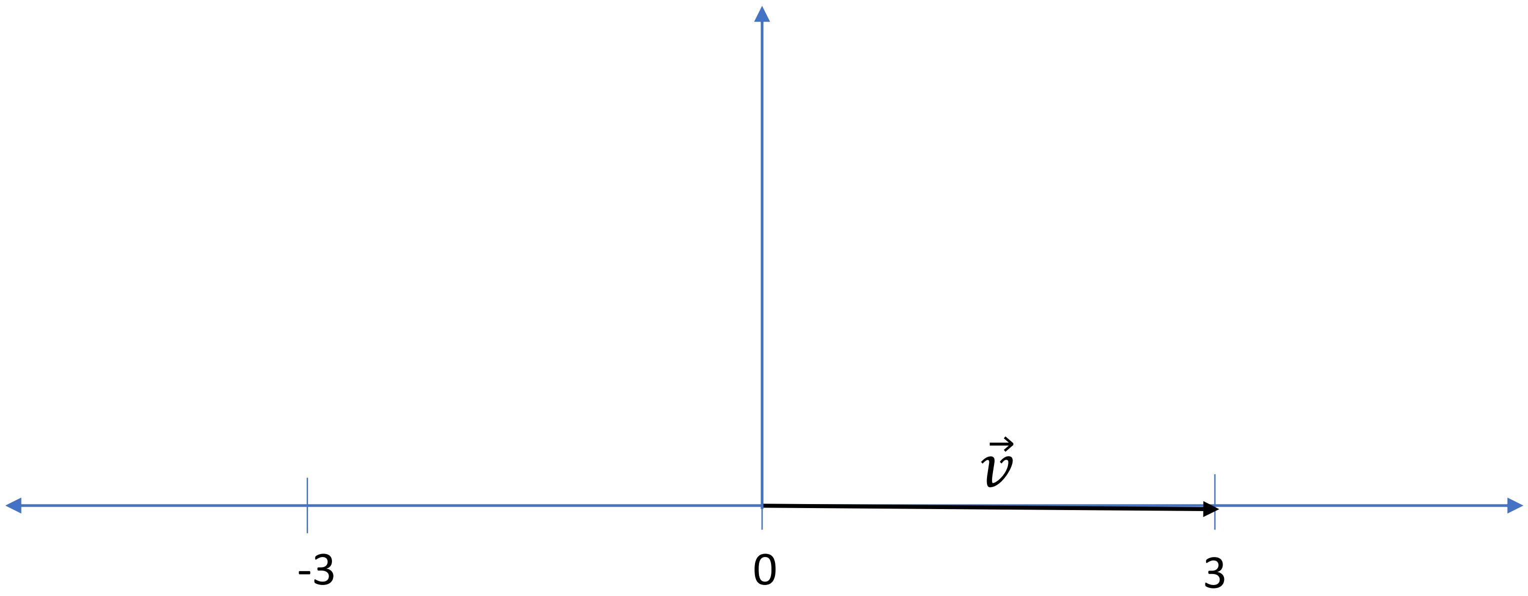
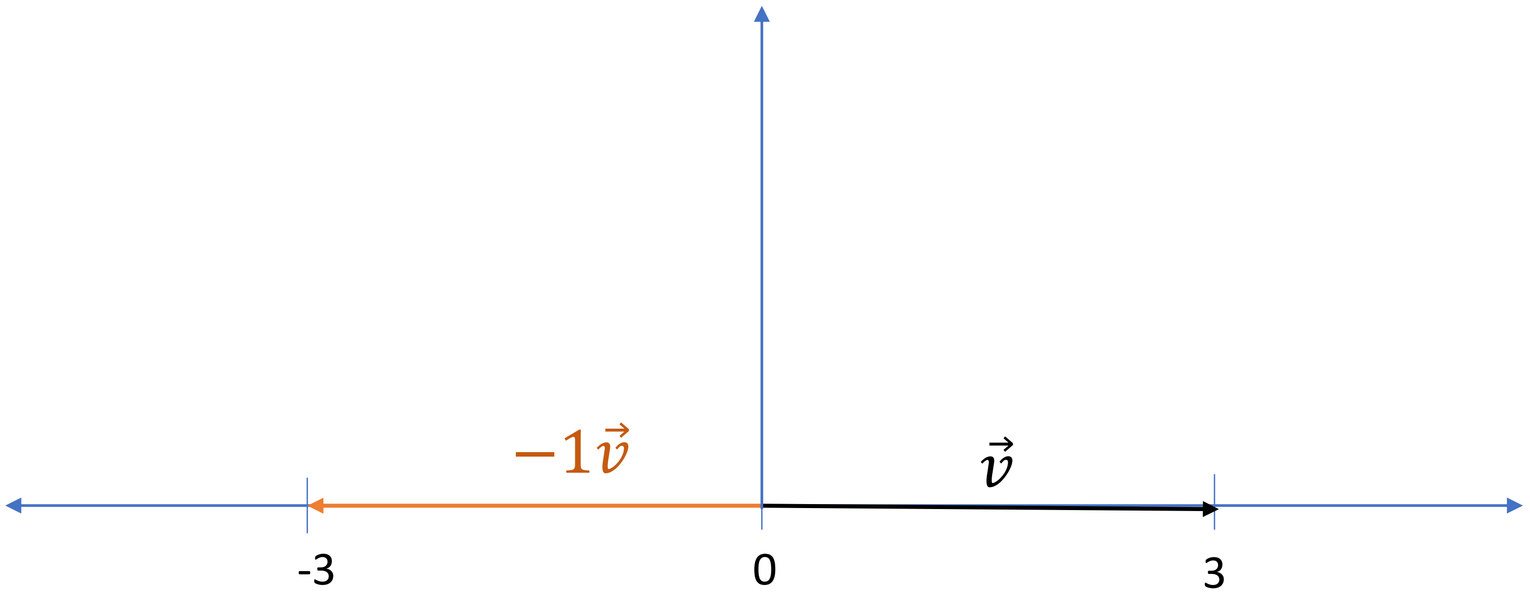
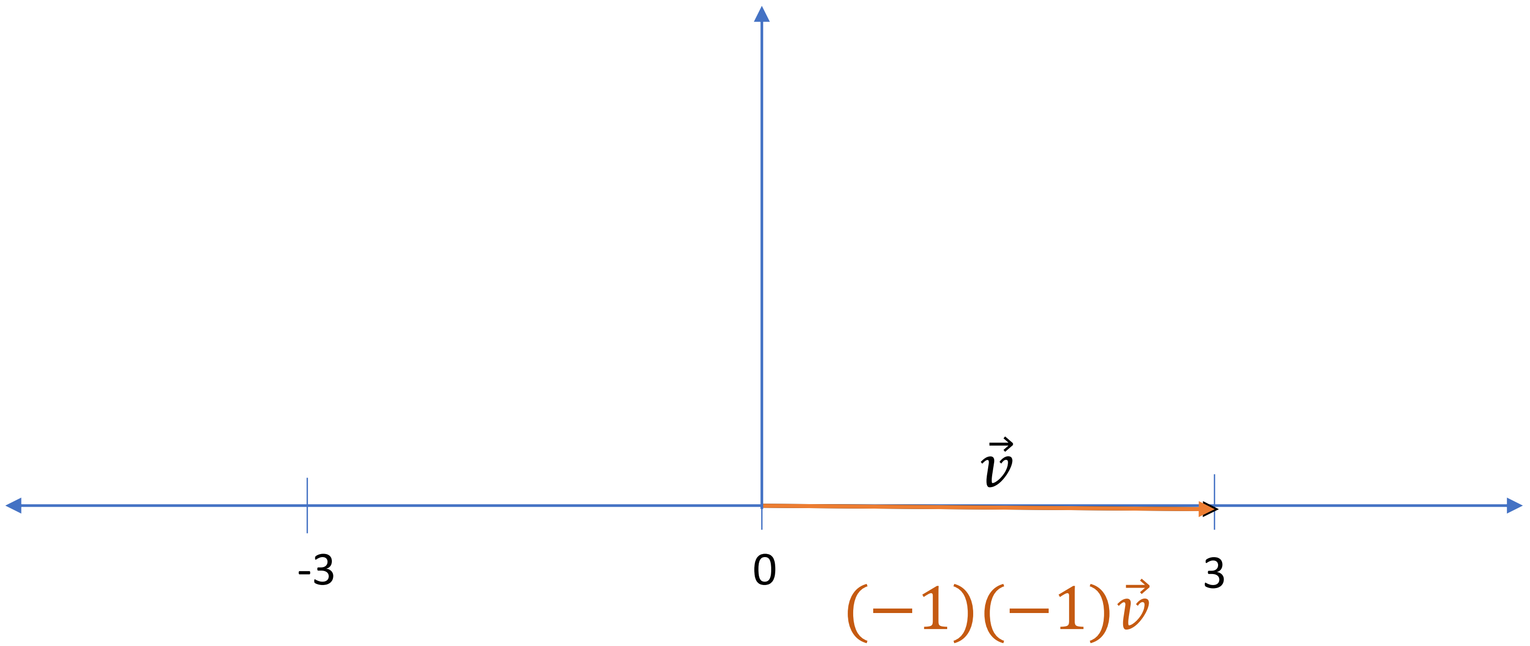


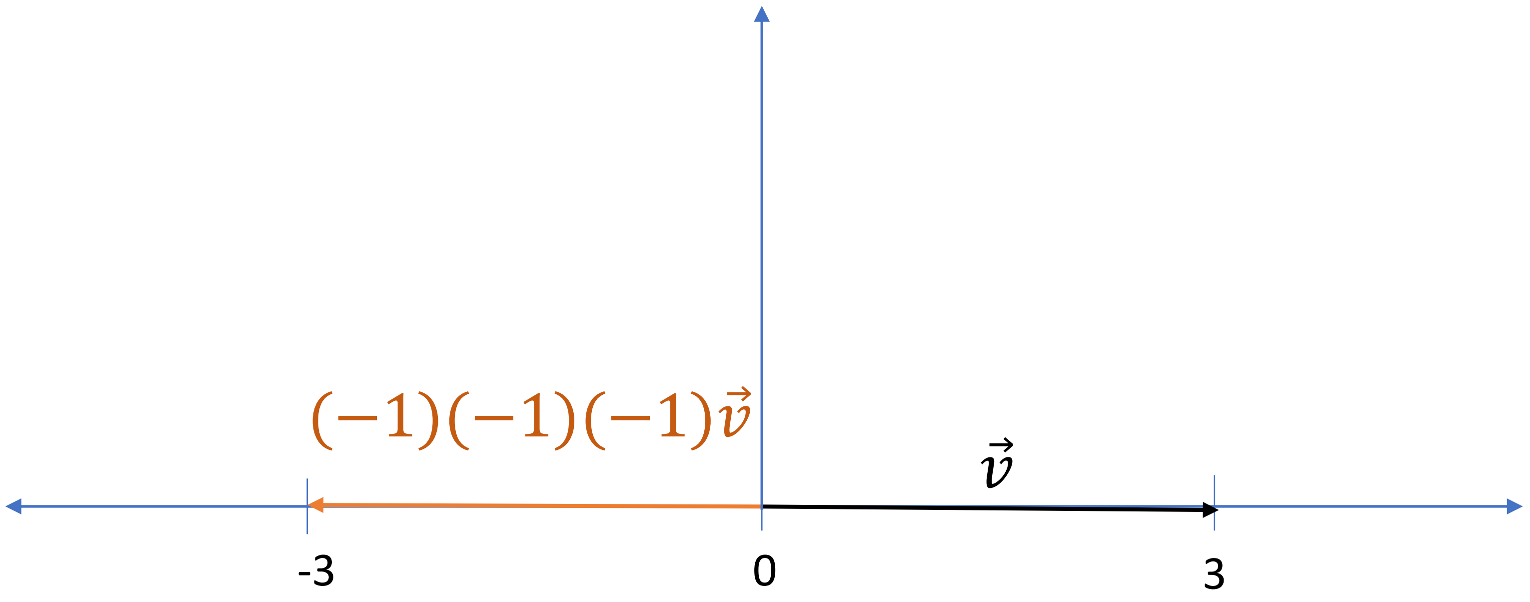
$\vec{v}$
-3
0
3

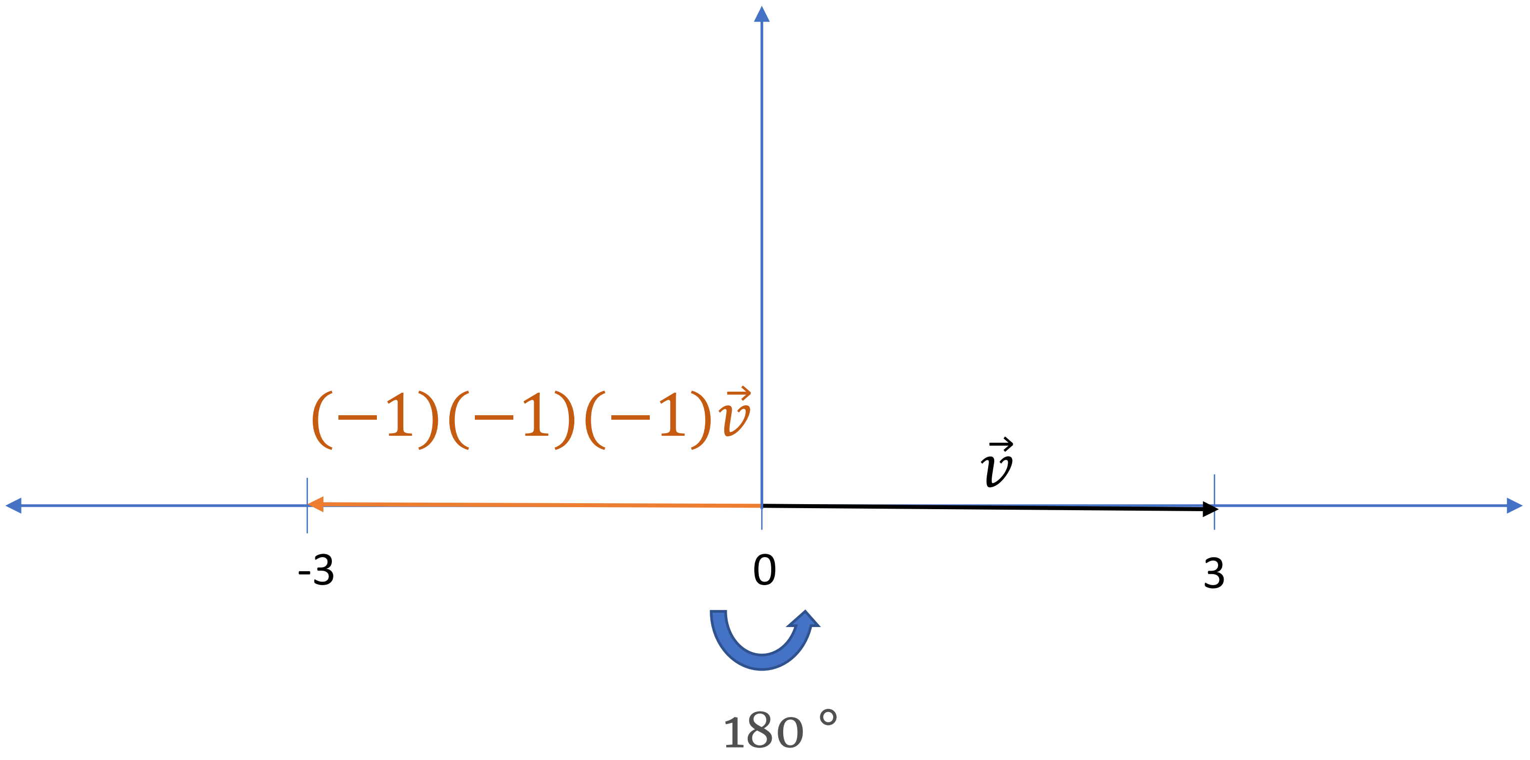


$-1\vec{v}$
$\vec{v}$
-3
0
3



$\vec{v}$
-3
0
3
$(-1)(-1)\vec{v}$

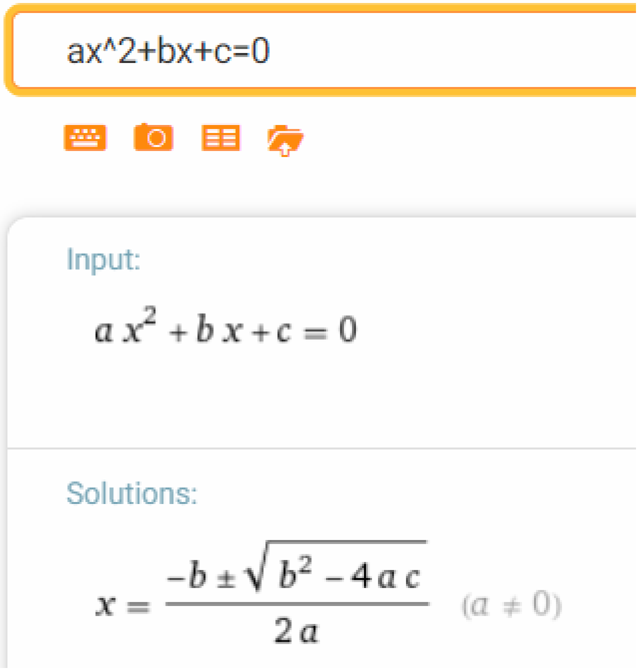

$(-1)(-1)(-1)\vec{v}$
$\vec{v}$
-3
0
3



$(-1)(-1)(-1)\vec{v}$
$\vec{v}$
-3
0
3
180 °

complex numbers

$$ax^2 + bx + c$$



complex numbers

$$ax^2 + bx + c$$

ax^2+bx+c=0

Input:

$$a x^2 + b x + c = 0$$

Solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4 a c}}{2 a} \quad (a \neq 0)$$

Alternate form:

$$n = m x + x^3$$

$$x^3 + mx - n$$

Solutions:

$$x = \frac{\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{\sqrt[3]{2}\, 3^{2/3}} - \frac{\sqrt[3]{\frac{2}{3}}\, m}{\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}$$

$$x = \frac{(1 + i\sqrt{3})\, m}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}} - \frac{(1 - i\sqrt{3})\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{2\sqrt[3]{2}\, 3^{2/3}}$$

$$x = \frac{(1 - i\sqrt{3})\, m}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}} - \frac{(1 + i\sqrt{3})\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{2\sqrt[3]{2}\, 3^{2/3}}$$

complex numbers

$ax^2 + bx + c$

ax^2+bx+c=0

Input:

$$a x^2 + b x + c = 0$$

Solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4 a c}}{2 a} \quad (a \neq 0)$$

Alternate form:

$$n = m x + x^3$$

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Solutions:

$$x = \frac{\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{\sqrt[3]{2}\, 3^{2/3}} - \frac{\sqrt[3]{\frac{2}{3}}\, m}{\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}$$

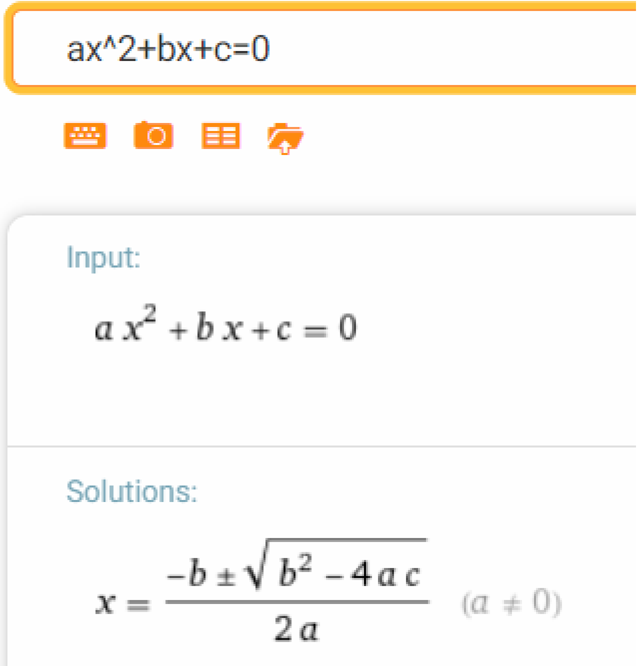
at least one real solution exists

$$x = \frac{(1 + i\sqrt{3})\, m}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}} - \frac{(1 - i\sqrt{3})\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{2\sqrt[3]{2}\, 3^{2/3}}$$

$$x = \frac{(1 - i\sqrt{3})\, m}{2^{2/3}\sqrt[3]{3}\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}} - \frac{(1 + i\sqrt{3})\sqrt[3]{\sqrt{3}\sqrt{4 m^3 + 27 n^2} + 9 n}}{2\sqrt[3]{2}\, 3^{2/3}}$$

complex numbers

$$ax^2 + bx + c$$



$$x^3 + mx = n$$

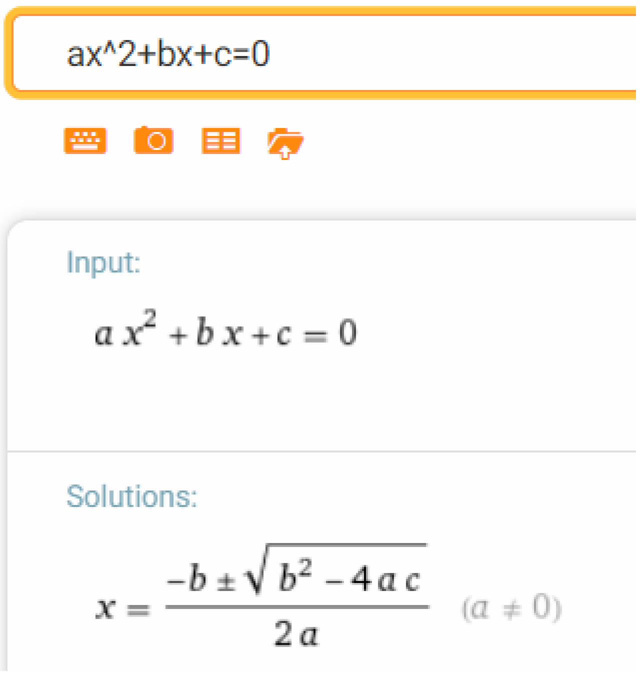
del Ferro’s solution:

$$\sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} + \sqrt[3]{\frac{n}{2} - \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

$$x^3 - 15x = 4$$

complex numbers

$ax^2 + bx + c$



$x^3 + mx = n$

del Ferro's solution:

$$\sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} + \sqrt[3]{\frac{n}{2} - \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

$$x^3 - 15x = 4$$

$$x = \sqrt[3]{2 + \sqrt{-121}} - \sqrt[3]{-2 + \sqrt{-121}}$$

complex numbers

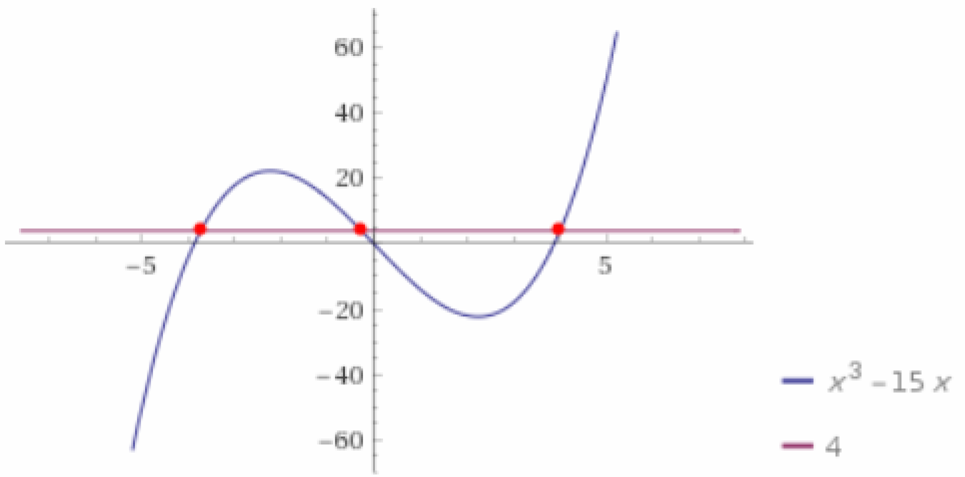
$x^3 - 15x = 4$

$x^3 + mx = n$

Input:

$x^3 - 15x = 4$

Plot:



del Ferro's solution:

$$\sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} + \sqrt[3]{\frac{n}{2} - \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

$$x = \sqrt[3]{2 + \sqrt{-121}} - \sqrt[3]{-2 + \sqrt{-121}}$$

complex numbers

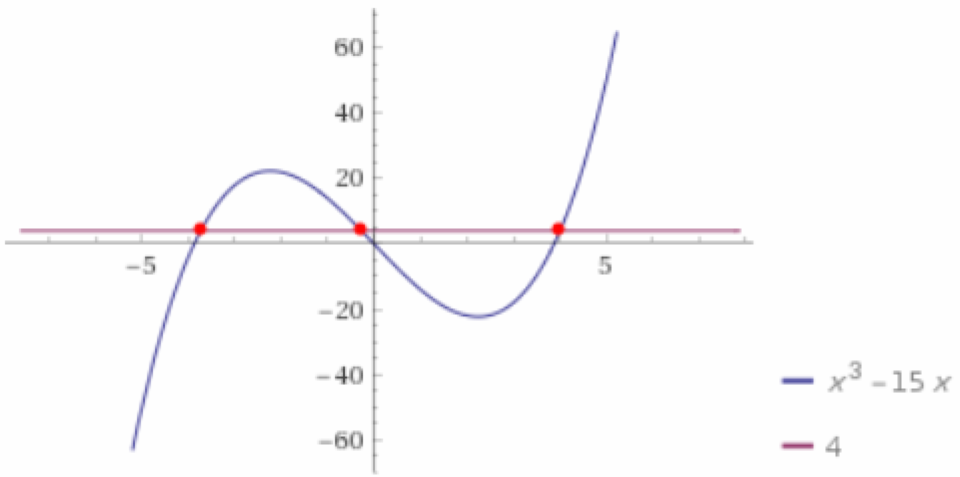
$$x^3 - 15x = 4$$

$$x^3 + mx = n$$

Input:

$x^3 - 15x = 4$

Plot:



del Ferro's solution: $$\sqrt[3]{\frac{n}{2}+\sqrt{\frac{n^2}{4}+\frac{m^3}{27}}}+\sqrt[3]{\frac{n}{2}-\sqrt{\frac{n^2}{4}+\frac{m^3}{27}}}$$

$$x = \sqrt[3]{2+\sqrt{-121}} - \sqrt[3]{-2+\sqrt{-121}}$$

$$x = \sqrt[3]{(2+\sqrt{-1})^3} - \sqrt[3]{(-2+\sqrt{-1})^3}$$

complex numbers

$x^3 - 15x = 4$

Alternate forms:

$x^3 = 15x + 4$

$x(x^2 - 15) = 4$

$x^3 - 15x - 4 = 0$

Solutions:

$x = 4$

$x = -2 - \sqrt{3}$

$x = \sqrt{3} - 2$

$x^3 + mx = n$

del Ferro's solution:

$$\sqrt[3]{\frac{n}{2} + \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}} + \sqrt[3]{\frac{n}{2} - \sqrt{\frac{n^2}{4} + \frac{m^3}{27}}}$$

$$x = \sqrt[3]{2 + \sqrt{-121}} - \sqrt[3]{-2 + \sqrt{-121}}$$

$$x = \sqrt[3]{(2 + \sqrt{-1})^3} - \sqrt[3]{(-2 + \sqrt{-1})^3}$$

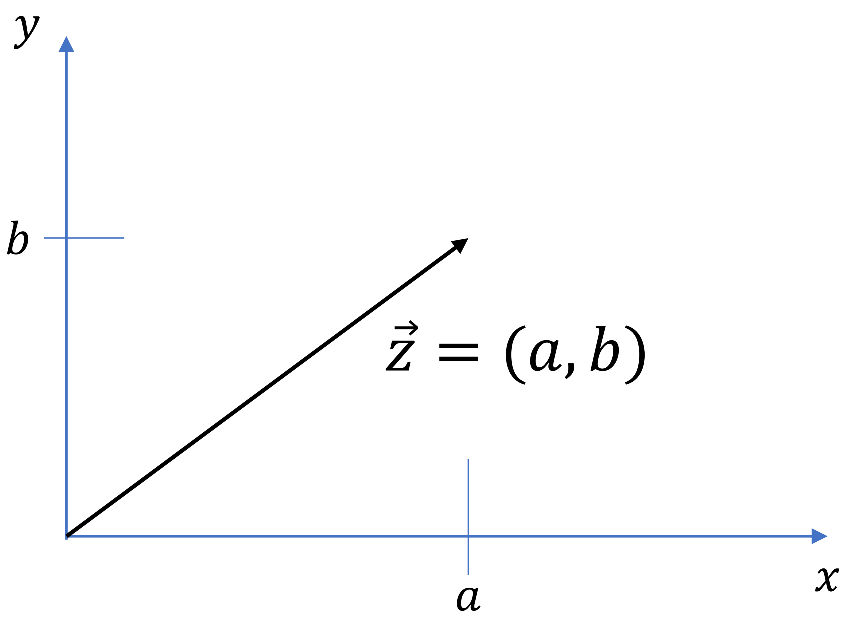
complex numbers

$i = \sqrt{-1}$

complex numbers

$i = \sqrt{-1}$

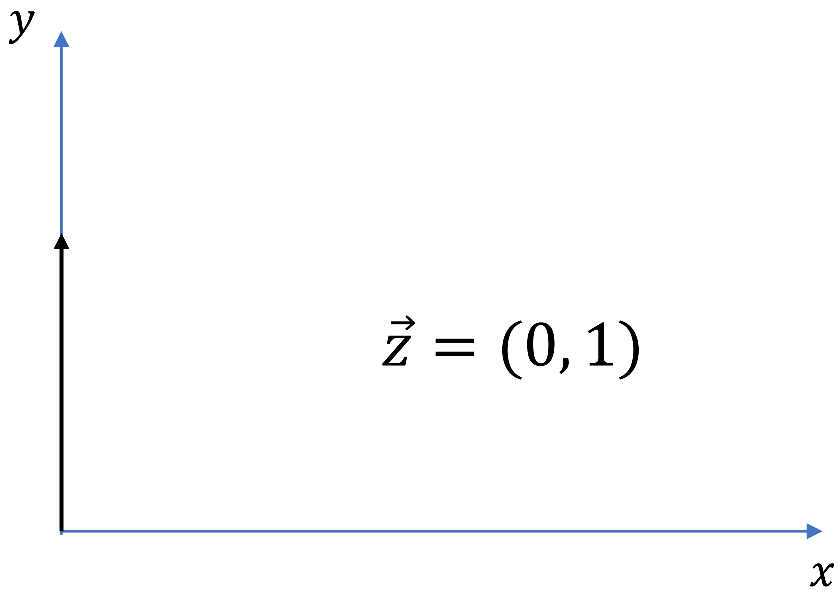
$a + bi = z$ $\quad a \in \mathbb{R}; b \in \mathbb{R}$



complex numbers

$i = \sqrt{-1}$

$0 + 1i = z$

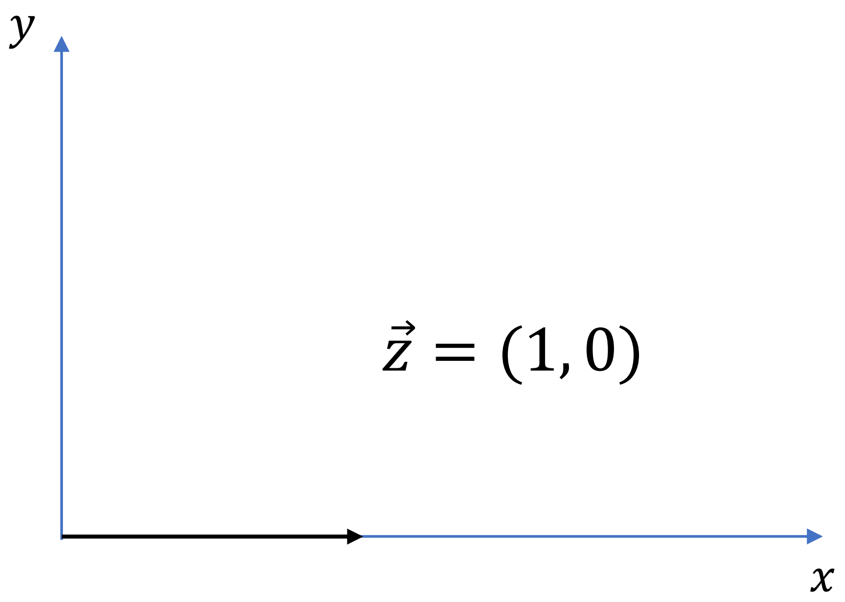


$\vec{z} = (0, 1)$

complex numbers

$i = \sqrt{-1}$

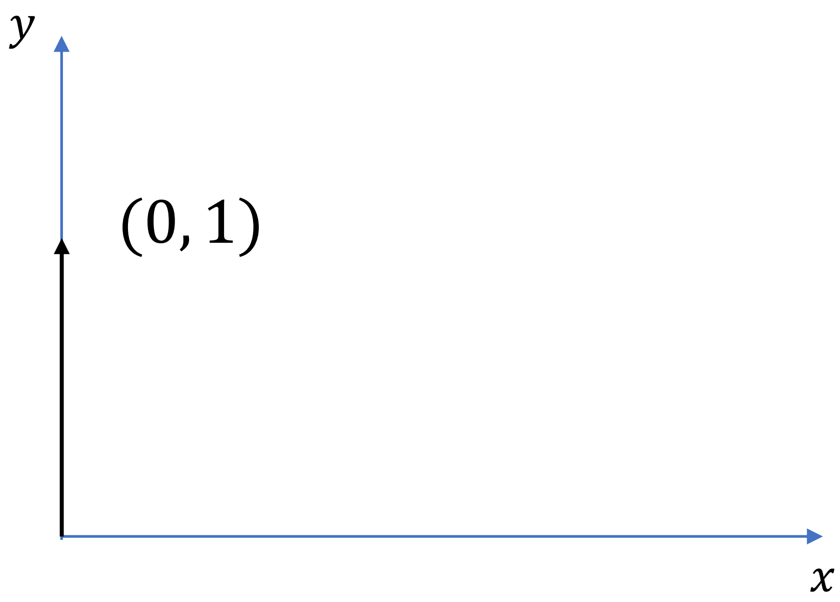
$1 + 0i = z$



complex numbers

$i = \sqrt{-1}$

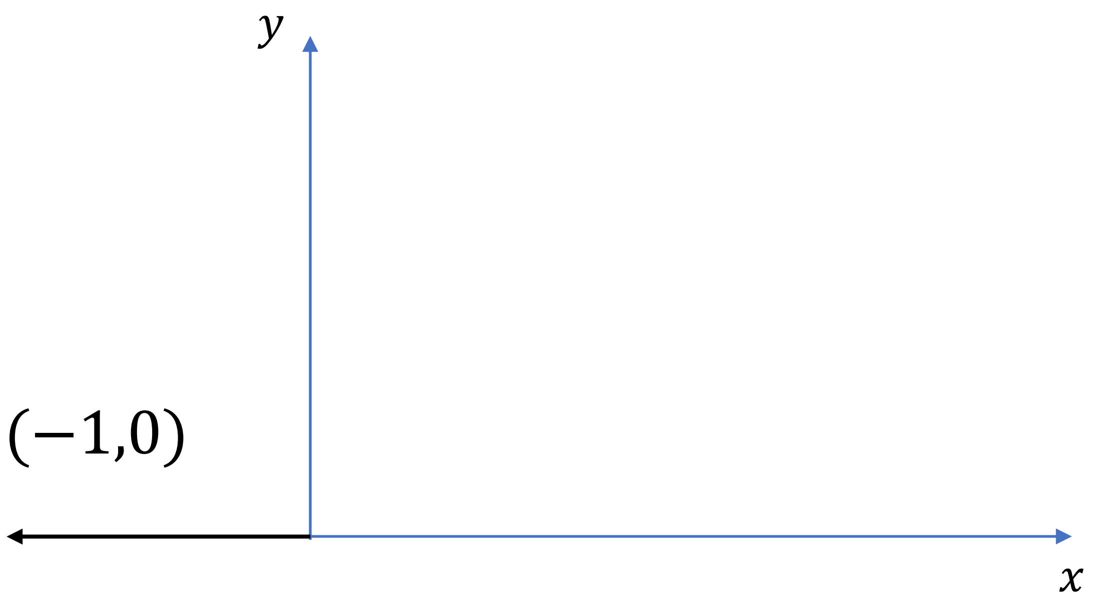
$1 + 0i = z$
$0 + 1i = zi$



complex numbers

$i = \sqrt{-1}$

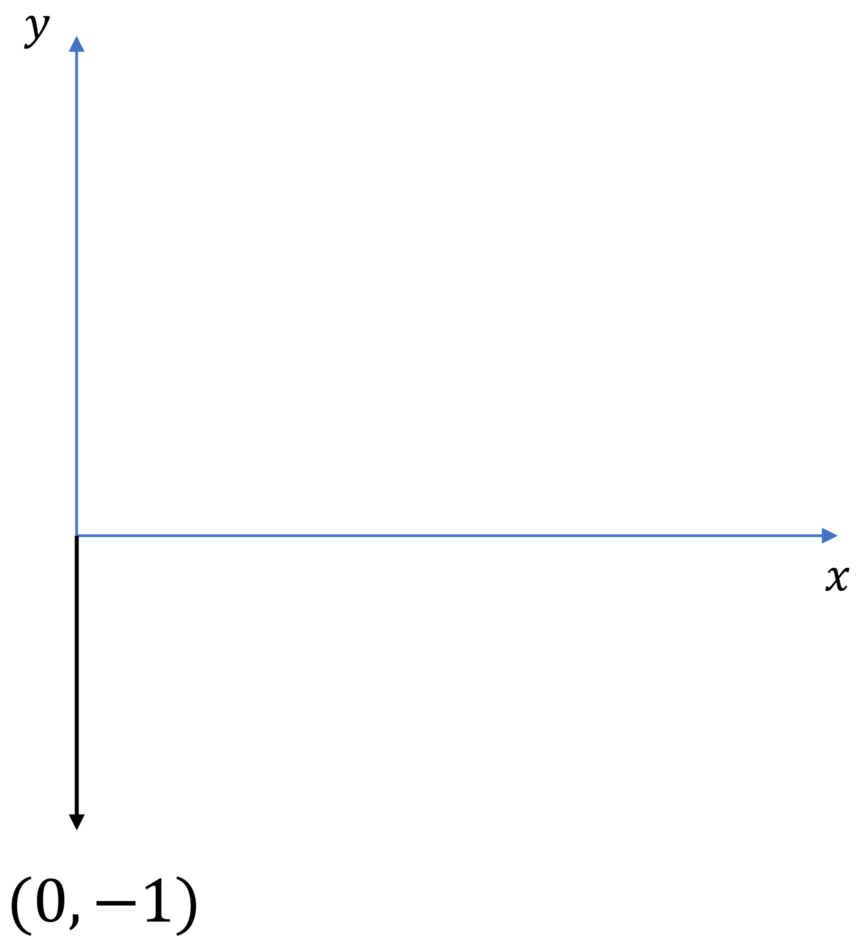
$$1 + 0i = z$$
$$0 + 1i = zi$$
$$-1 + 0i = zii$$



complex numbers

$i = \sqrt{-1}$

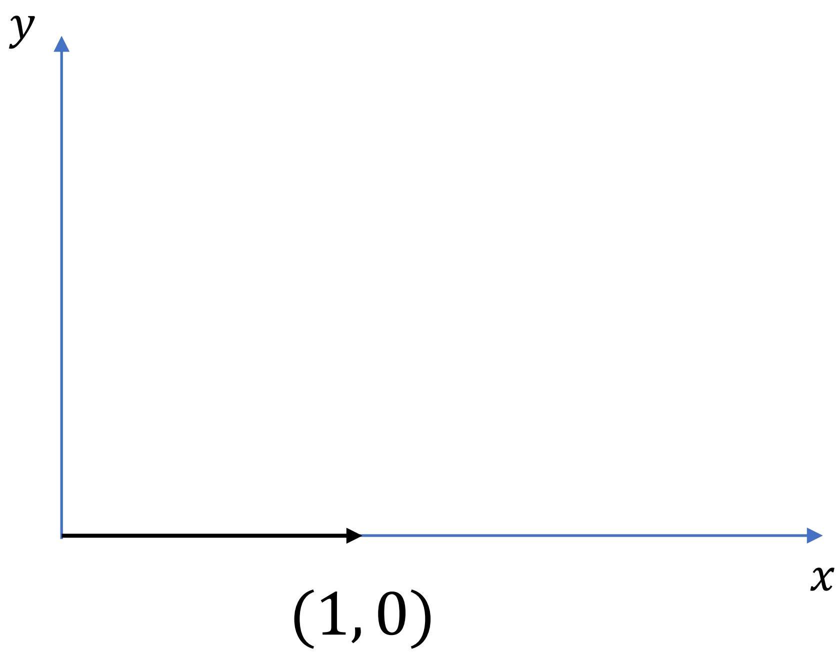
$$1 + 0i = z$$
$$0 + 1i = zi$$
$$-1 + 0i = zii$$
$$0 - 1i = ziii$$



complex numbers

$i = \sqrt{-1}$

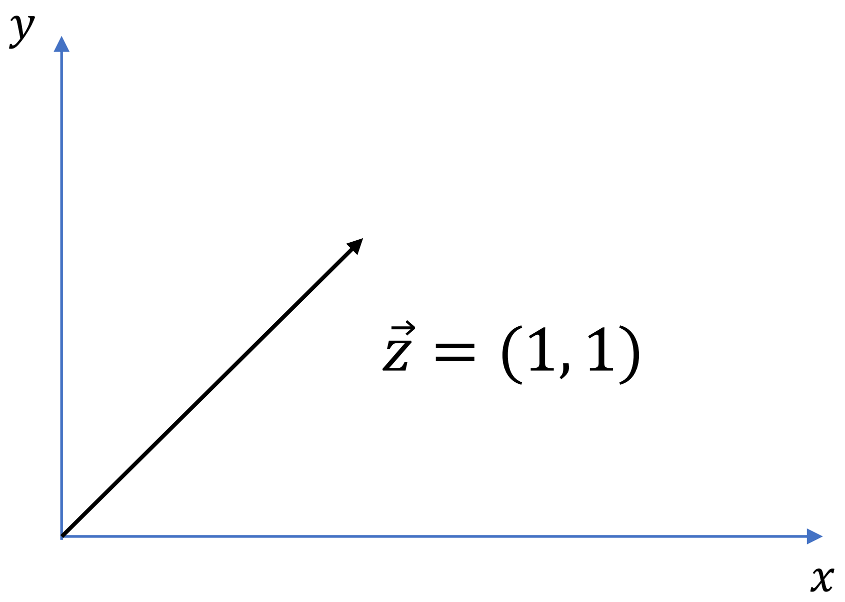
$$1 + 0i = z$$
$$0 + 1i = zi$$
$$-1 + 0i = zii$$
$$0 - 1i = ziii$$
$$1 + 0i = ziiii$$



complex numbers

$i = \sqrt{-1}$

$1 + 1i = z$



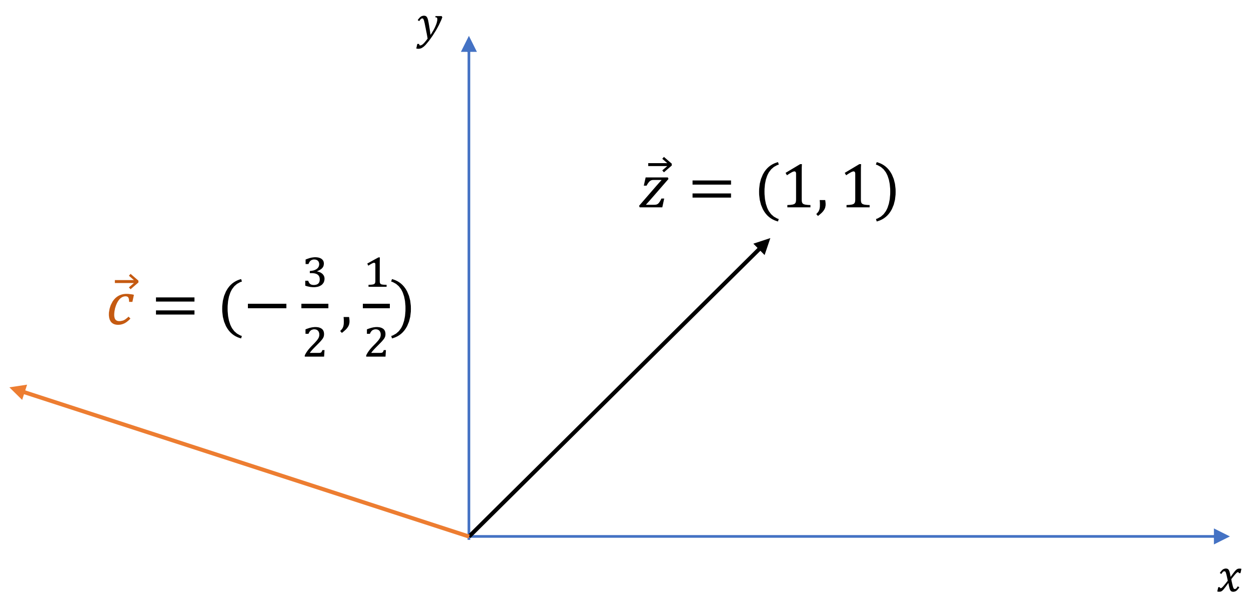
complex numbers

$$i = \sqrt{-1}$$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = ?$$



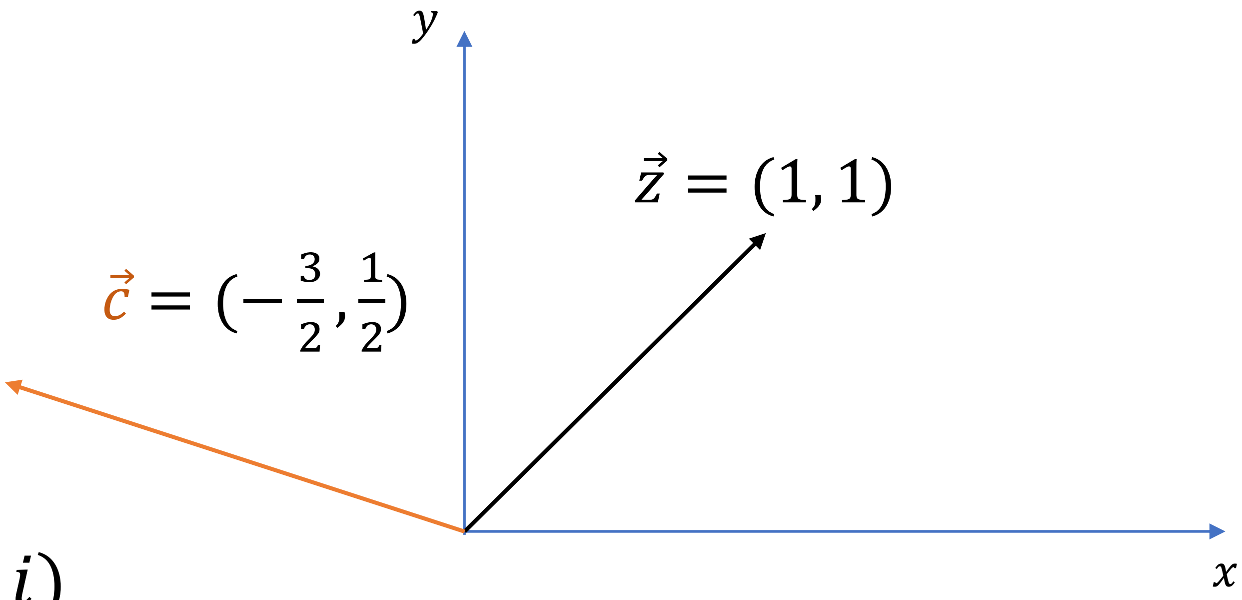
complex numbers

$i = \sqrt{-1}$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = (1 + i)(-\frac{3}{2} + \frac{1}{2}i)$$



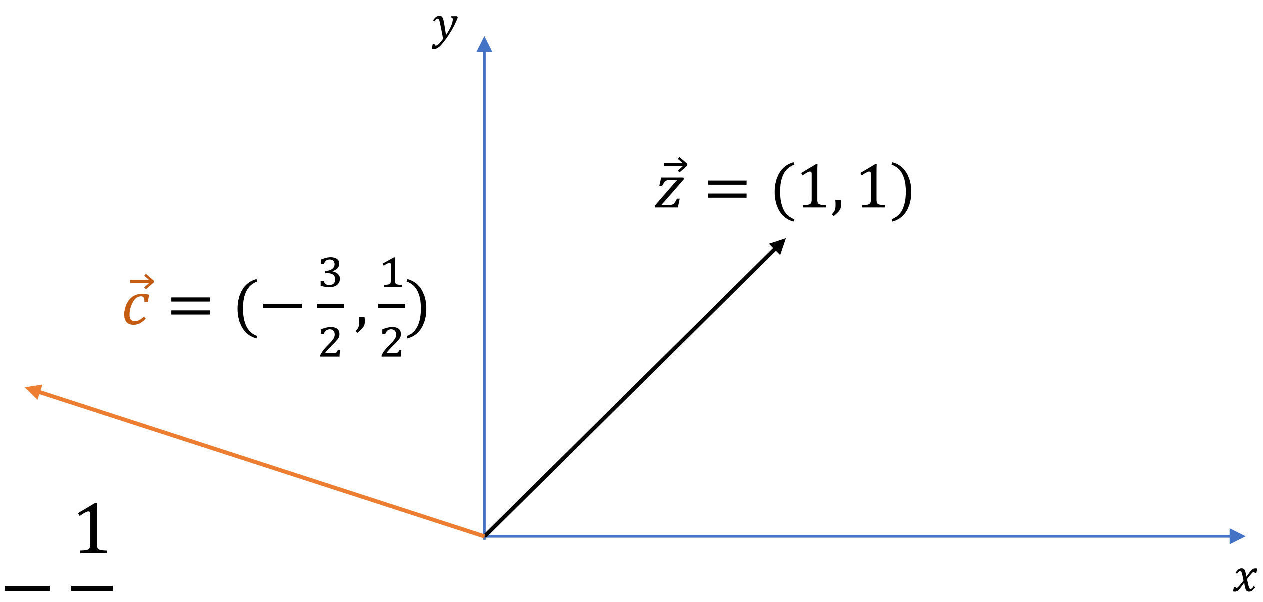
complex numbers

$$i = \sqrt{-1}$$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = -\frac{3}{2} + \frac{1}{2}i - \frac{3}{2}i - \frac{1}{2}$$



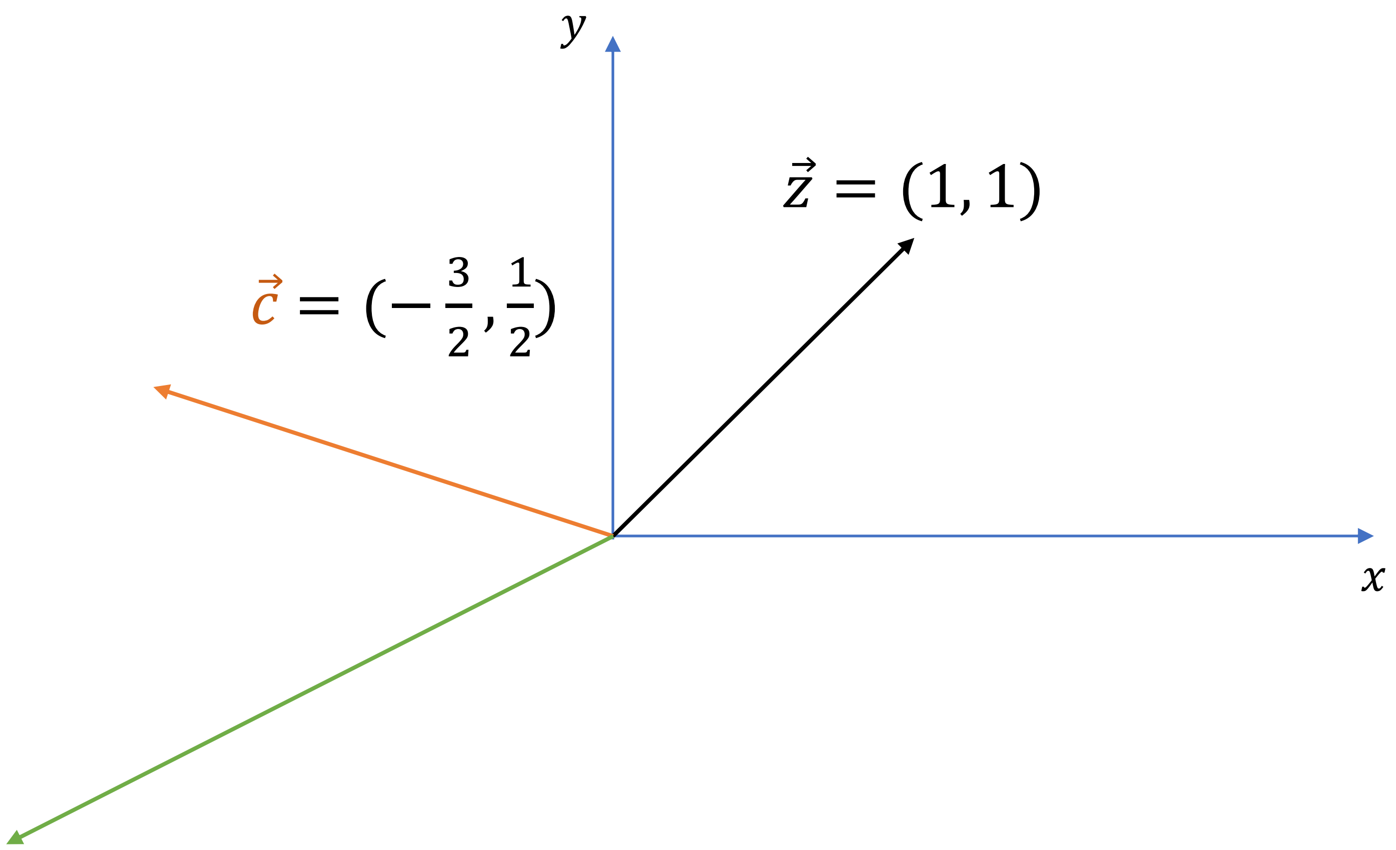
complex numbers

$i = \sqrt{-1}$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = -\frac{4}{2} - \frac{2}{2}i$$



complex numbers

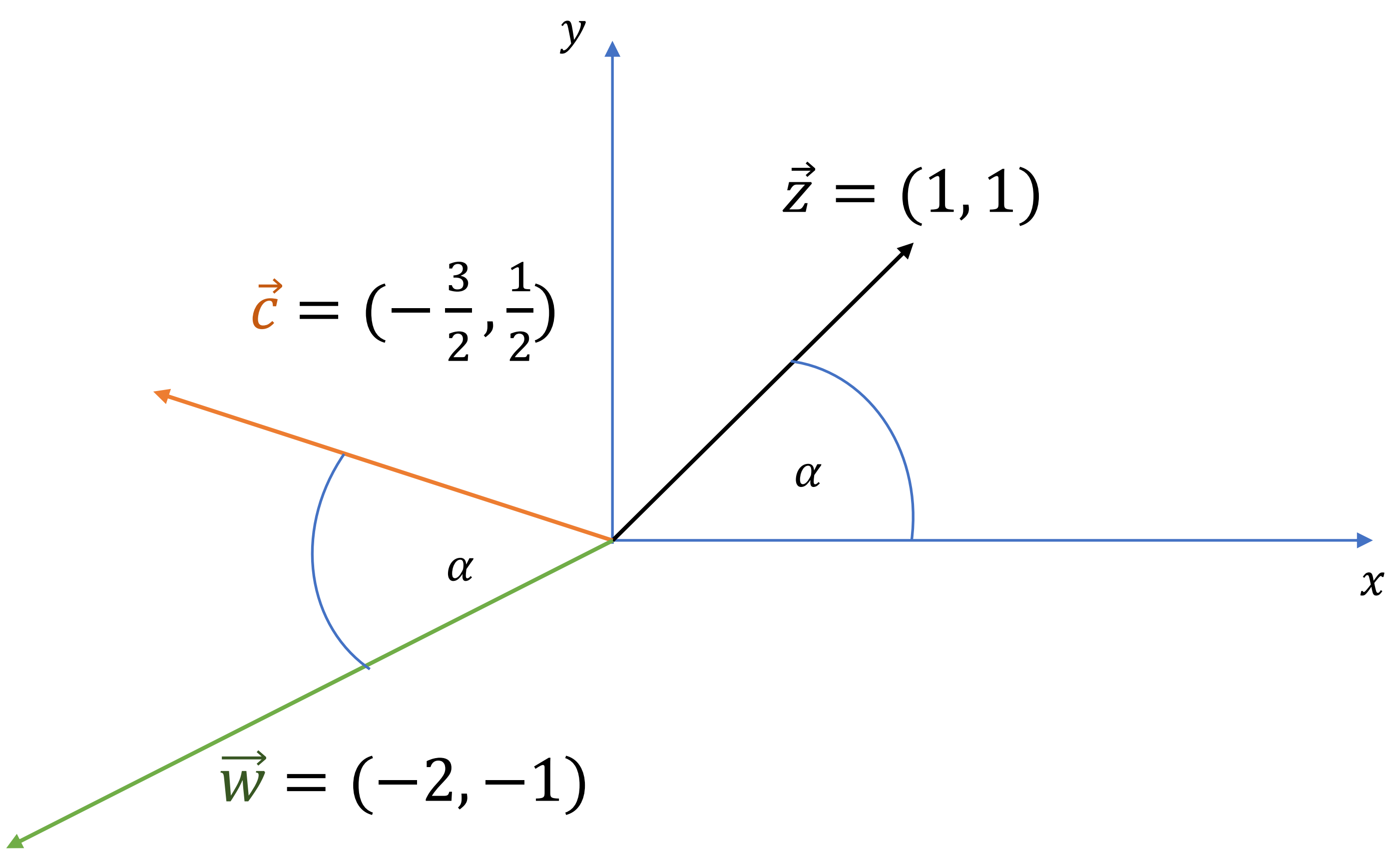
$i = \sqrt{-1}$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = -2 - 1i$$

$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$



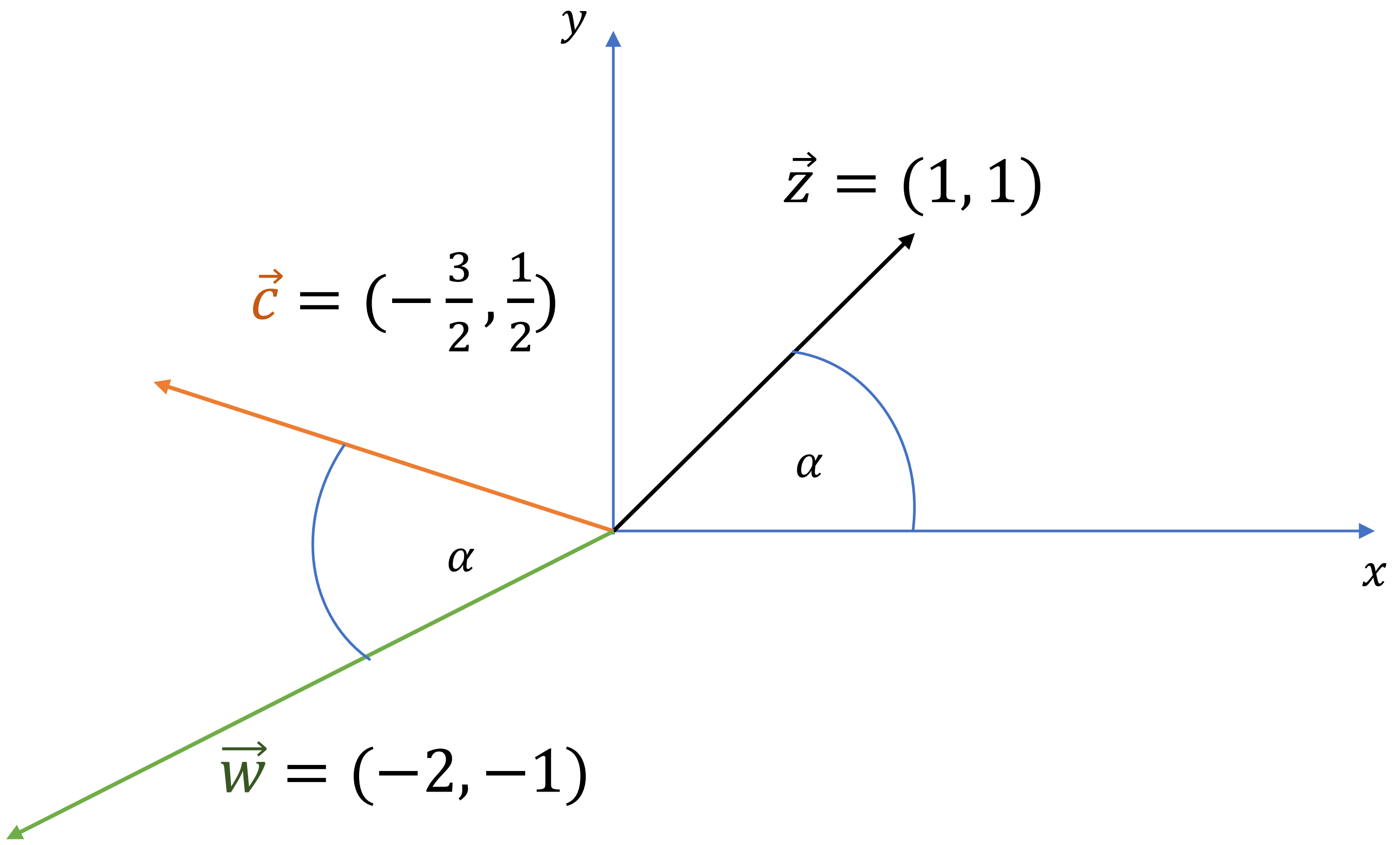
complex numbers

$i = \sqrt{-1}$

$1 + i = z$

$-\frac{3}{2} + \frac{1}{2}i = c$

$zc = -2 - 1i$



$|\vec{w}| = |\vec{z}|\,|\vec{c}|$

$|\vec{z}| = \sqrt{2}$

$|\vec{c}| = \sqrt{\frac{9}{4} + \frac{1}{4}} = \frac{\sqrt{10}}{2}$

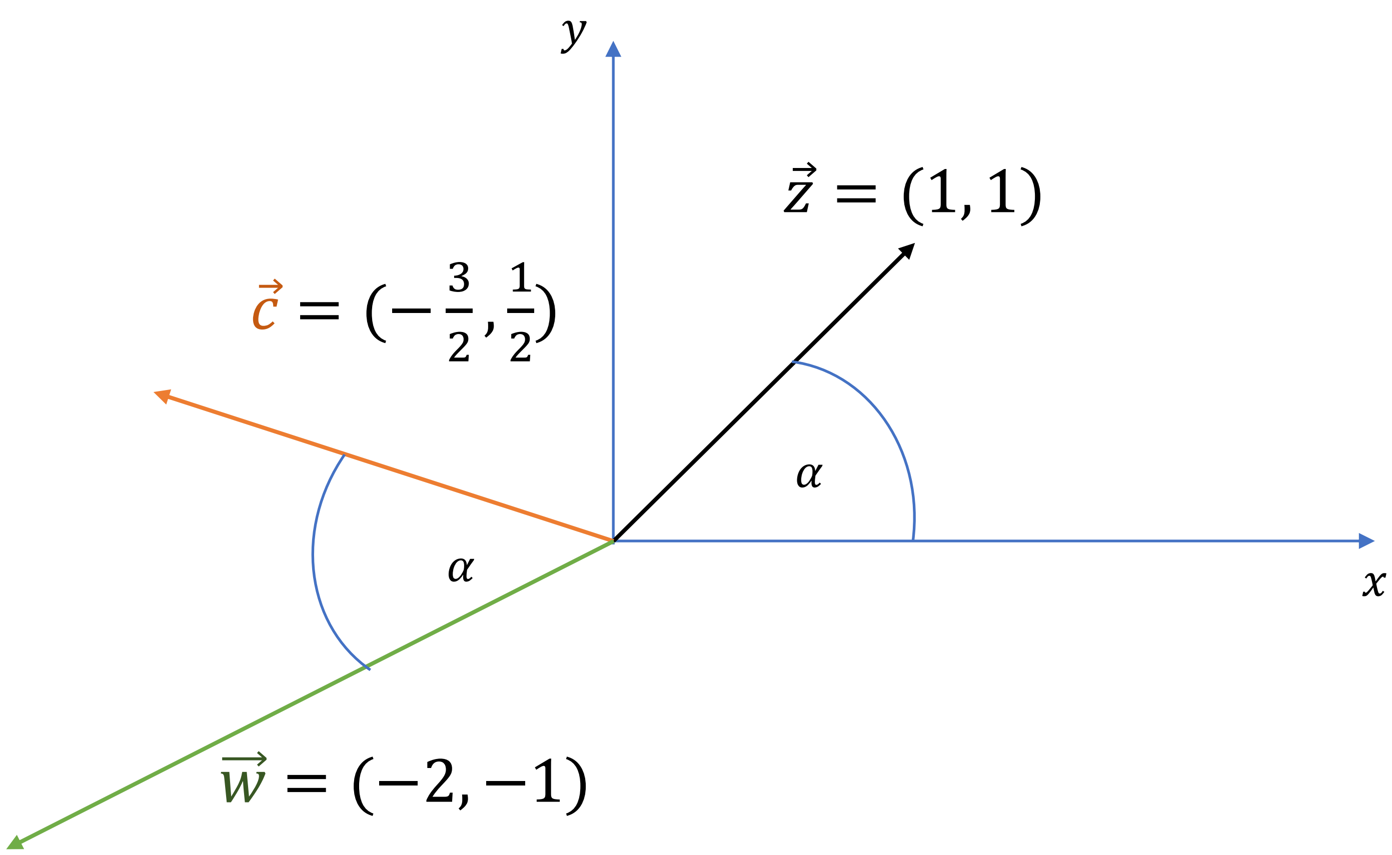
complex numbers

$i = \sqrt{-1}$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = -2 - 1i$$



$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = \sqrt{2}$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

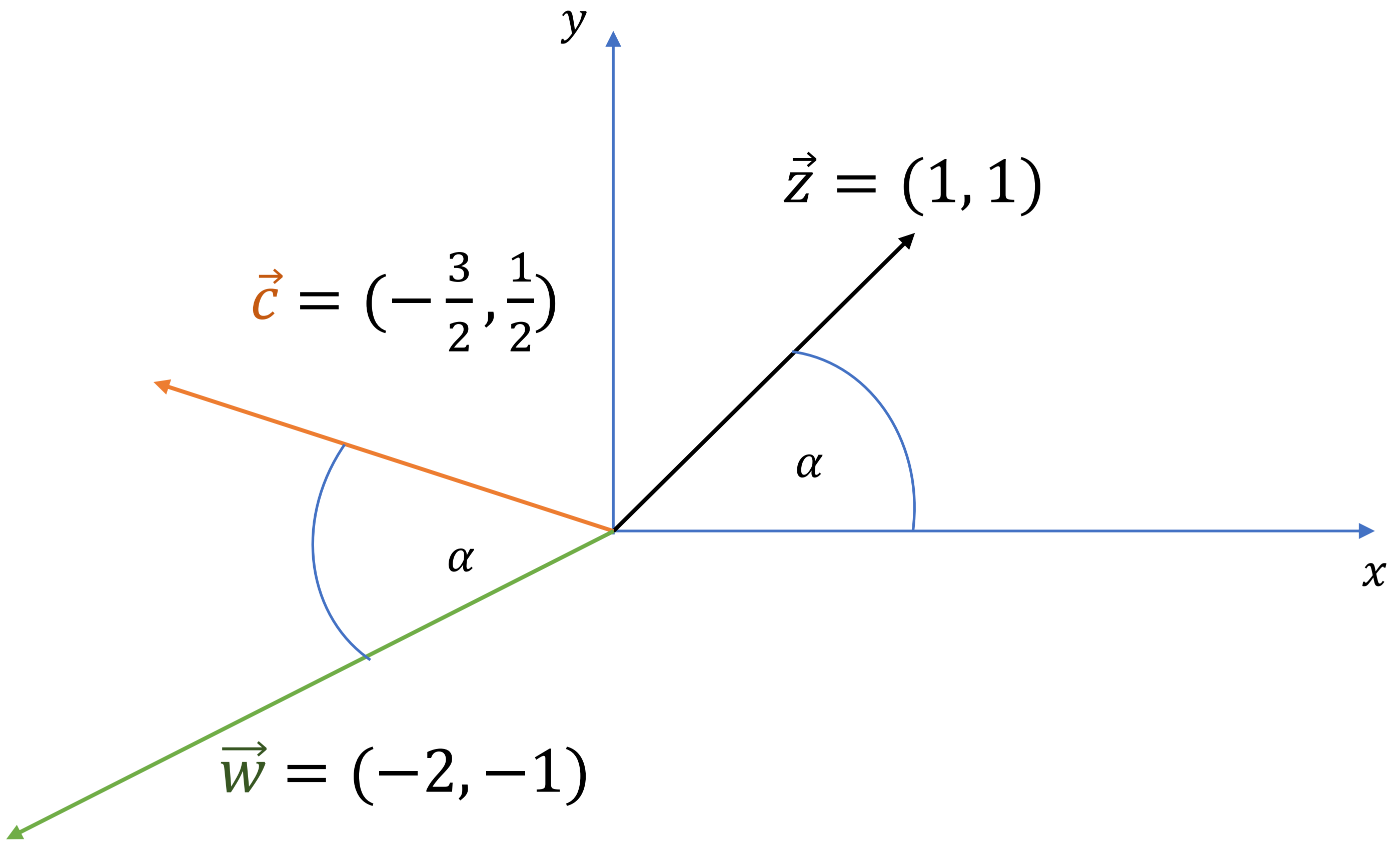
complex numbers

$i = \sqrt{-1}$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2}i = c$$

$$zc = -2 - 1i$$



$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = \sqrt{2}$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \sqrt{5}$$

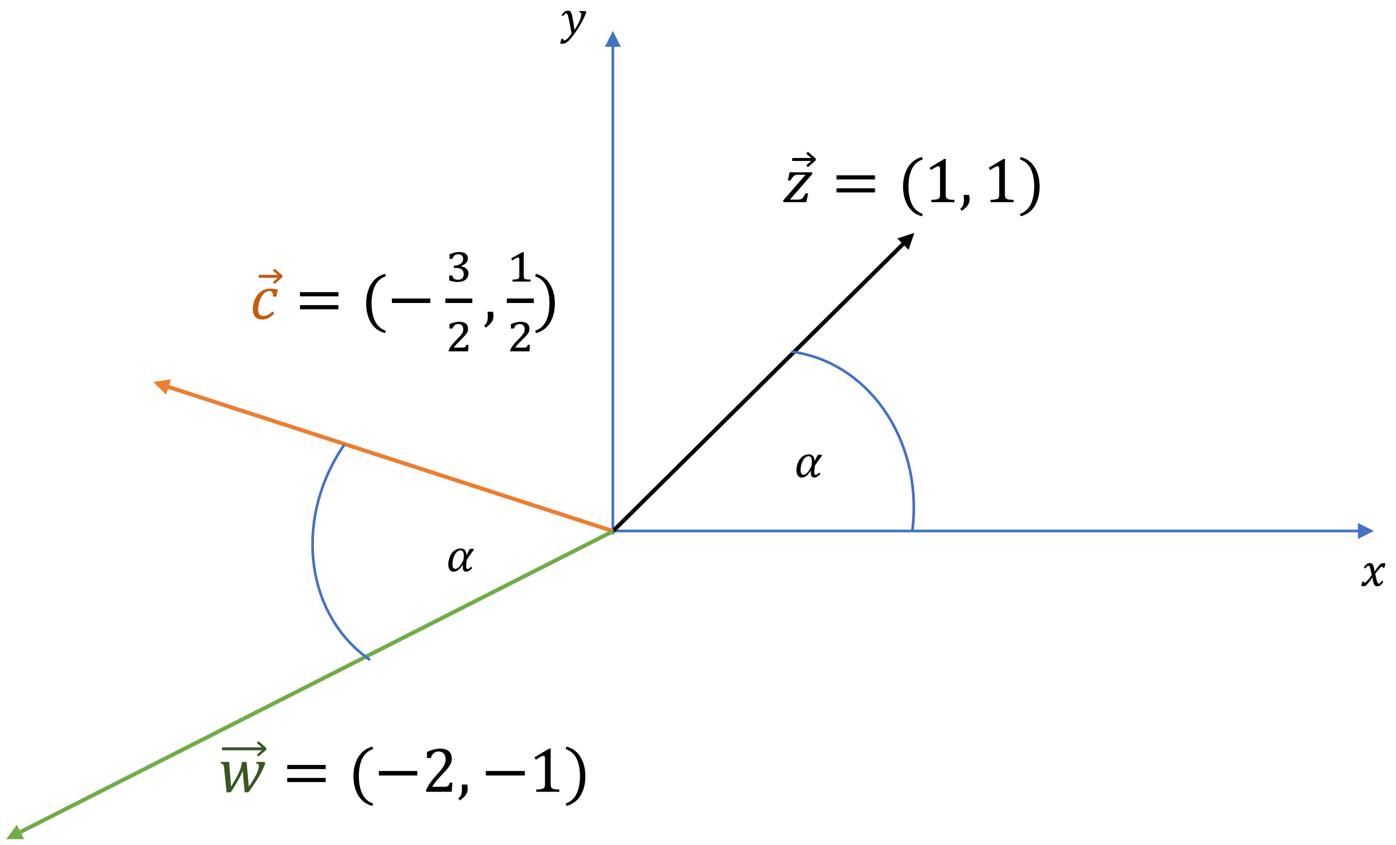
complex numbers

$$i = \sqrt{-1}$$

$$1 + i = z$$

$$-\frac{3}{2} + \frac{1}{2} i = c$$

$$zc = -2 - 1i$$



$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = \sqrt{2}$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \sqrt{5}$$

$$|\vec{w}| = \sqrt{4 + 1}$$

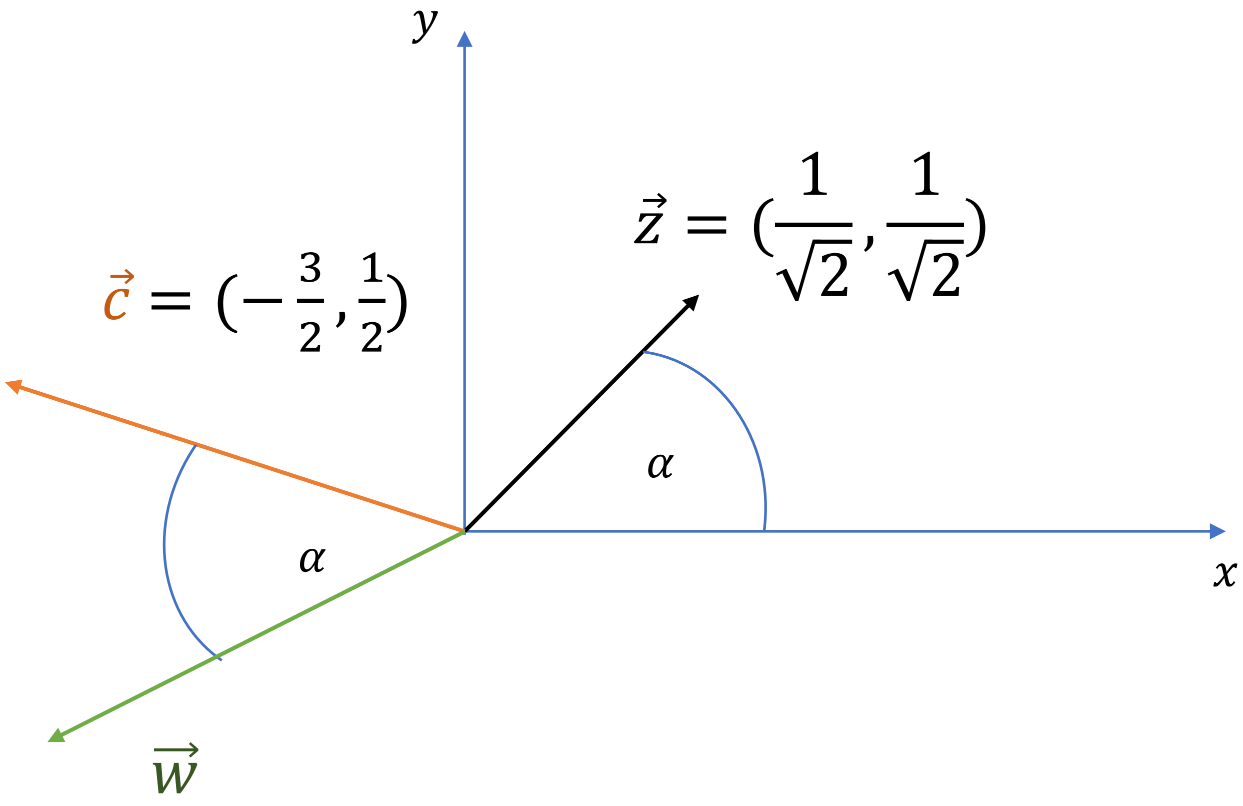
complex numbers

$i = \sqrt{-1}$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = z$$

$$-\frac{3}{2} + \frac{i}{2} = c$$

$$zc = \frac{-4 - 2i}{2\sqrt{2}}$$



$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = 1$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \frac{\sqrt{5}}{2}\sqrt{2}$$

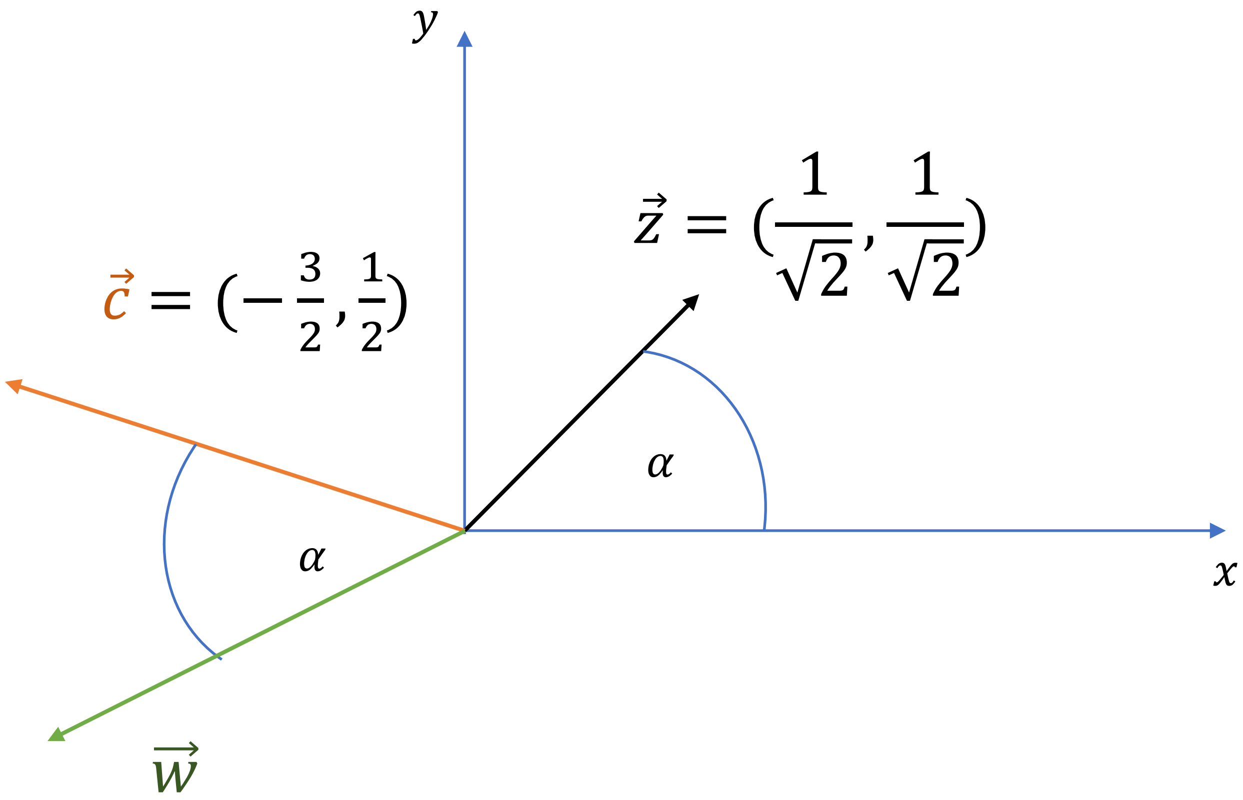
complex numbers

$$i = \sqrt{-1}$$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = z$$

$$-\frac{3}{2} + \frac{i}{2} = c$$

$$zc = \frac{-2}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$



$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = 1$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \frac{\sqrt{5}}{2}\sqrt{2}$$

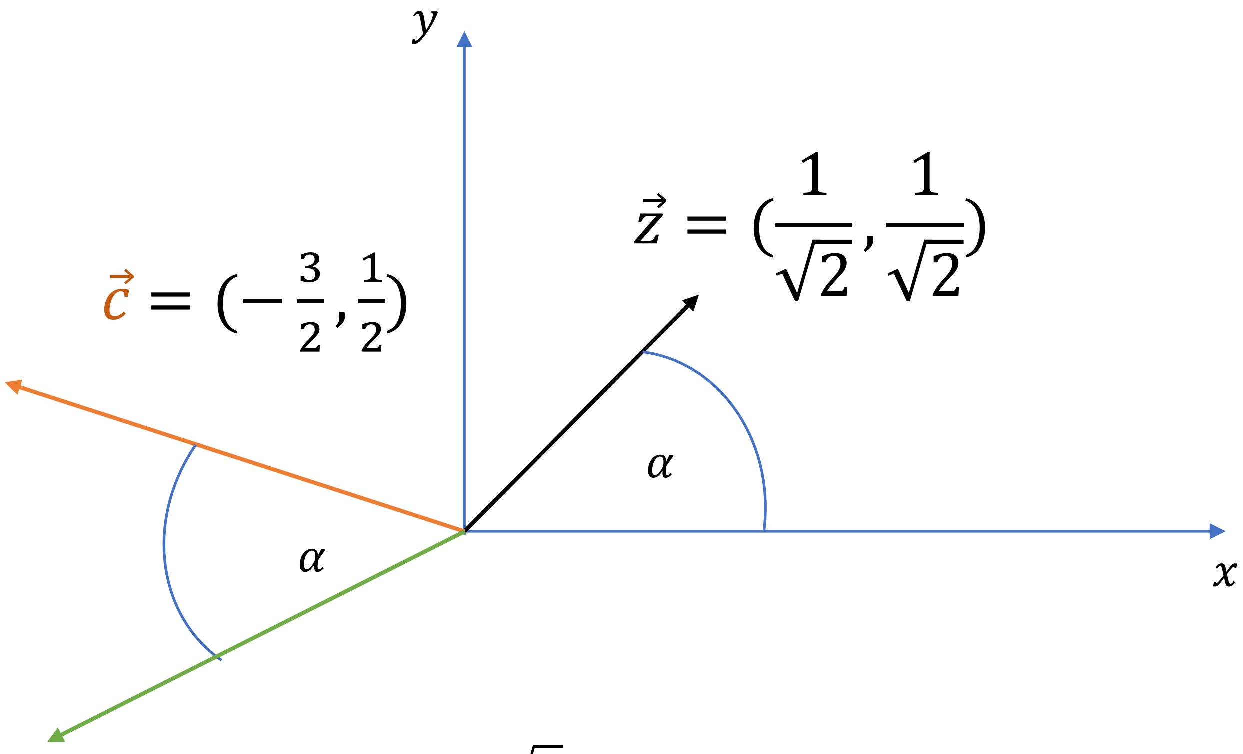
complex numbers

$i = \sqrt{-1}$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = z$$

$$-\frac{3}{2} + \frac{i}{2} = c$$

$$zc = -\sqrt{2} - \frac{\sqrt{2}}{2} i$$



$$\vec{c} = (-\tfrac{3}{2}, \tfrac{1}{2})$$

$$\vec{z} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$\vec{w} = (-\sqrt{2}, -\tfrac{\sqrt{2}}{2})$$

$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = 1$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \tfrac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{w}| = \sqrt{2 + \tfrac{2}{4}}$$

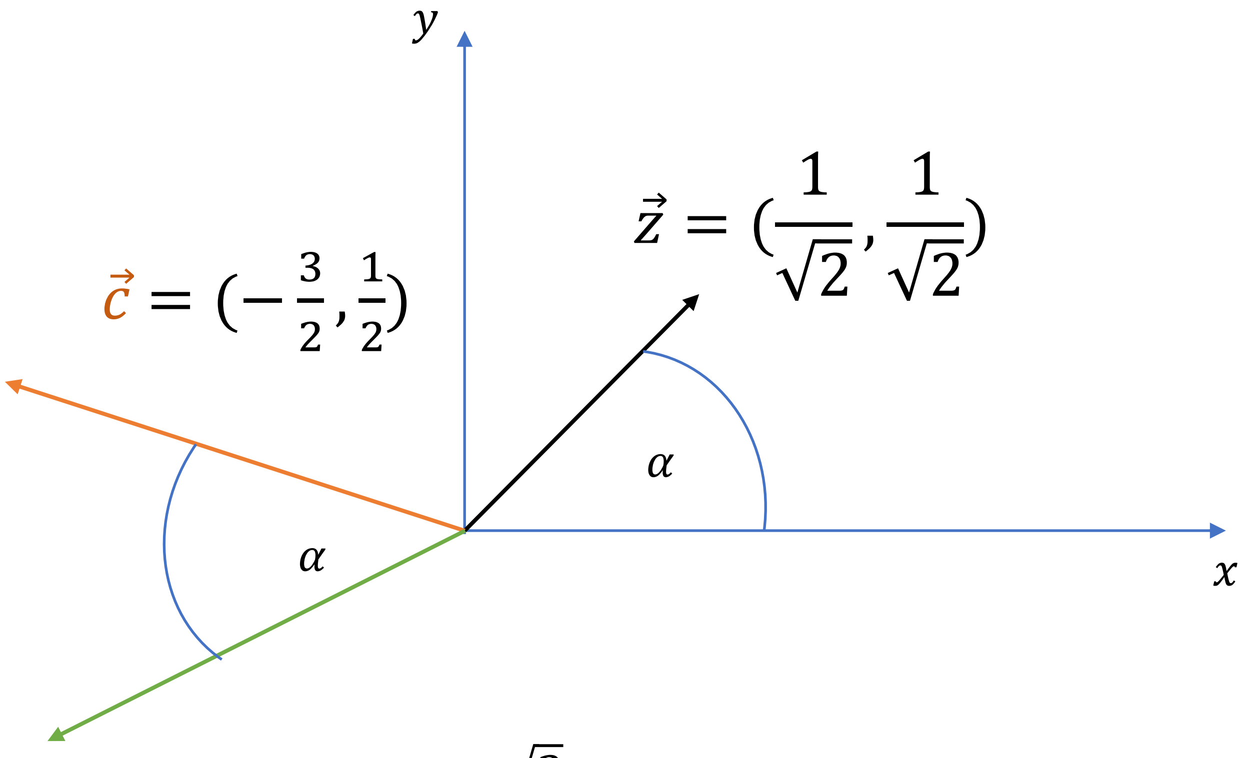
complex numbers

$$i = \sqrt{-1}$$

$$\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = z$$

$$-\frac{3}{2} + \frac{i}{2} = c$$

$$zc = -\sqrt{2} - \frac{\sqrt{2}}{2} i$$



$$\vec{c} = (-\tfrac{3}{2}, \tfrac{1}{2})$$

$$\vec{z} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$\vec{w} = (-\sqrt{2}, -\tfrac{\sqrt{2}}{2})$$

$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = 1$$

$$|\vec{c}| = \frac{\sqrt{10}}{2} = \frac{\sqrt{5}}{2}\sqrt{2}$$

$$|\vec{z}|\,|\vec{c}| = \frac{\sqrt{5}}{2}\sqrt{2}$$

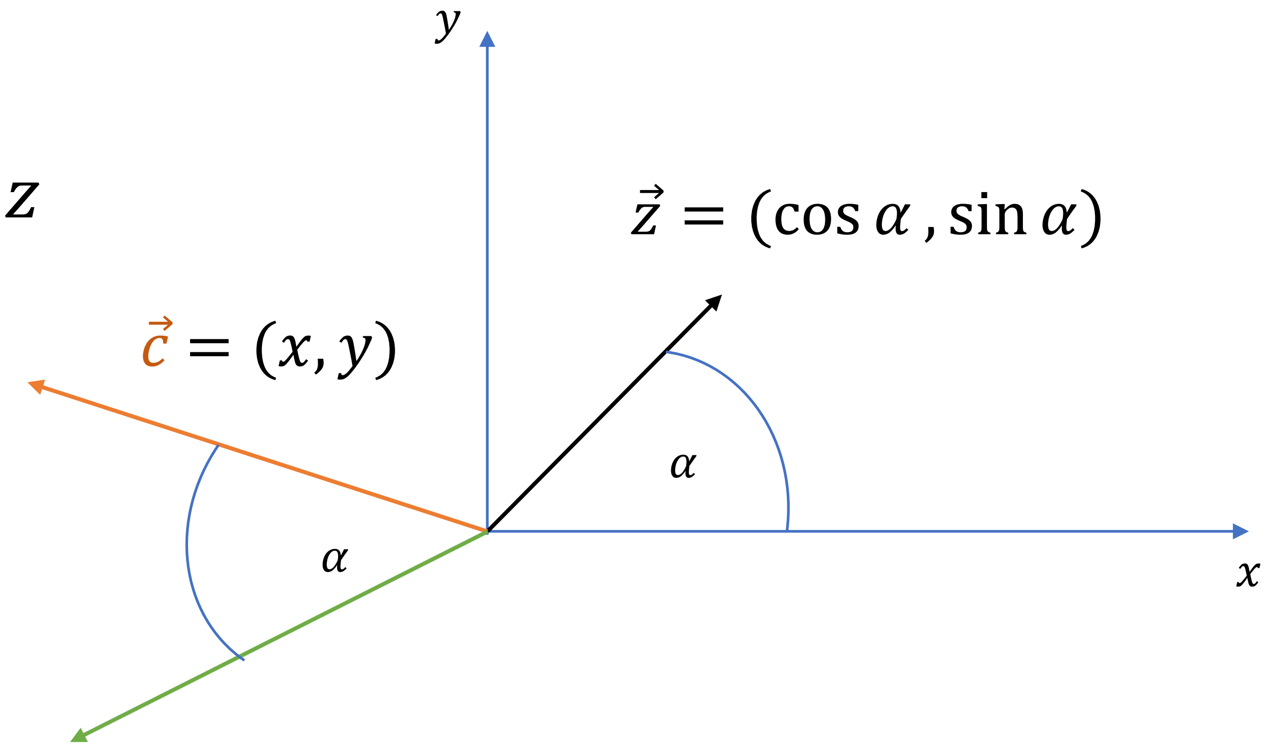
$$|\vec{w}| = \sqrt{\frac{10}{4}} = \frac{\sqrt{5}\sqrt{2}}{2}$$

complex numbers

$$i = \sqrt{-1}$$

$$\cos\alpha + (\sin\alpha)i = z$$

$$x + yi = c$$



$$\vec{w} = (x\cos\alpha - y\sin\alpha, x\sin\alpha + y\cos\alpha)$$

$$|\vec{w}| = |\vec{z}|\,|\vec{c}|$$

$$|\vec{z}| = 1$$

$$|\vec{c}| = \sqrt{x^2 + y^2}$$

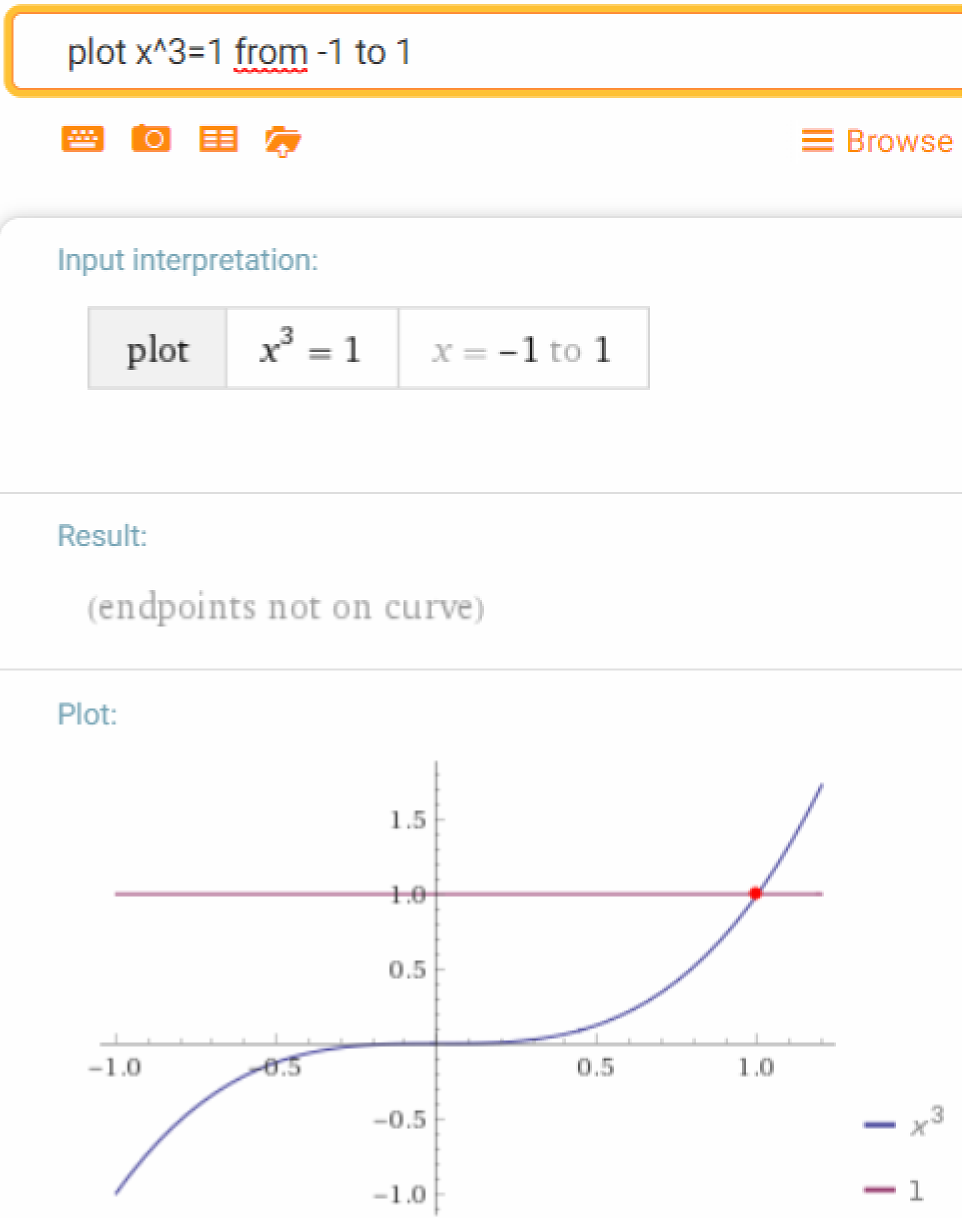
$$|\vec{z}|\,|\vec{c}| = \sqrt{x^2 + y^2}$$

$$|\vec{w}| = \sqrt{x^2 + y^2}$$

$$zc = x\cos\alpha - y\sin\alpha + (x\sin\alpha + y\cos\alpha)i$$

complex numbers

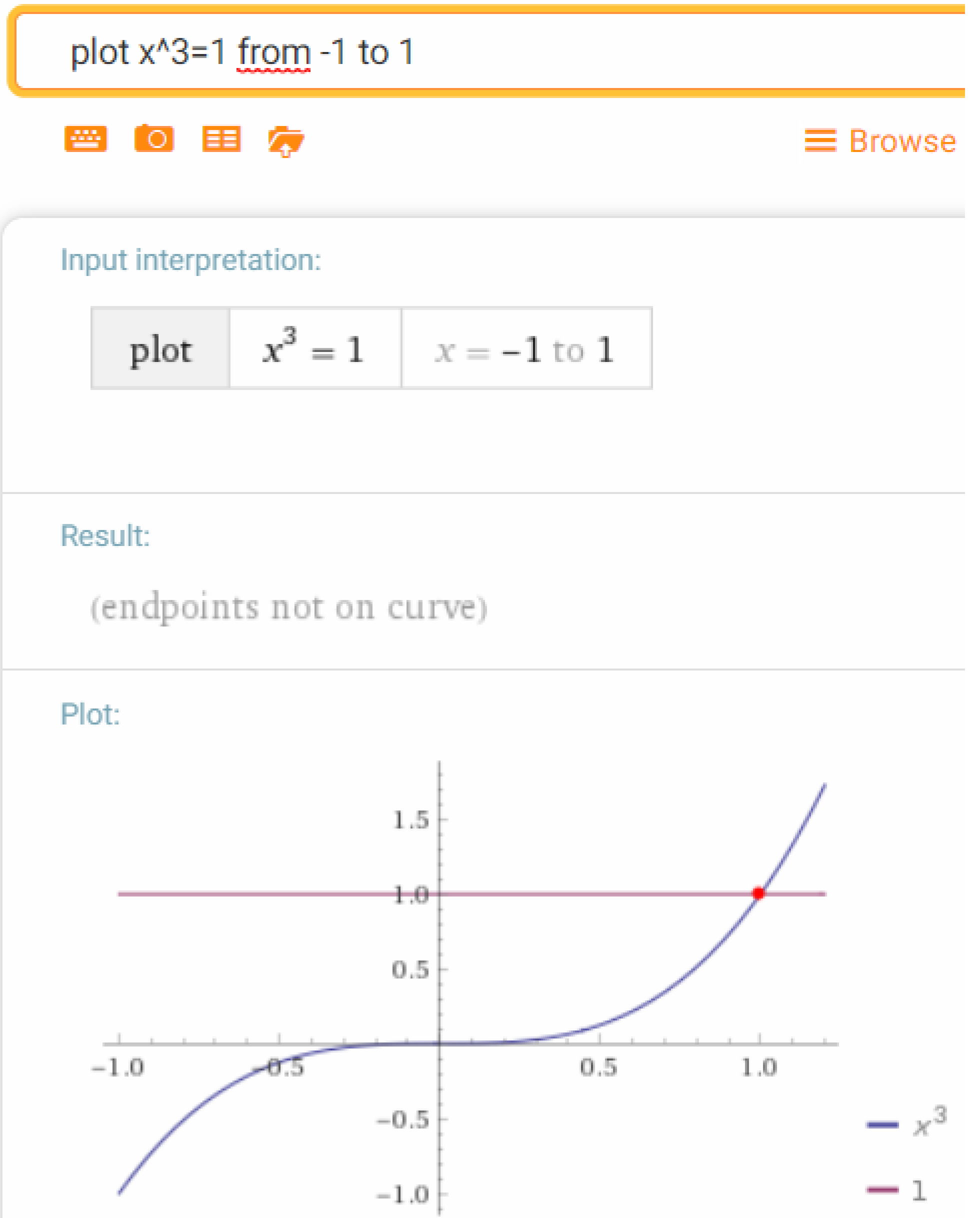
$x^3 = 1$



complex numbers

$x^3 = 1$

$x_1 = 1$



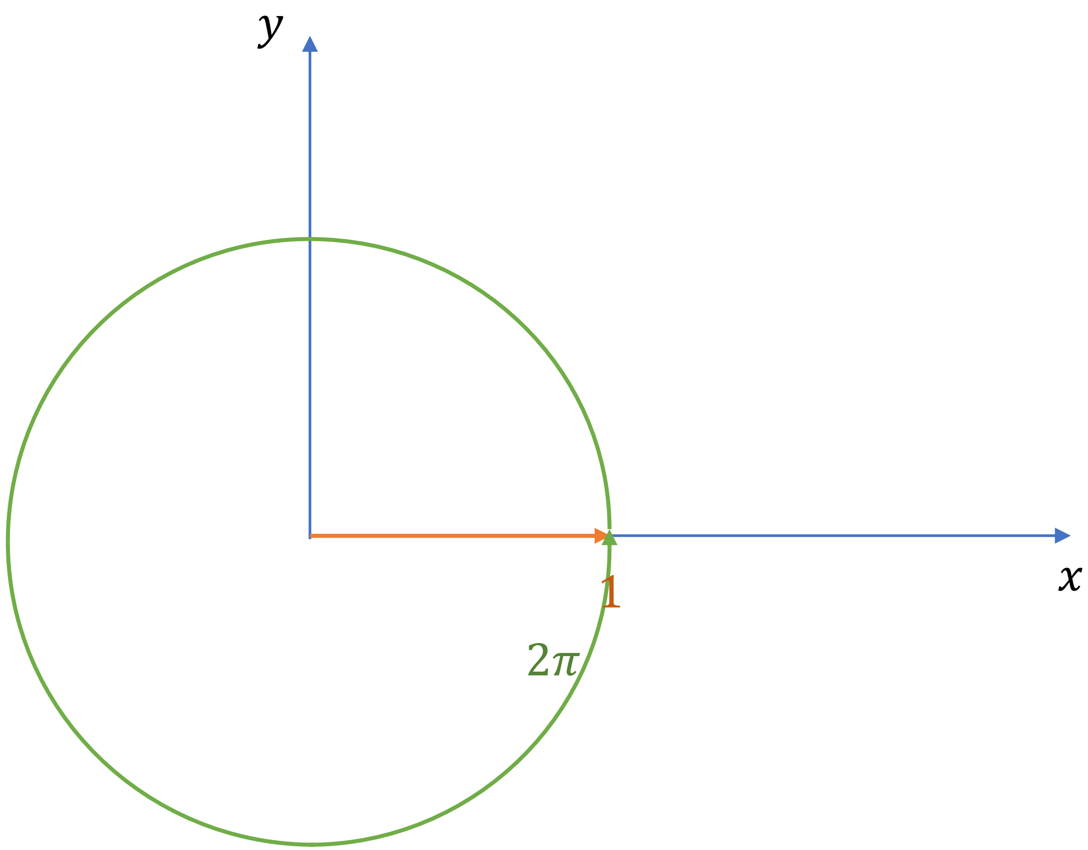
complex numbers

$x^3 = 1$

$x_1 = 1$

$x_2 = ?$

$x_3 = ?$



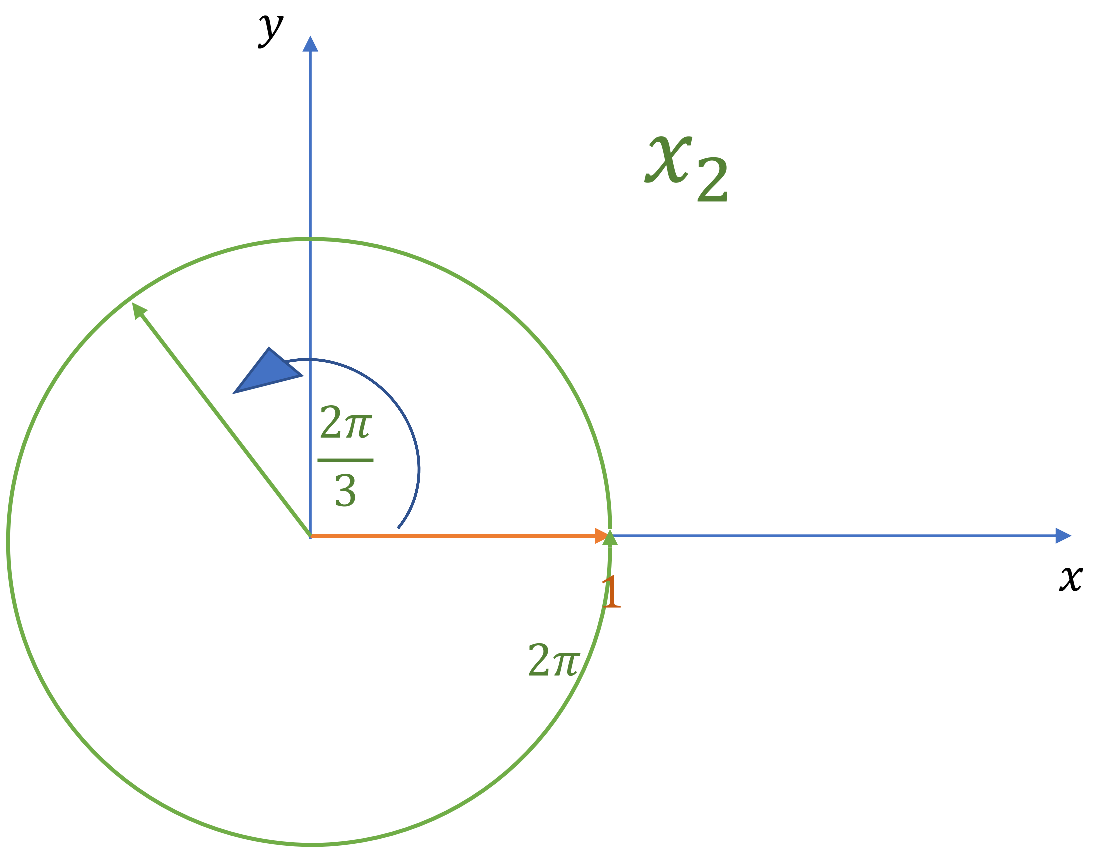
complex numbers

$x^3 = 1$

$x_1 = 1$

$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$

$x_3 = ?$



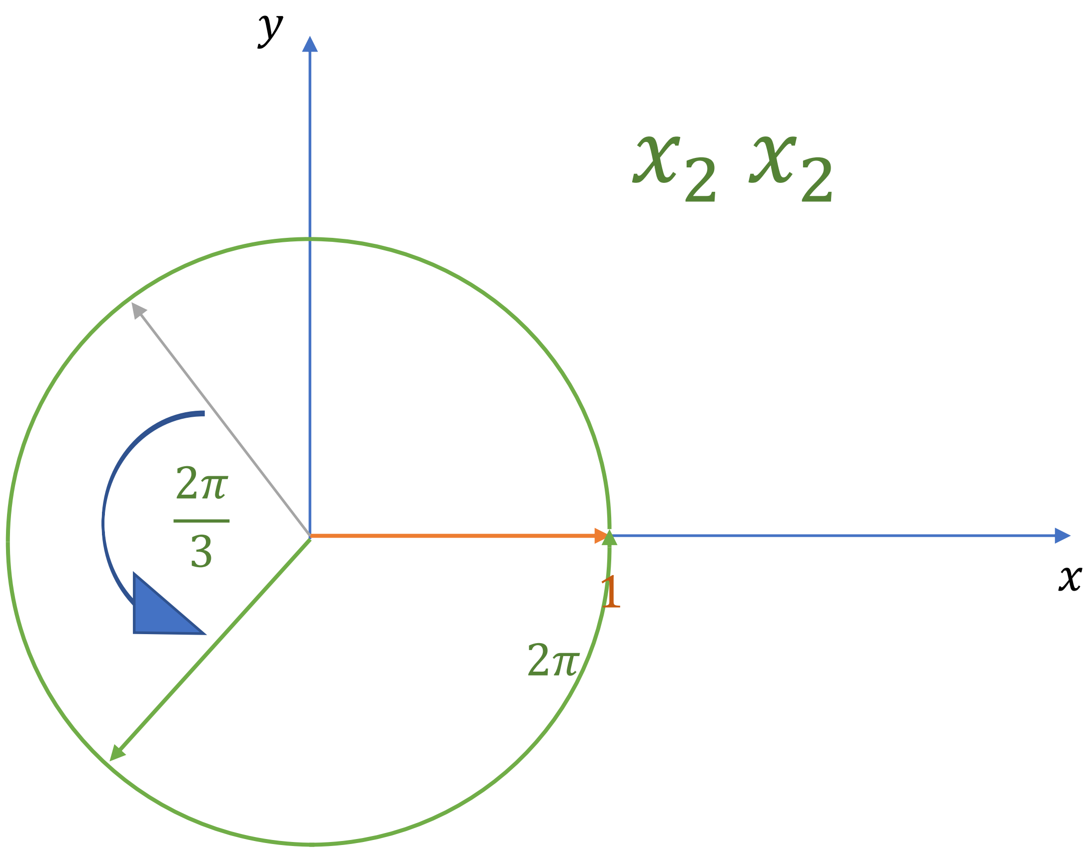
complex numbers

$x^3 = 1$

$x_1 = 1$

$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$

$x_3 = ?$



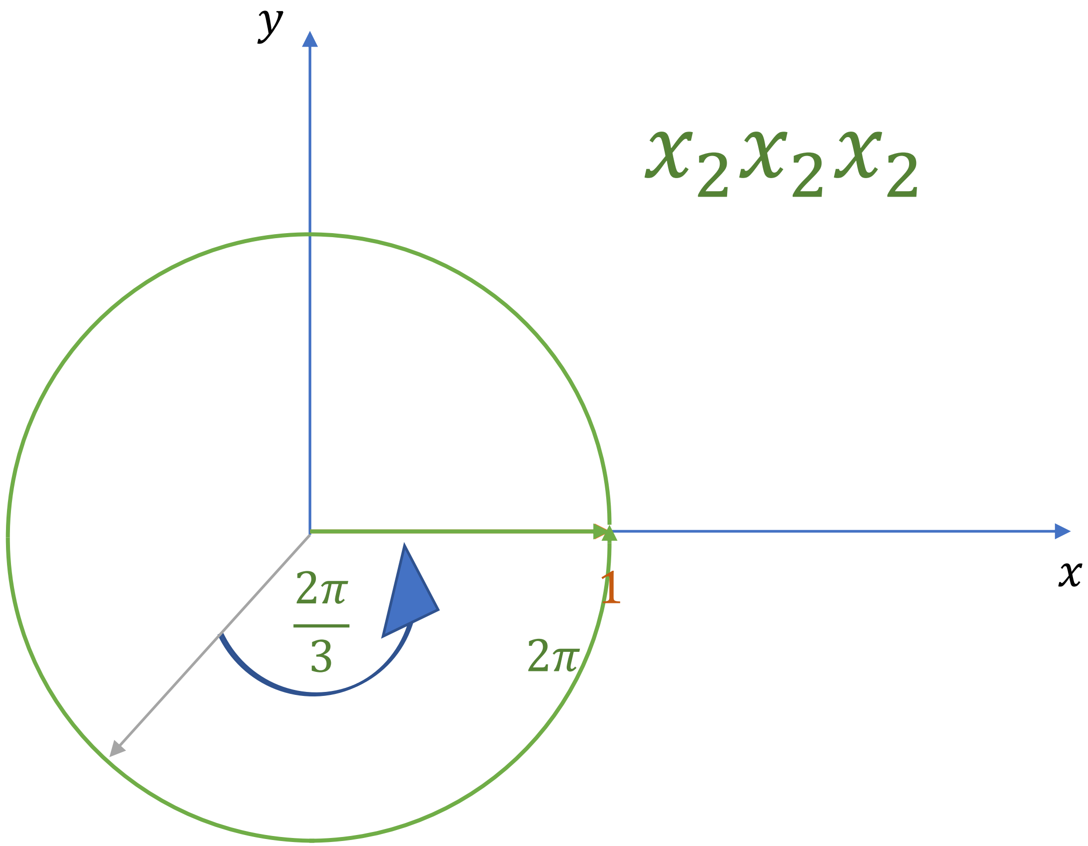
complex numbers

$x^3 = 1$

$x_1 = 1$

$$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$$

$x_3 = ?$



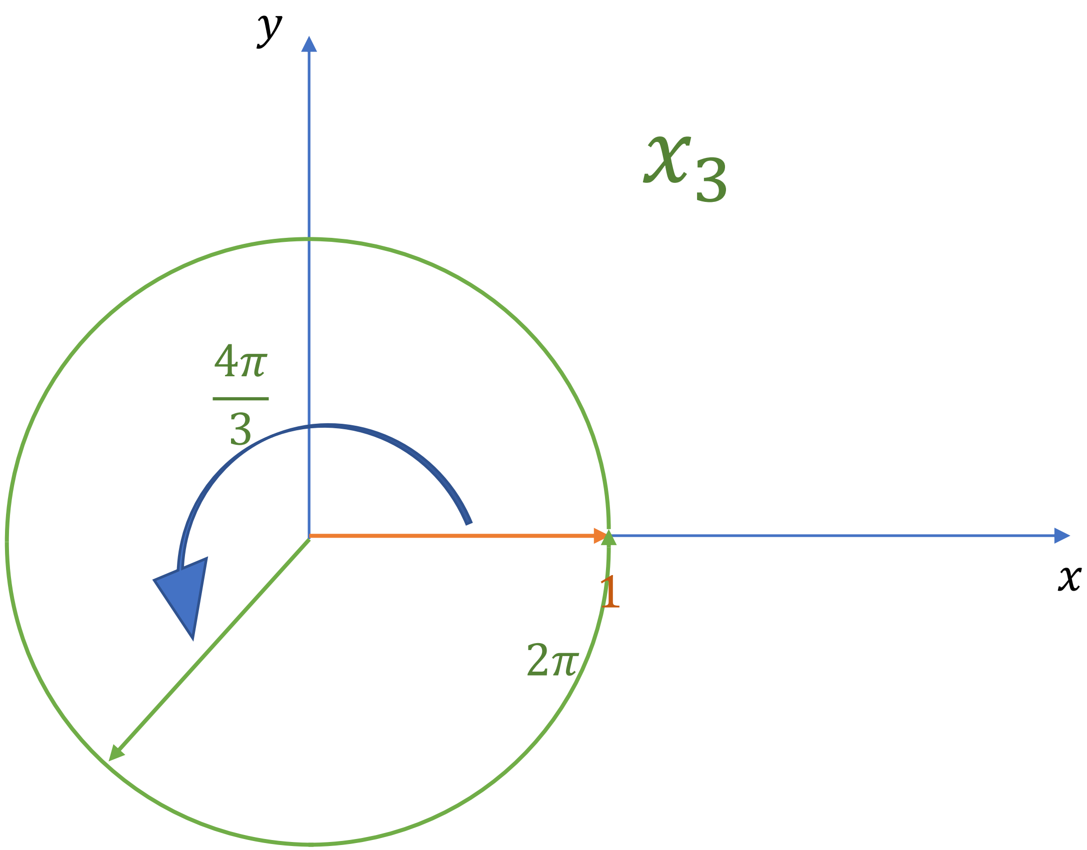
complex numbers

$x^3 = 1$

$x_1 = 1$

$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$

$x_3 = \cos\frac{4\pi}{3} + \mathrm{i}\sin\frac{4\pi}{3}$



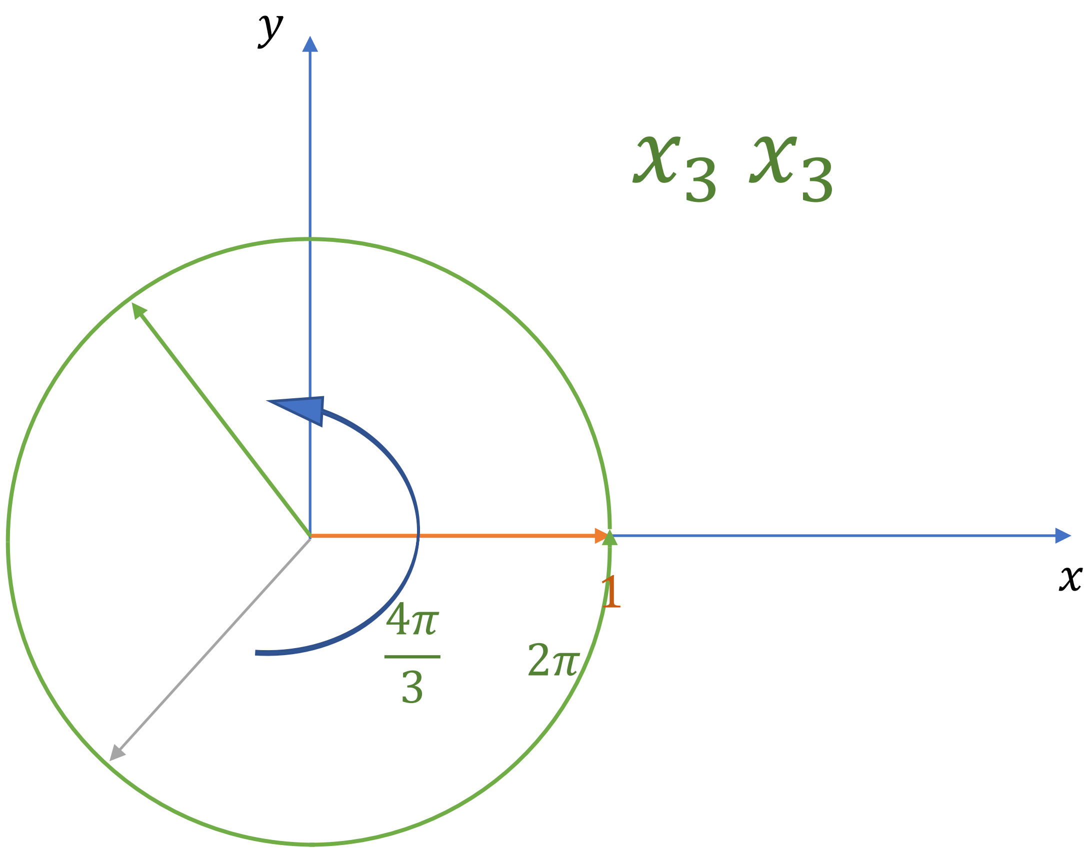
complex numbers

$x^3 = 1$

$x_1 = 1$

$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$

$x_3 = \cos\frac{4\pi}{3} + \mathrm{i}\sin\frac{4\pi}{3}$



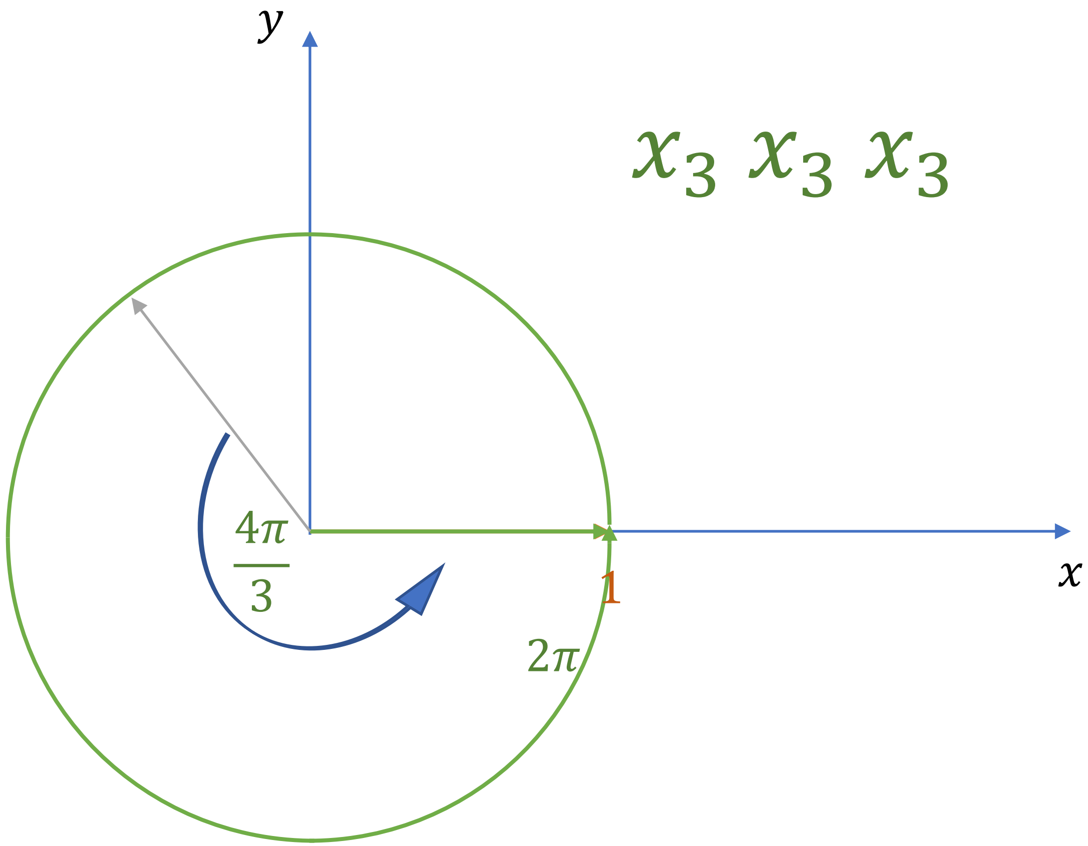
complex numbers

$x^3 = 1$

$x_1 = 1$

$$x_2 = \cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}$$

$$x_3 = \cos\frac{4\pi}{3} + \mathrm{i}\sin\frac{4\pi}{3}$$



quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$

$$i^2jk = -i$$

$$-jk = -i$$

$$jk = i$$

quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$
$$i^2jk = -i$$
$$-jk = -i$$
$$jk = i$$
$$j^2k = ji$$
$$-k = ji$$

quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$

$$i^2jk = -i$$

$$-jk = -i$$

$$jk = i$$

$$j^2k = ji$$

$$-k = ji$$

$$-ki = ji^2$$

$$ki = j$$

quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$
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$$-jk = -i$$
$$jk = i$$
$$j^2k = ji$$
$$-k = ji$$
$$-ki = ji^2$$
$$ki = j$$

$$kj = -i$$
$$ij = k$$
$$ik = -j$$

quaternions

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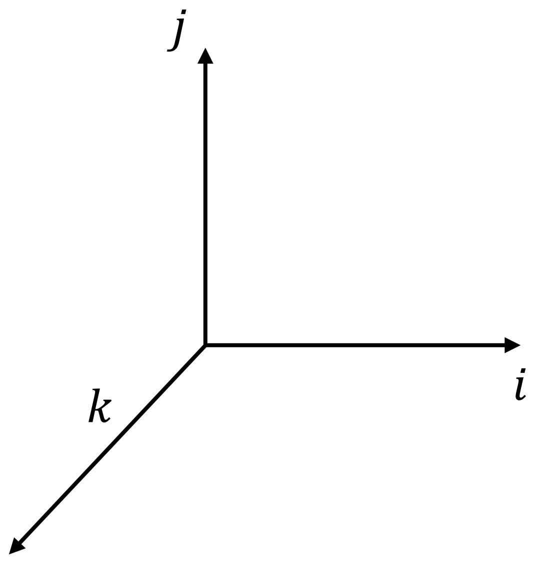
$$ki = j$$

$$kj = -i$$
$$ij = k$$
$$ik = -j$$



quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$ijk = -1$

$i^2jk = -i$

$-jk = -i$

$jk = i$

$j^2k = ji$

$-k = ji$

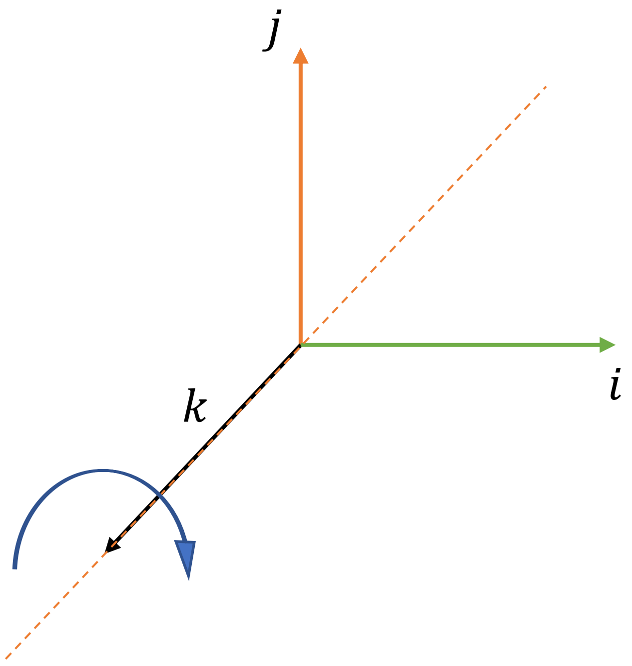
$-ki = ji^2$

$ki = j$

$kj = -i$

$ij = k$

$ik = -j$



quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$

$$i^2jk = -i$$

$$-jk = -i$$

$$jk = i$$

$$j^2k = ji$$

$$-k = ji$$

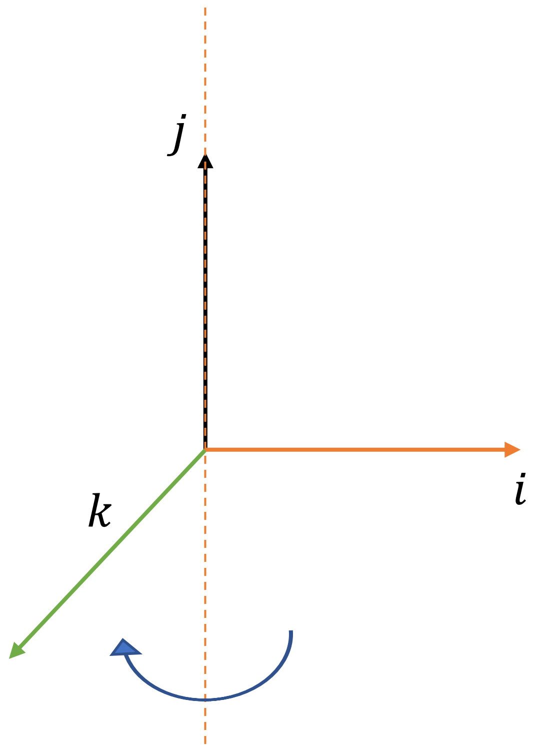
$$-ki = ji^2$$

$$ki = j$$

$$kj = -i$$

$$ij = k$$

$$ik = -j$$



quaternions

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ijk = -1$$
$$i^2jk = -i$$
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$$jk = i$$
$$j^2k = ji$$
$$-k = ji$$
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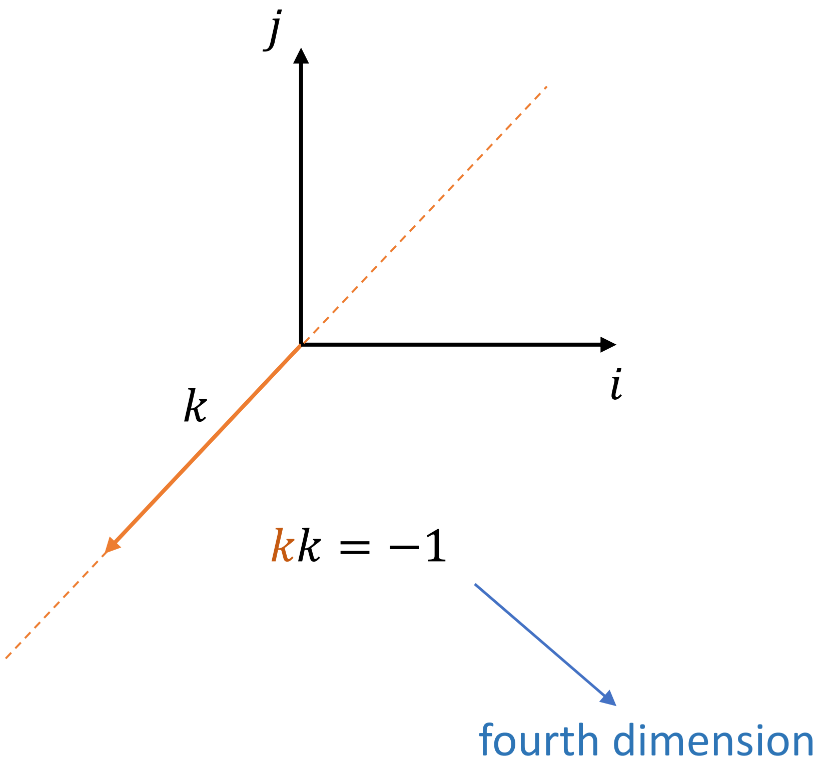

https://www.youtube.com/watch?v=d4EgbgTm0Bg