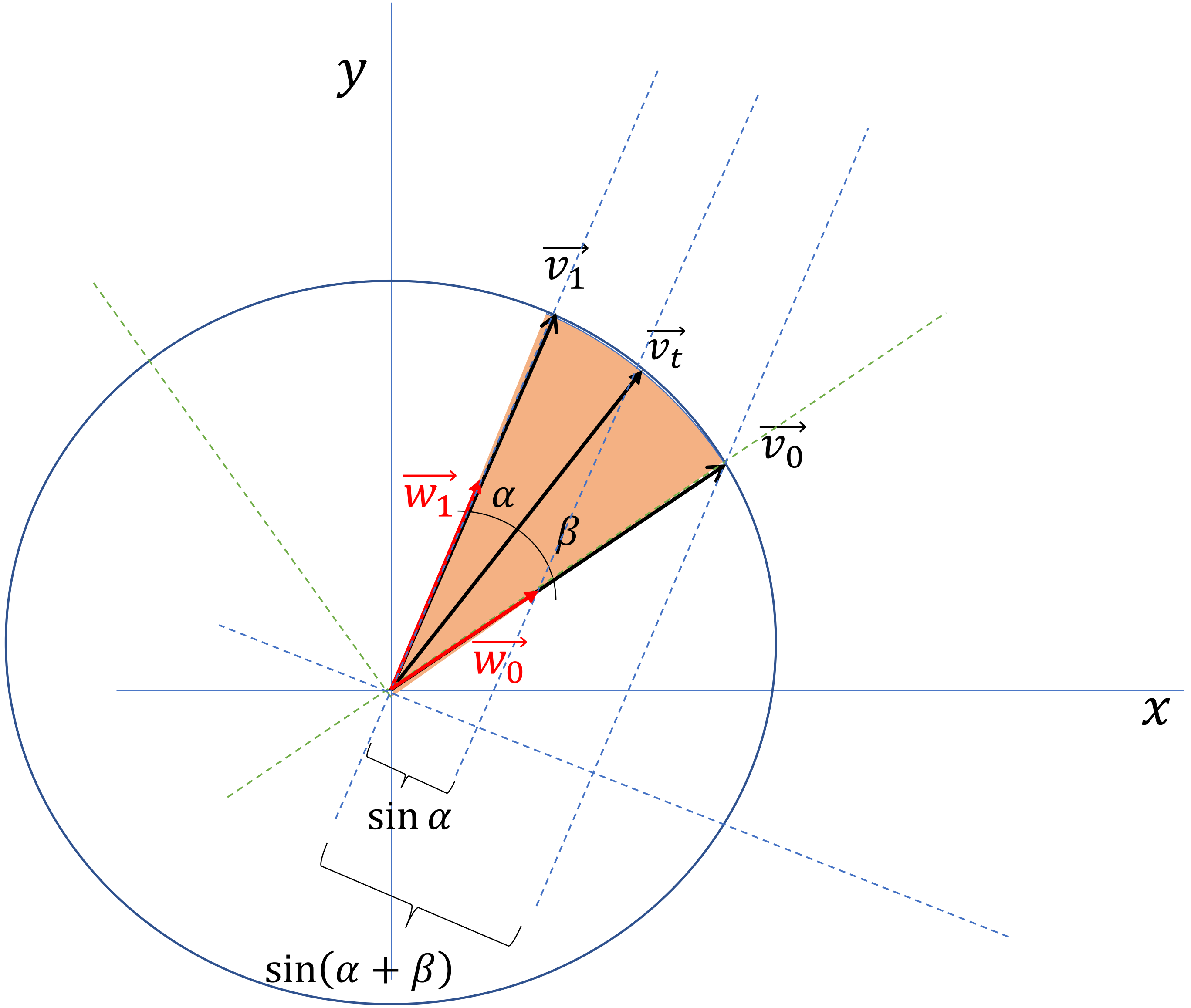
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin\beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$



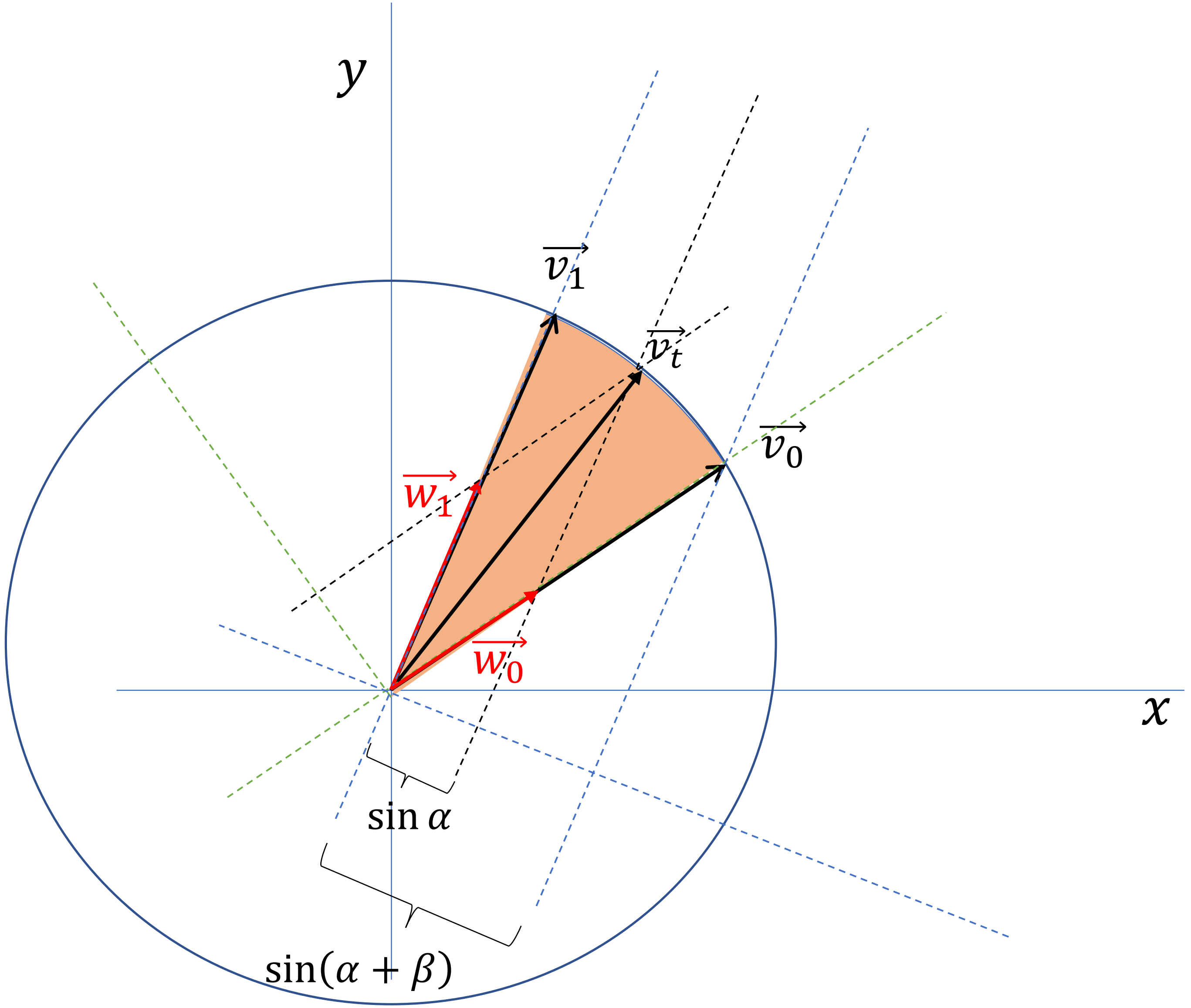
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin\beta}{\sin(\alpha+\beta)}\overrightarrow{v_1}$
  - $\overrightarrow{w_0} = \frac{\sin\alpha}{\sin(\alpha+\beta)}\overrightarrow{v_0}$



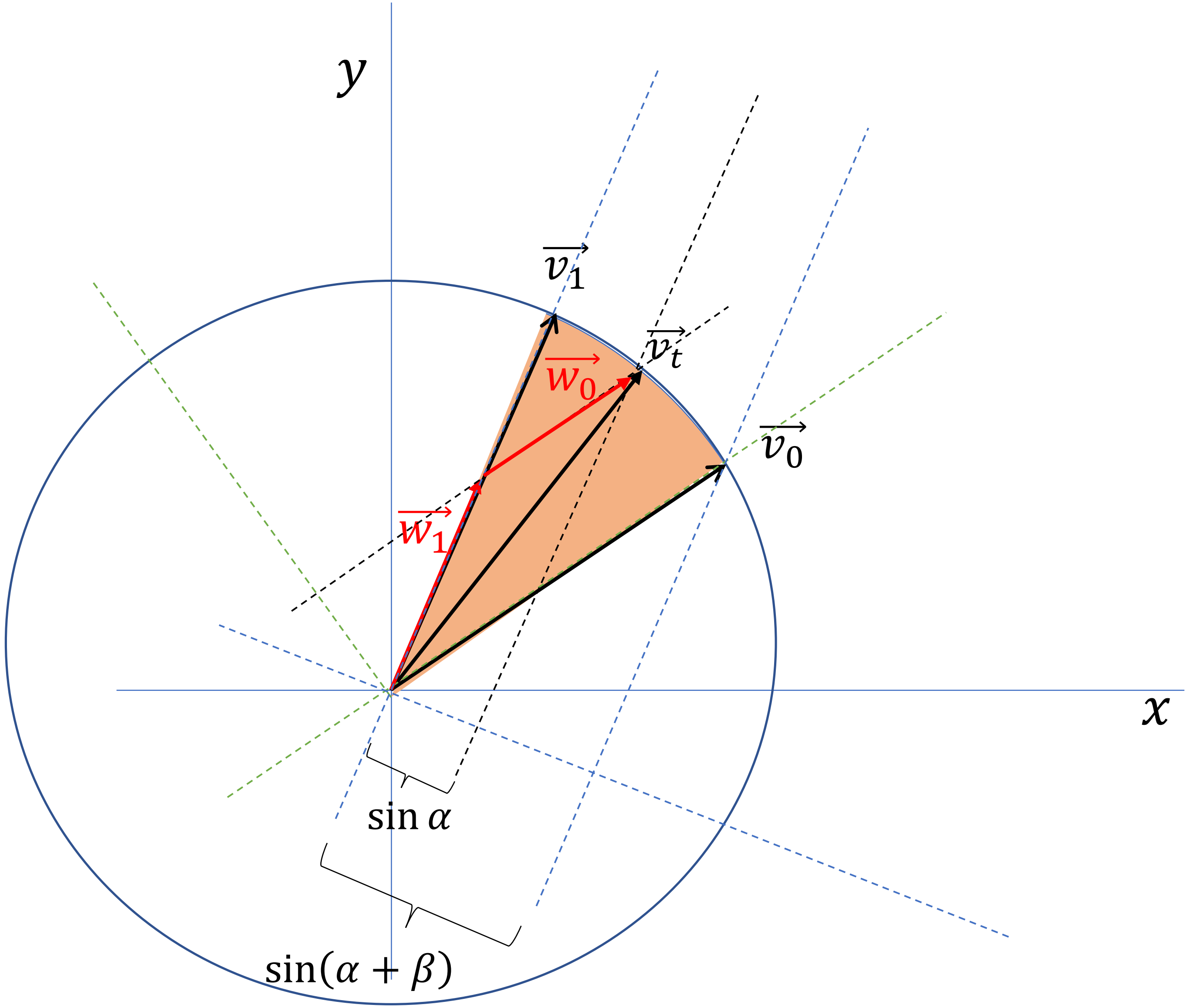
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin \beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$
  - $\overrightarrow{w_0} = \frac{\sin \alpha}{\sin(\alpha+\beta)} \overrightarrow{v_0}$



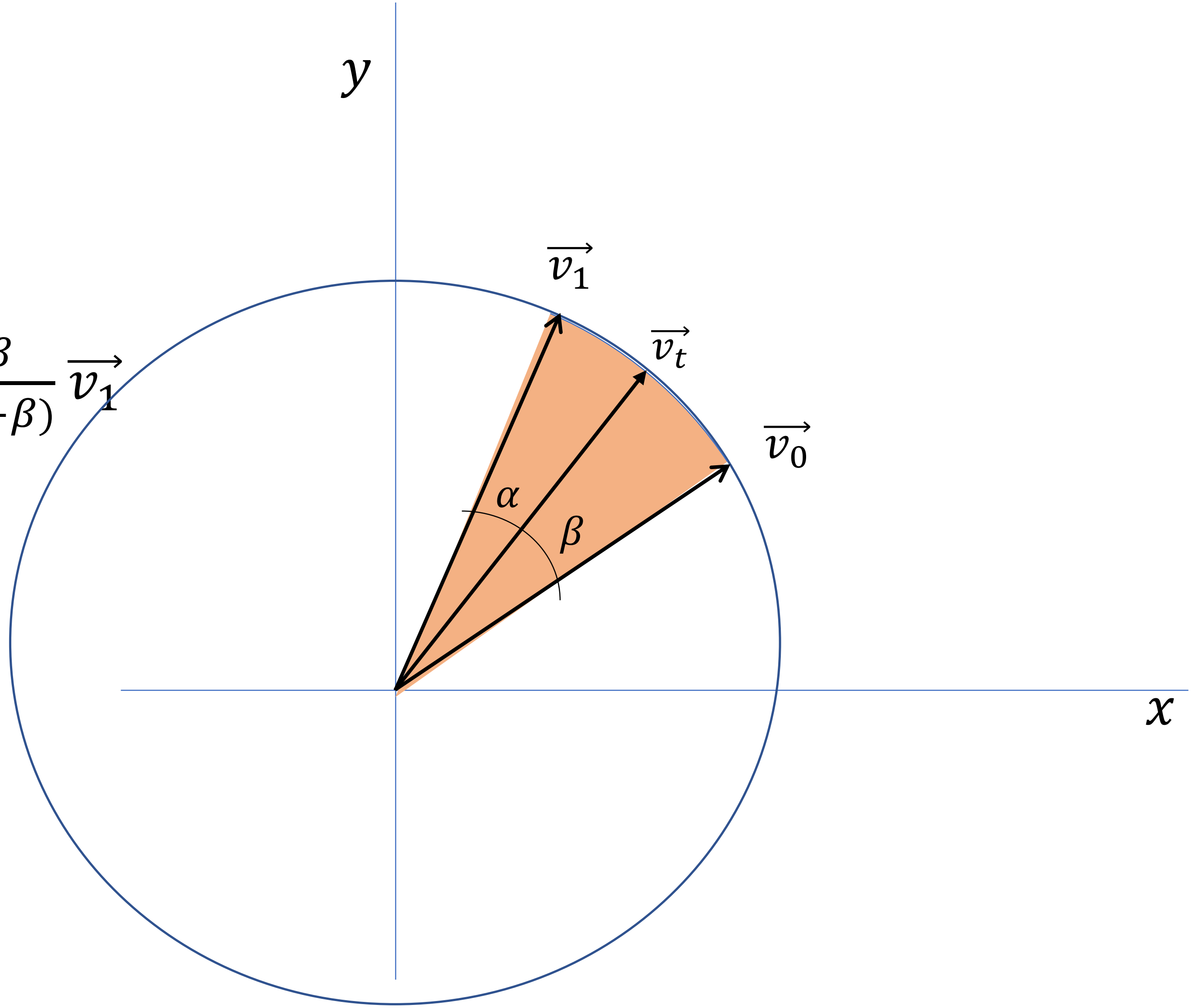
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{w_1} = \frac{\sin \beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$
  - $\overrightarrow{w_0} = \frac{\sin \alpha}{\sin(\alpha+\beta)} \overrightarrow{v_0}$
  - $\overrightarrow{v_t} = \overrightarrow{w_0} + \overrightarrow{w_1}$



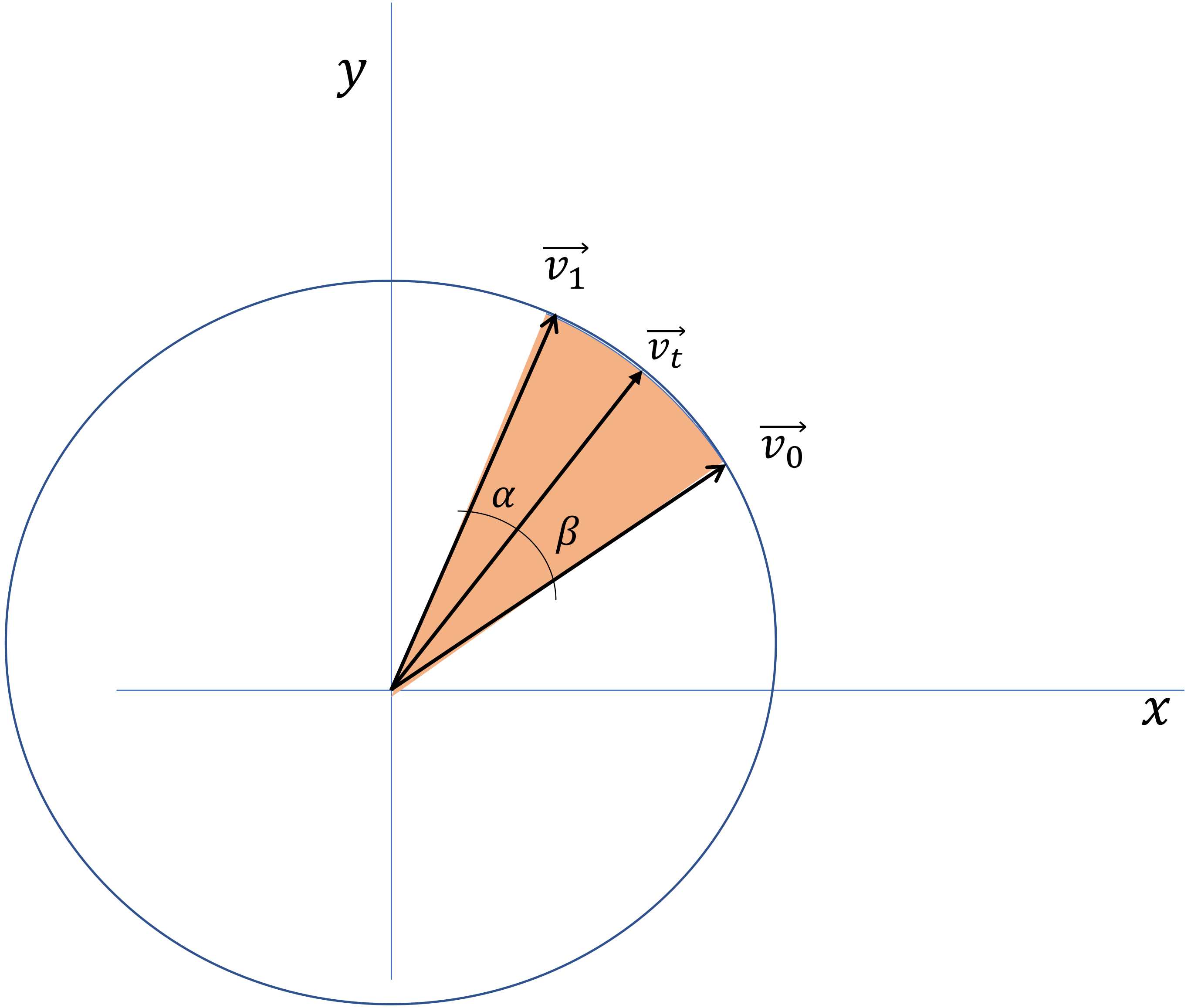
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{v_t} = \frac{\sin \alpha}{\sin(\alpha+\beta)} \overrightarrow{v_0} + \frac{\sin \beta}{\sin(\alpha+\beta)} \overrightarrow{v_1}$



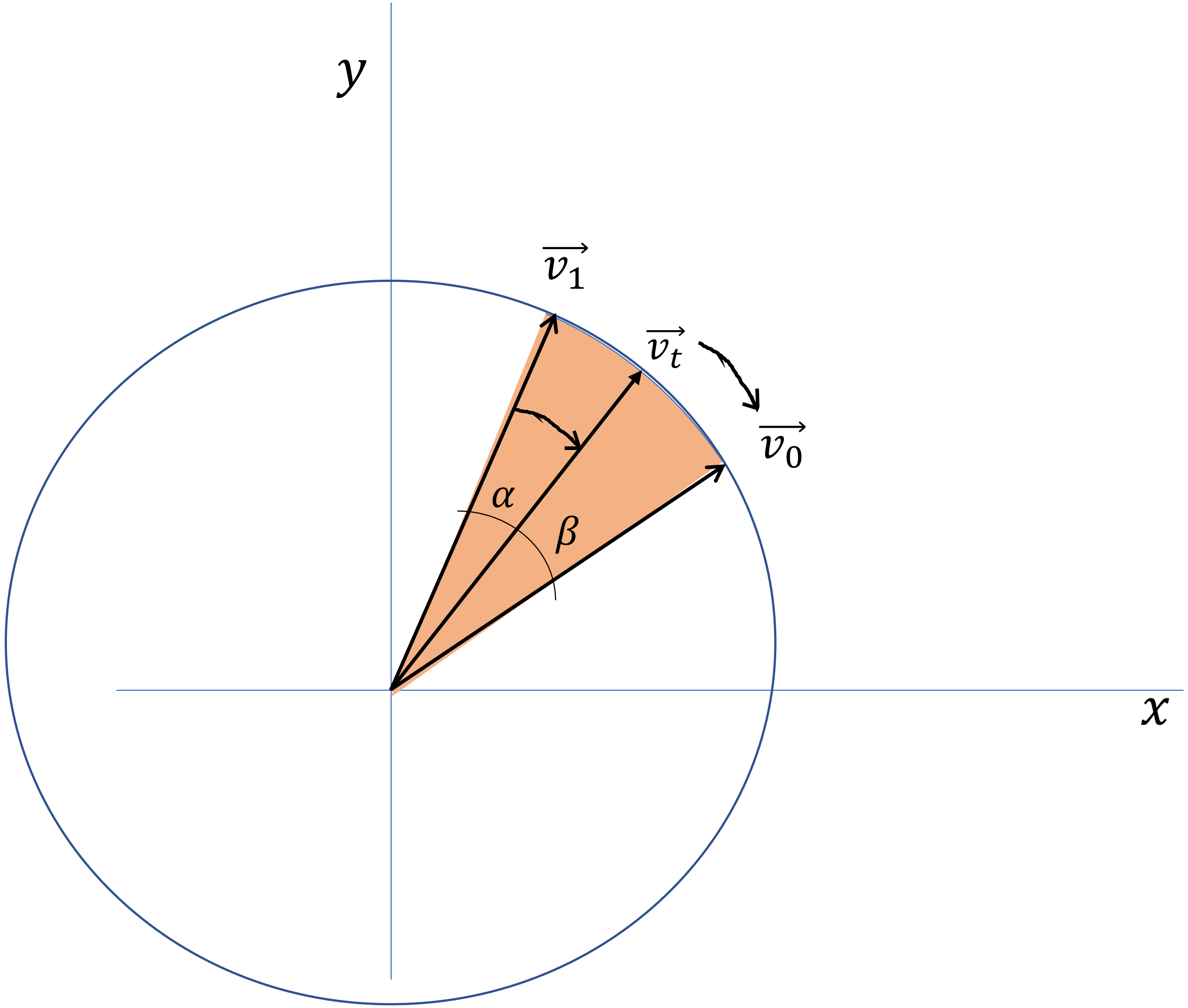
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $\overrightarrow{v_t} = \frac{\sin \alpha}{\sin \theta} \overrightarrow{v_0} + \frac{\sin \beta}{\sin \theta} \overrightarrow{v_1}$



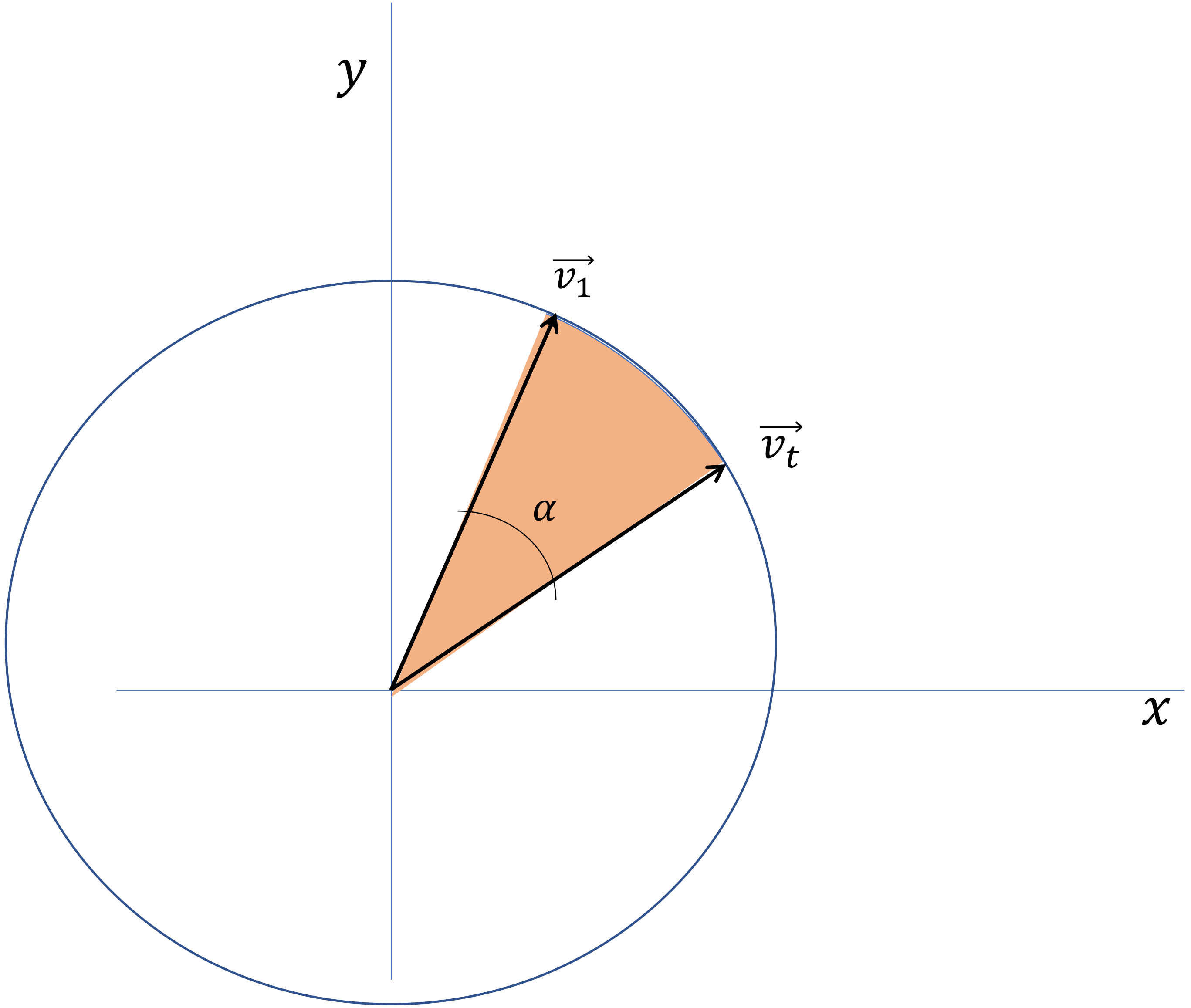
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = 0 \Rightarrow \overrightarrow{v_t} = \overrightarrow{v_0}$



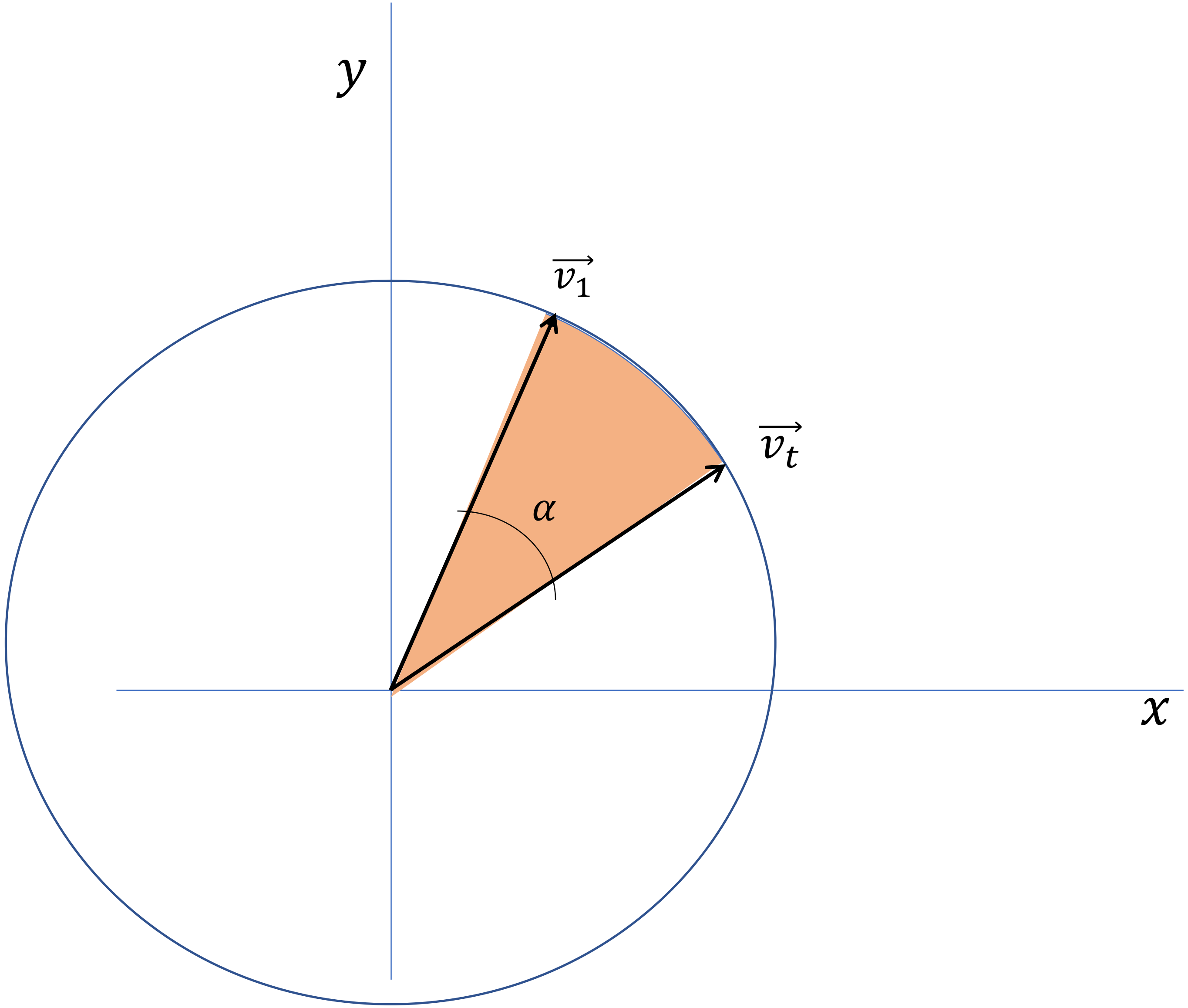
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = 0 \Rightarrow \theta = \alpha$
  - $\theta = \alpha \Rightarrow \beta = 0 = \theta t$
  - $\overrightarrow{v_t} = \frac{\sin \alpha}{\sin \theta} \overrightarrow{v_0} + \frac{\sin \beta}{\sin \theta} \overrightarrow{v_1}$



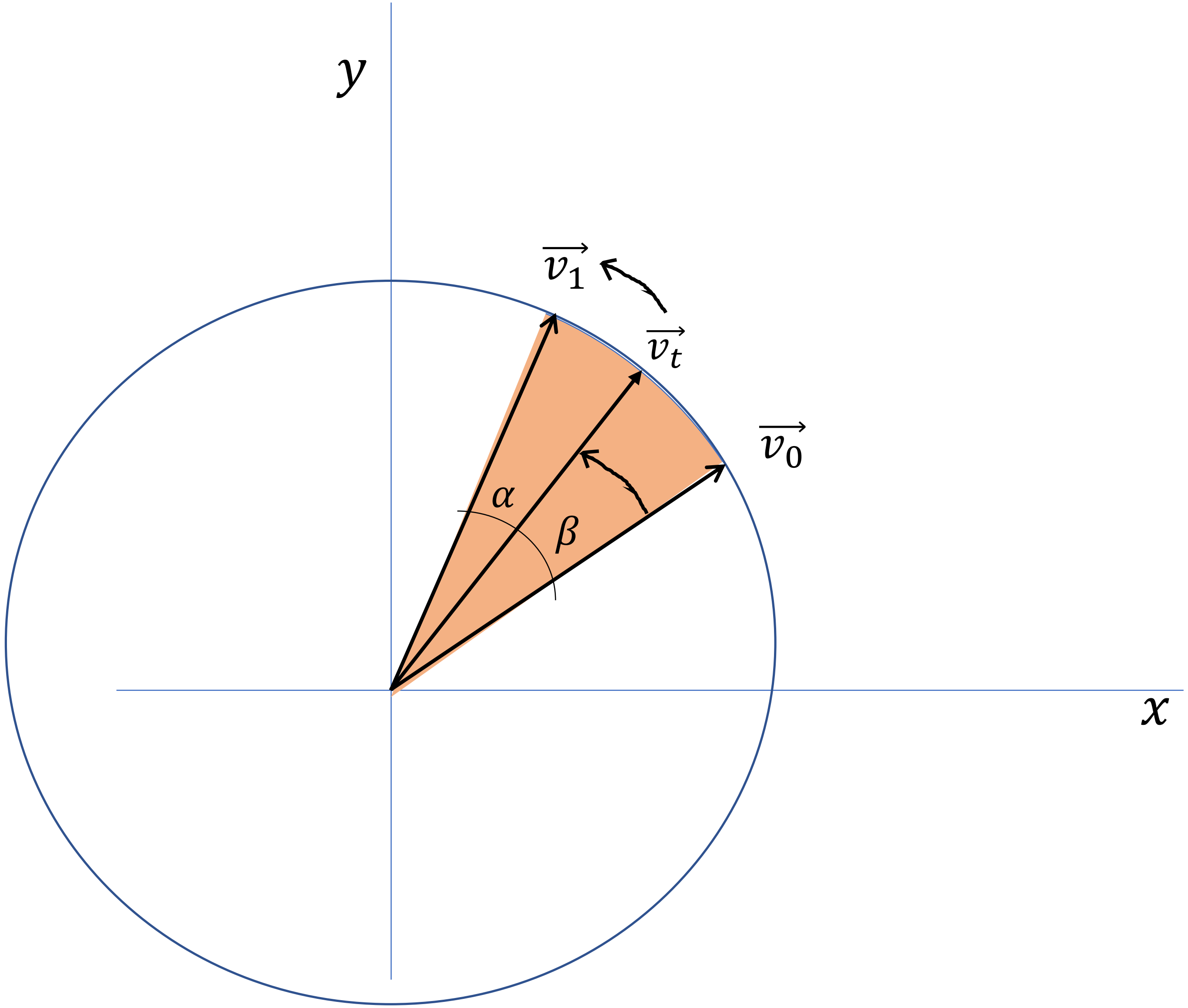
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = 0 \Rightarrow \theta = \alpha$
  - $\theta = \alpha \Rightarrow \beta = 0 = t\theta$
  - $\overrightarrow{v_t} = \frac{\sin \alpha}{\sin \theta} \overrightarrow{v_0} + \frac{\sin t\theta}{\sin \theta} \overrightarrow{v_1}$



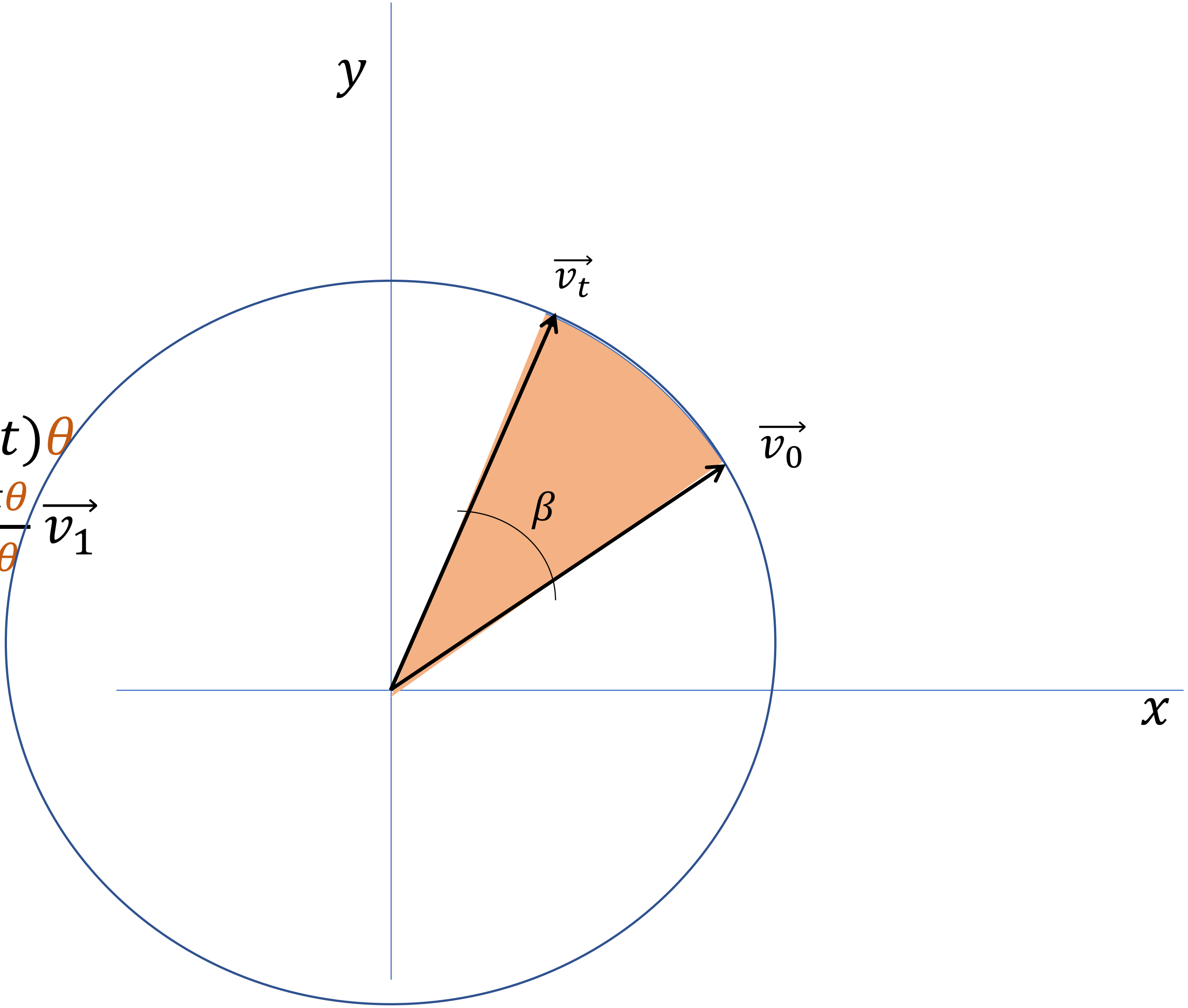
Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = 1 \Rightarrow \overrightarrow{v_t} = \overrightarrow{v_1}$
  - $\overrightarrow{v_t} = \frac{\sin \alpha}{\sin \theta} \overrightarrow{v_0} + \frac{\sin t\theta}{\sin \theta} \overrightarrow{v_1}$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = 1 \Rightarrow \theta = \beta$
  - $\theta = \beta \Rightarrow \alpha = 0 = (1 - t)\theta$
  - $\overrightarrow{v_t} = \frac{\sin((1-t)\theta)}{\sin \theta}\overrightarrow{v_0} + \frac{\sin t\theta}{\sin \theta}\overrightarrow{v_1}$



Spherical linear interpolation interpolation

- For $t \in [0, 1]$
  - $|\overrightarrow{v_0}| = |\overrightarrow{v_1}|$
  - $\theta = \alpha + \beta$
  - $t = \frac{1}{2} \Rightarrow \alpha = \beta$
  - $\alpha = \beta \Rightarrow \alpha = \frac{1}{2} \wedge \beta = \frac{1}{2}$
  - $\overrightarrow{v_t} = \frac{\sin\frac{\theta}{2}}{\sin\theta}\overrightarrow{v_0} + \frac{\sin\frac{\theta}{2}}{\sin\theta}\overrightarrow{v_1}$