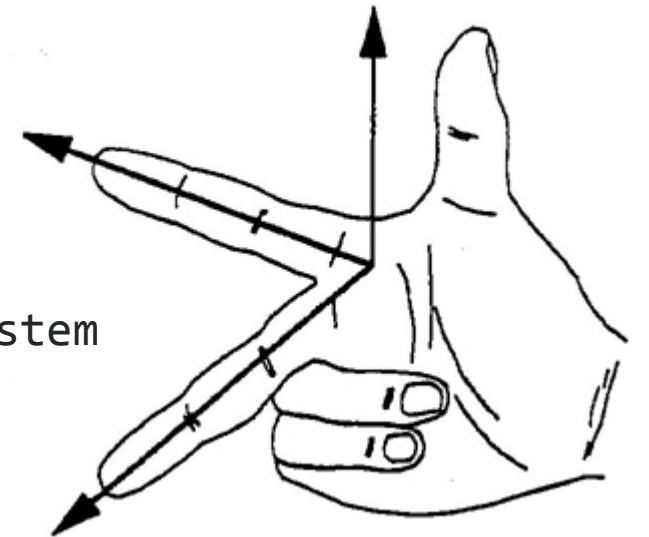
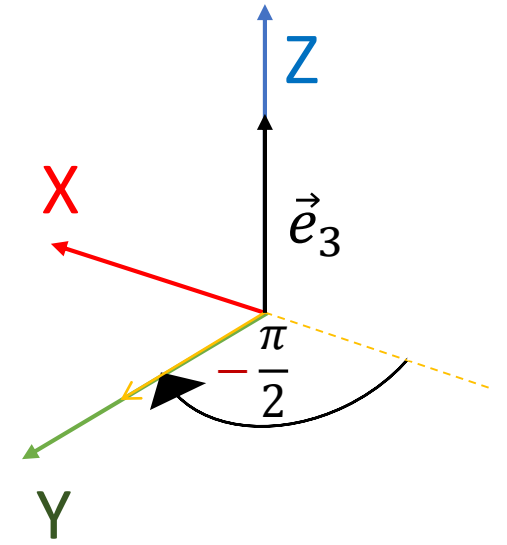
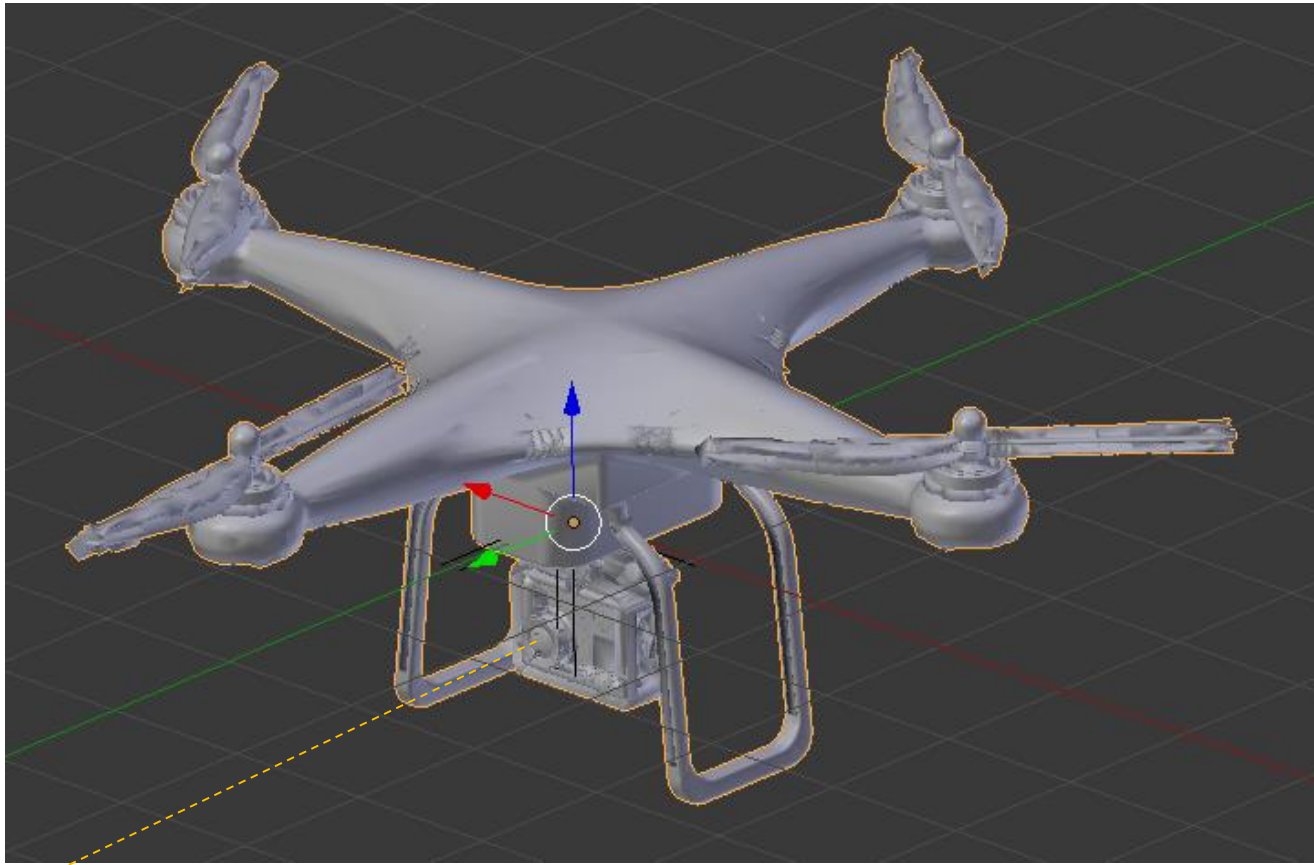
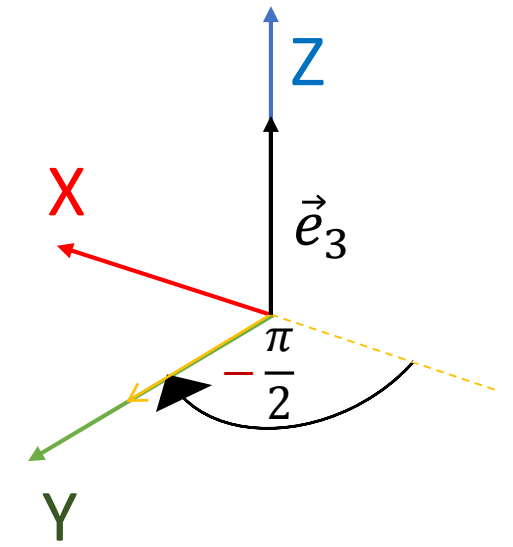


right-handed coordinate system

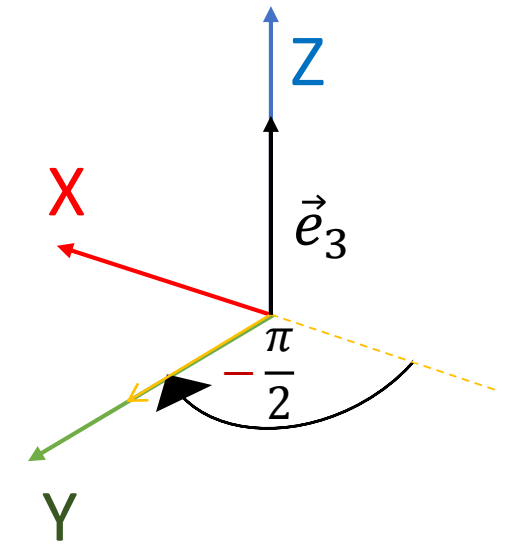




$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



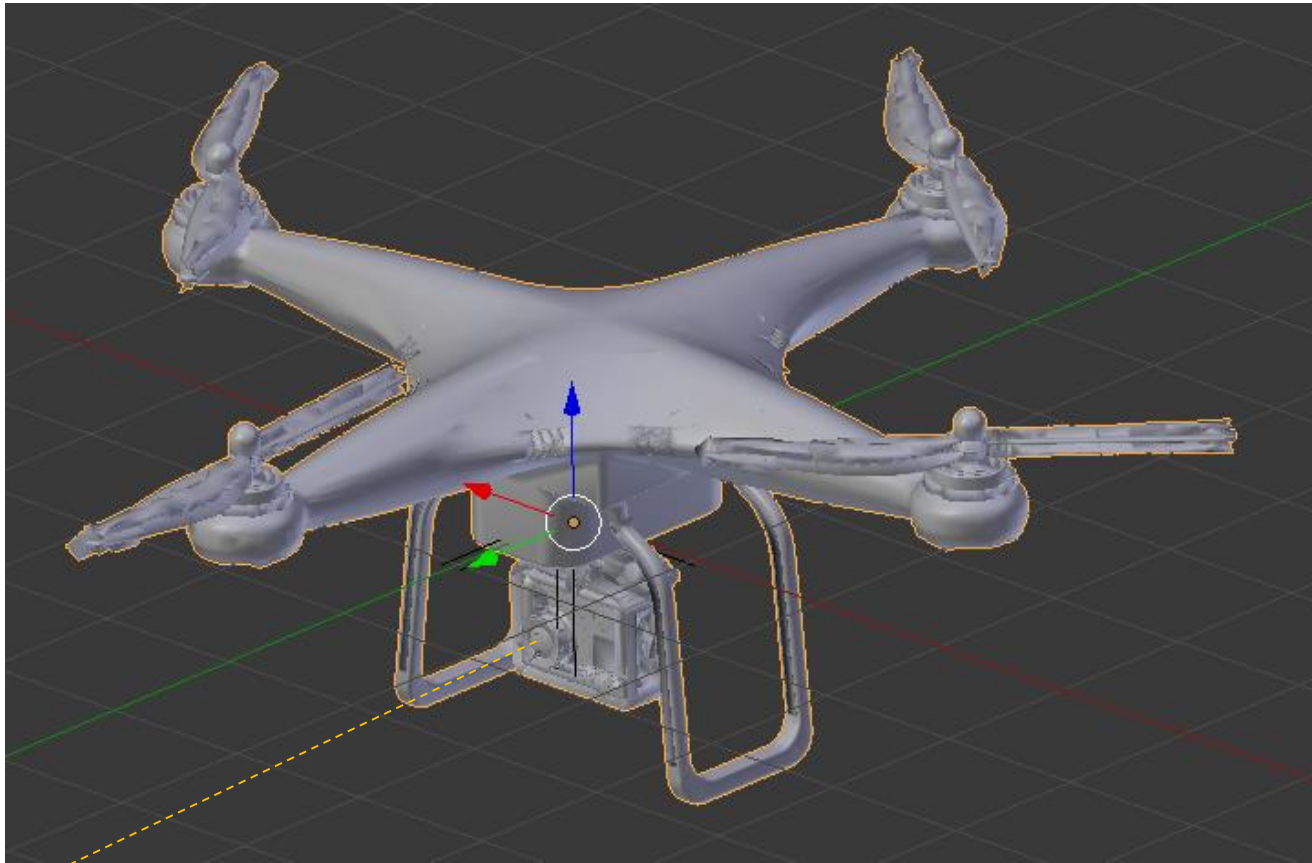
$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad \alpha = -\frac{\pi}{2}$$



$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

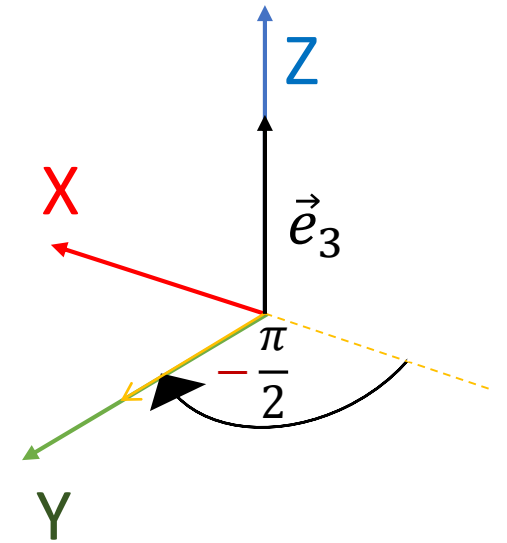
$$\alpha = -\frac{\pi}{2}$$

$$q_0 = \cos \frac{\alpha}{2} + (0i + 0j + 1k) \sin \frac{\alpha}{2}$$

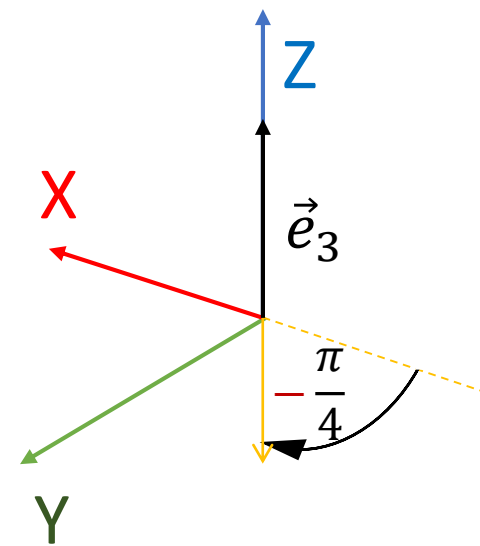
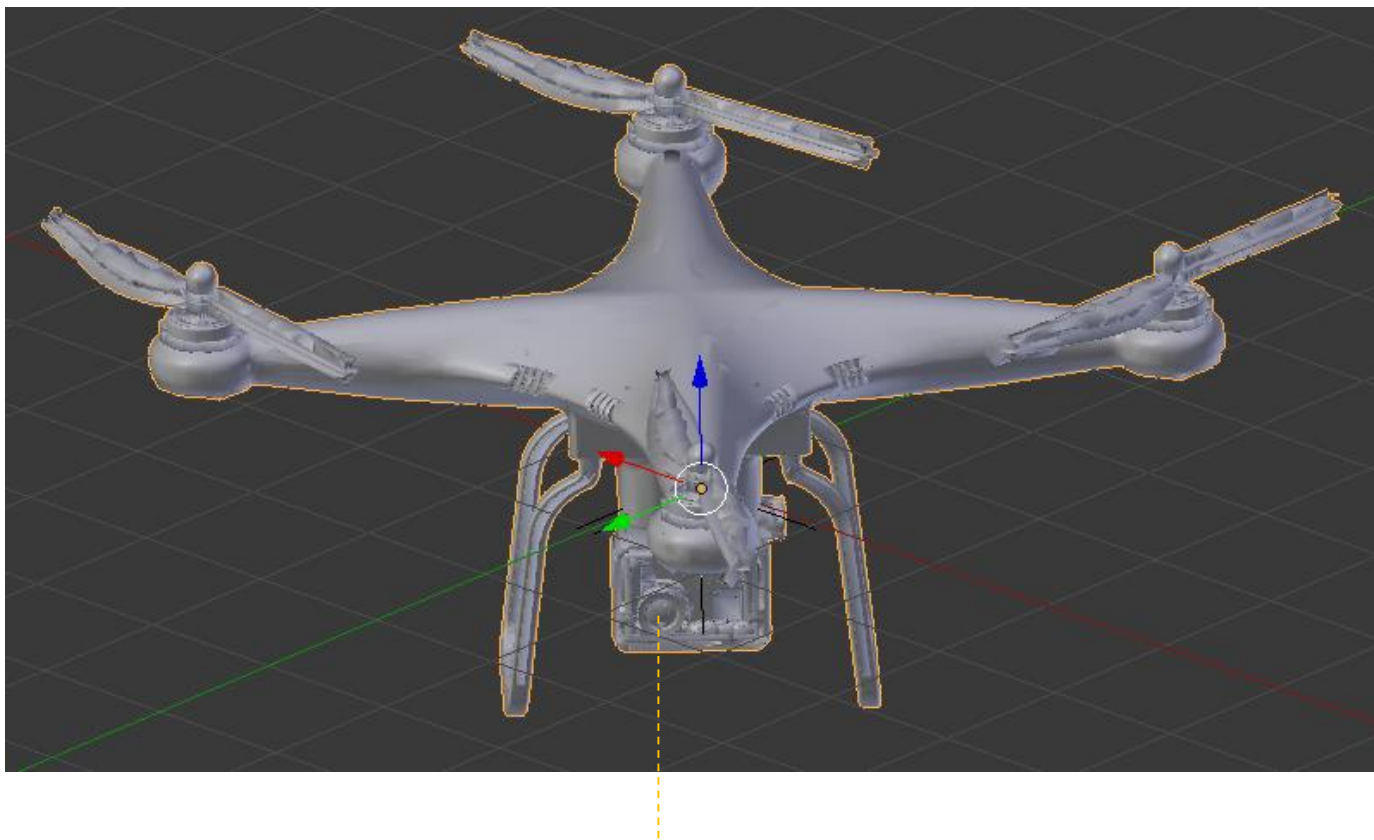


$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

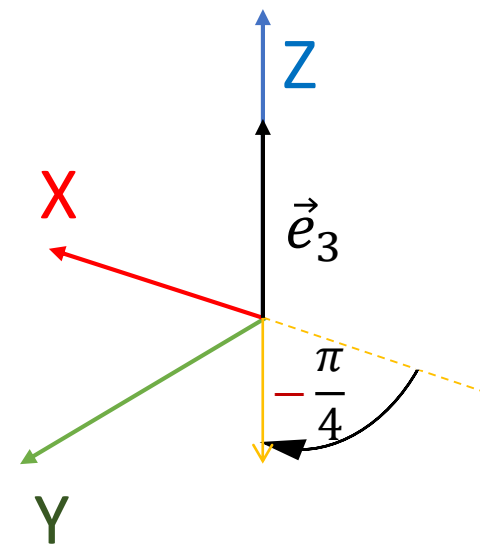
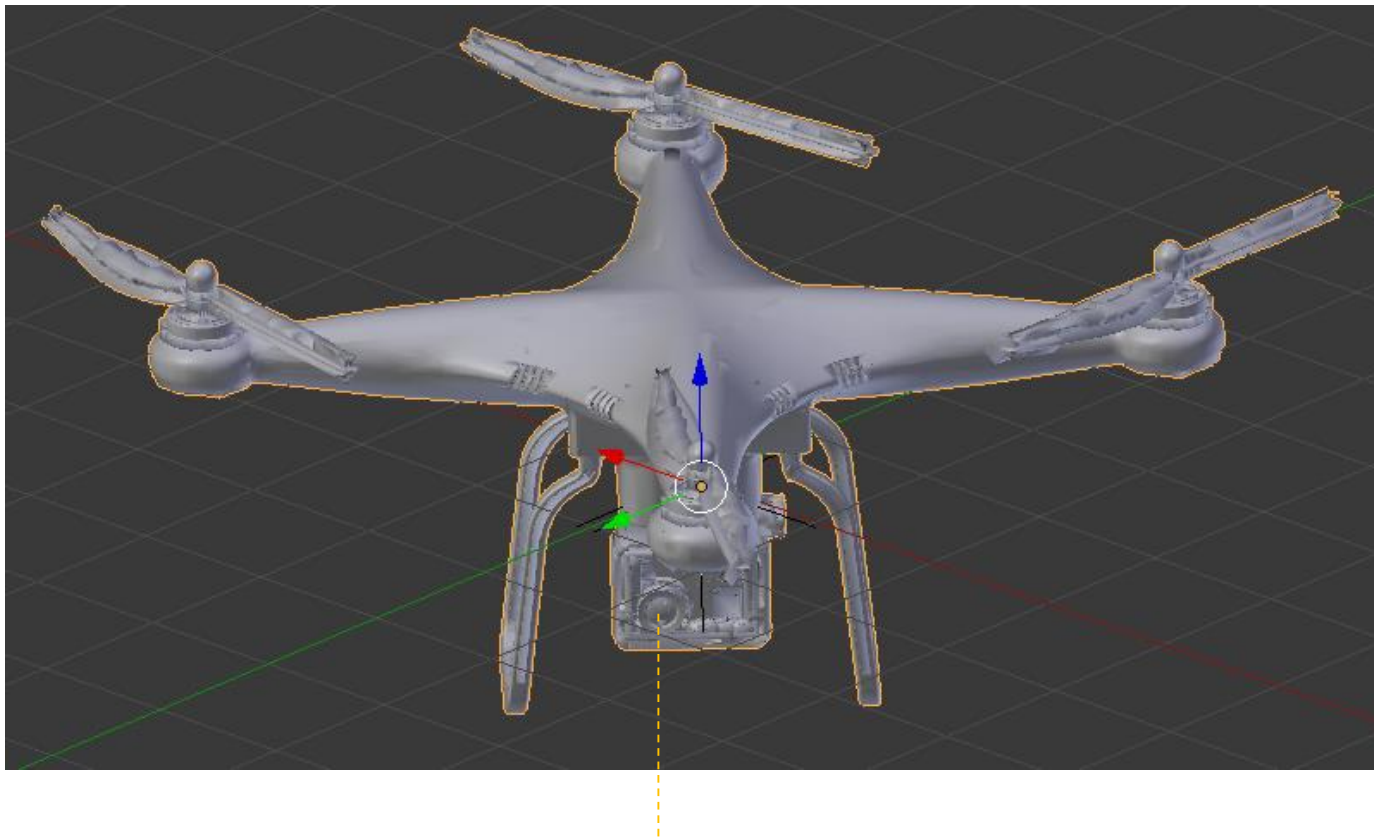
$$\alpha = -\frac{\pi}{2}$$



$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

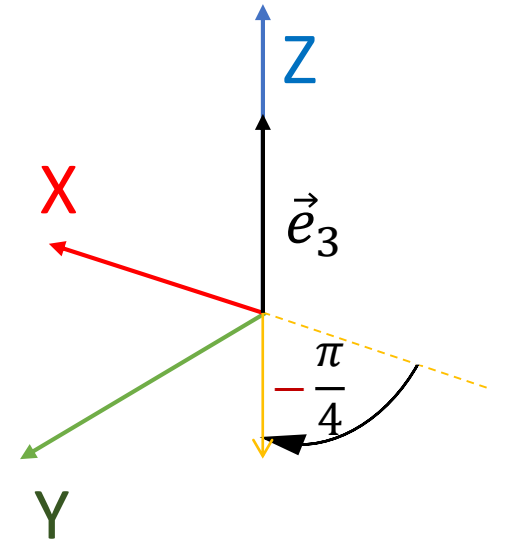
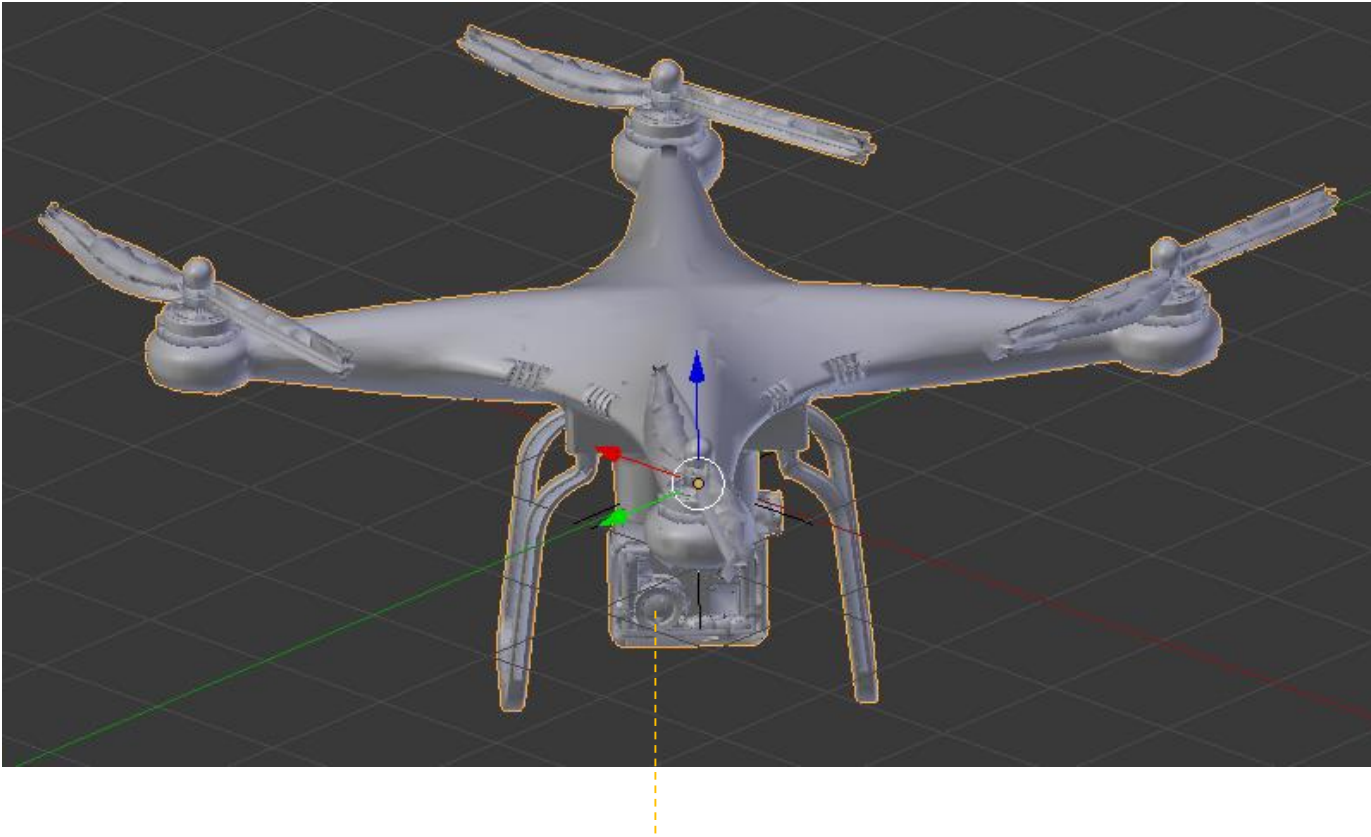


$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

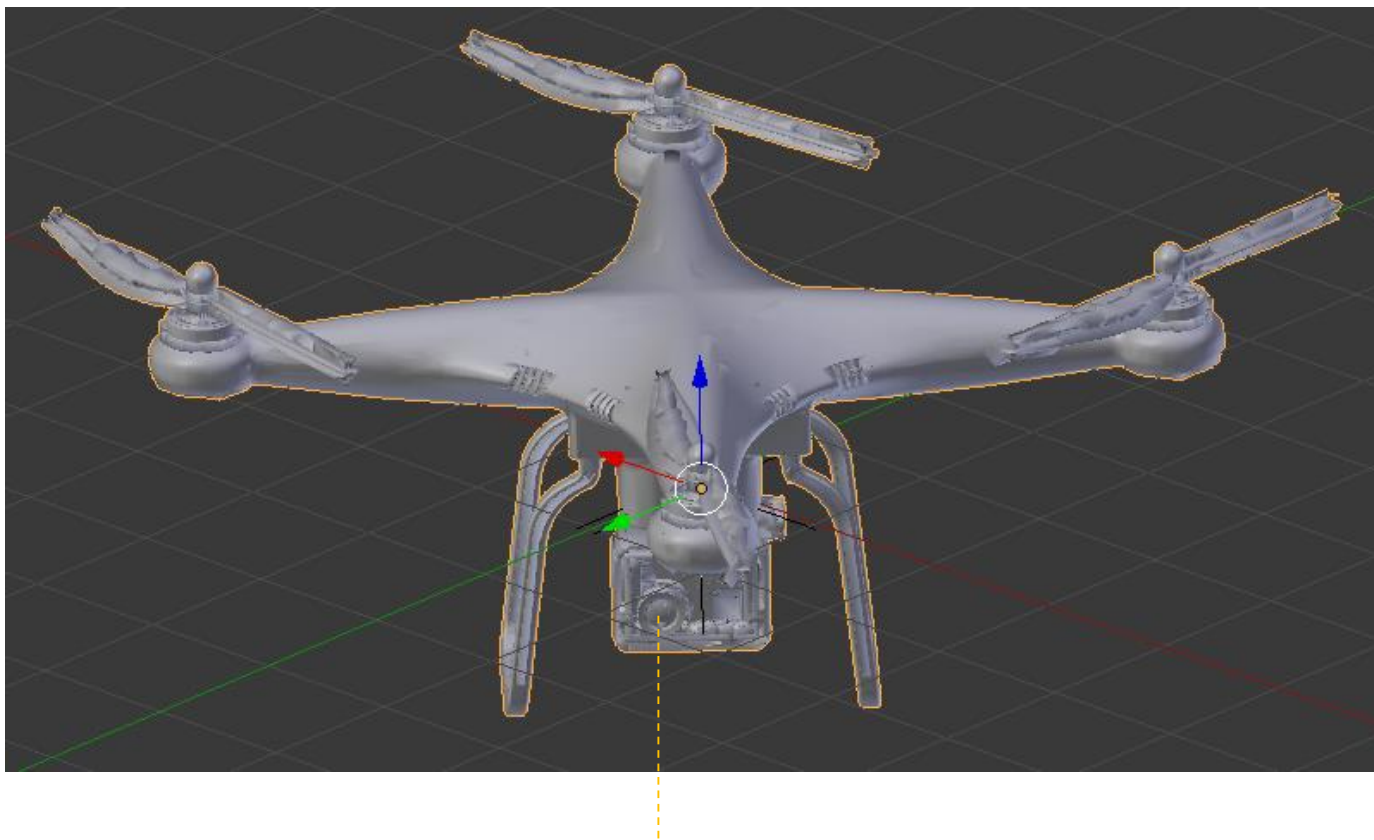
$$\alpha = -\frac{\pi}{4}$$



$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

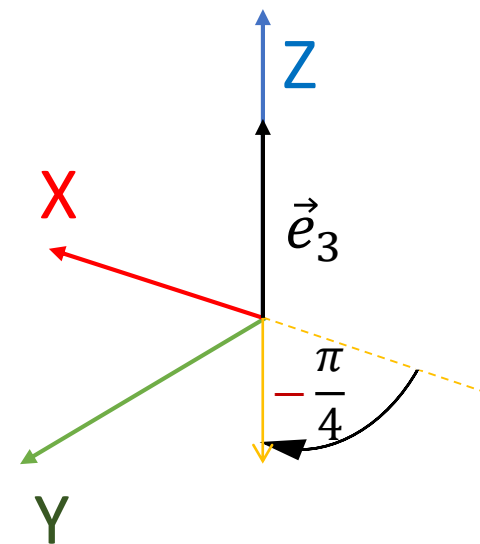
$$\alpha = -\frac{\pi}{4}$$

$$q_1 = \cos \frac{\alpha}{2} + (0i + 0j + 1k) \sin \frac{\alpha}{2}$$



$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\alpha = -\frac{\pi}{4}$$



$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$\alpha = -\frac{\pi}{2}$$

$$\vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\alpha = -\frac{\pi}{4}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 \cdot \vec{q}_1 = ?$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

dot product (cos(-pi/4),0,0,sin(-pi/4)) (cos(-pi/8),0,0,sin(-pi/8))



Input interpretation:

$$\left(\cos\left(-\frac{\pi}{4}\right), 0, 0, \sin\left(-\frac{\pi}{4}\right)\right) \cdot \left(\cos\left(-\frac{\pi}{8}\right), 0, 0, \sin\left(-\frac{\pi}{8}\right)\right)$$

Result:

$$\frac{\sin\left(\frac{\pi}{8}\right)}{\sqrt{2}} + \frac{\cos\left(\frac{\pi}{8}\right)}{\sqrt{2}}$$

Decimal approximation:

0.923879532511286756128183189396788286822416625863642486115...

Alternate forms:

$$\frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_0 \cdot \vec{q}_1 = ?$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 \cdot \vec{q}_1 = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 \cdot \vec{q}_1 = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\cos(\theta) = \frac{\sqrt{2 + \sqrt{2}}}{2} \Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{2 + \sqrt{2}}}{2} \right)$$

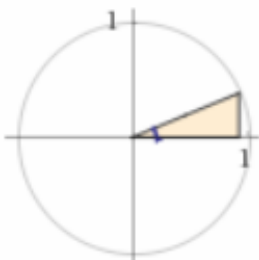
Input:

$$\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)$$

Conversion from radians to degrees:

$$22.5^\circ$$

Reference triangle for angle 0.3927 radians:



width	$\cos\left(\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)\right) = \frac{\sqrt{2+\sqrt{2}}}{2} \approx 0.92388$
height	$\sin\left(\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)\right) = \sqrt{1 + \frac{1}{4}(-2 - \sqrt{2})} \approx 0.382683$

Alternate forms:

$$\frac{\pi}{8}$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 \cdot \vec{q}_1 = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\cos(\theta) = \frac{\sqrt{2 + \sqrt{2}}}{2} \Rightarrow \theta = \cos^{-1} \left(\frac{\sqrt{2 + \sqrt{2}}}{2} \right) = \frac{\pi}{8}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \theta = \frac{\pi}{8}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp}(\vec{q}_1, \vec{q}_2, t) = \frac{\sin((1-t)\theta)}{\sin \theta} \vec{q}_0 + \frac{\sin(t\theta)}{\sin \theta} \vec{q}_1$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \theta = \frac{\pi}{8}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

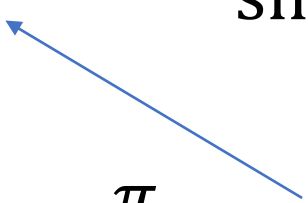
$$\text{slerp}(\vec{q}_1, \vec{q}_2, t) = \frac{\sin((1-t)\theta)}{\sin \theta} \vec{q}_1 + \frac{\sin(t\theta)}{\sin \theta} \vec{q}_2$$

$$t = \frac{1}{2}; \quad \frac{\sin\left(\left(1 - \frac{1}{2}\right)\theta\right)}{\sin \theta} = \frac{\sin \frac{\theta}{2}}{\sin \theta} = \frac{\sin \frac{\pi}{16}}{\sin \frac{\pi}{8}} = \sqrt{\frac{2 - \sqrt{2} + \sqrt{2}}{2 - \sqrt{2}}}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \theta = \frac{\pi}{8}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp}(\vec{q}_1, \vec{q}_2, t) = \sqrt{\frac{2 - \sqrt{2 + \sqrt{2}}}{2 - \sqrt{2}}} \vec{q}_1 + \frac{\sin(t\theta)}{\sin \theta} \vec{q}_2$$

$$t = \frac{1}{2}; \frac{\sin\left(\left(1 - \frac{1}{2}\right)\theta\right)}{\sin \theta} = \frac{\sin \frac{\theta}{2}}{\sin \theta} = \frac{\sin \frac{\pi}{16}}{\sin \frac{\pi}{8}} = \sqrt{\frac{2 - \sqrt{2 + \sqrt{2}}}{2 - \sqrt{2}}}$$


$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad \theta = \frac{\pi}{8} \quad \vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp}(\vec{q}_1, \vec{q}_2, t) = \sqrt{\frac{2 - \sqrt{2 + \sqrt{2}}}{2 - \sqrt{2}}} \vec{q}_1 + \sqrt{\frac{2 - \sqrt{2 + \sqrt{2}}}{2 - \sqrt{2}}} \vec{q}_2$$

$$t = \frac{1}{2}; \frac{\sin\left(\frac{1}{2}\theta\right)}{\sin\theta} = \frac{\sin\frac{\theta}{2}}{\sin\theta} = \frac{\sin\frac{\pi}{16}}{\sin\frac{\pi}{8}} = \sqrt{\frac{2 - \sqrt{2 + \sqrt{2}}}{2 - \sqrt{2}}}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp} \left(\vec{q}_1, \vec{q}_2, \frac{1}{2} \right) = \sqrt{\frac{2 - \sqrt{2} + \sqrt{2}}{2 - \sqrt{2}}} \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} + \sqrt{\frac{2 - \sqrt{2} + \sqrt{2}}{2 - \sqrt{2}}} \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp} \left(\vec{q}_1, \vec{q}_2, \frac{1}{2} \right) = \sqrt{\frac{2 - \sqrt{2} + \sqrt{2}}{2 - \sqrt{2}}} \begin{pmatrix} \cos \frac{-\pi}{4} + \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} + \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\text{slerp} \left(\vec{q}_1, \vec{q}_2, \frac{1}{2} \right) = \begin{pmatrix} \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \\ 0 \\ 0 \\ \frac{-\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \end{pmatrix}$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_t = \begin{pmatrix} \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \\ 0 \\ 0 \\ -\frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \end{pmatrix}$$

$$|\vec{q}_t| = \frac{2 + \sqrt{2 - \sqrt{2}}}{4} + \frac{2 - \sqrt{2 - \sqrt{2}}}{4} = 1$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix}$$

$$t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$\vec{q}_t = \begin{pmatrix} \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \\ 0 \\ 0 \\ \frac{-\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \end{pmatrix} \Rightarrow q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{q}_0 = \begin{pmatrix} \cos \frac{-\pi}{4} \\ 0 \\ 0 \\ \sin \frac{-\pi}{4} \end{pmatrix} \quad t = \frac{1}{2}$$

$$\vec{q}_1 = \begin{pmatrix} \cos \frac{-\pi}{8} \\ 0 \\ 0 \\ \sin \frac{-\pi}{8} \end{pmatrix}$$

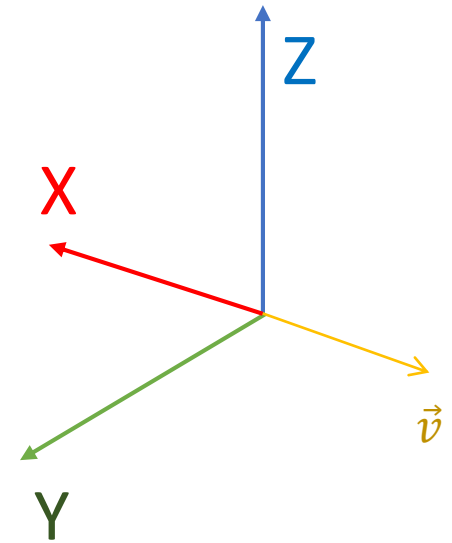
$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

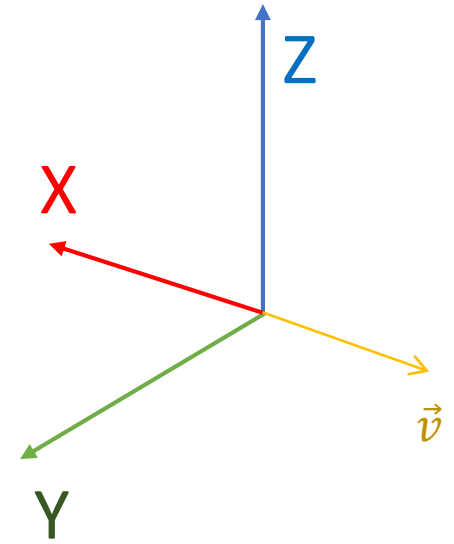
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$



$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

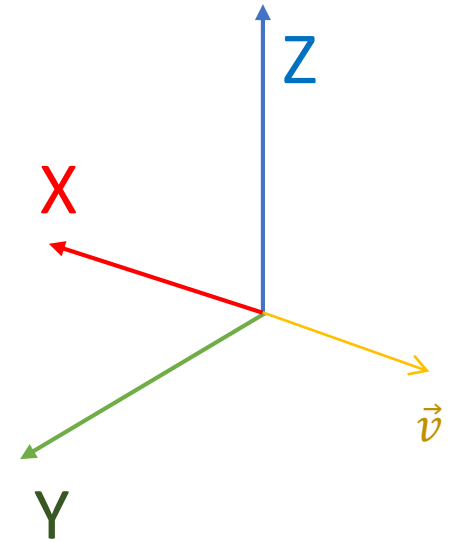


$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k \right) (-i) \right) q_t^{-1}$$

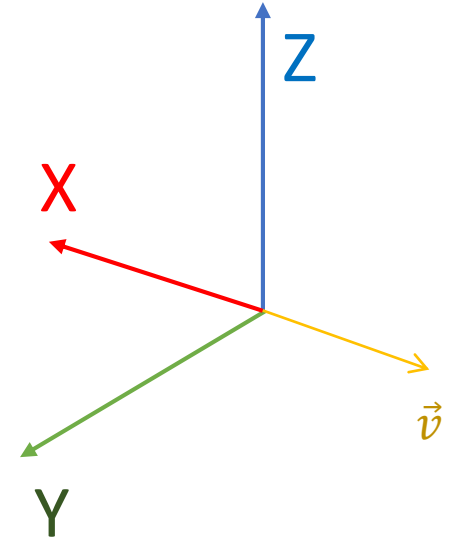


$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{-i}{2} - \left(\frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{(-ki)}{2} \right) q_t^{-1}$$

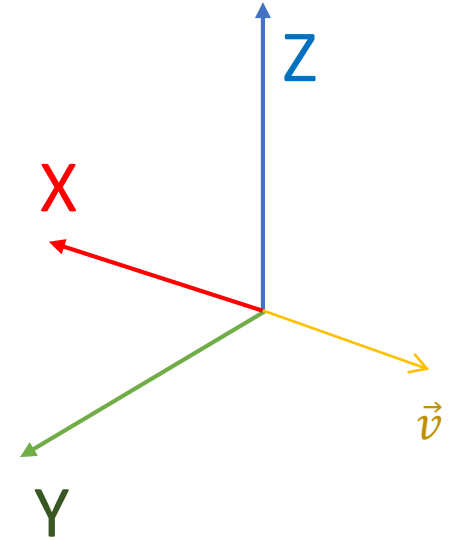


$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \right)^{-i} - \left(\frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \right)^{-j} \right) q_t^{-1}$$

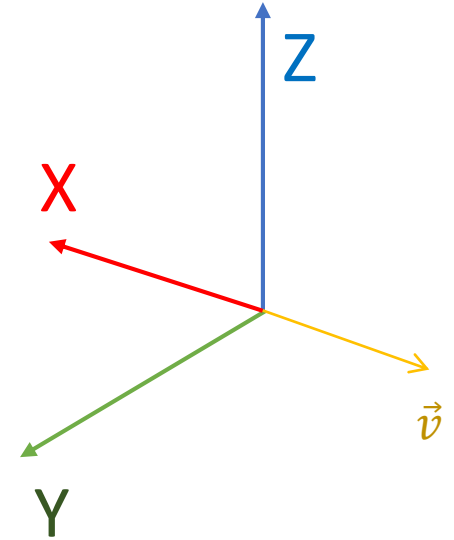


$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$q_t^{-1} = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} + \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{-i}{2} + \left(\frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{j}{2} \right) \left(\left(\frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{1}{2} + \left(\frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} \right) \frac{k}{2} \right)$$



$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \left(\left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}} \right)^{\frac{-i}{2}} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}} \right)^{\frac{j}{2}} \right) \left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}} \right)^{\frac{1}{2}} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}} \right)^{\frac{k}{2}} \right) \right)$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \left(\left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}} \right)^{-i} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}} \right)^{j} \right) \frac{1}{2} + \left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}} \right)^{\frac{1}{2}} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}} \right)^{\frac{k}{2}} \right) \right)$$

$$q_t q_v q_t^{-1} = (-ai + bj)(a + bk)$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = (-ai + bj)(a + bk) = -a^2 i - abik + abj + b^2 jk$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab \mathbf{ik} + abj + b^2 \mathbf{jk}$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab(-j) + abj + b^2 i$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab(-j) + abj + b^2 i = (-a^2 + b^2)i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = (-a^2 + b^2)i + 2abj = \frac{-2 - \sqrt{2 - \sqrt{2}} + 2 - \sqrt{2 - \sqrt{2}}}{4}i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \frac{-2 - \sqrt{2 - \sqrt{2}} + 2 - \sqrt{2 - \sqrt{2}}}{4} i + 2abj = \frac{-2\sqrt{2 - \sqrt{2}}}{4} i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \frac{-2\sqrt{2 - \sqrt{2}}}{4} i + 2abj = \frac{-\sqrt{2 - \sqrt{2}}}{2} i + 2abj$$

$$\frac{\sqrt{2 - \sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$q_t q_v q_t^{-1} = \frac{-\sqrt{2 - \sqrt{2}}}{2} i + \sqrt{2 + \sqrt{2 - \sqrt{2}}} \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} j$$

$$\frac{\sqrt{2 - \sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$\frac{\sqrt{2 + \sqrt{2}}}{2} = \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$q_t q_v q_t^{-1} = \frac{-\sqrt{2 - \sqrt{2}}}{2} i + \frac{\sqrt{2 + \sqrt{2}}}{2} j$$

$$\frac{\sqrt{2 - \sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

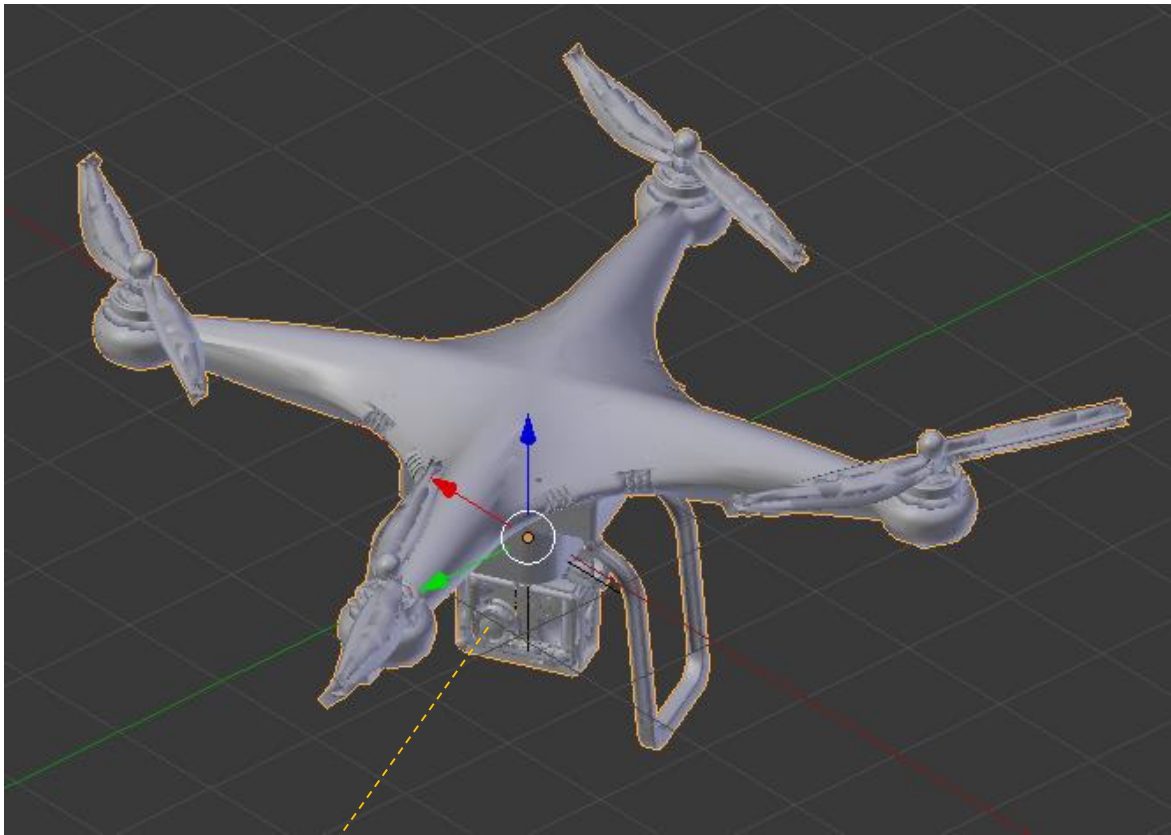
$$\frac{\sqrt{2 + \sqrt{2}}}{2} = \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$q_t q_v q_t^{-1} = \frac{-\sqrt{2 - \sqrt{2}}}{2} i + \frac{\sqrt{2 + \sqrt{2}}}{2} j \Rightarrow \vec{v}' = \begin{pmatrix} \frac{-\sqrt{2 - \sqrt{2}}}{2} \\ \frac{\sqrt{2 + \sqrt{2}}}{2} \\ 2 \\ 0 \end{pmatrix}$$

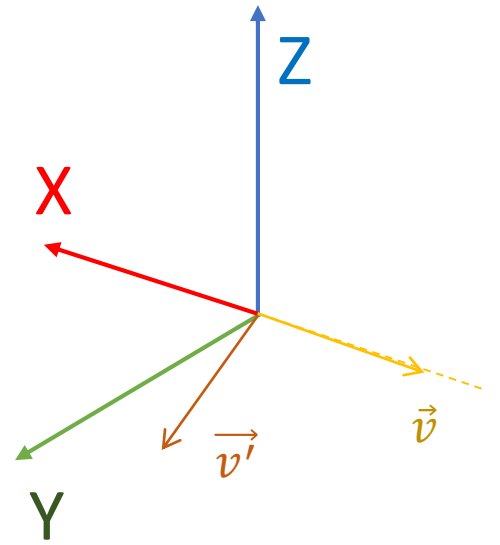
$$\frac{\sqrt{2 - \sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$\frac{\sqrt{2 + \sqrt{2}}}{2} = \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$\vec{v}' = \begin{pmatrix} -\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$



$$\vec{v}' = \begin{pmatrix} -\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$



$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$t = \frac{1}{2}$$

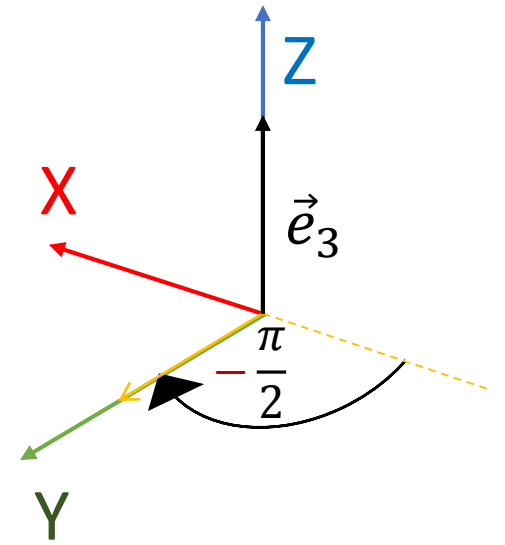
$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$t = 0$

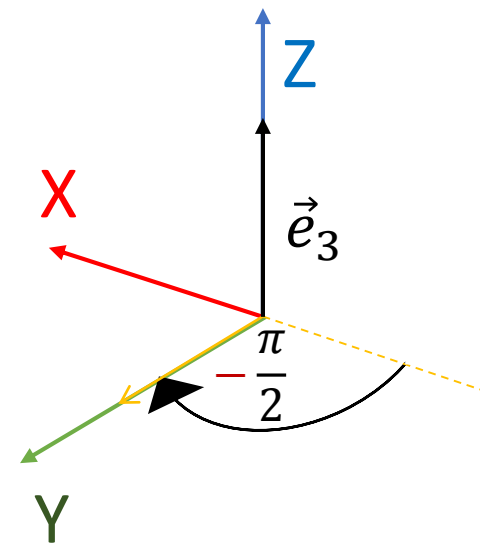
$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$



$t = 0$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

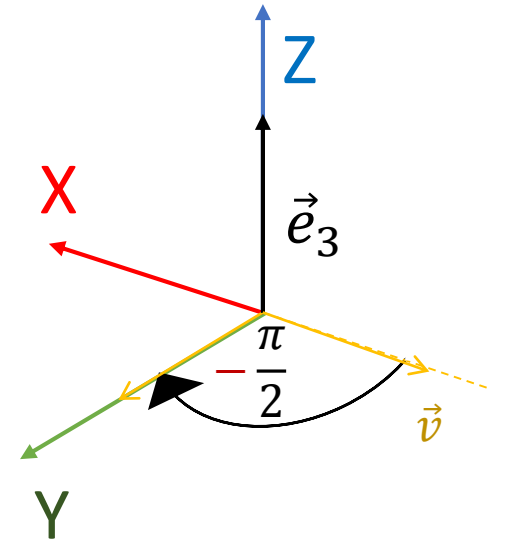


$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



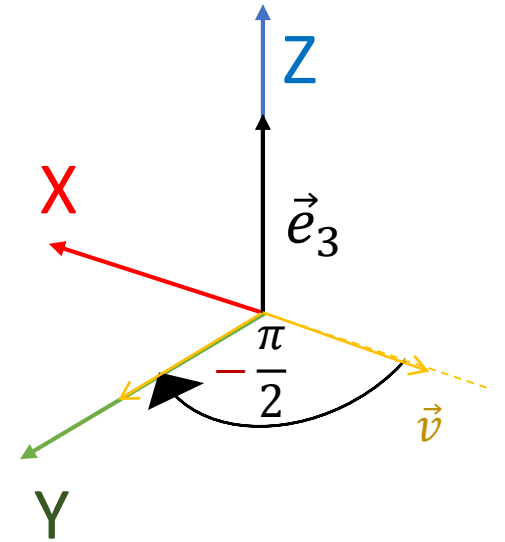
$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

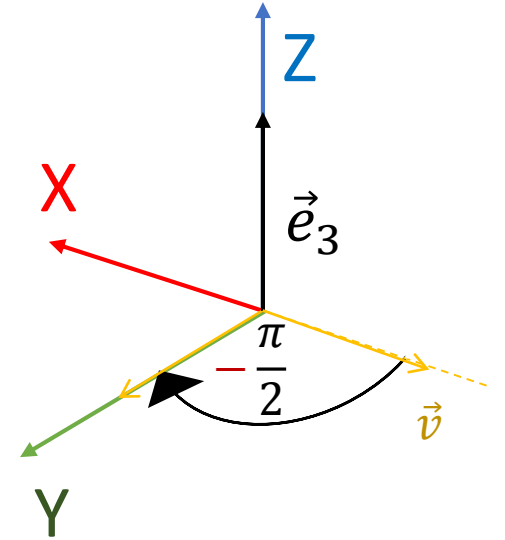
$$q_0 q_v q_0^{-1} = \left(\left(\cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4} \right) (-i) \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right)$$



$t = 0$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = \left(-i \cos \frac{-\pi}{4} + \sin \frac{-\pi}{4} (-ki) \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right)$$

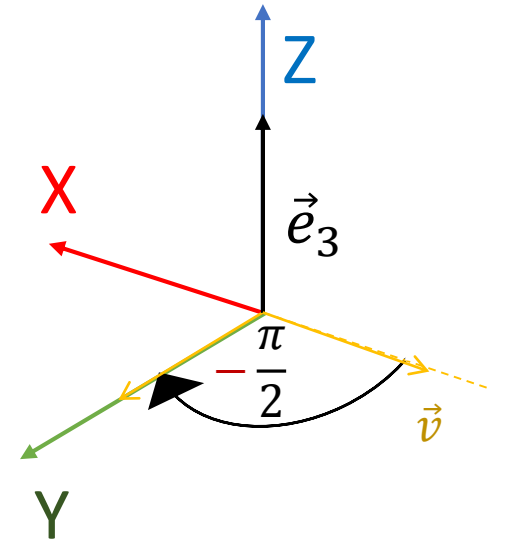
$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

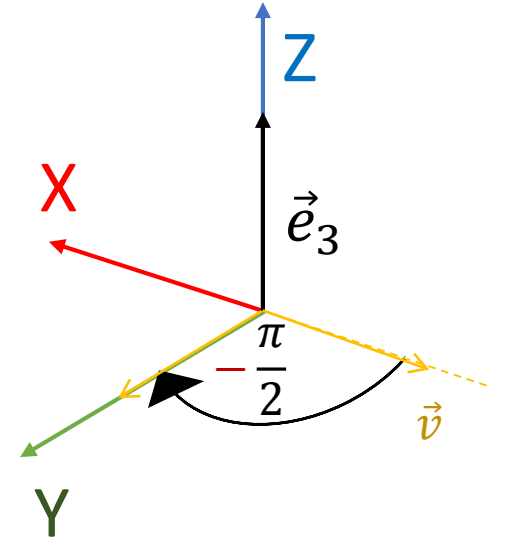
$$q_0 q_v q_0^{-1} = \left(-i \cos \frac{-\pi}{4} + \sin \frac{-\pi}{4} (-j) \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right)$$



$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$



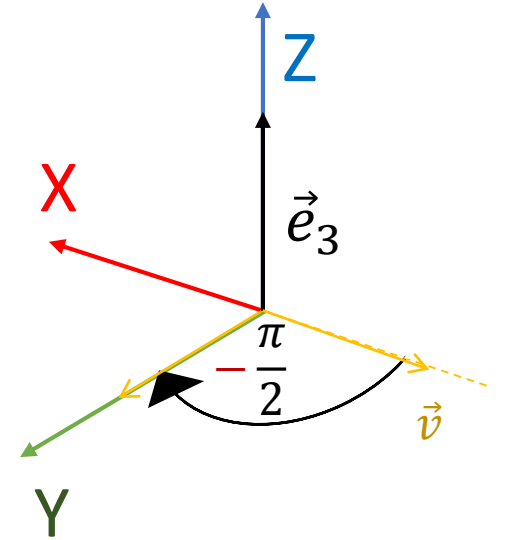
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= -i \cos^2 \left(\frac{-\pi}{4} \right) + \mathbf{ik} \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} - j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} + \mathbf{jk} \sin^2 \left(\frac{-\pi}{4} \right) \end{aligned}$$

$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

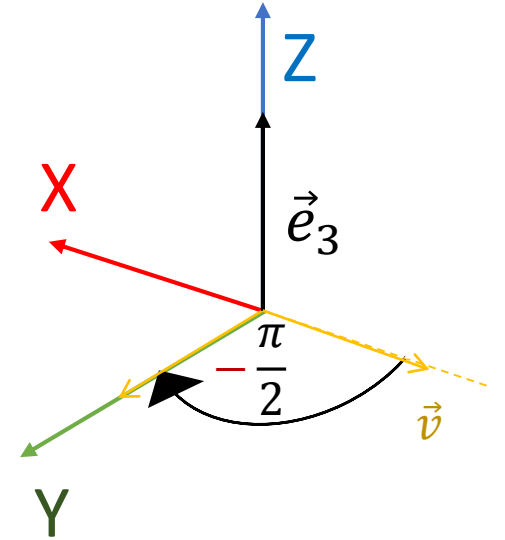
$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= -i \cos^2 \left(\frac{-\pi}{4} \right) - j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} - j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} + i \sin^2 \left(\frac{-\pi}{4} \right) \end{aligned}$$

$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

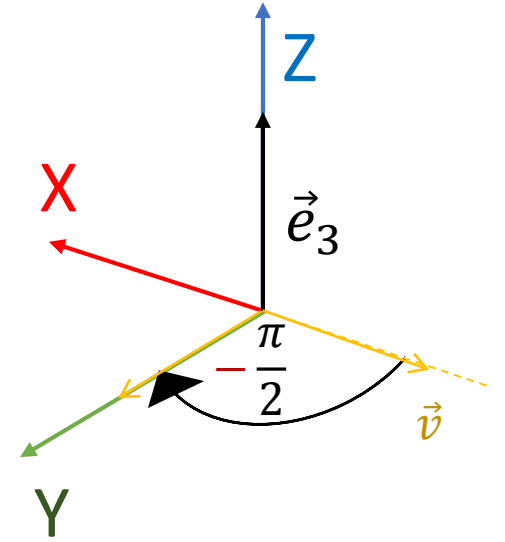


$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= -i \cos^2 \left(\frac{-\pi}{4} \right) - 2 j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} + i \sin^2 \left(\frac{-\pi}{4} \right) \end{aligned}$$

$t = 0$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

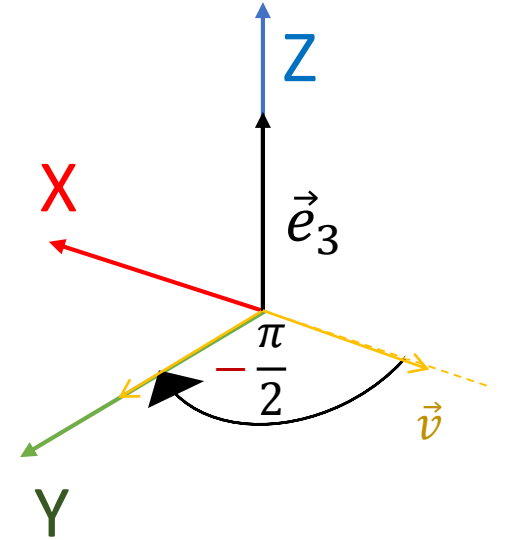
$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= i \sin^2 \left(\frac{-\pi}{4} \right) - i \cos^2 \left(\frac{-\pi}{4} \right) - 2 j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} \end{aligned}$$

$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= i \left(\sin^2 \left(\frac{-\pi}{4} \right) - \cos^2 \left(\frac{-\pi}{4} \right) \right) - 2 j \cos \frac{-\pi}{4} \sin \frac{-\pi}{4} \end{aligned}$$

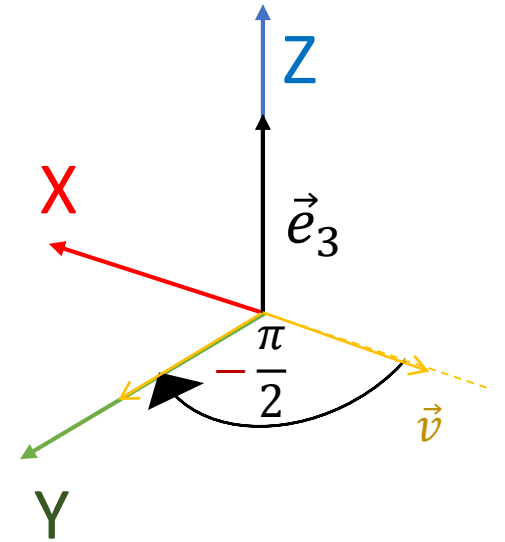
$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$\begin{aligned} q_0 q_v q_0^{-1} &= \left(-i \cos \frac{-\pi}{4} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} \right) = \\ &= i \left(\sin^2 \left(\frac{-\pi}{4} \right) - \sin^2 \left(\frac{-\pi}{4} \right) \right) - 2 \left(-\frac{1}{2} \right) j \end{aligned}$$



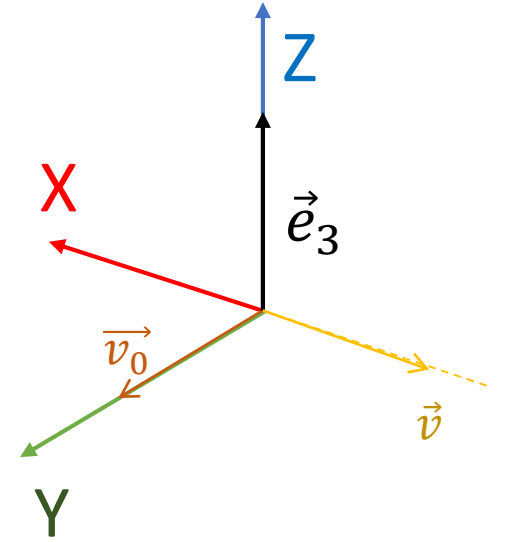
$$t = 0$$

$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

$$q_0^{-1} = \cos \frac{-\pi}{4} - k \sin \frac{-\pi}{4} k$$

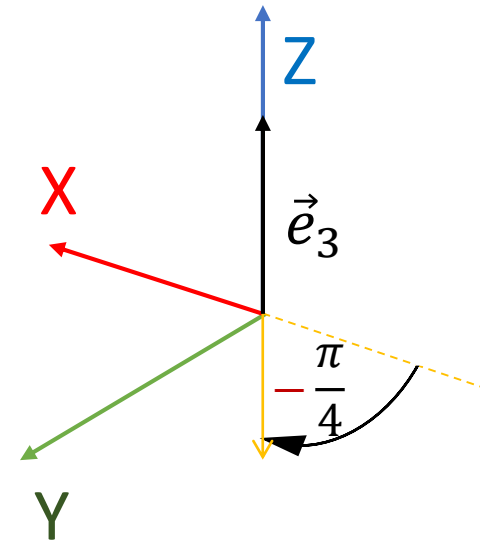
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = j \Rightarrow \vec{v}_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

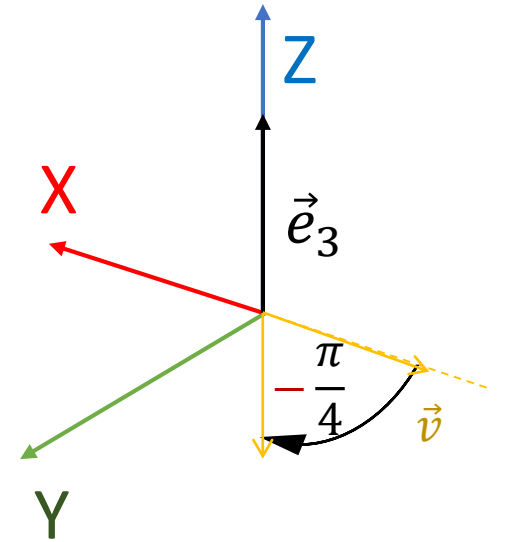


$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$

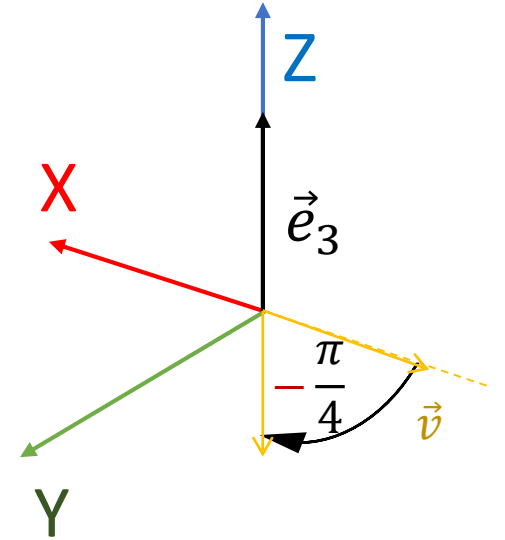
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$



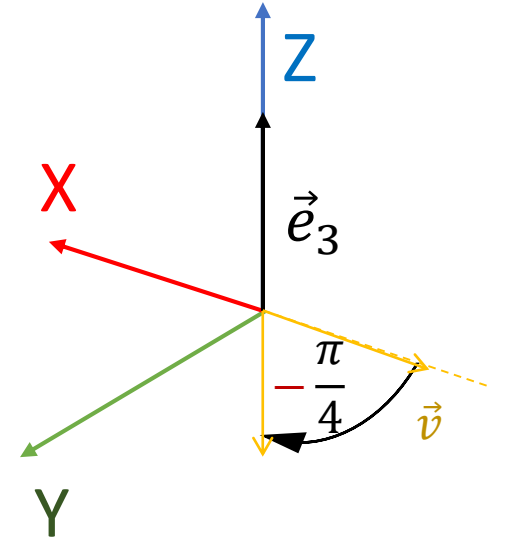
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(\left(\cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8} \right) (-i) \right) \left(\cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} \right)$$

$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(-i \cos \frac{-\pi}{8} + \sin \frac{-\pi}{8} (-ki) \right) \left(\cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} \right)$$

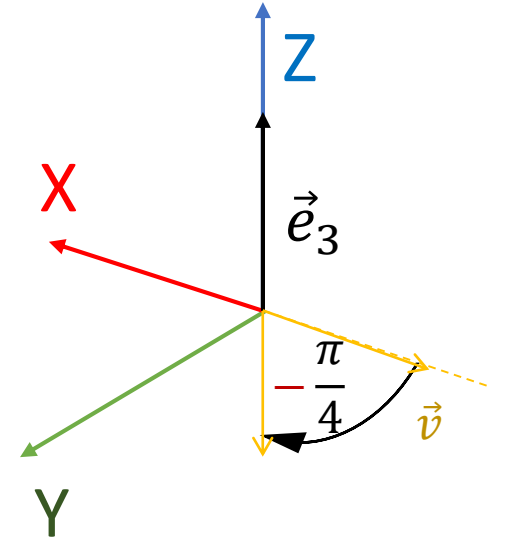
$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

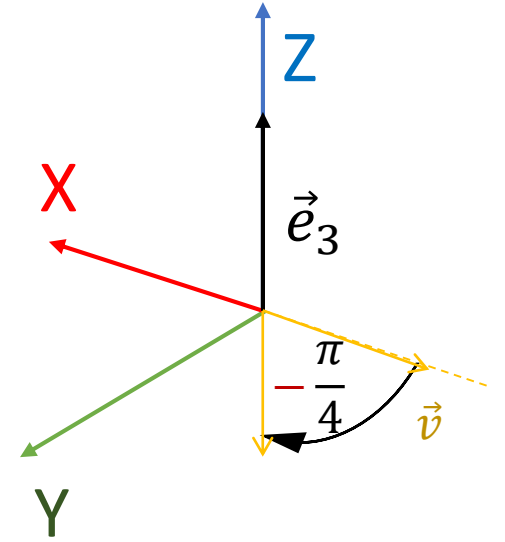
$$q_1 q_v q_1^{-1} = \left(-i \cos \frac{-\pi}{8} + \sin \frac{-\pi}{8} (-j) \right) \left(\cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} \right)$$



$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$\begin{aligned} q_1 q_v q_1^{-1} &= \left(-i \cos \frac{-\pi}{8} - j \sin \frac{-\pi}{8} \right) \left(\cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} \right) = \\ &= i \left(\sin^2 \left(\frac{-\pi}{8} \right) - \cos^2 \left(\frac{-\pi}{8} \right) \right) - 2 j \cos \frac{-\pi}{8} \sin \frac{-\pi}{8} \end{aligned}$$

$$(\sin(-\pi/8))^2 - (\cos(-\pi/8))^2$$



Input:

$$\sin^2\left(-\frac{\pi}{8}\right) - \cos^2\left(-\frac{\pi}{8}\right)$$

Exact result:

$$\sin^2\left(\frac{\pi}{8}\right) - \cos^2\left(\frac{\pi}{8}\right)$$

Decimal approximation:

-0.70710678118654752440084436210484903

Alternate forms:

$$-\frac{1}{\sqrt{2}}$$

$$\sin(-\pi/8) * \cos(-\pi/8)$$



Input:

$$\sin\left(-\frac{\pi}{8}\right) \cos\left(-\frac{\pi}{8}\right)$$

Exact result:

$$\sin\left(\frac{\pi}{8}\right) \left(-\cos\left(\frac{\pi}{8}\right)\right)$$

Decimal approximation:

-0.353553390593273762200422181052424519642417968844237018

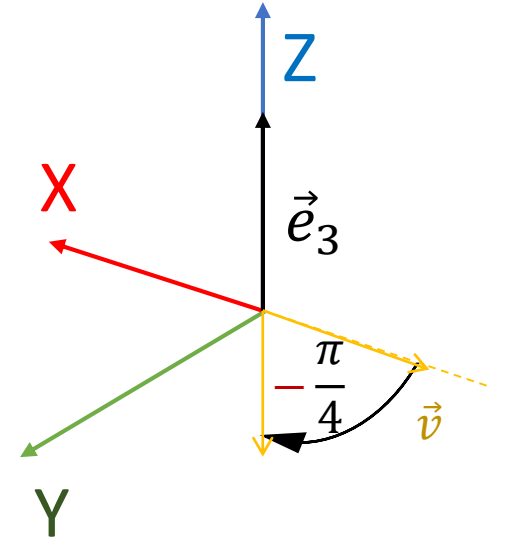
Alternate forms:

$$-\frac{1}{2\sqrt{2}}$$

$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$



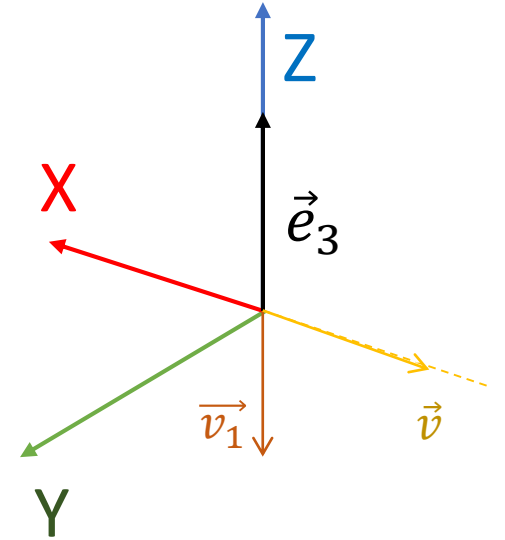
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$\begin{aligned} q_1 q_v q_1^{-1} &= \left(-i \cos \frac{-\pi}{8} - j \sin \frac{-\pi}{4} \right) \left(\cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} \right) = \\ &= i \left(-\frac{1}{\sqrt{2}} \right) - 2j \left(-\frac{1}{2\sqrt{2}} \right) \end{aligned}$$

$$t = 1$$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

$$q_1^{-1} = \cos \frac{-\pi}{8} - k \sin \frac{-\pi}{8} k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = -i \left(\frac{1}{\sqrt{2}} \right) + j \left(\frac{1}{\sqrt{2}} \right) \Rightarrow \vec{v}_1 = \begin{pmatrix} -\cos \frac{\pi}{4} \\ \sin \frac{\pi}{4} \\ 0 \end{pmatrix}$$

$t = 0$

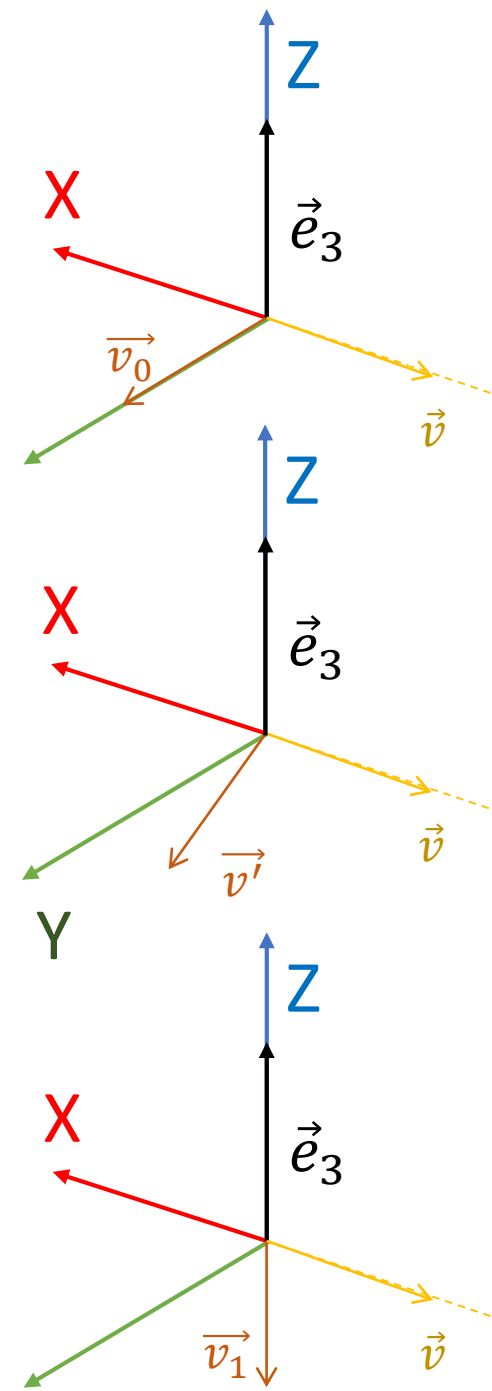
$$q_0 = \cos \frac{-\pi}{4} + k \sin \frac{-\pi}{4}$$

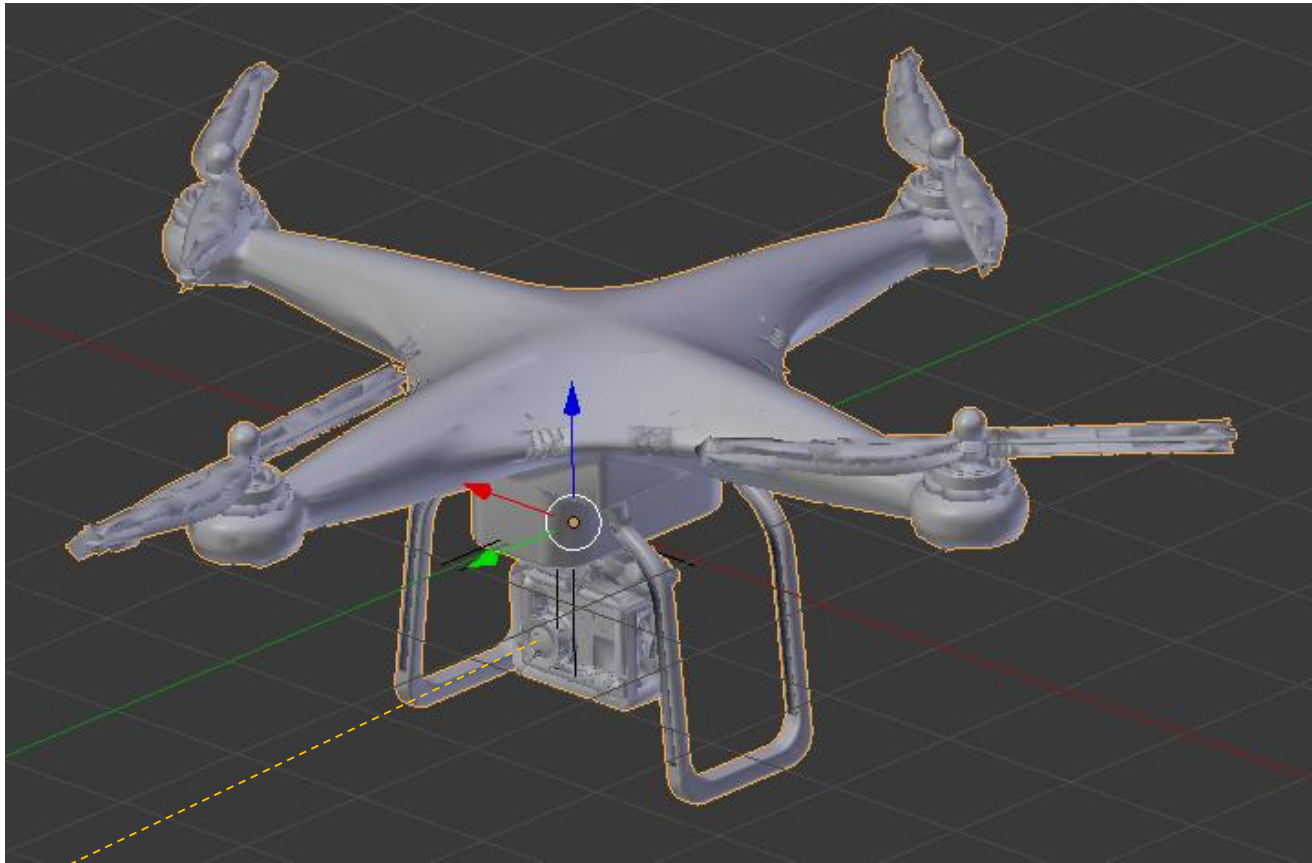
$t = \frac{1}{2}$

$$q_t = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2} - \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2} k$$

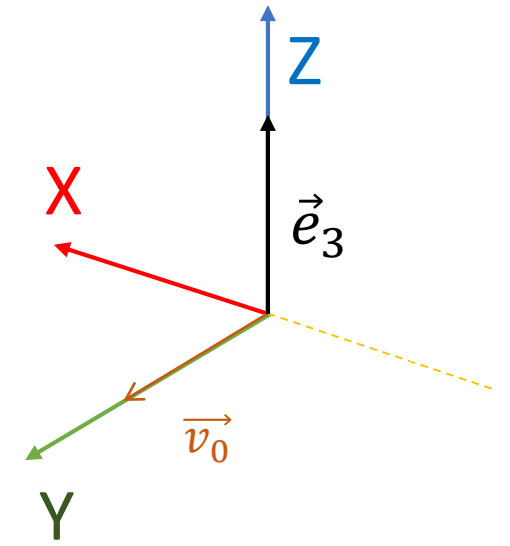
$t = 1$

$$q_1 = \cos \frac{-\pi}{8} + k \sin \frac{-\pi}{8}$$

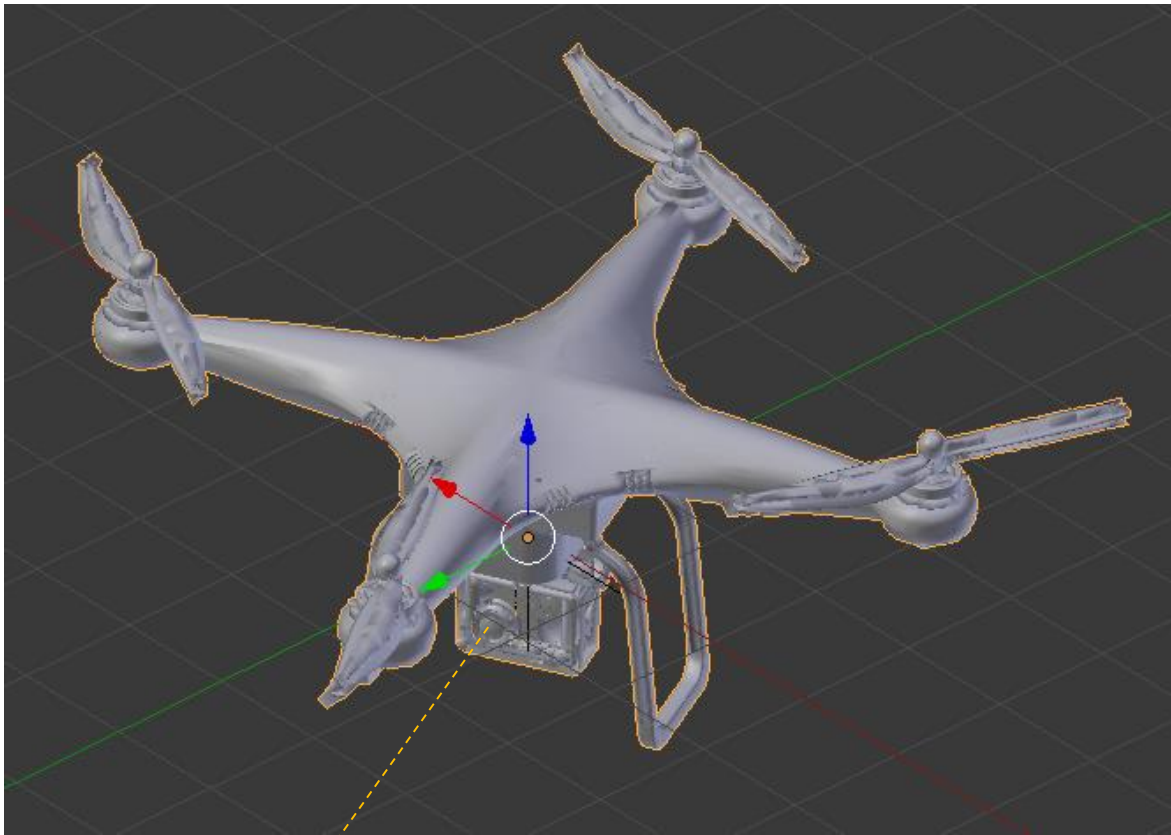




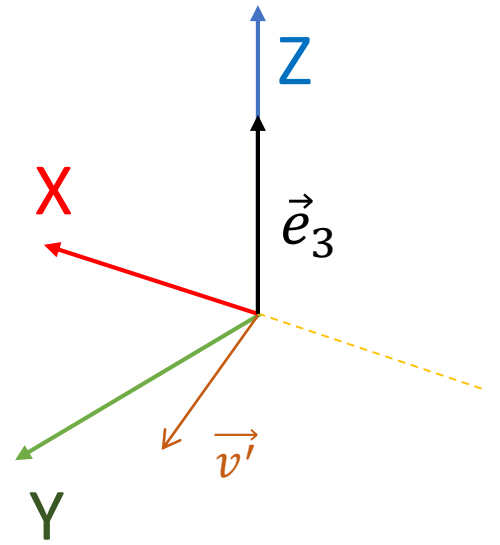
$t = 0$



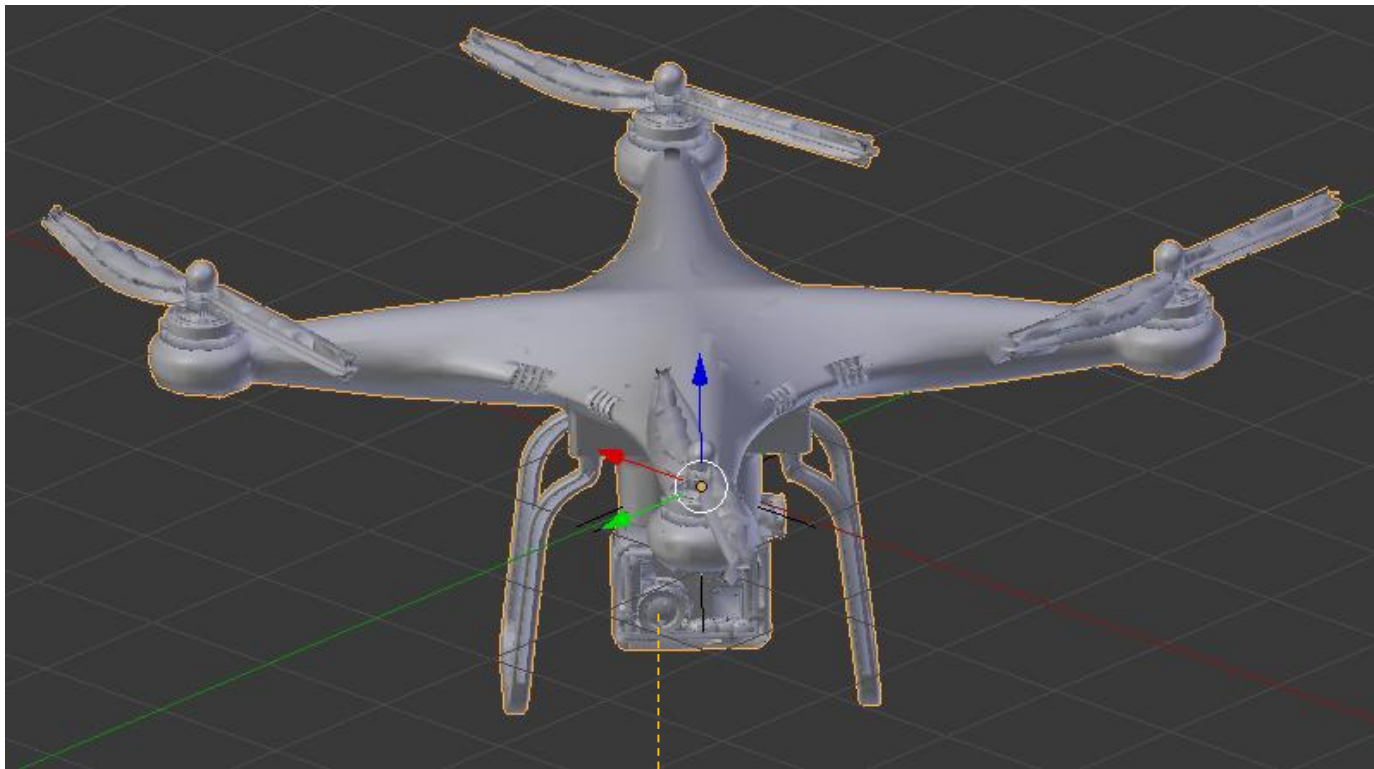
$$\vec{v}_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



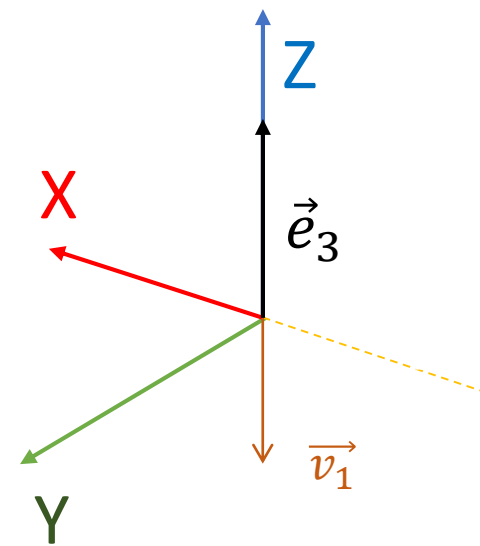
$$t = \frac{1}{2}$$



$$\vec{v}' = \begin{pmatrix} -\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$



$t = 1$



$$\vec{v}_1 = \begin{pmatrix} -\cos \frac{\pi}{4} \\ \sin \frac{\pi}{4} \\ 0 \end{pmatrix}$$