Systems of Differential Equations

WolframAlpha

dy(t)/dt = 1

Extended Keyboard Upload

Input interpretation:

$$\frac{\partial y(t)}{\partial t} = 1$$

ODE names:

Separable equation

$$y'(t) = 1$$

Homogeneous equation

$$y'(t) = 1$$

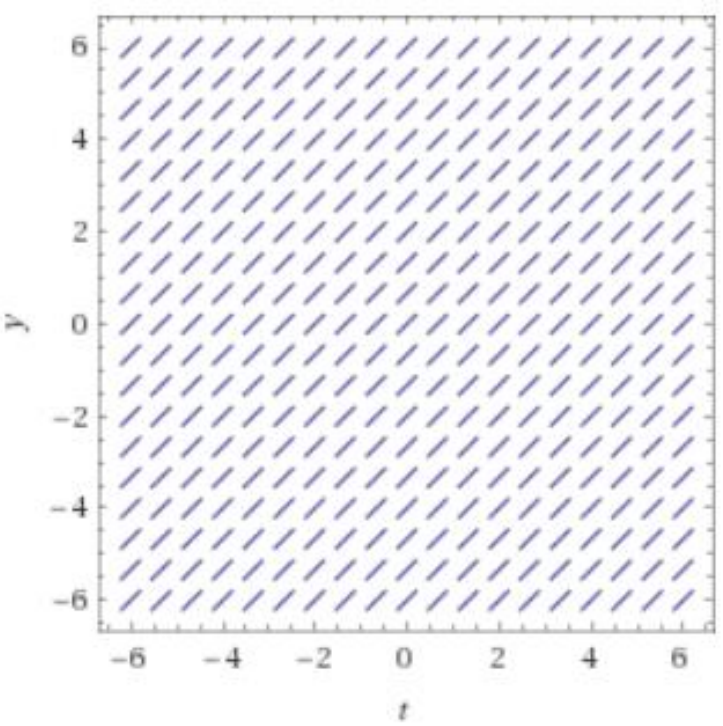
ODE classification:

first-order linear ordinary differential equation

Differential equation solution:

$$y(t) = c_1 + t$$

Slope field:



dy(t)/dt = 1, d(x)/dt = y(t)

Extended Keyboard Upload

Input interpretation:

$$\left\{\frac{\partial y(t)}{\partial t} = 1,\ \frac{\partial x(t)}{\partial t} = y(t)\right\}$$

ODE classification:

First-order system of linear differential equations

Alternate form:

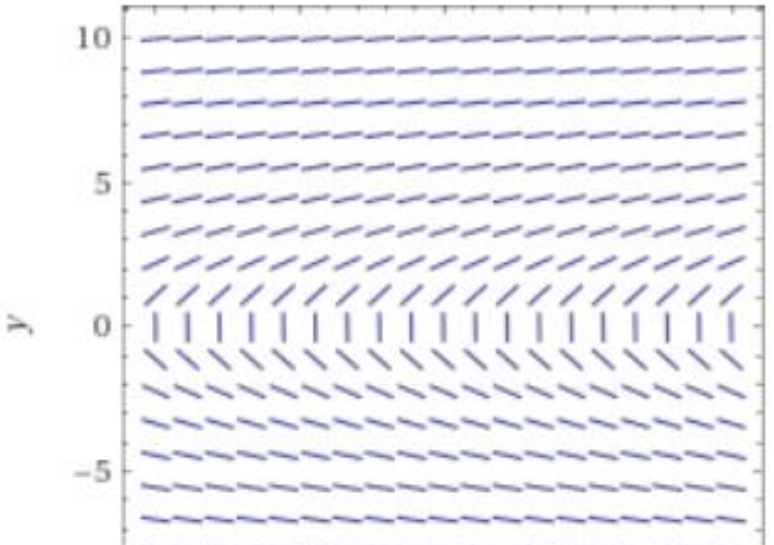
$$\{y'(t) = 1,\ y(t) = x'(t)\}$$

Differential equation solutions:

$$x(t) = c_2\, t + c_1 + \frac{t^2}{2}$$

$$y(t) = c_2 + t$$

Slope field:



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$$y(t) = \int \mathrm{d}t$$

$$y(t) = t + c$$

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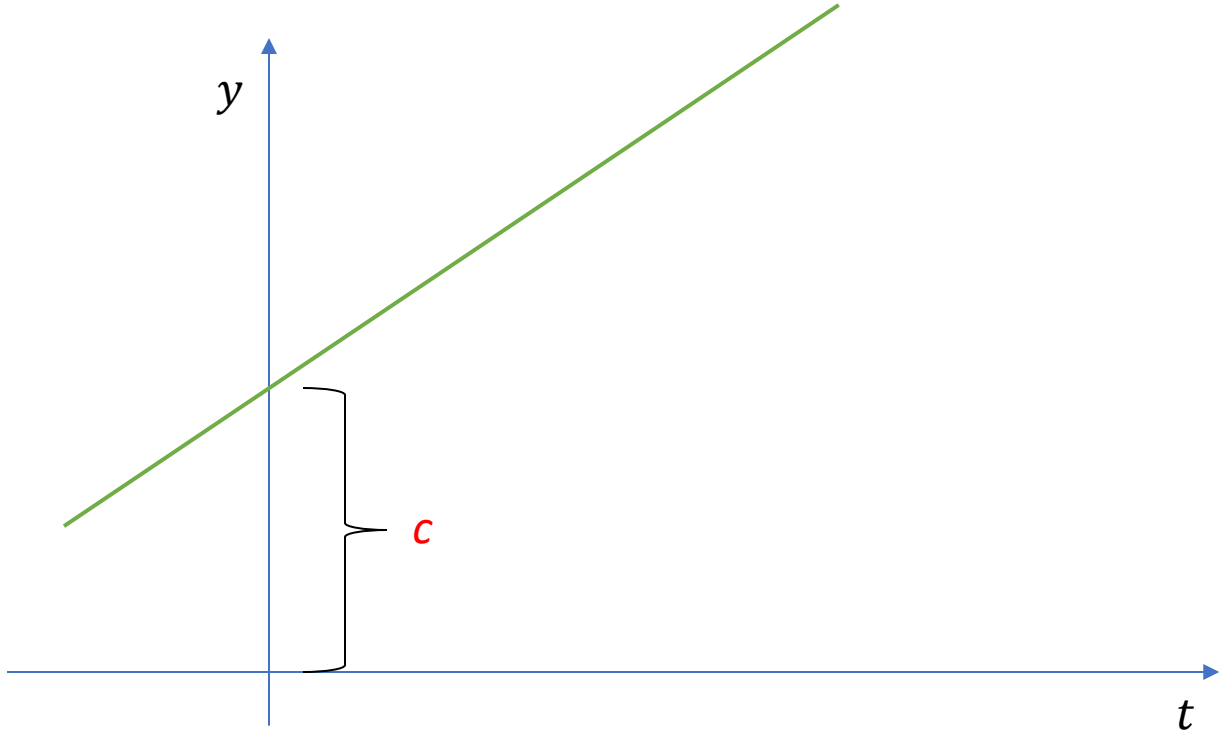

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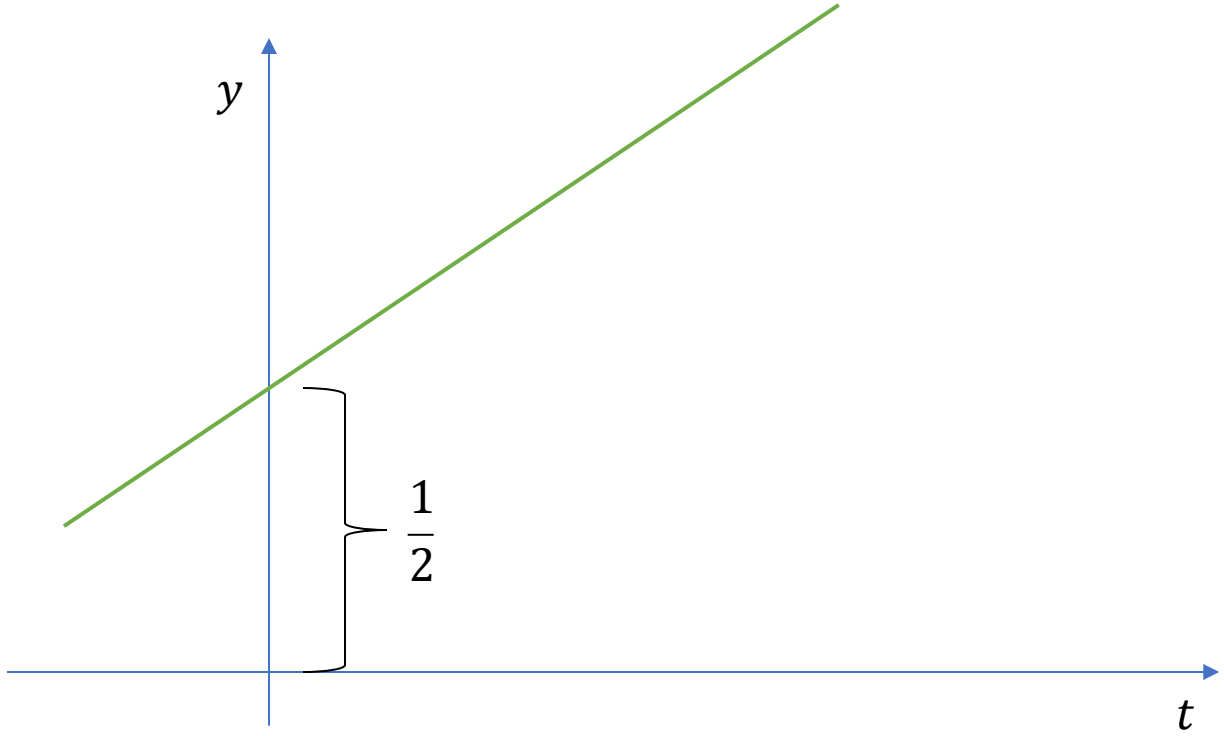

$y(0)=\frac{1}{2}$ $\frac{dy(t)}{dt}=1$ $t \in [0,2]$ $\Delta t = 0.01$

https://octave-online.net/

```
y0 = 0.5;
t = 0:.01:2

%write all the  differential equations
%in a single function
function xdot = f(y, t)

  xdot = zeros (1,1);%return a 1-dimensional array whose elements are all 0

  xdot (1) = 1; %dy/dt=1
endfunction

%compute the solution
y=lsode("f",[y0],t);

%plot
subplot(1,1,1)
plot(t,y(:,1))
xlabel("t")
ylabel("y(t)")
box()
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$$\frac{dy(t)}{dt} = 1 \qquad t \in [0,2] \qquad \Delta t = 0.01$$

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$t \in [0,2]$

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$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 1$$

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Columns 197 through 201:

   1.96000    1.97000    1.98000    1.99000    2.00000
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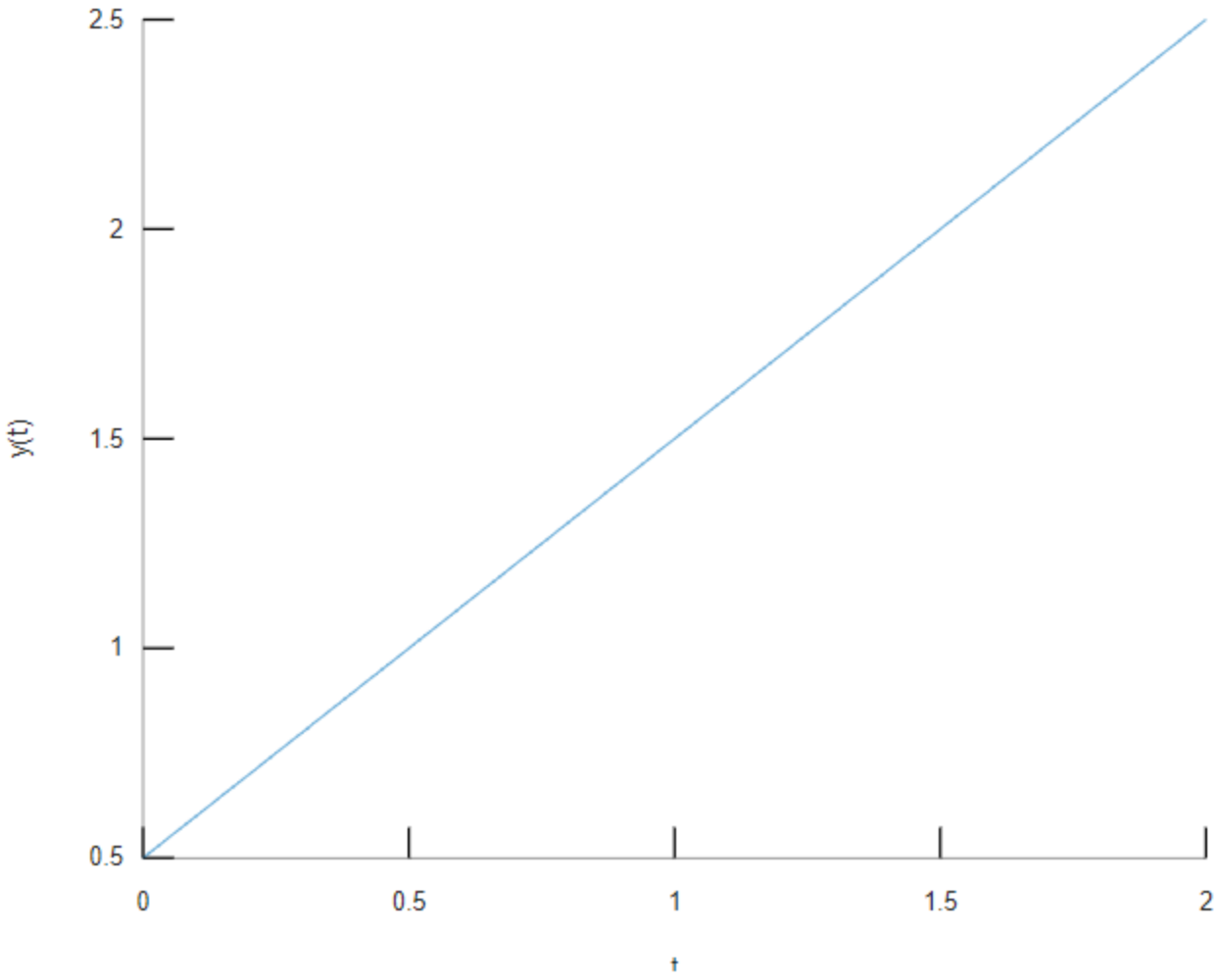

Systems of Differential Equations

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = y(t)$$

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 1$$

$$t = 0 \Rightarrow y(t) = \frac{1}{2}; x(t) = 0$$

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$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = 1$$

$$\Downarrow$$

$$y(t) = t + c_1$$

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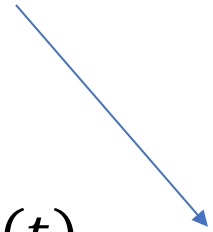

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$$x(t) = \int (t + c_1)\,\mathrm{d}t$$

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$$x(t) = \frac{t^2}{2} + tc_1 + c_2$$

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$$x(t) = \frac{t^2}{2} + t\frac{1}{2} + 0$$

$$y(t) = t + \frac{1}{2}$$

$x(0)= 0$ $\quad \frac{dx(t)}{dt} = y(t)$ $\quad y(0)= \frac{1}{2}$ $\quad \frac{dy(t)}{dt} = 1$ $\quad t \in [0,2]$ $\quad \Delta t = 0.01$

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y0 = 0.5;
t = 0:.01:2

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%compute the solution
y=lsode("f",[x0 y0],t);

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%plot
subplot(2,1,1)
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$\{x(1), x(2)\}$

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$\{x(1), x(2)\}$

$x(1) \approx x(t)$

$x(2) \approx y(t)$

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$\{x(1), x(2)\}$

$x(1) \approx x(t)$

$x(2) \approx y(t)$

t	*x(t)*	*y(t)*
t_0	$x(t_0)$	$y(t_0)$
$t_0 + \Delta t$	$x(t_0 + \Delta t)$	$y(t_0 + \Delta t)$
…	…	…

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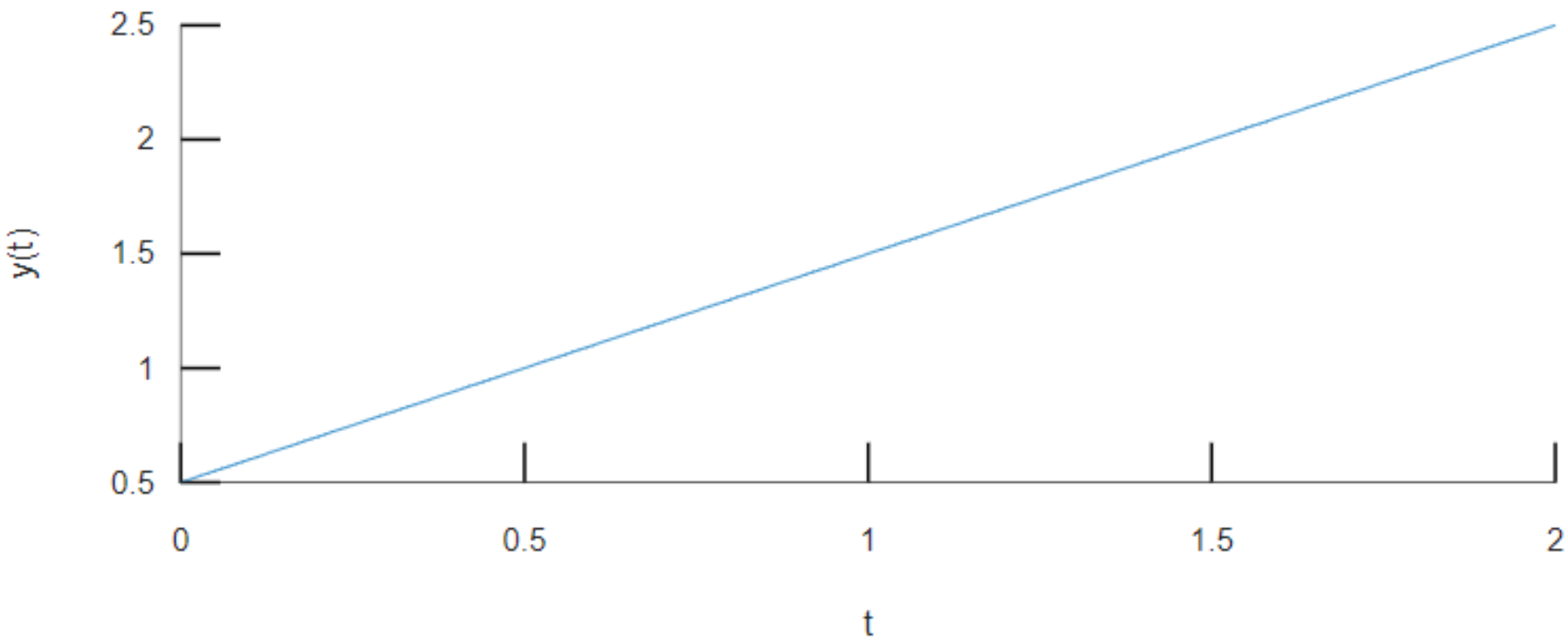
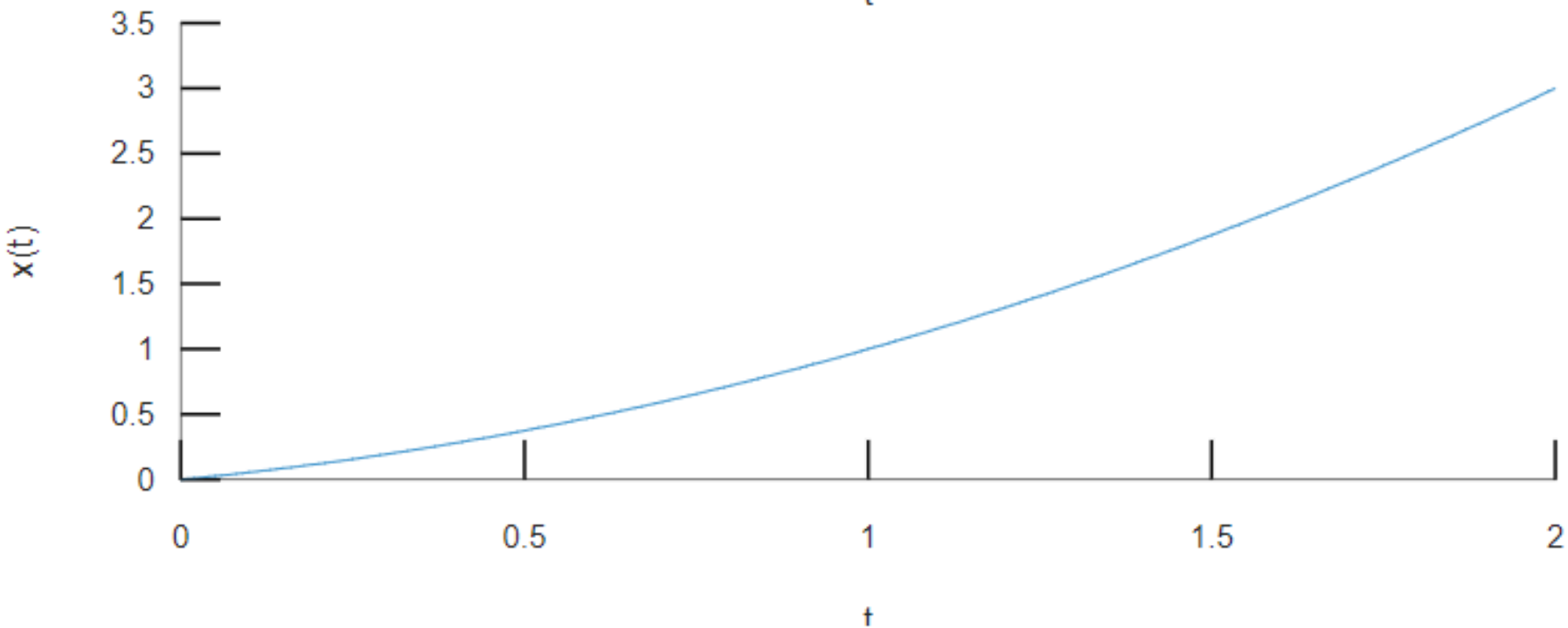
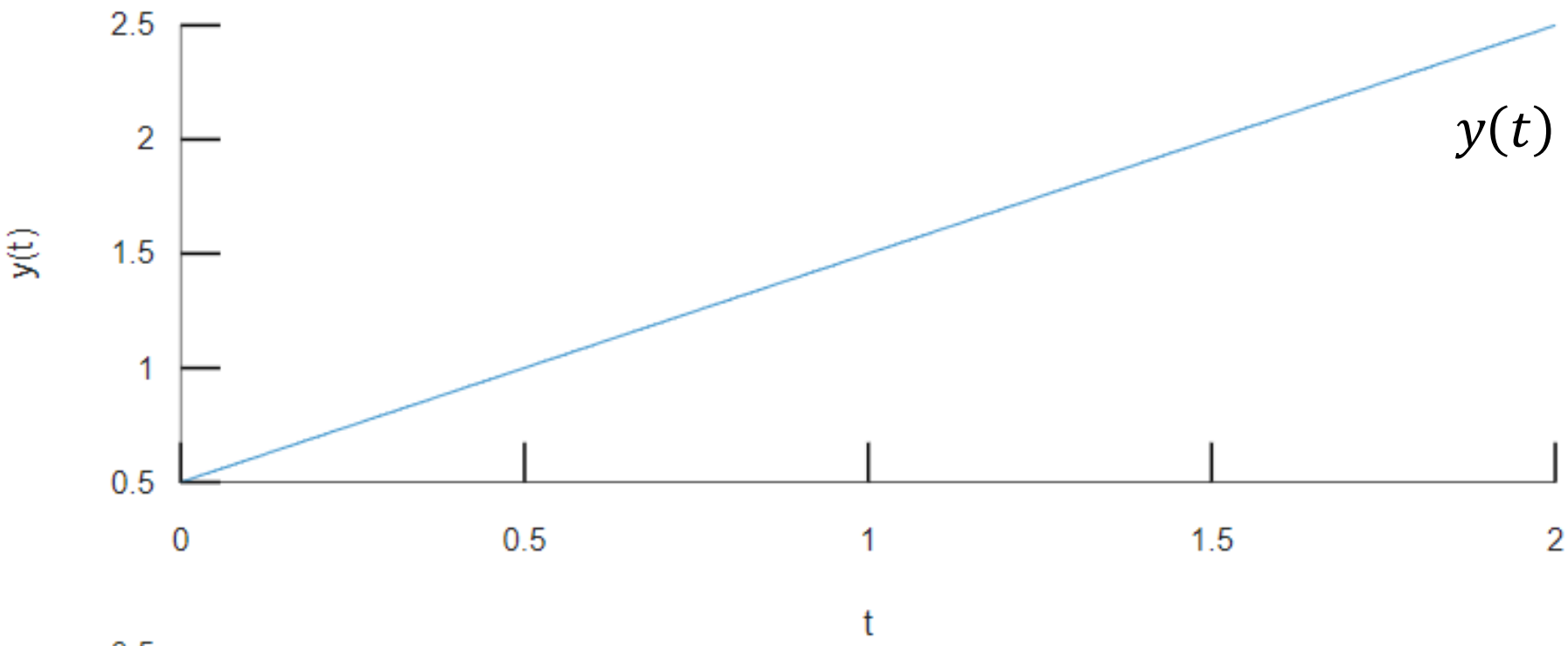
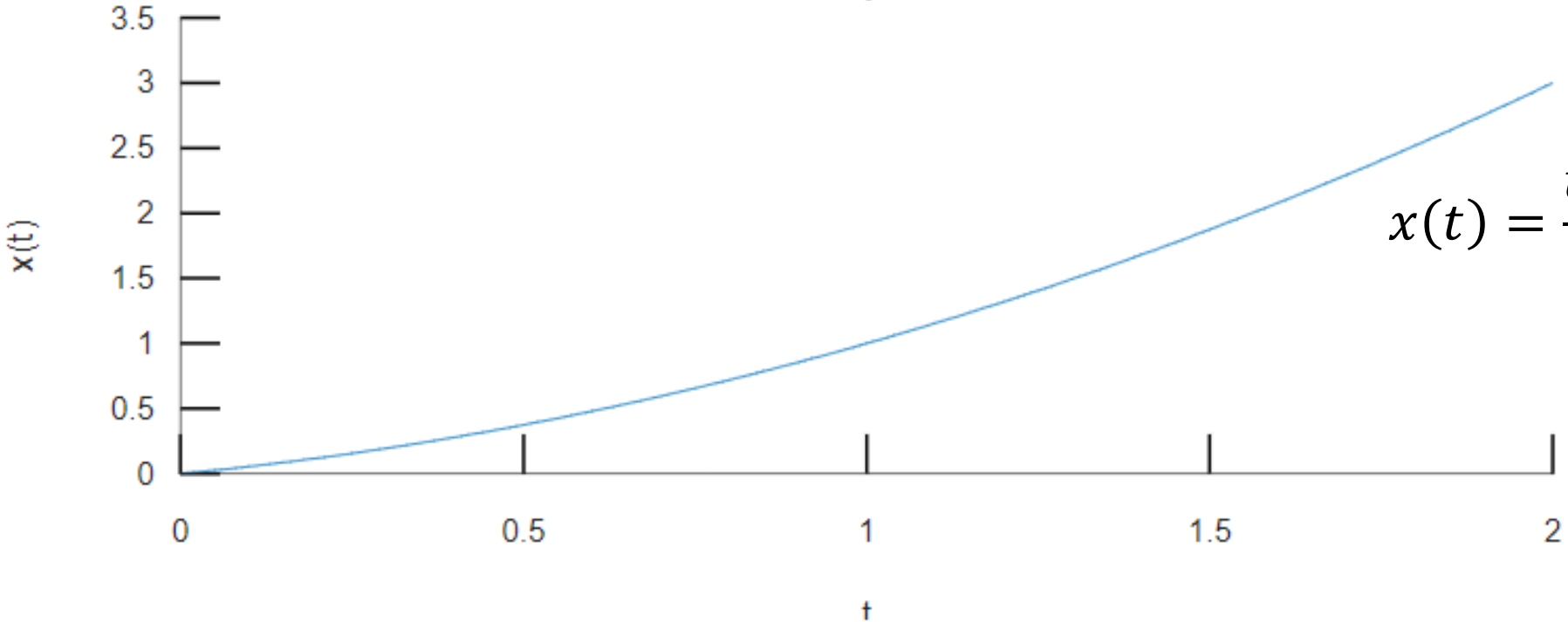

2.5
2
1.5
1
0.5
y(t)
0
0.5
1
1.5
2
t


3.5
3
2.5
2
1.5
1
0.5
0
x(t)
0
0.5
1
1.5
2
t



$$y(t) = t + \frac{1}{2}$$



$$x(t) = \frac{t^2}{2} + \frac{t}{2}$$