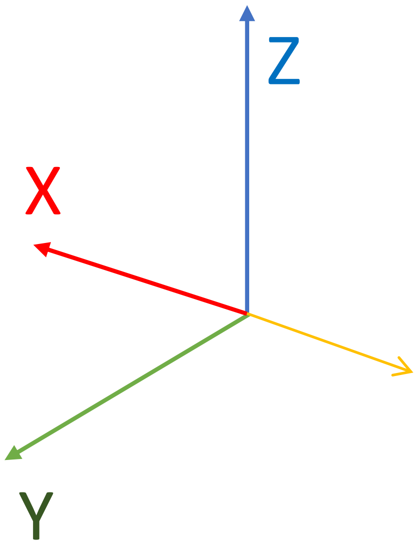
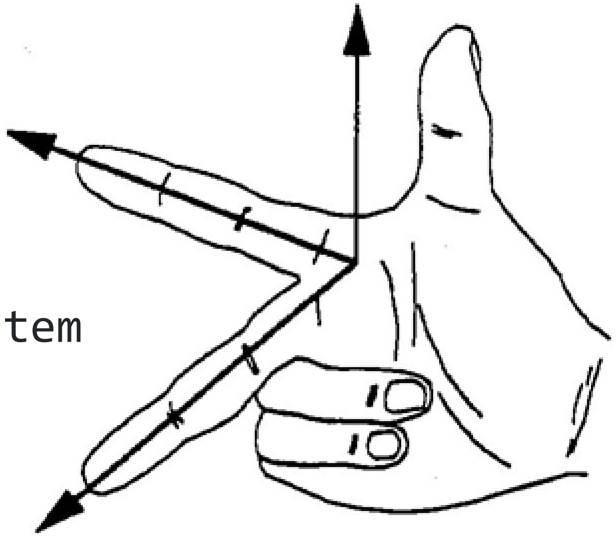
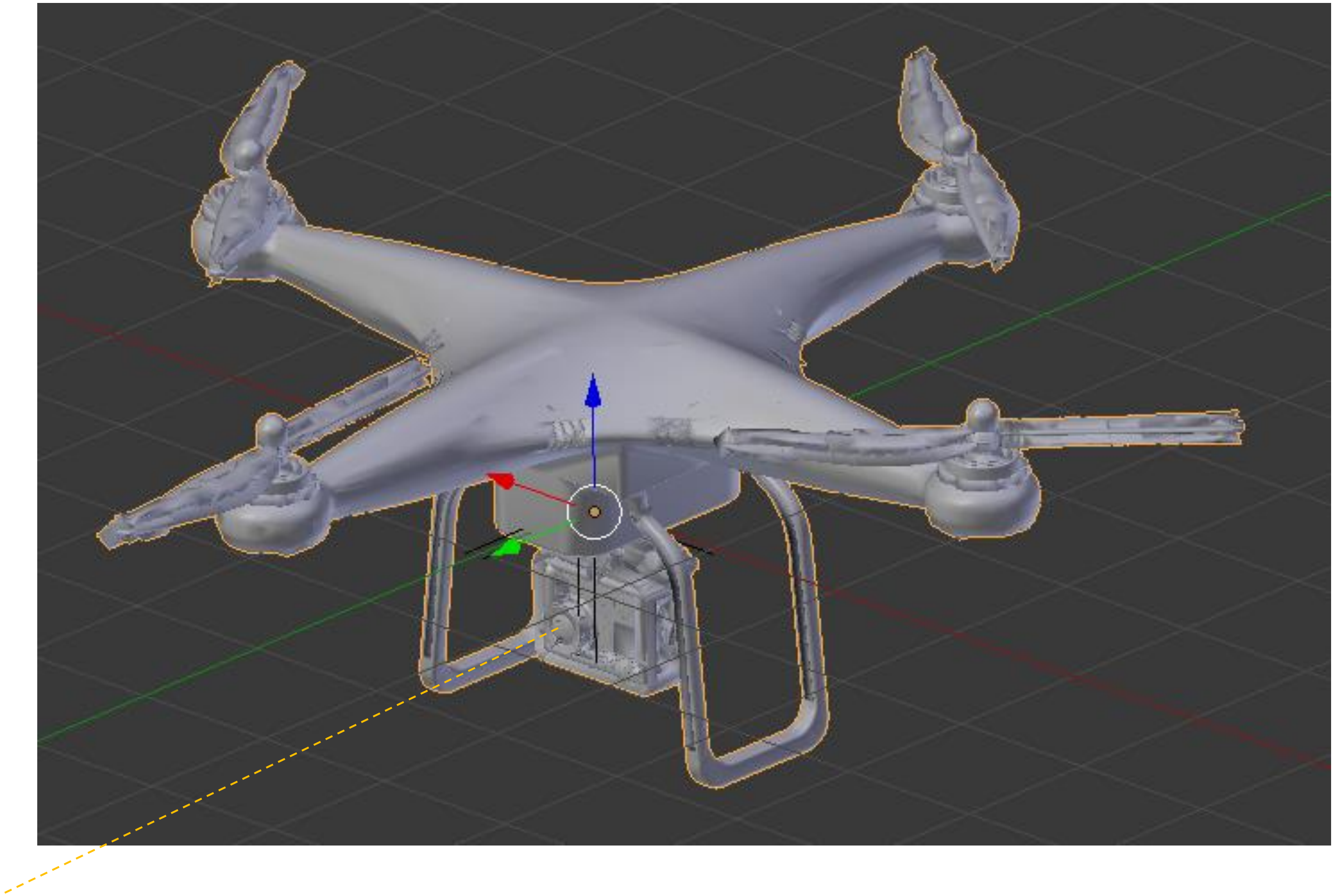
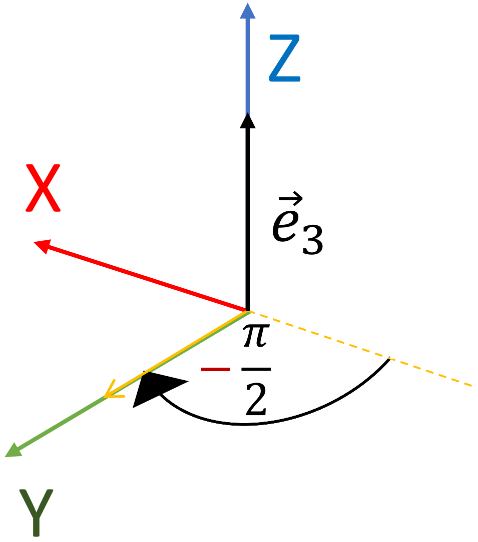


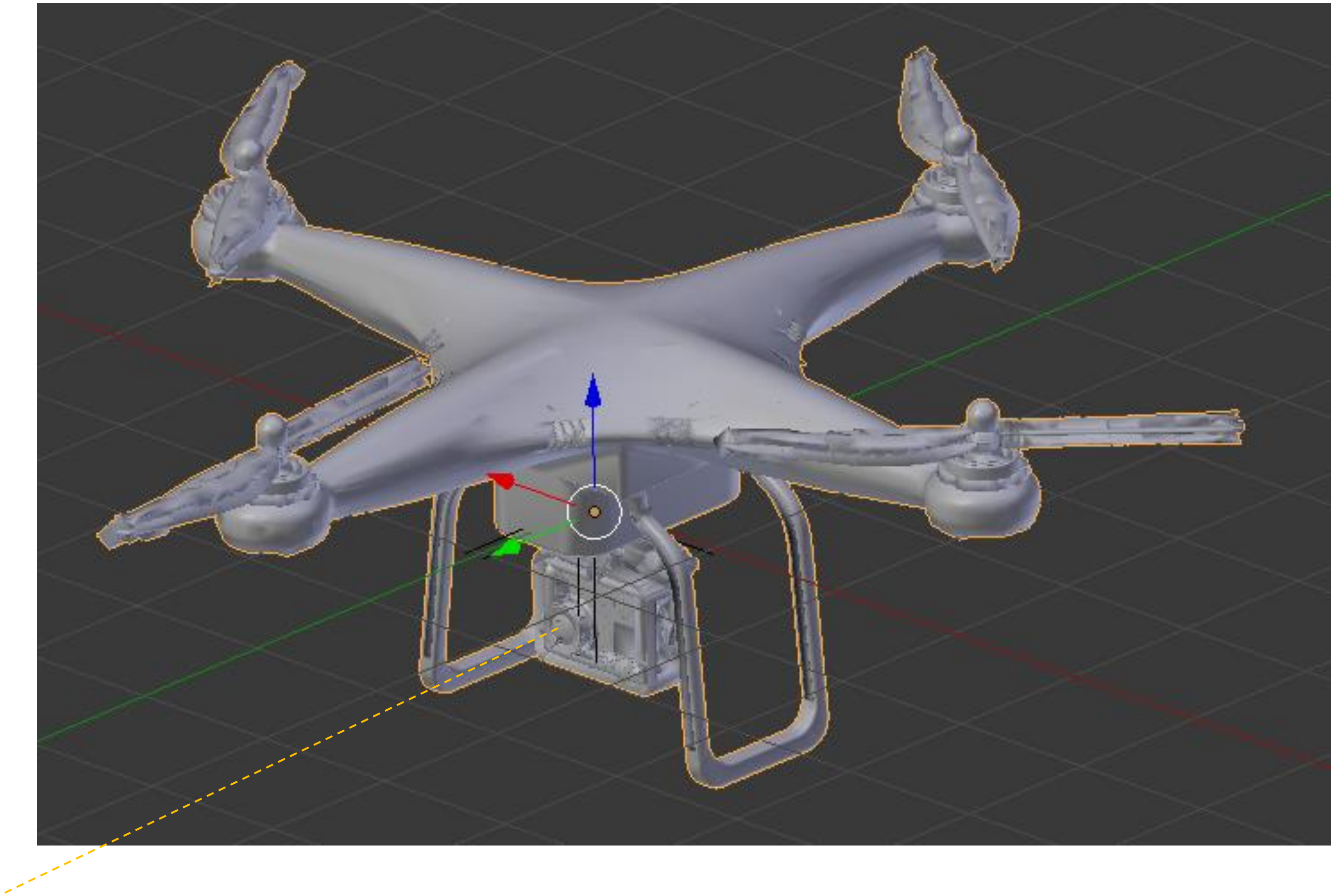
right-handed coordinate system

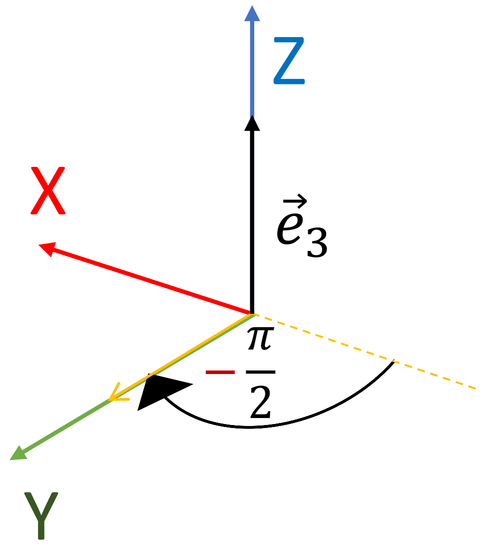




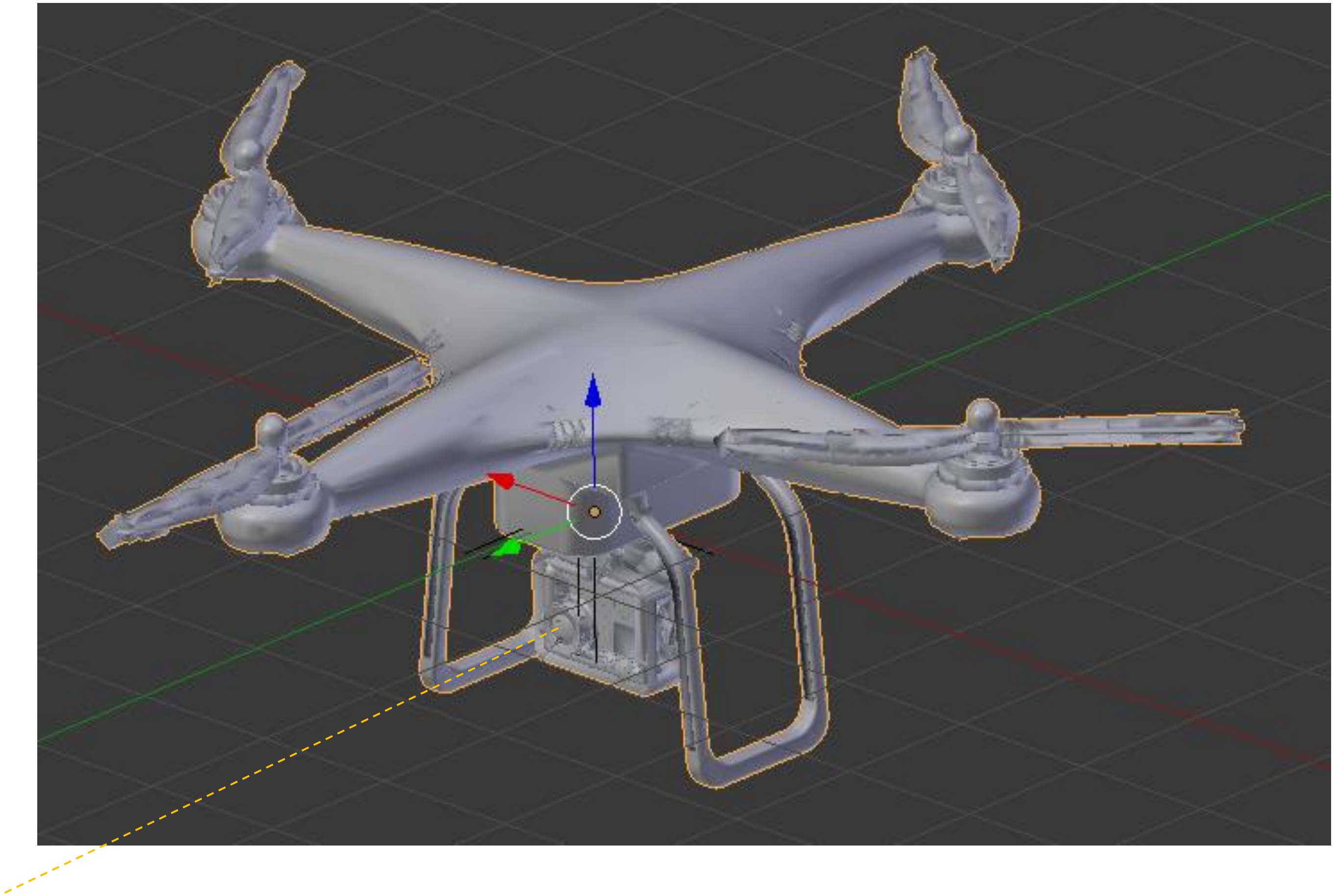


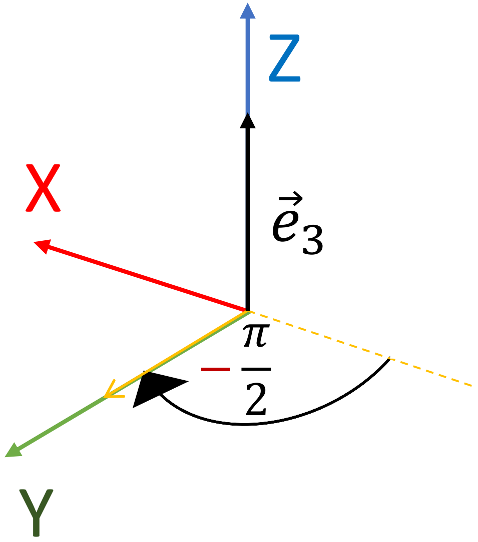
$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



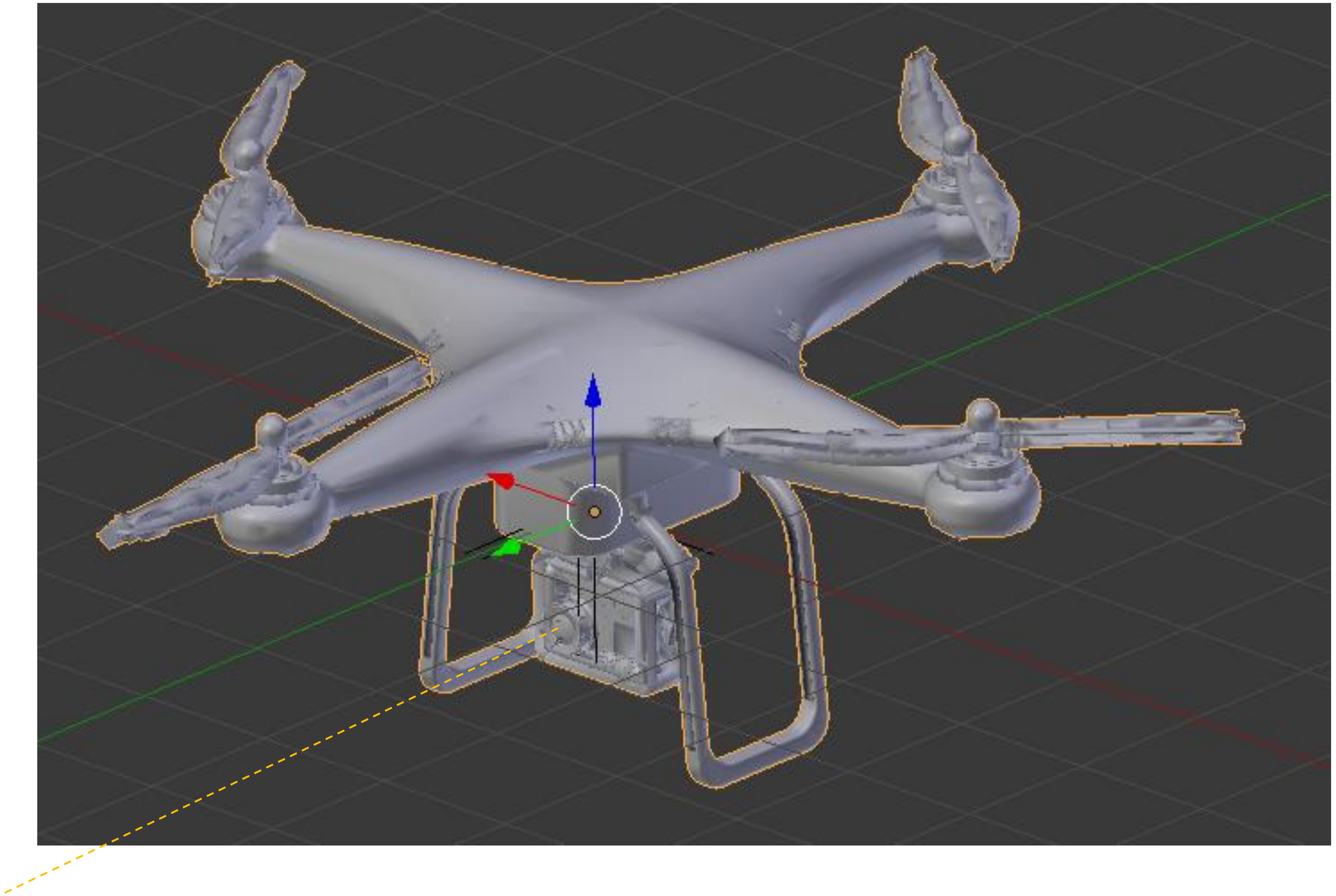


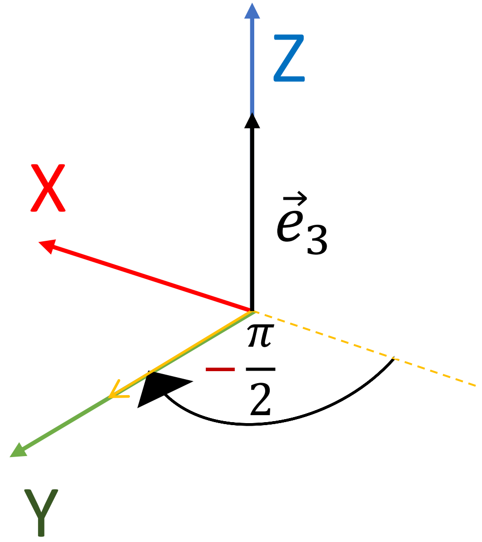
$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \alpha = -\frac{\pi}{2}$$





$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \alpha = -\frac{\pi}{2} \qquad q_0 = \cos\frac{\alpha}{2} + (0\mathrm{i} + 0j + 1k)\sin\frac{\alpha}{2}$$

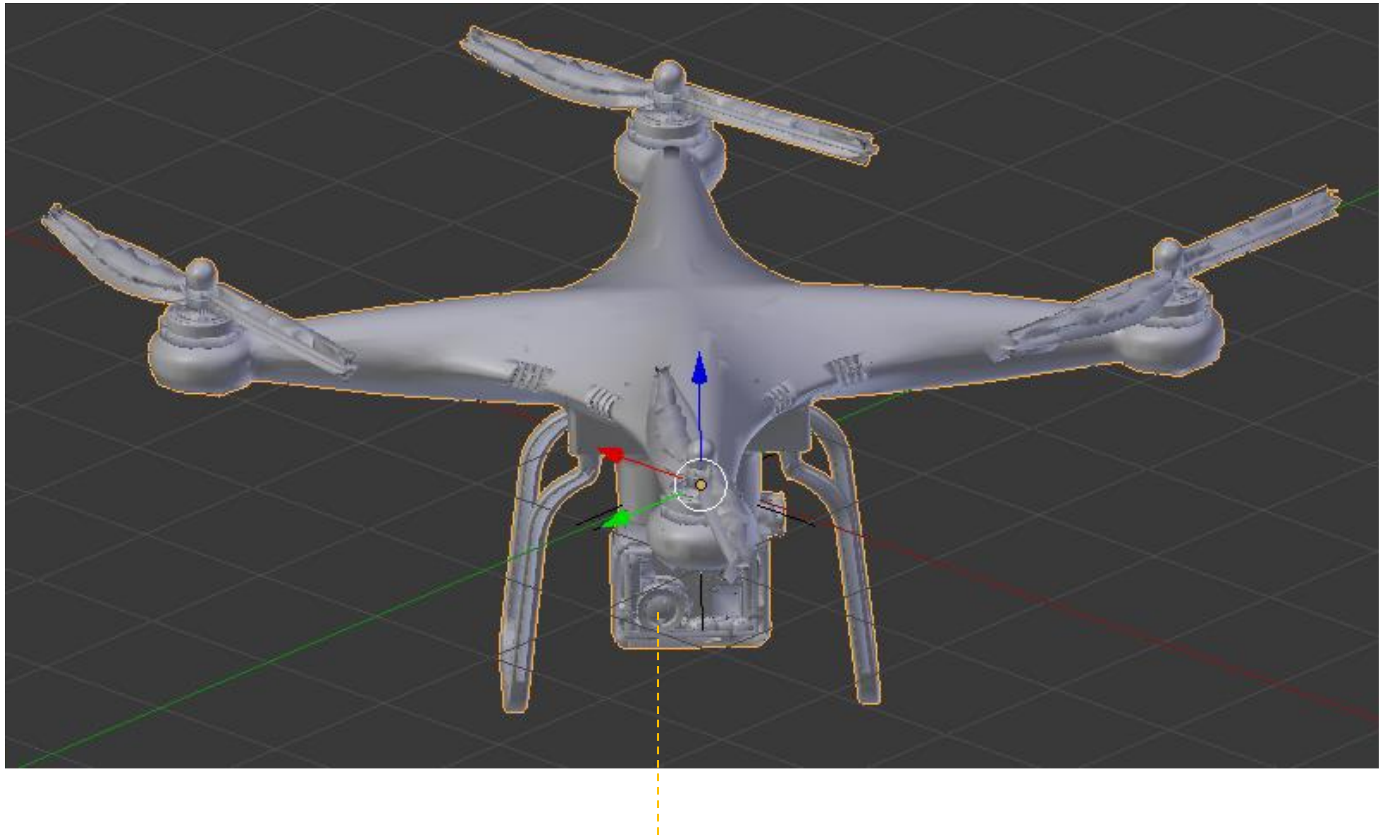


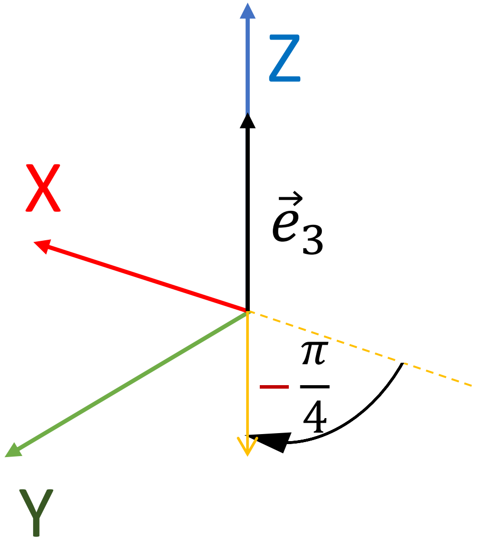


$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

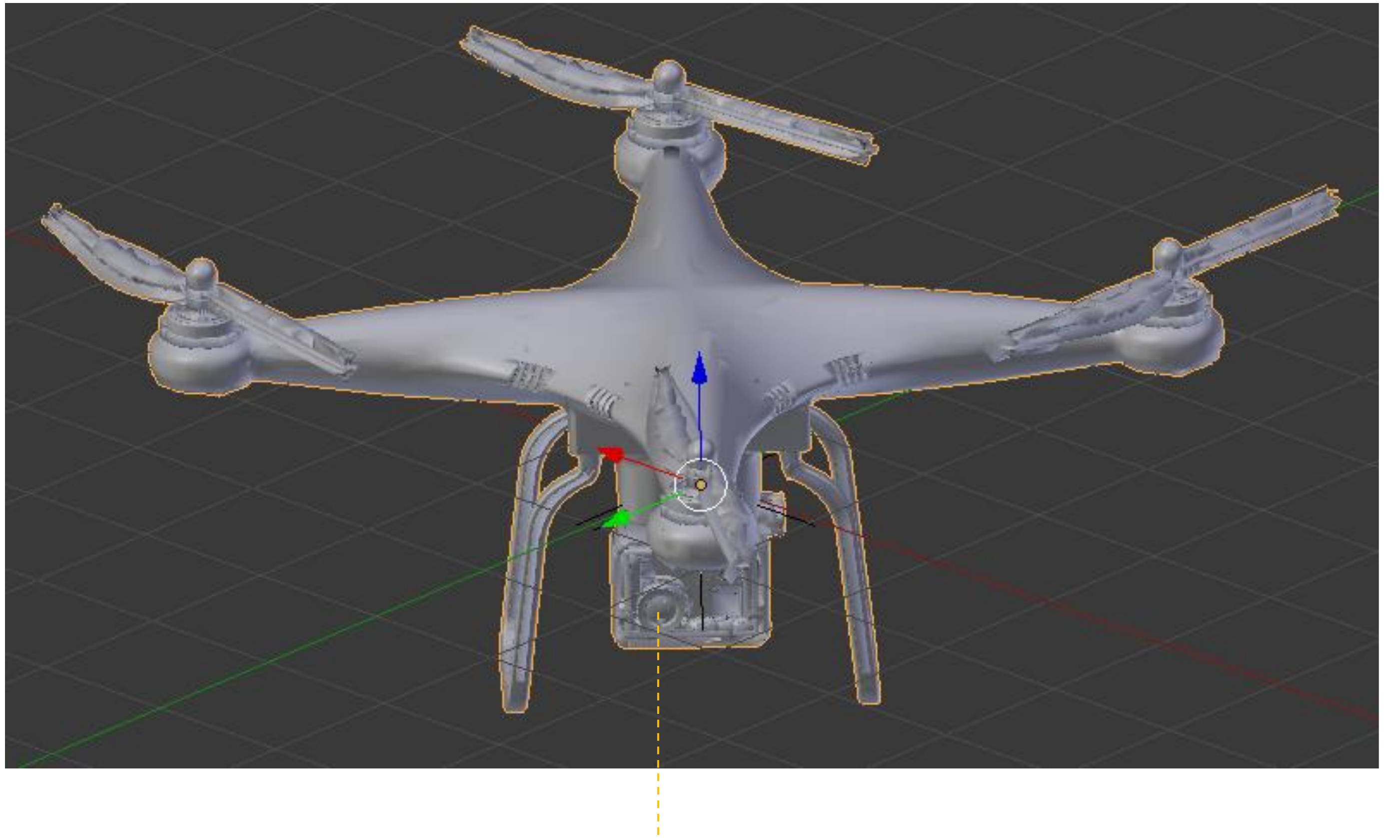
$$\alpha = -\frac{\pi}{2}$$

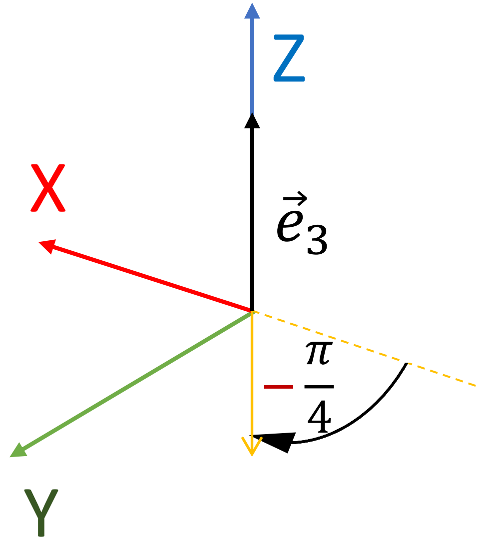
$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$



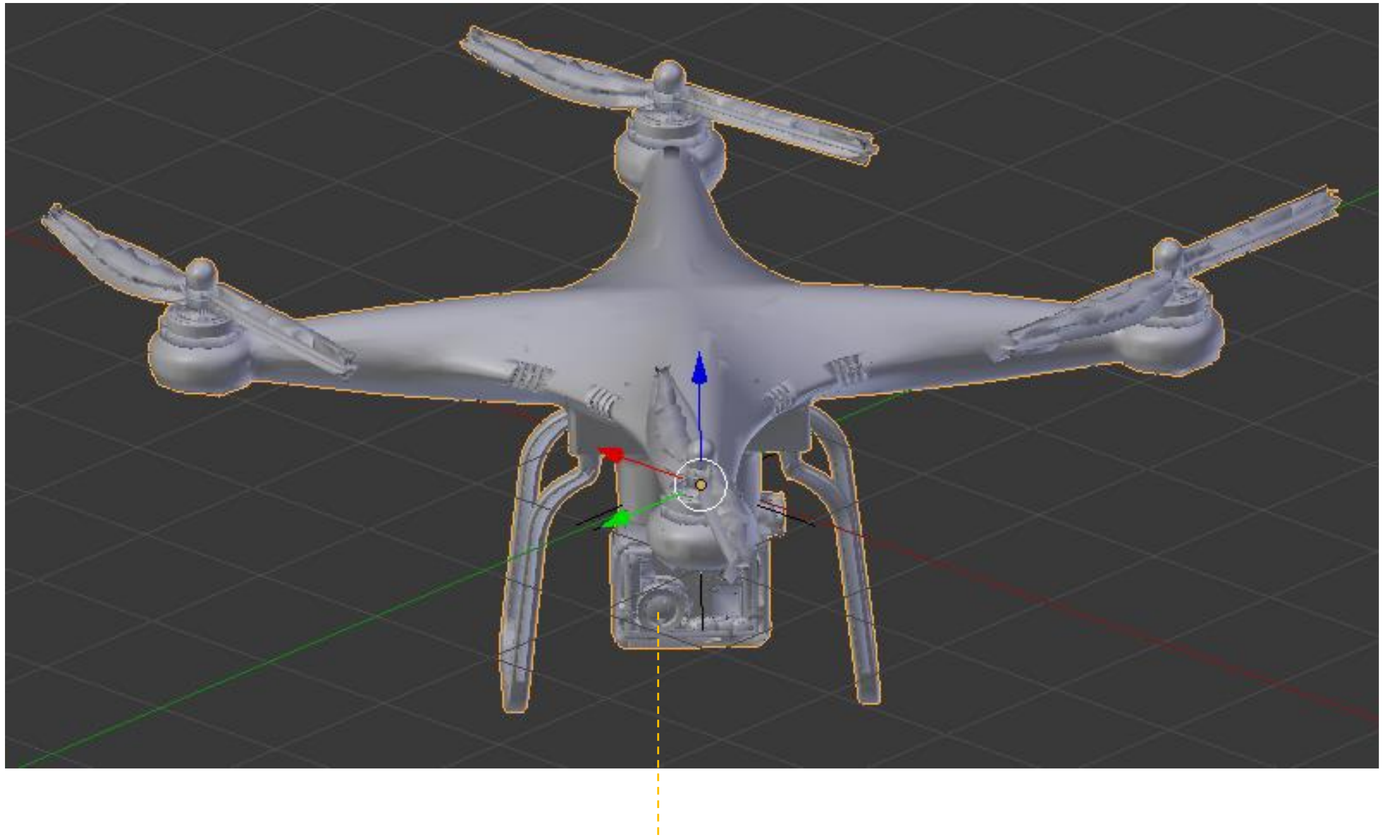


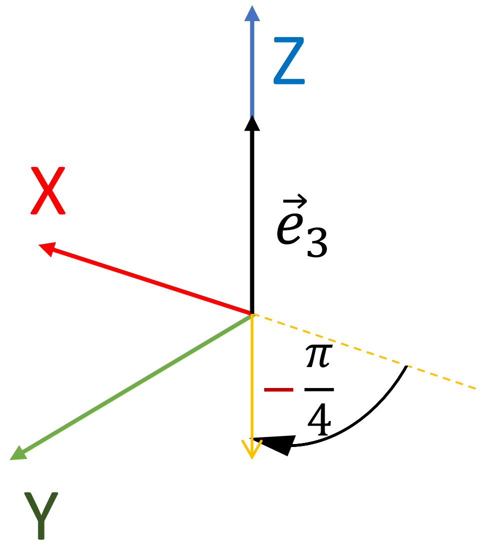
$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$



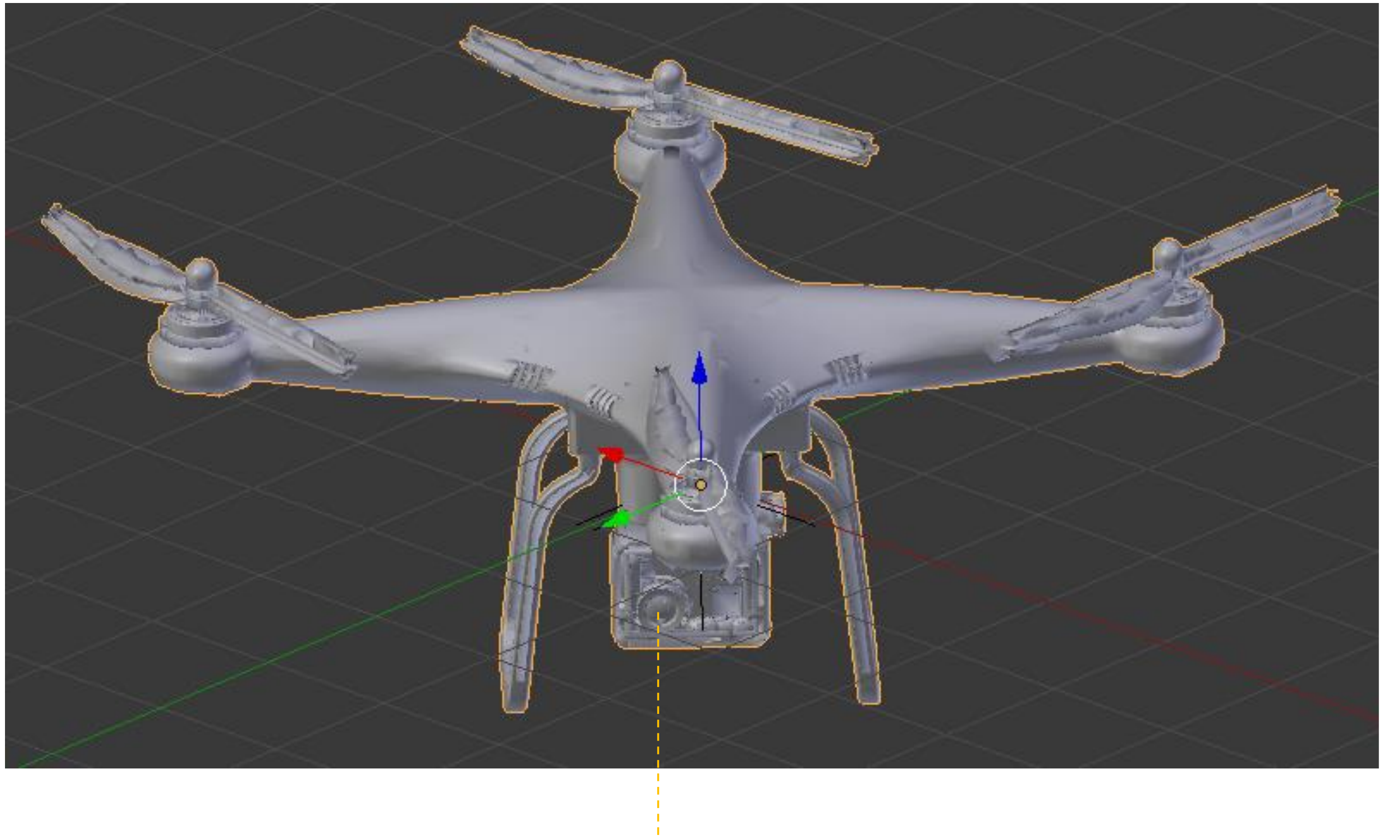


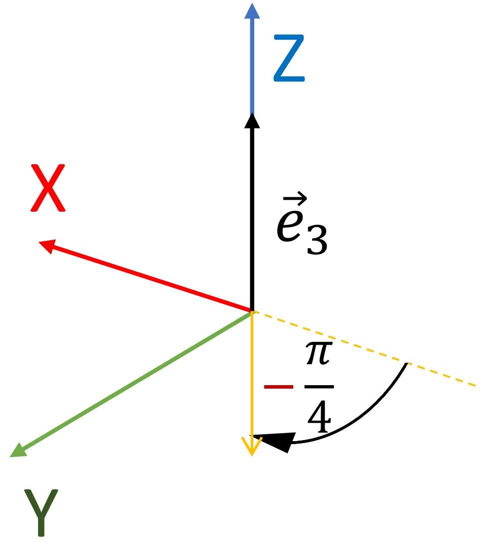
$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \alpha = -\frac{\pi}{4}$$





$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \alpha = -\frac{\pi}{4} \qquad q_1 = \cos\frac{\alpha}{2} + (0\mathrm{i} + 0j + 1k)\sin\frac{\alpha}{2}$$





$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \alpha = -\frac{\pi}{4} \qquad q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\alpha = -\frac{\pi}{2}$$

$$\overrightarrow{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\alpha = -\frac{\pi}{4}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix}$$

$$\overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \quad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} \cdot \overrightarrow{q_1} = ?$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

dot product (cos(-pi/4),0,0,sin(-pi/4)) (cos(-pi/8),0,0,sin(-pi/8))

Input interpretation:

$$\left(\cos\left(-\frac{\pi}{4}\right), 0, 0, \sin\left(-\frac{\pi}{4}\right)\right).\left(\cos\left(-\frac{\pi}{8}\right), 0, 0, \sin\left(-\frac{\pi}{8}\right)\right)$$

Result:

$$\frac{\sin\left(\frac{\pi}{8}\right)}{\sqrt{2}} + \frac{\cos\left(\frac{\pi}{8}\right)}{\sqrt{2}}$$

Decimal approximation:

0.92387953251128675612818318939678828682241662586364248611 5...

Alternate forms:

$$\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\vec{q_0} \cdot \vec{q_1} = ?$$

$$\vec{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \quad \vec{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \quad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} \cdot \overrightarrow{q_1} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \quad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} \cdot \overrightarrow{q_1} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\cos(\theta) = \frac{\sqrt{2+\sqrt{2}}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)$$

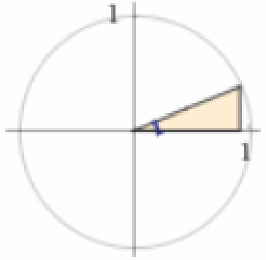
Input:

$$\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)$$

Conversion from radians to degrees:

22.5°

Reference triangle for angle 0.3927 radians:



width	$\cos\left(\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)\right)=\frac{\sqrt{2+\sqrt{2}}}{2}\approx 0.92388$
height	$\sin\left(\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right)\right)=\sqrt{1+\frac{1}{4}\left(-2-\sqrt{2}\right)}\approx 0.382683$

Alternate forms:

$$\frac{\pi}{8}$$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \quad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} \cdot \overrightarrow{q_1} = \frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\cos(\theta) = \frac{\sqrt{2+\sqrt{2}}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right) = \frac{\pi}{8}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad \theta = \frac{\pi}{8} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp(\overrightarrow{q_0}, \overrightarrow{q_1}, t) = \frac{\sin\big((1-t)\theta\big)}{\sin\theta}\overrightarrow{q_0} + \frac{\sin(t\theta)}{\sin\theta}\overrightarrow{q_1}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad \theta = \frac{\pi}{8} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp(\overrightarrow{q_0}, \overrightarrow{q_1}, t) = \frac{\sin\big((1-t)\theta\big)}{\sin\theta}\overrightarrow{q_1} + \frac{\sin(t\theta)}{\sin\theta}\overrightarrow{q_2}$$

$$t = \frac{1}{2};\ \frac{\sin\left(\left(1-\frac{1}{2}\right)\theta\right)}{\sin\theta} = \frac{\sin\frac{\theta}{2}}{\sin\theta} = \frac{\sin\frac{\pi}{16}}{\sin\frac{\pi}{8}} = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad \theta = \frac{\pi}{8} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp(\overrightarrow{q_0}, \overrightarrow{q_1}, t) = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}\,\overrightarrow{q_1} + \frac{\sin(t\theta)}{\sin\theta}\,\overrightarrow{q_2}$$

$$t = \frac{1}{2};\ \frac{\sin\left(\left(1-\frac{1}{2}\right)\theta\right)}{\sin\theta} = \frac{\sin\frac{\theta}{2}}{\sin\theta} = \frac{\sin\frac{\pi}{16}}{\sin\frac{\pi}{8}} = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad \theta = \frac{\pi}{8} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp(\overrightarrow{q_0}, \overrightarrow{q_1}, t) = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}\,\overrightarrow{q_1} + \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}\,\overrightarrow{q_2}$$

$$t = \frac{1}{2};\ \frac{\sin\left(\frac{1}{2}\theta\right)}{\sin\theta} = \frac{\sin\frac{\theta}{2}}{\sin\theta} = \frac{\sin\frac{\pi}{16}}{\sin\frac{\pi}{8}} = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp\left(\overrightarrow{q_0}, \overrightarrow{q_1}, \frac{1}{2}\right) = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}} \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} + \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}} \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp\left(\overrightarrow{q_0}, \overrightarrow{q_1}, \frac{1}{2}\right) = \sqrt{\frac{2-\sqrt{2+\sqrt{2}}}{2-\sqrt{2}}} \begin{pmatrix} \cos\frac{-\pi}{4} + \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} + \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$slerp\left(\overrightarrow{q_0}, \overrightarrow{q_1}, \frac{1}{2}\right) = \begin{pmatrix} \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} \\ 0 \\ 0 \\ \frac{-\sqrt{2-\sqrt{2-\sqrt{2}}}}{2} \end{pmatrix}$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_t} = \begin{pmatrix} \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} \\ 0 \\ 0 \\ \frac{-\sqrt{2-\sqrt{2-\sqrt{2}}}}{2} \end{pmatrix} \qquad |\overrightarrow{q_t}| = \frac{2+\sqrt{2-\sqrt{2}}}{4} + \frac{2-\sqrt{2-\sqrt{2}}}{4} = 1$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$\overrightarrow{q_t} = \begin{pmatrix} \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} \\ 0 \\ 0 \\ \frac{-\sqrt{2-\sqrt{2-\sqrt{2}}}}{2} \end{pmatrix} \Rightarrow q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$\overrightarrow{q_0} = \begin{pmatrix} \cos\frac{-\pi}{4} \\ 0 \\ 0 \\ \sin\frac{-\pi}{4} \end{pmatrix} \qquad t = \frac{1}{2} \qquad \overrightarrow{q_1} = \begin{pmatrix} \cos\frac{-\pi}{8} \\ 0 \\ 0 \\ \sin\frac{-\pi}{8} \end{pmatrix}$$

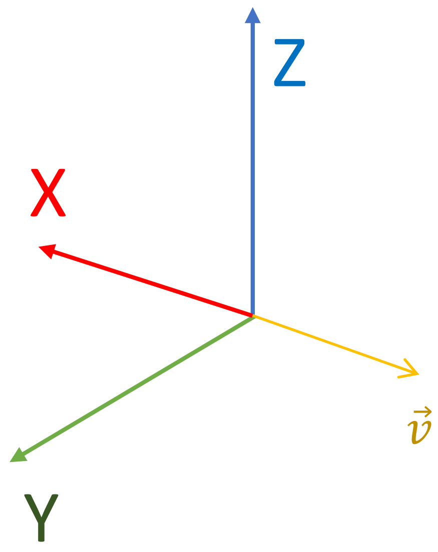
$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

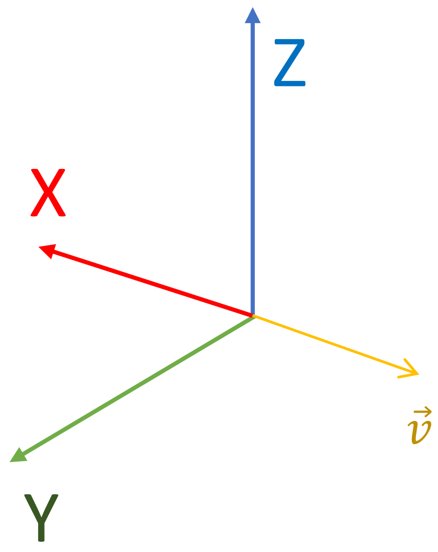
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$



$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

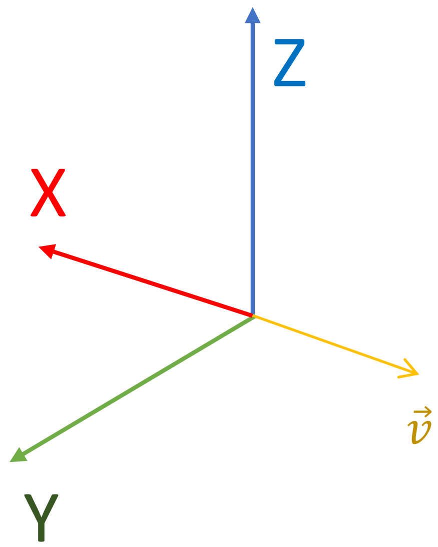
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

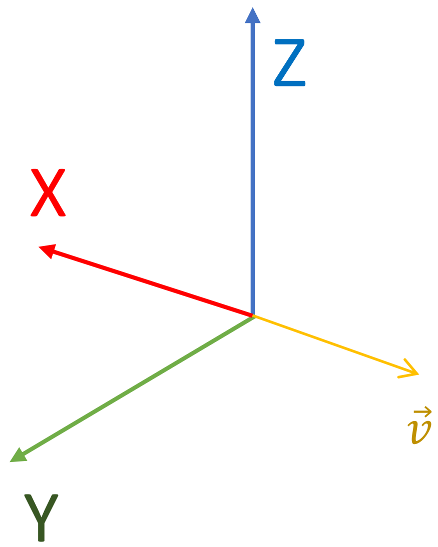
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



$$q_t q_v q_t^{-1} = \left(\left(\frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k\right)(-i)\right) q_t^{-1}$$

$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$



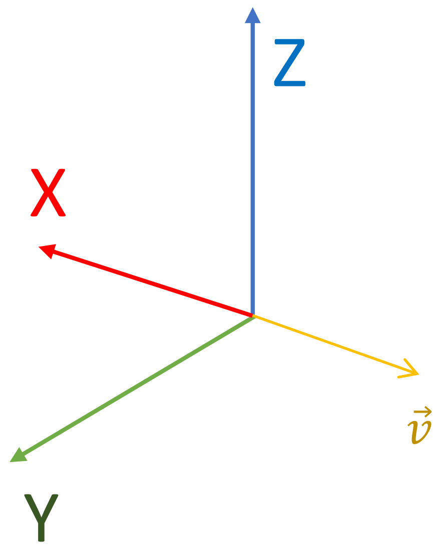
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\sqrt{2+\sqrt{2-\sqrt{2}}} \right) \frac{-i}{2} - \left(\sqrt{2-\sqrt{2-\sqrt{2}}} \right) \frac{(-ki)}{2} \right) q_t^{-1}$$

$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$



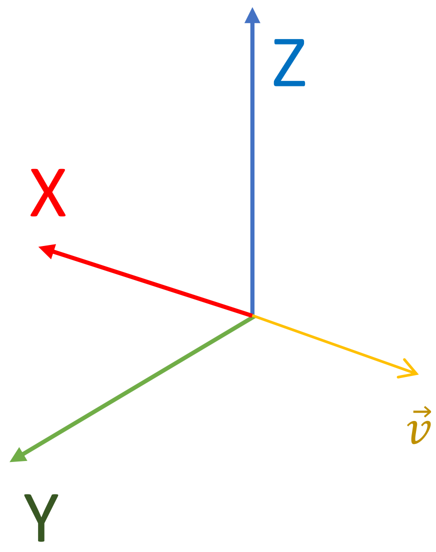
$$q_t q_v q_t^{-1} = \left(\left(\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\frac{-i}{2} - \left(\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\frac{-j}{2}\right) q_t^{-1}$$

$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$q_t^{-1} = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} + \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_t q_v q_t^{-1} = \left(\left(\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\frac{-i}{2} + \left(\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\frac{j}{2}\right)\left(\left(\sqrt{2+\sqrt{2-\sqrt{2}}}\right)\frac{1}{2} + \left(\sqrt{2-\sqrt{2-\sqrt{2}}}\right)\frac{k}{2}\right)$$



$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}}\right)\frac{-i}{2} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}}\right)\frac{j}{2}\right)\left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}}\right)\frac{1}{2} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}}\right)\frac{k}{2}\right)$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}}\right)\frac{-i}{2} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}}\right)\frac{j}{2}\right)\left(\left(\sqrt{2 + \sqrt{2 - \sqrt{2}}}\right)\frac{1}{2} + \left(\sqrt{2 - \sqrt{2 - \sqrt{2}}}\right)\frac{k}{2}\right)$$

$$q_t q_v q_t^{-1} = (-ai + bj)(a + bk)$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = (-ai + bj)(a + bk) = -a^2 i - abik + abj + b^2\, jk$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab ik + abj + b^2\ jk$$

$$a = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab(-\mathrm{j}) + abj + b^2\ i$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = -a^2 i - ab(-\mathrm{j}) + abj + b^2\, i = (-a^2 + b^2)i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = (-a^2 + b^2)i + 2abj = \frac{-2 - \sqrt{2 - \sqrt{2}} + 2 - \sqrt{2 - \sqrt{2}}}{4} i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \frac{-2 - \sqrt{2 - \sqrt{2}} + 2 - \sqrt{2 - \sqrt{2}}}{4} i + 2abj = \frac{-2\sqrt{2 - \sqrt{2}}}{4} i + 2abj$$

$$a = \frac{\sqrt{2 + \sqrt{2 - \sqrt{2}}}}{2}$$

$$b = \frac{\sqrt{2 - \sqrt{2 - \sqrt{2}}}}{2}$$

$$q_t q_v q_t^{-1} = \frac{-2\sqrt{2 - \sqrt{2}}}{4} i + 2abj = \frac{-\sqrt{2 - \sqrt{2}}}{2} i + 2abj$$

$$\frac{\sqrt{2-\sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$q_t q_v q_t^{-1} = \frac{-\sqrt{2-\sqrt{2}}}{2} i + \sqrt{2+\sqrt{2-\sqrt{2}}}\,\frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2} j$$

$$\frac{\sqrt{2-\sqrt{2}}}{2} = \cos\left(\frac{\pi}{4}+\frac{\pi}{8}\right)$$

$$\frac{\sqrt{2+\sqrt{2}}}{2} = \sin\left(\frac{\pi}{4}+\frac{\pi}{8}\right)$$

$$q_t q_v q_t^{-1} = \frac{-\sqrt{2-\sqrt{2}}}{2} i + \frac{\sqrt{2+\sqrt{2}}}{2} j$$

$$\frac{\sqrt{2-\sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

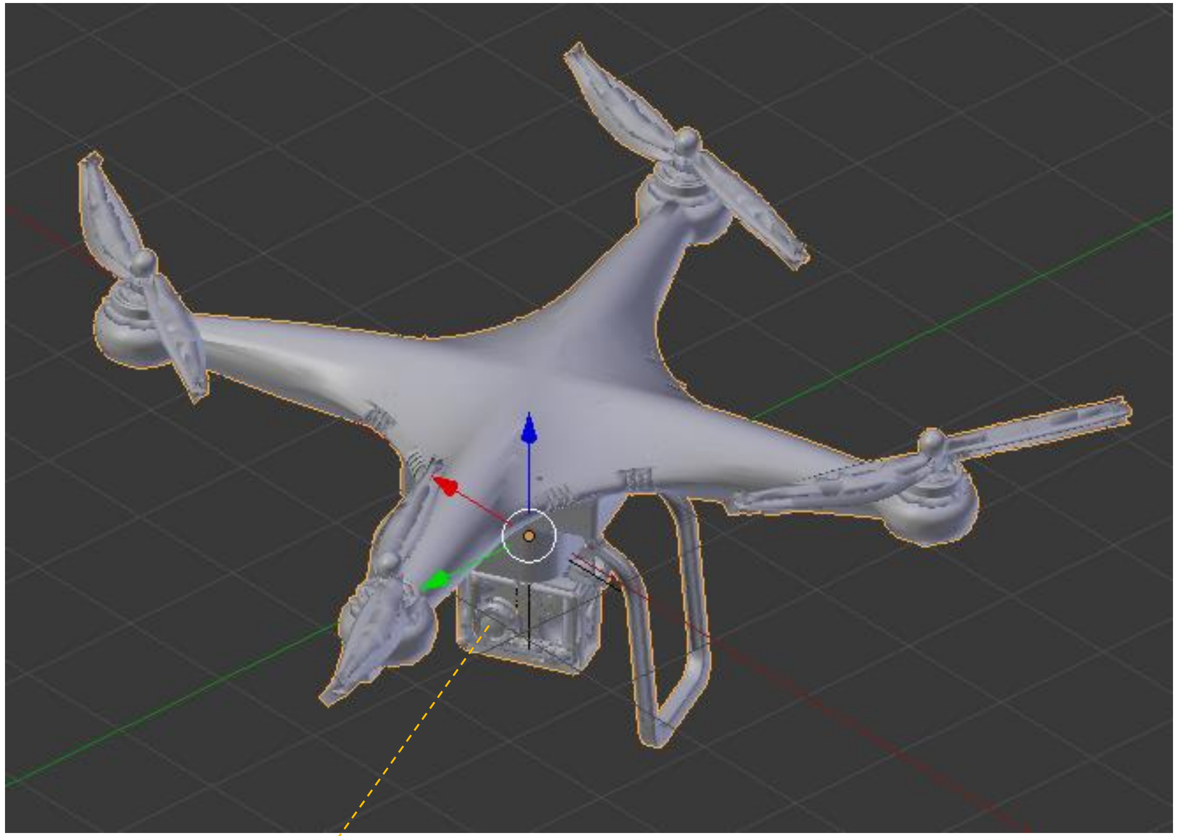
$$\frac{\sqrt{2+\sqrt{2}}}{2} = \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

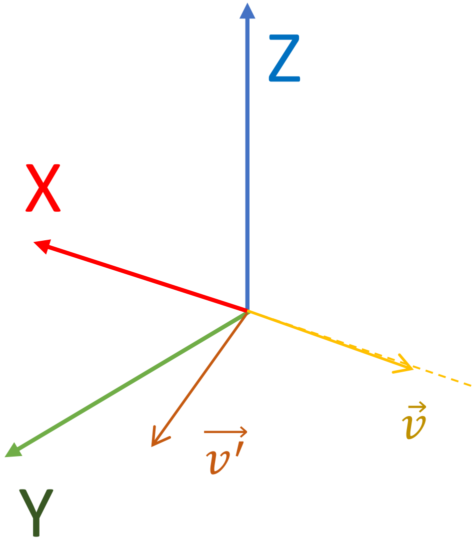
$$q_t q_v q_t^{-1} = \frac{-\sqrt{2-\sqrt{2}}}{2} i + \frac{\sqrt{2+\sqrt{2}}}{2} j \Rightarrow \overrightarrow{v'} = \begin{pmatrix} \frac{-\sqrt{2-\sqrt{2}}}{2} \\ \frac{\sqrt{2+\sqrt{2}}}{2} \\ 0 \end{pmatrix}$$

$$\frac{\sqrt{2-\sqrt{2}}}{2} = \cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$\frac{\sqrt{2+\sqrt{2}}}{2} = \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right)$$

$$\overrightarrow{v'} = \begin{pmatrix} -\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$





$$\overrightarrow{v'} = \begin{pmatrix} -\cos\left(\frac{\pi}{4}+\frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4}+\frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$t = \frac{1}{2}$

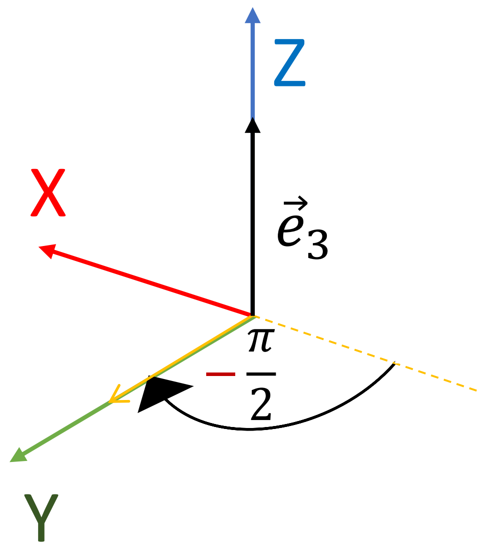
$$q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$t = 0$

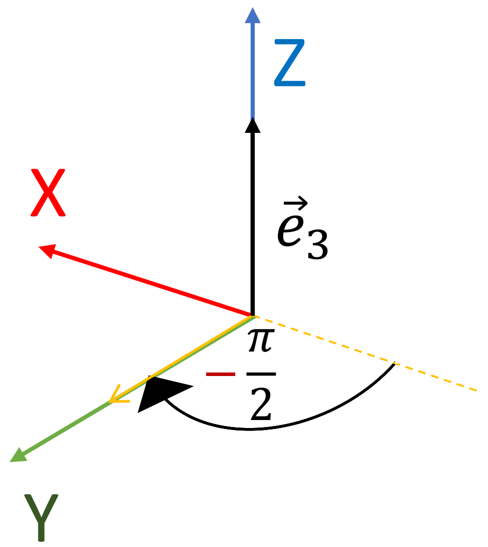
$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$



$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

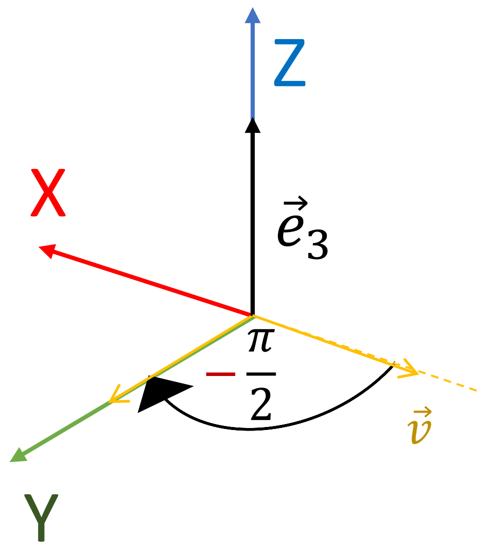
$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k \sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k \sin\frac{-\pi}{4} k$$

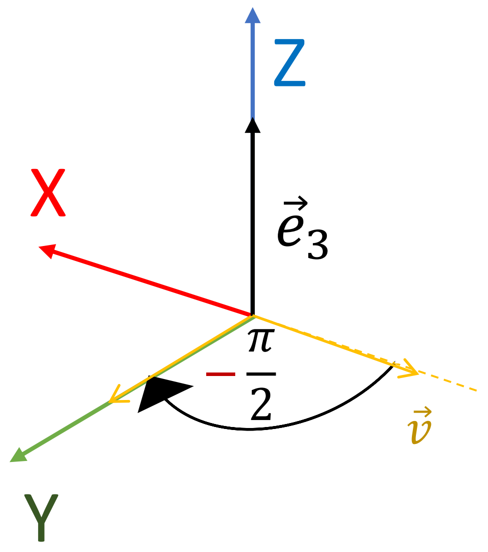


$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



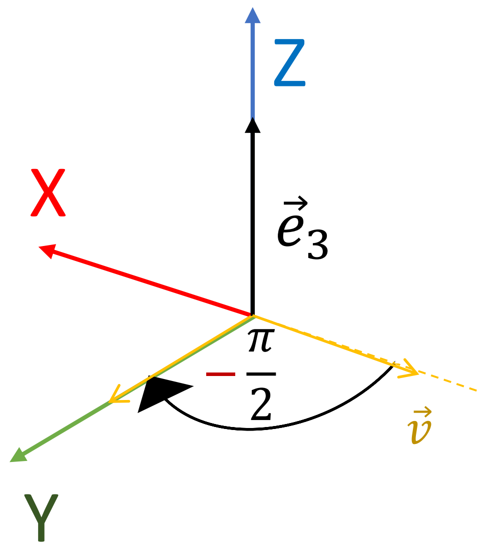
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = \left(\left(\cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}\right)(-i)\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



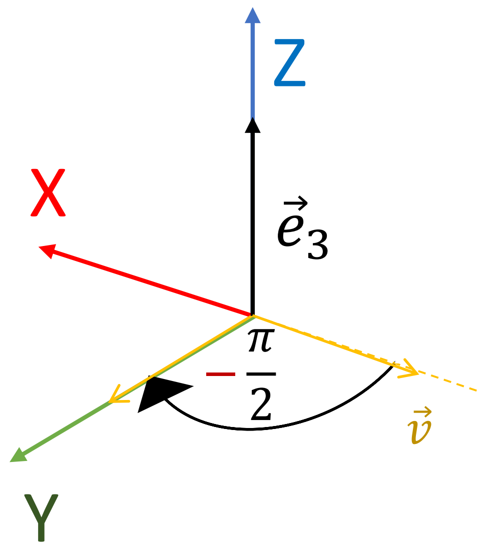
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} + \sin\frac{-\pi}{4}(-ki)\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



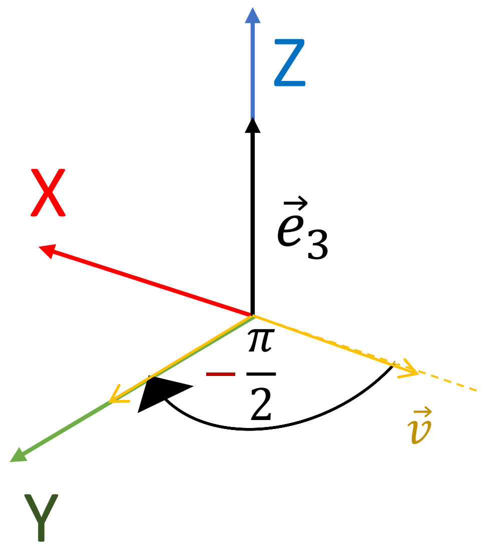
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} + \sin\frac{-\pi}{4}(-j)\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

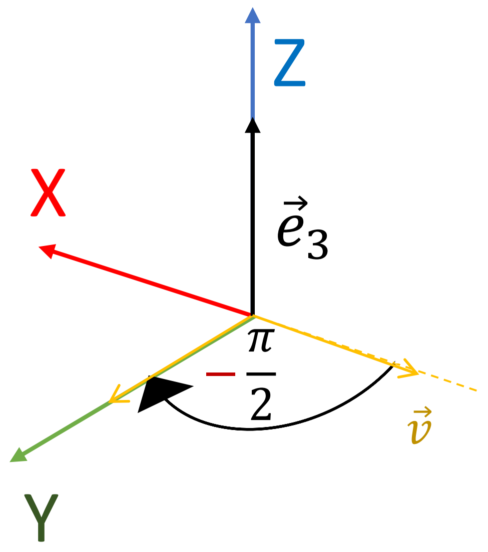
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= -i\cos^2\left(\frac{-\pi}{4}\right) + ik\cos\frac{-\pi}{4}\sin\frac{-\pi}{4} - j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4} + jk\sin^2\left(\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

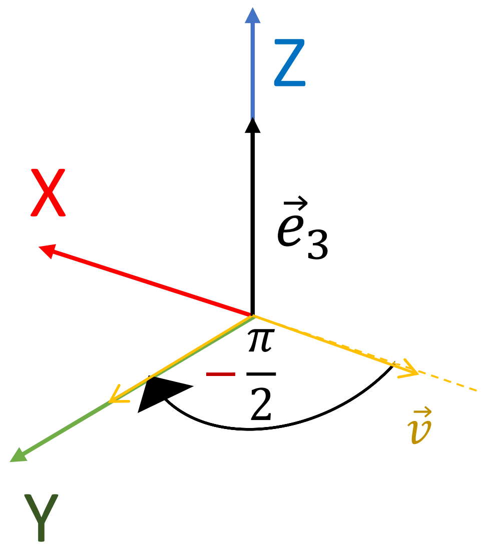
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= -i\cos^2\left(\frac{-\pi}{4}\right) - j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4} - j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4} + i\ \sin^2\left(\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

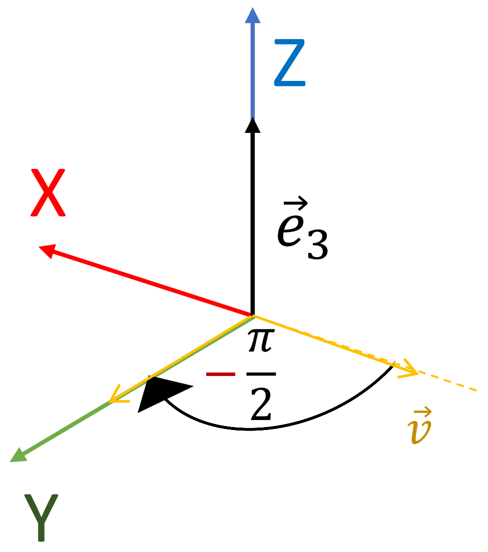
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= -i\cos^2\left(\frac{-\pi}{4}\right) - 2\,j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4} + i\ \sin^2\left(\frac{-\pi}{4}\right)$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

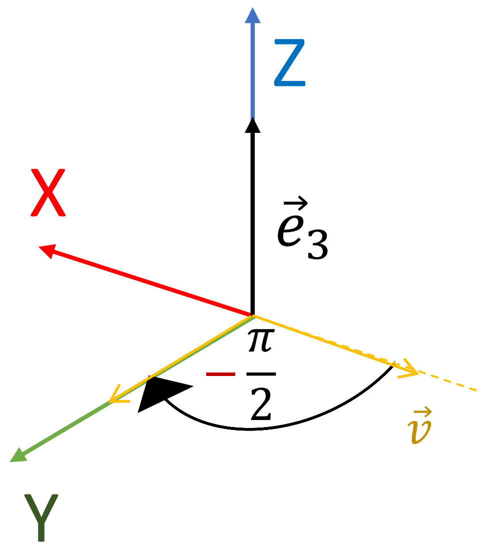
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= i\ \sin^2\left(\frac{-\pi}{4}\right) - i\cos^2\left(\frac{-\pi}{4}\right) - 2\,j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4}$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

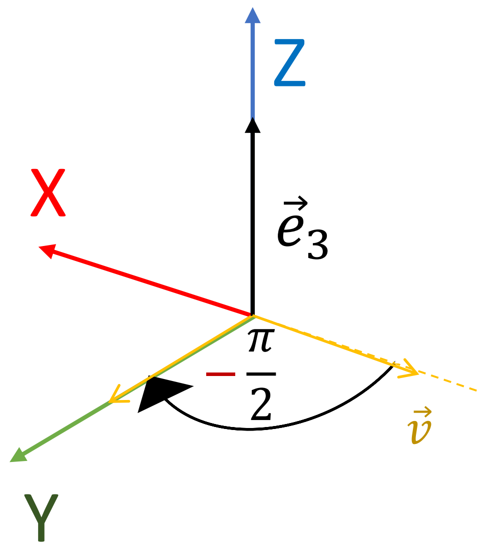
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= i\left(\sin^2\left(\frac{-\pi}{4}\right) - \cos^2\left(\frac{-\pi}{4}\right)\right) \quad - \quad 2\,j\cos\frac{-\pi}{4}\sin\frac{-\pi}{4}$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

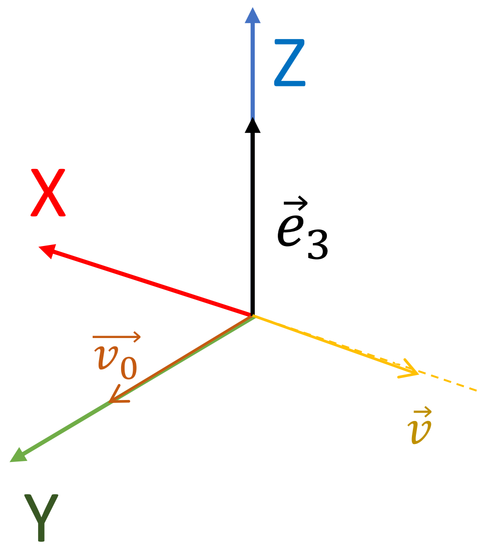
$$q_0 q_v q_0^{-1} = \left(-i\cos\frac{-\pi}{4} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}\right) =$$

$$= i\left(\sin^2\left(\frac{-\pi}{4}\right) - \sin^2\left(\frac{-\pi}{4}\right)\right) \quad - \quad 2\left(-\frac{1}{2}\right)j$$

$t = 0$

$$q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$q_0^{-1} = \cos\frac{-\pi}{4} - k\sin\frac{-\pi}{4}k$$

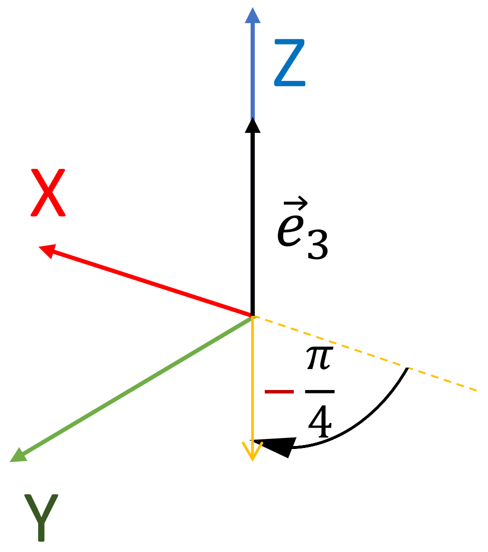


$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_0 q_v q_0^{-1} = j \Rightarrow \overrightarrow{v_0} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$t = 1$

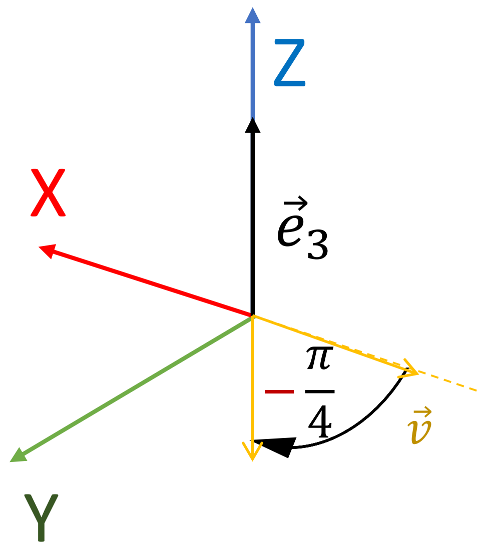
$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$



$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$

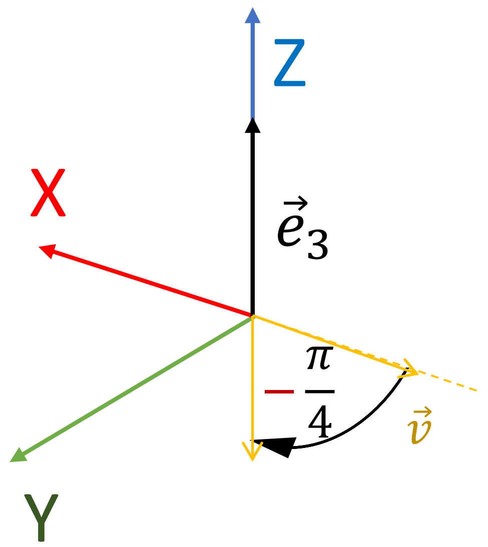


$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



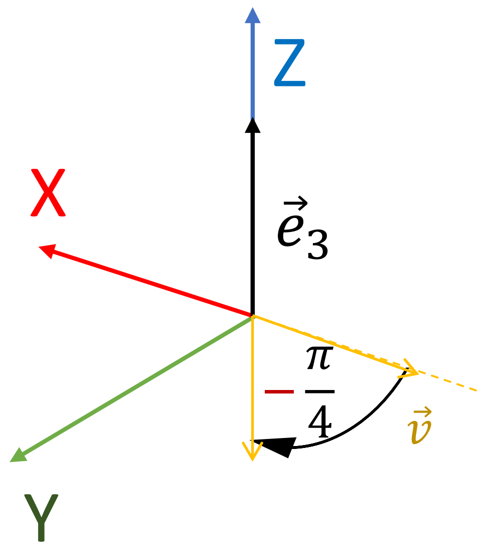
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(\left(\cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}\right)(-i)\right)\left(\cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}\right)$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



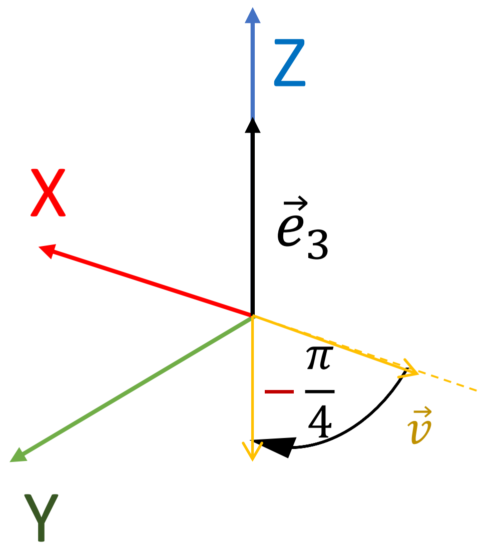
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(-i\cos\frac{-\pi}{8} + \sin\frac{-\pi}{8}(-ki)\right)\left(\cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}\right)$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



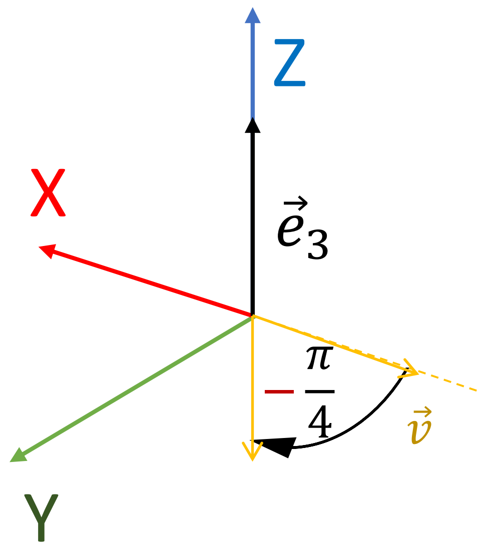
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(-i\cos\frac{-\pi}{8} + \sin\frac{-\pi}{8}(-j)\right)\left(\cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}\right)$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

$$q_1 q_v q_1^{-1} = \left(-i\cos\frac{-\pi}{8} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}\right) =$$

$$= i\left(\sin^2\left(\frac{-\pi}{8}\right) - \cos^2\left(\frac{-\pi}{8}\right)\right) \quad - \quad 2\,j\cos\frac{-\pi}{8}\sin\frac{-\pi}{8}$$

(sin(-pi/8))^2 - (cos(-pi/8))^2

Input:

$$\sin^2\left(-\frac{\pi}{8}\right) - \cos^2\left(-\frac{\pi}{8}\right)$$

Exact result:

$$\sin^2\left(\frac{\pi}{8}\right) - \cos^2\left(\frac{\pi}{8}\right)$$

Decimal approximation:

$$-0.70710678118654752440084436210484903$$

Alternate forms:

$$-\frac{1}{\sqrt{2}}$$

sin(-pi/8) * cos(-pi/8)

Input:

$$\sin\left(-\frac{\pi}{8}\right)\cos\left(-\frac{\pi}{8}\right)$$

Exact result:

$$\sin\left(\frac{\pi}{8}\right)\left(-\cos\left(\frac{\pi}{8}\right)\right)$$

Decimal approximation:

$$-0.3535533905932737622004221810524245196424179688442370 18$$

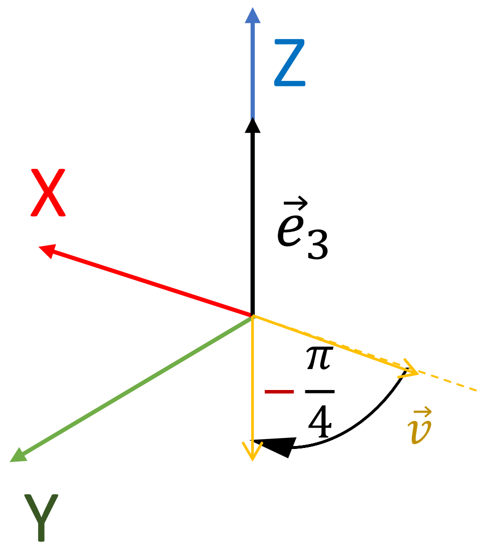
Alternate forms:

$$-\frac{1}{2\sqrt{2}}$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

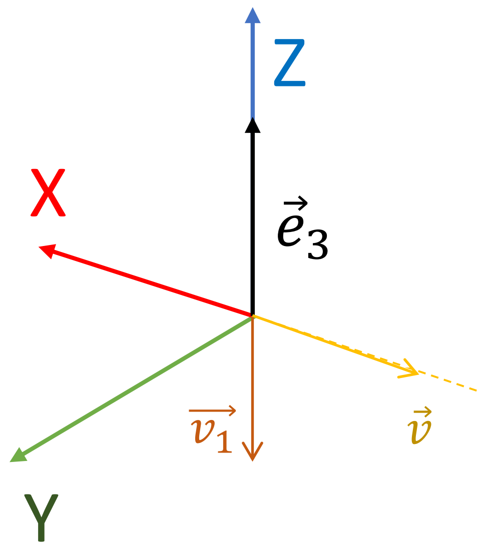
$$q_1 q_v q_1^{-1} = \left(-i\cos\frac{-\pi}{8} - j\sin\frac{-\pi}{4}\right)\left(\cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}\right) =$$

$$= i\left(-\frac{1}{\sqrt{2}}\right) \quad - \quad 2\,j\left(-\frac{1}{2\sqrt{2}}\right)$$

$t = 1$

$$q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

$$q_1^{-1} = \cos\frac{-\pi}{8} - k\sin\frac{-\pi}{8}k$$



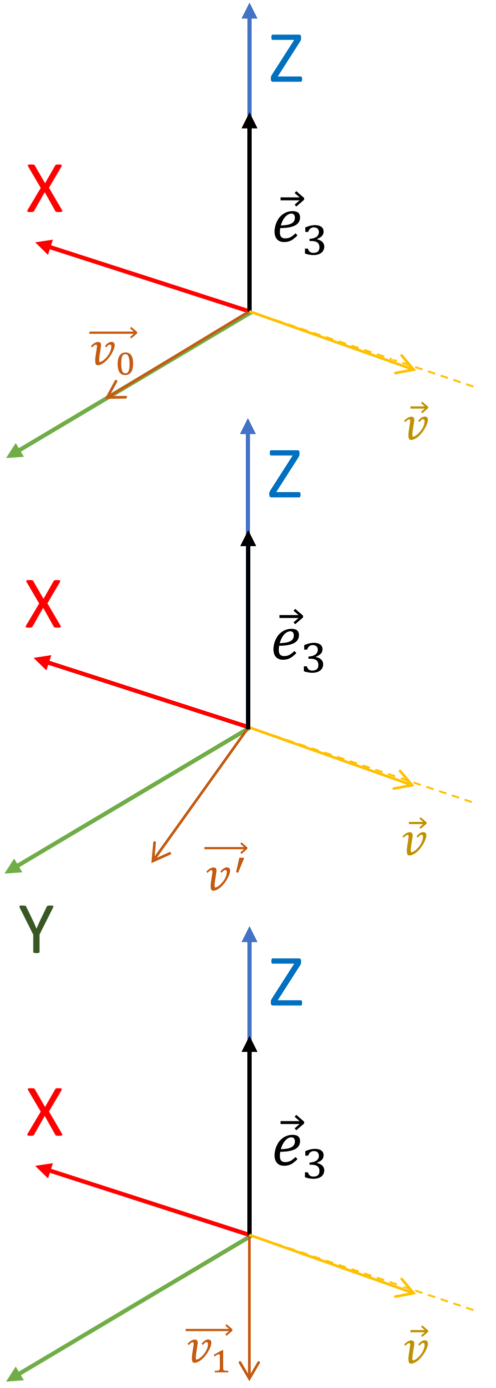
$$\vec{v} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow q_v = 0 - 1i + 0j + 0k = -i$$

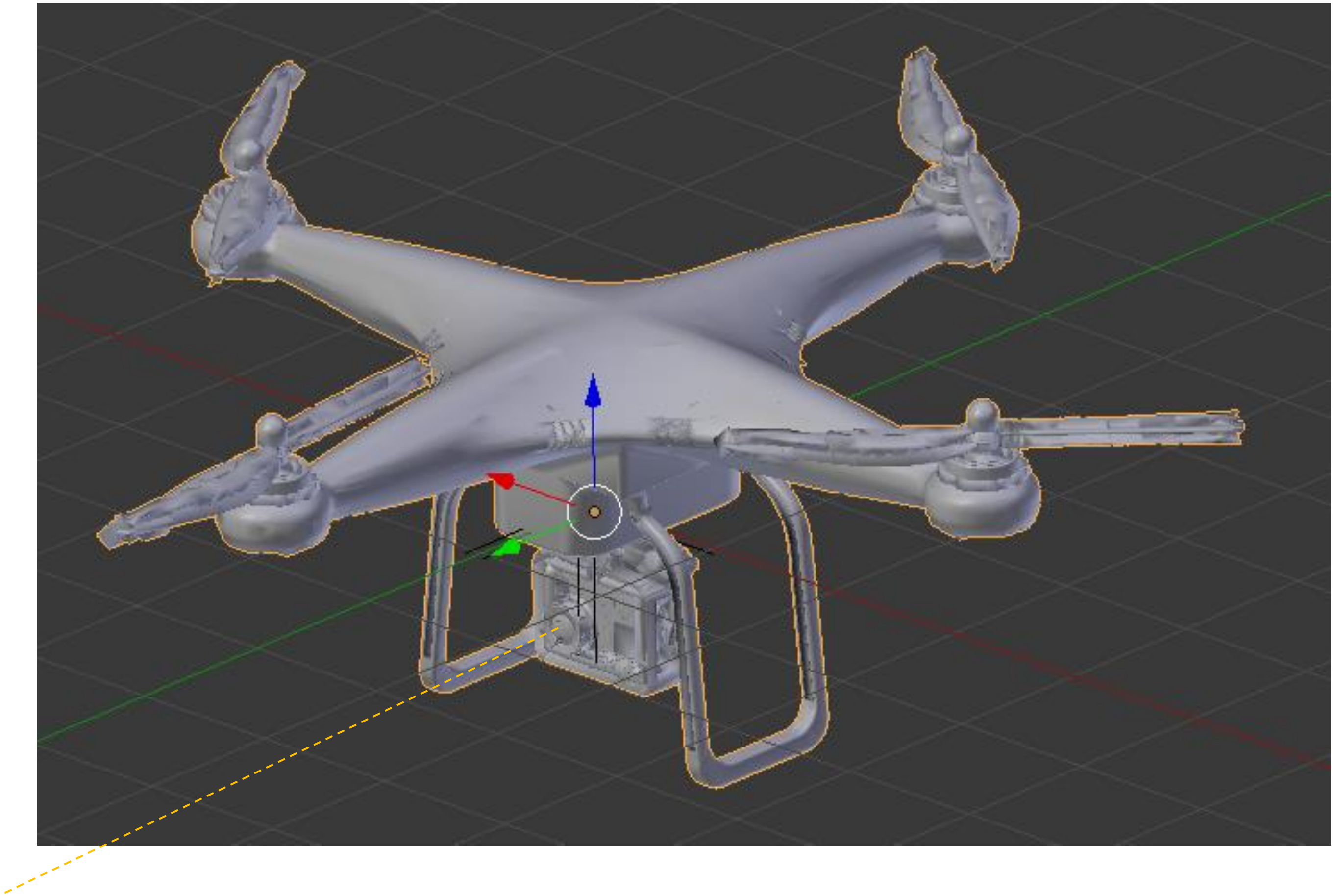
$$q_1 q_v q_1^{-1} = \text{-}i\left(\frac{1}{\sqrt{2}}\right) \quad + \quad j\left(\frac{1}{\sqrt{2}}\right) \Rightarrow \overrightarrow{v_1} = \begin{pmatrix} -\cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \\ 0 \end{pmatrix}$$

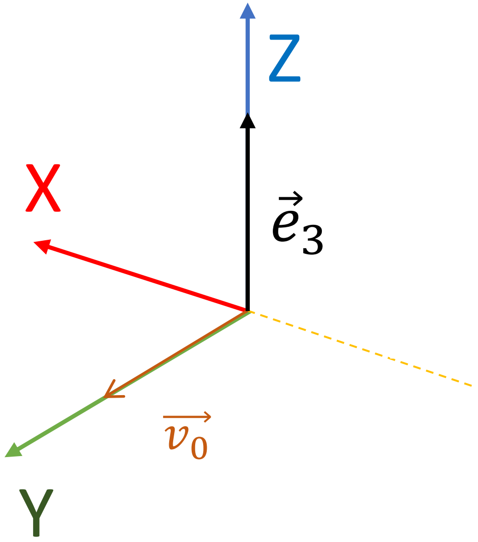
$$t = 0 \qquad q_0 = \cos\frac{-\pi}{4} + k\sin\frac{-\pi}{4}$$

$$t = \frac{1}{2} \qquad q_t = \frac{\sqrt{2+\sqrt{2-\sqrt{2}}}}{2} - \frac{\sqrt{2-\sqrt{2-\sqrt{2}}}}{2}k$$

$$t = 1 \qquad q_1 = \cos\frac{-\pi}{8} + k\sin\frac{-\pi}{8}$$

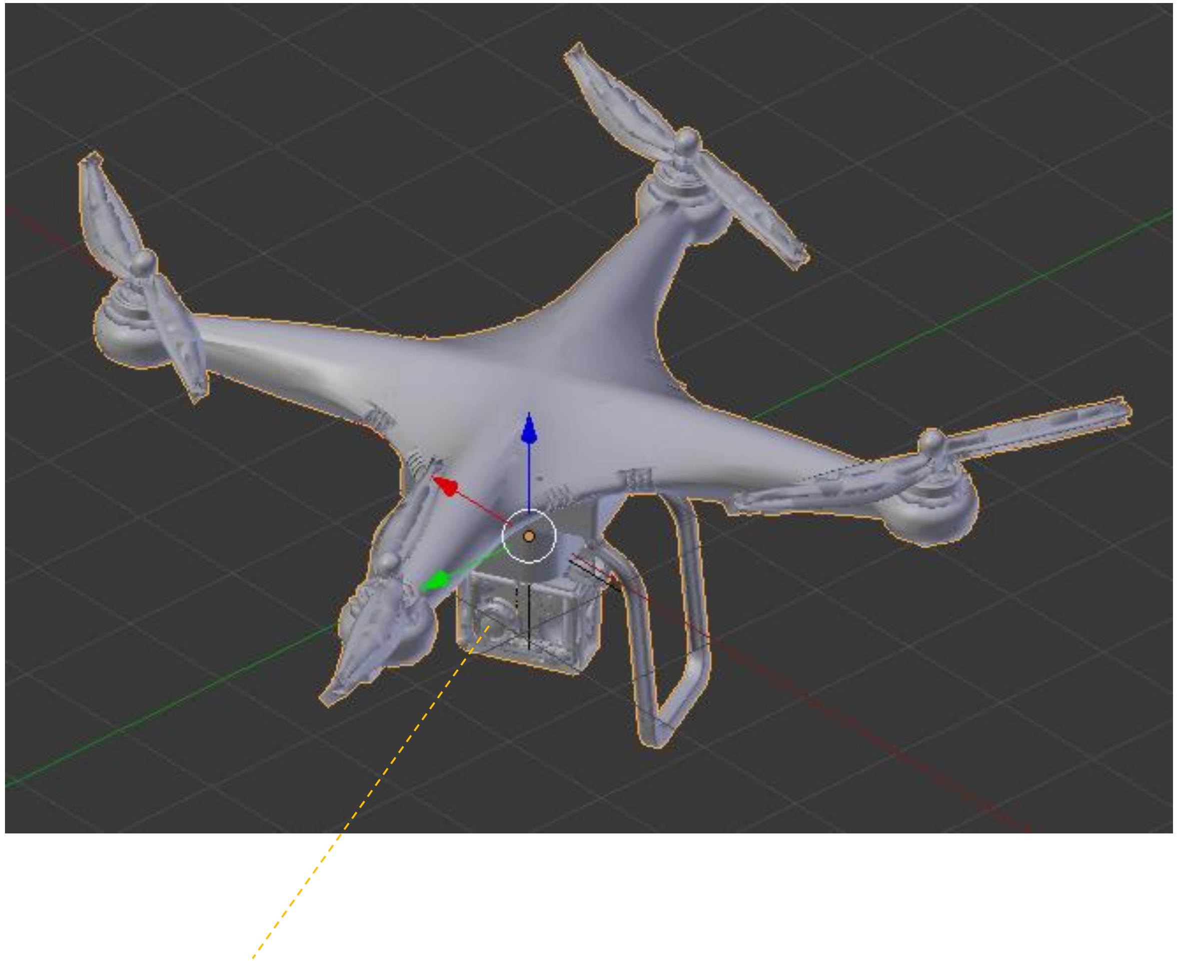


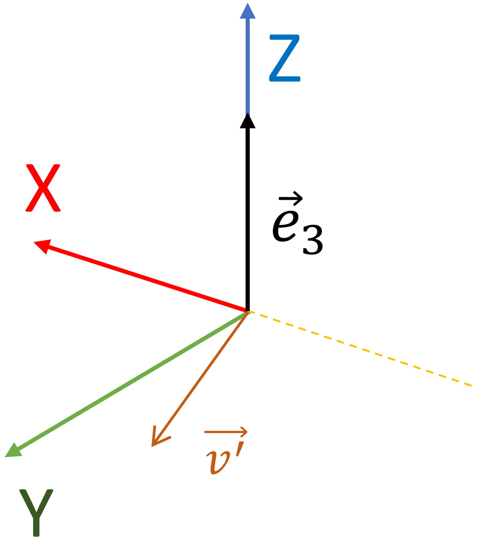




$t = 0$

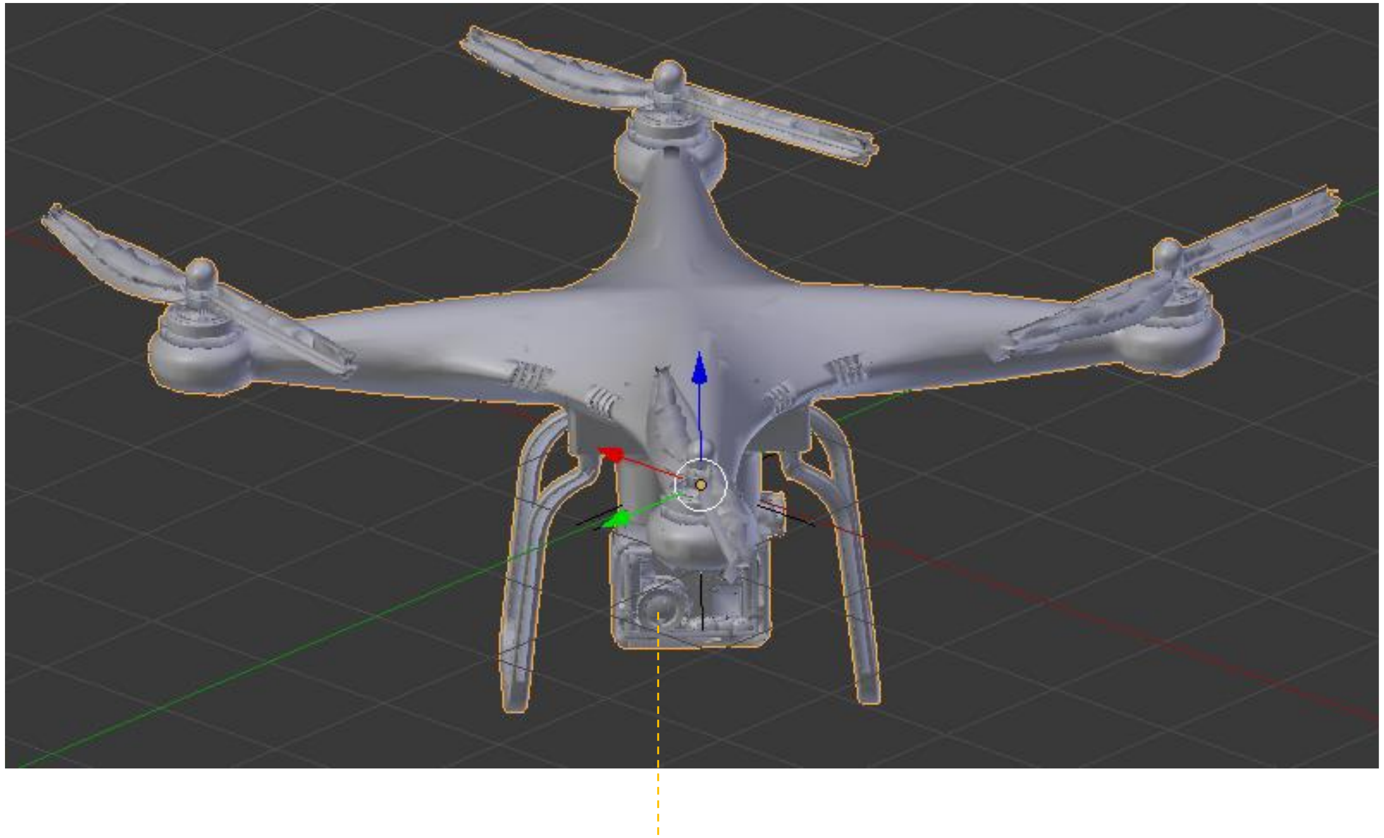
$$\vec{v_0} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



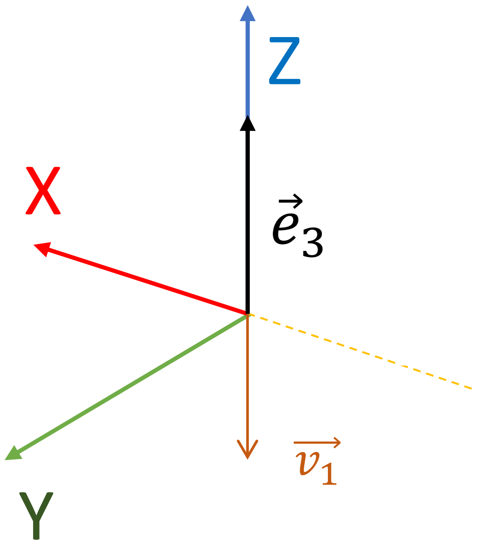


$$t = \frac{1}{2}$$

$$\overrightarrow{v'} = \begin{pmatrix} -\cos\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ \sin\left(\frac{\pi}{4} + \frac{\pi}{8}\right) \\ 0 \end{pmatrix}$$



$$t = 1$$



$$\overrightarrow{v_1} = \begin{pmatrix} -\cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \\ 0 \end{pmatrix}$$