Lecture 8: Abstract Argumentation Frameworks

2-AIN-108 Computational Logic

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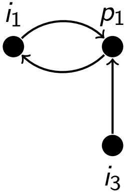
Example

Israeli:	My government cannot negotiate with your government because your government doesn't recognize my government.
Palestinian:	Your government doesn't recognize my government either.
Israeli:	But your government is a terrorist government.



Abstract Argumentation Framework

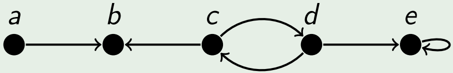
Definition (Abstract Argumentation Framework)

An abstract argumentation framework is a pair $AF = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a set of arguments and $\mathcal{R}$ is an attack relation.

An argument A attacks an argument B iff $(A, B) \in \mathcal{R}$. An set S of arguments attacks an argument B iff B is attacked by an argument in S.

Example (Abstract Argumentation Framework)

Let $AF = (\mathcal{A}, \mathcal{R})$ be an argumentation framework where
$\mathcal{A} = \{a, b, c, d, e\}$ and
$\mathcal{R} = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}$.



Conflict Freeness and Admissibility

Definition (Conflict-Free)

A set S of arguments is conflict-free if there are no arguments A and B in S such that A attacks B.

Definition (Acceptability)

An argument A is acceptable with respect to a set S of arguments (resp. S defends A) iff each argument B attacking A is attacked by S.

Definition (Admissibility)

A set S of arguments is admissible iff each argument in S is acceptable with respect to S (is defended by S).

Extensions

Definition (Complete Extension)

An admissible set S of arguments is called complete extension iff each argument acceptable with respect to S belongs to S.

Definition (Grounded Extension)

The grounded extension is the least complete extension.

Definition (Preferred Extension)

A preferred extension is a maximal complete extension.

Definition (Stable Extension)

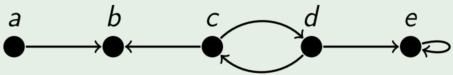
A conflict free set S of arguments is a stable extension iff S attacks each argument which does not belong to S.

Proposition

Let AF be an abstract argumentation framework. Then

- $\text{Stable}(AF) \subseteq \text{Preferred}(AF) \subseteq \text{Complete}(AF)$
- $\text{Grounded}(AF) \subseteq \text{Complete}(AF)$

Example (Continued)



$$
\begin{aligned}
\text{Admissible}(AF) &= \{\emptyset, \{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\} \\
\text{Complete}(AF) &= \{\{a\}, \{a, c\}, \{a, d\}\} \\
\text{Grounded}(AF) &= \{\{a\}\} \\
\text{Preferred}(AF) &= \{\{a, c\}, \{a, d\}\} \\
\text{Stable}(AF) &= \{\{a, d\}\}
\end{aligned}
$$

Characteristic Function

Definition (Characteristic Function)

The characteristic function $F_{AF} : 2^{\mathcal{A}} \mapsto 2^{\mathcal{A}}$ of an abstract argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ is defined as follows:

$$F_{AF}(S) = \{A \in \mathcal{A} \mid S \text{ defends } A\}$$

Definition (Iteration of F_{AF})

The iteration F^i_{AF} of the characteristic function F_{AF} is defined as follows:

$$\begin{array}{rcl} F^0_{AF} & = & \emptyset \\ F^{i+1}_{AF} & = & F_{AF}(F^i_{AF}) \\ F^{\infty}_{AF} & = & \bigcup_{i \geq 0} F^i_{AF} \end{array}$$

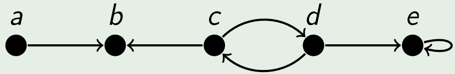
Properties

Proposition

A conflict-free set S of arguments is

1. *admissible iff S is a post-fixpoint of F (i.e. $S \subseteq F(S)$)*
2. *complete extension iff S is a fixpoint of F (i.e. $S = F(S)$)*
3. *grounded extension iff S is the least fixpoint of F*
4. *preferred extension iff S is a maximal fixpoint of F*

Example (Continued)



$$\begin{aligned} F^0 &= \emptyset \\ F^1 &= F(F^0) = F(\emptyset) = \{a\} \\ F^2 &= F(F^1) = F(\{a\}) = \{a\} = F^1 \end{aligned}$$

Computation of Grounded Extension

Definition (Finitary Argumentation Framework)

An abstract argumentation frametowk $AF = (\mathcal{A}, \mathcal{R})$ is finitary iff for each argument A, there exist only finitely-many arguments which attack A.

Proposition

Let S be the grounded extension of an argumentation framework AF. Then

1. $F_{AF}^{\infty} \subseteq S$
2. *If AF is finitary then* $S \subseteq F_{AF}^{\infty}$

Labelings

Definition (Labeling)

A labeling for an abstract argumentation framework $AF = (\mathcal{A}, \mathcal{R})$ is a function $\mathcal{L} \colon \mathcal{A} \mapsto \{\text{In}, \text{Out}, \text{UnDec}\}$. We define

$$\begin{aligned}
\text{In}(\mathcal{L}) &= \{A \in \mathcal{A} \mid \mathcal{L}(A) = \text{In}\} \\
\text{Out}(\mathcal{L}) &= \{A \in \mathcal{A} \mid \mathcal{L}(A) = \text{Out}\} \\
\text{UnDec}(\mathcal{L}) &= \{A \in \mathcal{A} \mid \mathcal{L}(A) = \text{UnDec}\}
\end{aligned}$$

Definition (Legal Argument)

Let $\mathcal{L}$ be a labeling for $AF = (\mathcal{A}, \mathcal{R})$. An argument A is legal iff

- if $\mathcal{L}(A) = \text{In}$ then $\forall B \in \mathcal{A} \colon (B, A) \in \mathcal{R} \Rightarrow \mathcal{L}(B) = \text{Out}$
- if $\mathcal{L}(A) = \text{Out}$ then $\exists B \in \mathcal{A} \colon (B, A) \in \mathcal{R} \wedge \mathcal{L}(B) = \text{In}$
- if $\mathcal{L}(A) = \text{UnDec}$ then $\exists B \in \mathcal{A} \colon (B, A) \in \mathcal{R} \wedge \mathcal{L}(B) \neq \text{Out}$ and $\forall B \in \mathcal{A} \colon (B, A) \in \mathcal{R} \Rightarrow \mathcal{L}(B) \neq \text{In}$

Labelings

Definition (Admissible and Complete Labeling)

A labeling $\mathcal{L}$ is

- admissible iff all arguments in $\mathsf{In}(\mathcal{L}) \cup \mathsf{Out}(\mathcal{L})$ are legal.
- complete iff all arguments in $\mathsf{In}(\mathcal{L}) \cup \mathsf{Out}(\mathcal{L}) \cup \mathsf{UnDec}(\mathcal{L})$ are legal.

Definition (Grounded, Preferred, and Stable Labeling)

A complete labeling $\mathcal{L}$ is

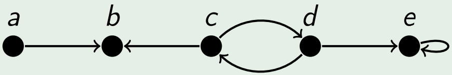
- grounded iff there does not exist a complete labeling $\mathcal{L}'$ with $\mathsf{In}(\mathcal{L}') \subset \mathsf{In}(\mathcal{L})$.
- preferred iff there does not exist a complete labeling $\mathcal{L}'$ with $\mathsf{In}(\mathcal{L}') \supset \mathsf{In}(\mathcal{L})$.
- stable iff $\mathsf{UnDec}(\mathcal{L}) = \emptyset$.

Properties

Proposition

Let $AF = (\mathcal{A}, \mathcal{R})$ be an abstract argumentation framework and S be a set of arguments. Then S is a complete, grounded, preferred, resp. stable extension of AF iff there exists a complete, grounded, preferred, resp. stable labeling $\mathcal{L}$ for AF with $\mathrm{In}(\mathcal{L}) = S$.

Example (Continued)



In	Out	UnDec	UnDec	UnDec
In	Out	In	Out	UnDec
In	Out	Out	In	Out