Lecture 11: Extensions and Applications of ASP

2-AIN-108 Computational Logic

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Constraints

Definition (Constraint)

A normal constraint is a rule of the form

$$\leftarrow L_1, \dots, L_n$$

where $0 \leq n$ and each L_i, $1 \leq i \leq n$, is a literal.

Definition (Stable Model)

An interpretation I is a stable model of a normal logic program P with a set of constraints C iff I is a stable model of P and satisfies C.

Default Negation in the Head

Definition (Generalized Logic Program)

A generalized logic program is a finite set of rules of the form

$$L_0 \leftarrow L_1, \ldots, L_n$$

where $0 \leq n$ and each L_i, $0 \leq i \leq n$, is a literal.

The rule

$$\sim q \leftarrow p$$

can be viewed as a constraint

$$\leftarrow p, q$$

Semantics of Generalized Logic Programs

Definition (Program Reduct)

Let I be an interpretation. A program reduct of a generalized logic program P is a definite logic program P^I with constraints obtained from P by

- deleting all rules with a default literal L in the body not satisfied by I
- deleting all rules with a default literal L in the head satisfied by I
- deleting all default literals from remaining rules

Definition (Stable Model)

An interpretation I is a stable model of a generalized logic program P iff I is the least model of the program reduct P^I.

Explicit Negation

$\sim train$

$train \qquad \emptyset \qquad \neg\, train$

$\sim \neg\, train$

$$cross \leftarrow \sim train$$

versus

$$cross \leftarrow \neg\, train$$

Extended Logic Programs

Definition (Literal)

A classical literal is an atom or an atom preceded by classical negation. A default literal is a classical literal preceded by default negation. A literal is either classical or default literal.

Definition (Extended Logic Program)

An extended logic program is a finite set of rules

$$L_0 \leftarrow L_1, \ldots, L_m, \sim L_{m+1}, \ldots, \sim L_n$$

where $0 \leq m \leq n$ and each L_i, $0 \leq i \leq n$, is a classical literal.

Semantics of Extended Logic Programs

Definition (Herbrand Interpretation)

An extended Herbrand base is a set of ground classical literals. A set of classical literals is consistent if it does not contain an atom A and its classical negation $\neg A$. A Herbrand interpretation is a consistent subset of the extended Herbrand base.

Definition (Stable Model)

An interpretation I is a stable model of an extended logic program P iff I is a stable model of the same logic program P with classical literals interpreted as new atoms.

Aggregates

Example

$$
\begin{aligned}
disk(d1) &\leftarrow \\
disk(d2) &\leftarrow \\
disk(d3) &\leftarrow \\
capacity(d1, 250) &\leftarrow \\
capacity(d2, 500) &\leftarrow \\
capacity(d3, 1000) &\leftarrow \\
raid(D) &\leftarrow disk(D), \sim \neg raid(D) \\
\neg raid(D) &\leftarrow disk(D), \sim raid(D) \\
&\leftarrow \#\mathrm{sum}\{C : raid(D), capacity(D, C)\} < 1200
\end{aligned}
$$

Aggregates

An aggregate atom is an expression

$$Lg \prec_1 f(S) \prec_2 Rg$$

where

- Lg, Rg are terms (left guard, right guard);
- $\prec_1, \prec_2 \in \{=, <, \leq, >, \geq\}$;
- $f \in \{\#\mathrm{count}, \#\mathrm{min}, \#\mathrm{max}, \#\mathrm{sum}, \#\mathrm{times}\}$ is an aggregate function;
- S is a set of the form $\{Vars : Conj\}$ where $Conj$ is a set of literals and $Vars$ are variables in $Conj$

Sudoku (Generate)

- domain predicates

$$d(0) \leftarrow \quad d(1) \leftarrow \quad d(2) \leftarrow$$

$$n(1) \leftarrow \quad \dots \quad n(9) \leftarrow$$

- each cell contains or does not contain a number

$$s(A, B, X, Y, N) \leftarrow d(A), d(B), d(X), d(Y), n(N), \sim \neg s(A, B, X, Y, N)$$

$$\neg s(A, B, X, Y, N) \leftarrow d(A), d(B), d(X), d(Y), n(N), \sim s(A, B, X, Y, N)$$

- if a cell contains a number, it is filled

$$f(A, B, X, Y) \leftarrow s(A, B, X, Y, N)$$

Sudoku (Test)

- each cell is filled

$$\leftarrow d(A), d(B), d(X), d(Y), \sim f(X, Y, A, B)$$

- each number appears at most once in each column

$$\leftarrow s(A_1, B, X_1, Y, N), s(A_2, B, X_2, Y, N), (A_1, X_1) < (A_2, X_2)$$

- each number appears at most once in each row

$$\leftarrow s(A, B_1, X, Y_1, N), s(A, B_2, X, Y_2, N), (B_1, Y_1) < (B_2, Y_2)$$

- each number appears at most once in each box

$$\leftarrow s(A, B, X_1, Y_1, N), s(A, B, X_2, Y_2, N), (X_1, Y_1) < (X_2, Y_2)$$

Reaction Control System (RCS) of the Space Shuttle

- RCS is controlled by computer during takeoff and landing
- In orbit, however, astronauts have the primary control
- For normal situations there are pre-scripted plans to achieve certain goals
- The number of possible failures is too large to pre-plan all exceptional situations
- An intelligent system to verify and generate plans would be helpful

Nogueira, M. et al. An A-Prolog Decision Support System for the Space Shuttle. In Practical Aspects of Declarative Languages (pp. 169–183). Springer Berlin Heidelberg.

And More...

http://www.cs.uni-potsdam.de/~torsten/asp/