

# CFGs and PCFGs

# (Probabilistic) Context-Free Grammars



# A phrase structure grammar

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

$N \rightarrow people$

$N \rightarrow fish$

$N \rightarrow tanks$

$N \rightarrow rods$

$V \rightarrow people$

$V \rightarrow fish$

$V \rightarrow tanks$

$P \rightarrow with$

*people fish tanks*

*people fish with rods*



# Phrase structure grammars = context-free grammars (CFGs)

- $G = (T, N, S, R)$ 
  - $T$  is a set of terminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $R$  is a set of rules/productions of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup T)^*$
- A grammar  $G$  generates a language  $L$ .



# Phrase structure grammars in NLP

- $G = (T, C, N, S, L, R)$ 
  - T is a set of terminal symbols
  - C is a set of preterminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ( $S \in N$ )
  - L is the lexicon, a set of items of the form  $X \rightarrow x$ 
    - $X \in P$  and  $x \in T$
  - R is the grammar, a set of items of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup C)^*$
- By usual convention, S is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write  $e$  for an empty sequence, rather than nothing



# Probabilistic – or stochastic – context-free grammars (PCFGs)

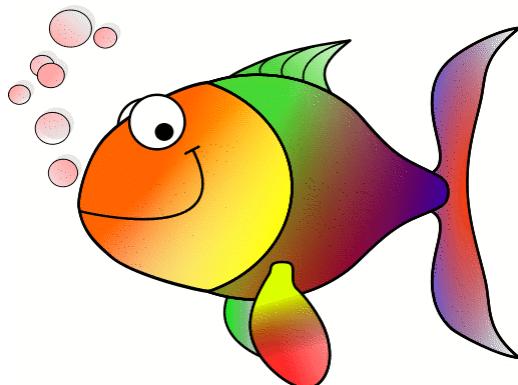
- $G = (T, N, S, R, P)$ 
  - $T$  is a set of terminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $R$  is a set of rules/productions of the form  $X \rightarrow \gamma$
  - $P$  is a probability function
    - $P: R \rightarrow [0,1]$
    - $\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$
- A grammar  $G$  generates a language model  $L$ .

$$\sum_{\gamma \in T^*} P(\gamma) = 1$$



# A PCFG

$S \rightarrow NP\ VP$	1.0	$N \rightarrow people$	0.5
$VP \rightarrow V\ NP$	0.6	$N \rightarrow fish$	0.2
$VP \rightarrow V\ NP\ PP$	0.4	$N \rightarrow tanks$	0.2
$NP \rightarrow NP\ NP$	0.1	$N \rightarrow rods$	0.1
$NP \rightarrow NP\ PP$	0.2	$V \rightarrow people$	0.1
$NP \rightarrow N$	0.7	$V \rightarrow fish$	0.6
$PP \rightarrow P\ NP$	1.0	$V \rightarrow tanks$	0.3
		$P \rightarrow with$	1.0



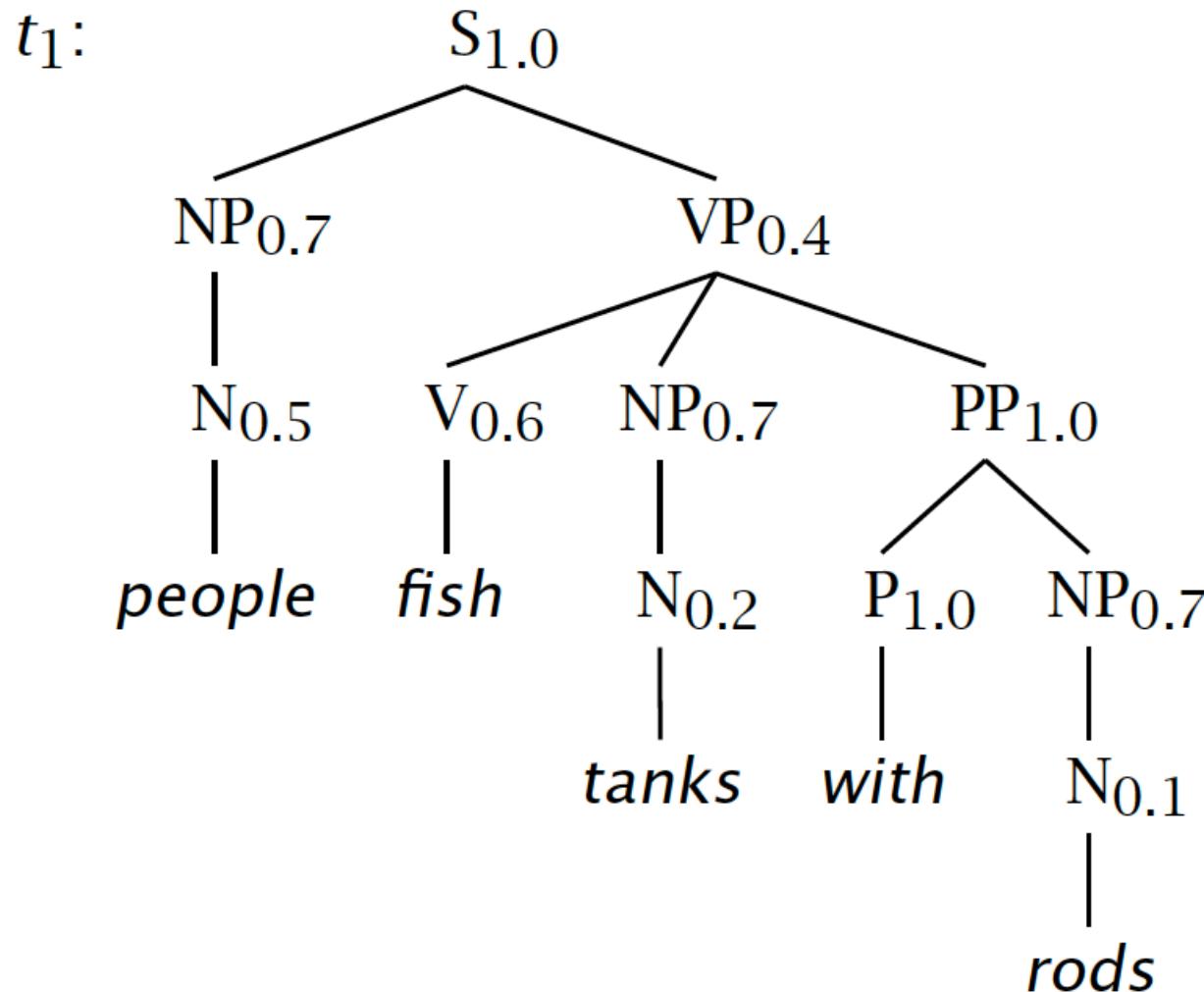
[With empty NP removed  
so less ambiguous]

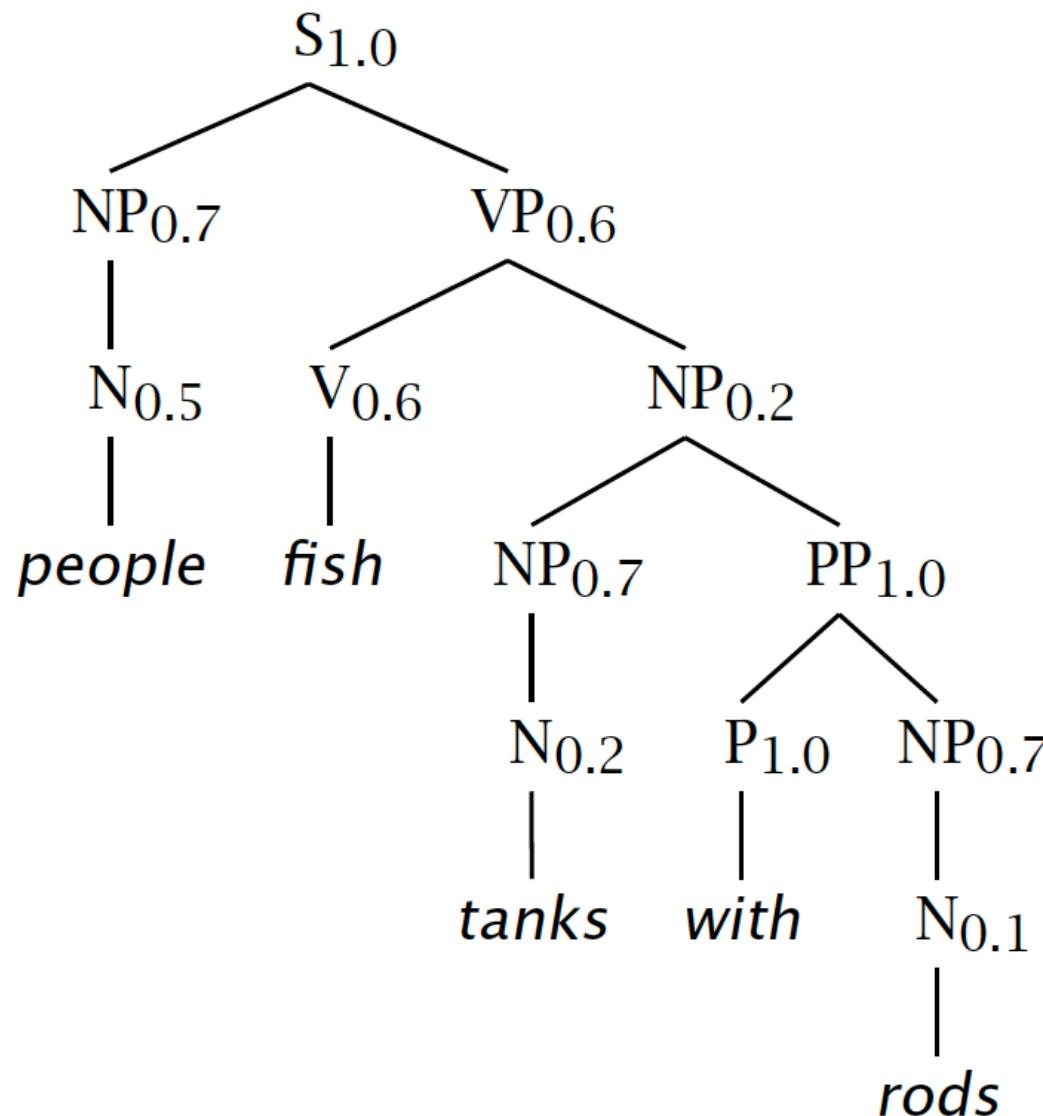


# The probability of trees and strings

- $P(t)$  – The probability of a tree  $t$  is the product of the probabilities of the rules used to generate it.
- $P(s)$  – The probability of the string  $s$  is the sum of the probabilities of the trees which have that string as their yield

$$\begin{aligned} P(s) &= \sum_j P(s, t) \text{ where } t \text{ is a parse of } s \\ &= \sum_j P(t) \end{aligned}$$



 $t_2:$ 

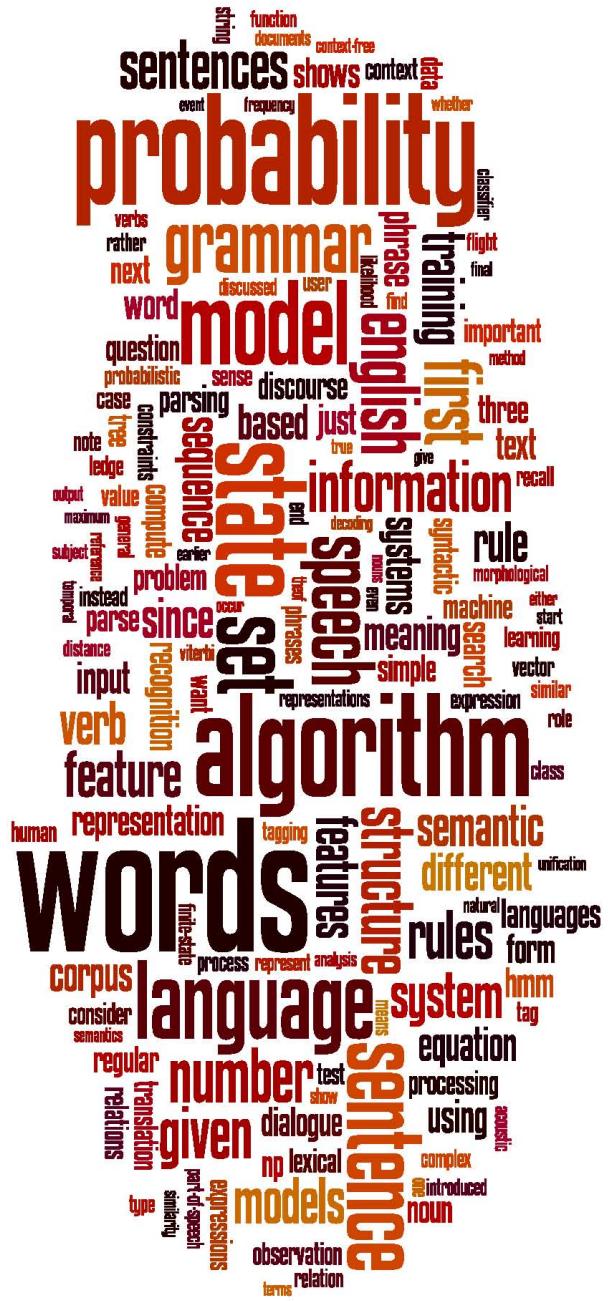


# Tree and String Probabilities

- $s = \text{people fish tanks with rods}$
- $P(t_1) = 1.0 \times 0.7 \times 0.4 \times 0.5 \times 0.6 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$ 
$$= 0.0008232$$
- $P(t_2) = 1.0 \times 0.7 \times 0.6 \times 0.5 \times 0.6 \times 0.2 \times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$ 
$$= 0.00024696$$
- $P(s) = P(t_1) + P(t_2)$ 
$$= 0.0008232 + 0.00024696$$
$$= 0.00107016$$

Verb attach

Noun attach



# Grammar Transforms

Restricting the  
grammar form for  
efficient parsing



# Chomsky Normal Form

- All rules are of the form  $X \rightarrow Y Z$  or  $X \rightarrow w$ 
  - $X, Y, Z \in N$  and  $w \in T$
- A transformation to this form doesn't change the weak generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- $n$ -ary rules are divided by introducing new nonterminals ( $n > 2$ )



# A phrase structure grammar

$S \rightarrow NP\ VP$

$VP \rightarrow V\ NP$

$VP \rightarrow V\ NP\ PP$

$NP \rightarrow NP\ NP$

$NP \rightarrow NP\ PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P\ NP$

$N \rightarrow people$

$N \rightarrow fish$

$N \rightarrow tanks$

$N \rightarrow rods$

$V \rightarrow people$

$V \rightarrow fish$

$V \rightarrow tanks$

$P \rightarrow with$



# Chomsky Normal Form steps

 $S \rightarrow NP\ VP$  $S \rightarrow VP$  $VP \rightarrow V\ NP$  $VP \rightarrow V$  $VP \rightarrow V\ NP\ PP$  $VP \rightarrow V\ PP$  $NP \rightarrow NP\ NP$  $NP \rightarrow NP$  $NP \rightarrow NP\ PP$  $NP \rightarrow PP$  $NP \rightarrow N$  $PP \rightarrow P\ NP$  $PP \rightarrow P$  $N \rightarrow people$  $N \rightarrow fish$  $N \rightarrow tanks$  $N \rightarrow rods$  $V \rightarrow people$  $V \rightarrow fish$  $V \rightarrow tanks$  $P \rightarrow with$



# Chomsky Normal Form steps

 $S \rightarrow NP\ VP$  $VP \rightarrow V\ NP$  $S \rightarrow V\ NP$  $VP \rightarrow V$  $S \rightarrow V$  $VP \rightarrow V\ NP\ PP$  $S \rightarrow V\ NP\ PP$  $VP \rightarrow V\ PP$  $S \rightarrow V\ PP$  $NP \rightarrow NP\ NP$  $NP \rightarrow NP$  $NP \rightarrow NP\ PP$  $NP \rightarrow PP$  $NP \rightarrow N$  $PP \rightarrow P\ NP$  $PP \rightarrow P$  $N \rightarrow people$  $N \rightarrow fish$  $N \rightarrow tanks$  $N \rightarrow rods$  $V \rightarrow people$  $V \rightarrow fish$  $V \rightarrow tanks$  $P \rightarrow with$



# Chomsky Normal Form steps

 $S \rightarrow NP\ VP$  $VP \rightarrow V\ NP$  $S \rightarrow V\ NP$  $VP \rightarrow V$  $VP \rightarrow V\ NP\ PP$  $S \rightarrow V\ NP\ PP$  $VP \rightarrow V\ PP$  $S \rightarrow V\ PP$  $NP \rightarrow NP\ NP$  $NP \rightarrow NP$  $NP \rightarrow NP\ PP$  $NP \rightarrow PP$  $NP \rightarrow N$  $PP \rightarrow P\ NP$  $PP \rightarrow P$  $N \rightarrow people$  $N \rightarrow fish$  $N \rightarrow tanks$  $N \rightarrow rods$  $V \rightarrow people$  $S \rightarrow people$  $V \rightarrow fish$  $S \rightarrow fish$  $V \rightarrow tanks$  $S \rightarrow tanks$  $P \rightarrow with$



# Chomsky Normal Form steps

 $S \rightarrow NP\ VP$  $VP \rightarrow V\ NP$  $S \rightarrow V\ NP$  $VP \rightarrow V\ NP\ PP$  $S \rightarrow V\ NP\ PP$  $VP \rightarrow V\ PP$  $S \rightarrow V\ PP$  $NP \rightarrow NP\ NP$  $NP \rightarrow NP$  $NP \rightarrow NP\ PP$  $NP \rightarrow PP$  $NP \rightarrow N$  $PP \rightarrow P\ NP$  $PP \rightarrow P$  $N \rightarrow people$  $N \rightarrow fish$  $N \rightarrow tanks$  $N \rightarrow rods$  $V \rightarrow people$  $S \rightarrow people$  $VP \rightarrow people$  $V \rightarrow fish$  $S \rightarrow fish$  $VP \rightarrow fish$  $V \rightarrow tanks$  $S \rightarrow tanks$  $VP \rightarrow tanks$  $P \rightarrow with$



# Chomsky Normal Form steps

$S \rightarrow NP\ VP$	$NP \rightarrow people$
$VP \rightarrow V\ NP$	$NP \rightarrow fish$
$S \rightarrow V\ NP$	$NP \rightarrow tanks$
$VP \rightarrow V\ NP\ PP$	$NP \rightarrow rods$
$S \rightarrow V\ NP\ PP$	$V \rightarrow people$
$VP \rightarrow V\ PP$	$S \rightarrow people$
$S \rightarrow V\ PP$	$VP \rightarrow people$
$NP \rightarrow NP\ NP$	$V \rightarrow fish$
$NP \rightarrow NP\ PP$	$S \rightarrow fish$
$NP \rightarrow P\ NP$	$VP \rightarrow fish$
$PP \rightarrow P\ NP$	$V \rightarrow tanks$
	$S \rightarrow tanks$
	$VP \rightarrow tanks$
	$P \rightarrow with$
	$PP \rightarrow with$



# Chomsky Normal Form steps

$S \rightarrow NP\ VP$	$NP \rightarrow people$
$VP \rightarrow V\ NP$	$NP \rightarrow fish$
$S \rightarrow V\ NP$	$NP \rightarrow tanks$
$VP \rightarrow V\ @VP\_V$	$NP \rightarrow rods$
$@VP\_V \rightarrow NP\ PP$	$V \rightarrow people$
$S \rightarrow V\ @S\_V$	$S \rightarrow people$
$@S\_V \rightarrow NP\ PP$	$VP \rightarrow people$
$VP \rightarrow V\ PP$	$V \rightarrow fish$
$S \rightarrow V\ PP$	$S \rightarrow fish$
$NP \rightarrow NP\ NP$	$VP \rightarrow fish$
$NP \rightarrow NP\ PP$	$V \rightarrow tanks$
$NP \rightarrow P\ NP$	$S \rightarrow tanks$
$PP \rightarrow P\ NP$	$VP \rightarrow tanks$
	$P \rightarrow with$
	$PP \rightarrow with$



# A phrase structure grammar

 $S \rightarrow NP\ VP$  $N \rightarrow people$  $VP \rightarrow V\ NP$  $N \rightarrow fish$  $VP \rightarrow V\ NP\ PP$  $N \rightarrow tanks$  $NP \rightarrow NP\ NP$  $N \rightarrow rods$  $NP \rightarrow NP\ PP$  $V \rightarrow people$  $NP \rightarrow N$  $V \rightarrow fish$  $NP \rightarrow e$  $V \rightarrow tanks$  $PP \rightarrow P\ NP$  $P \rightarrow with$



# Chomsky Normal Form steps

$S \rightarrow NP\ VP$	$NP \rightarrow people$
$VP \rightarrow V\ NP$	$NP \rightarrow fish$
$S \rightarrow V\ NP$	$NP \rightarrow tanks$
$VP \rightarrow V\ @VP\_V$	$NP \rightarrow rods$
$@VP\_V \rightarrow NP\ PP$	$V \rightarrow people$
$S \rightarrow V\ @S\_V$	$S \rightarrow people$
$@S\_V \rightarrow NP\ PP$	$VP \rightarrow people$
$VP \rightarrow V\ PP$	$V \rightarrow fish$
$S \rightarrow V\ PP$	$S \rightarrow fish$
$NP \rightarrow NP\ NP$	$VP \rightarrow fish$
$NP \rightarrow NP\ PP$	$V \rightarrow tanks$
$NP \rightarrow P\ NP$	$S \rightarrow tanks$
$PP \rightarrow P\ NP$	$VP \rightarrow tanks$
	$P \rightarrow with$
	$PP \rightarrow with$

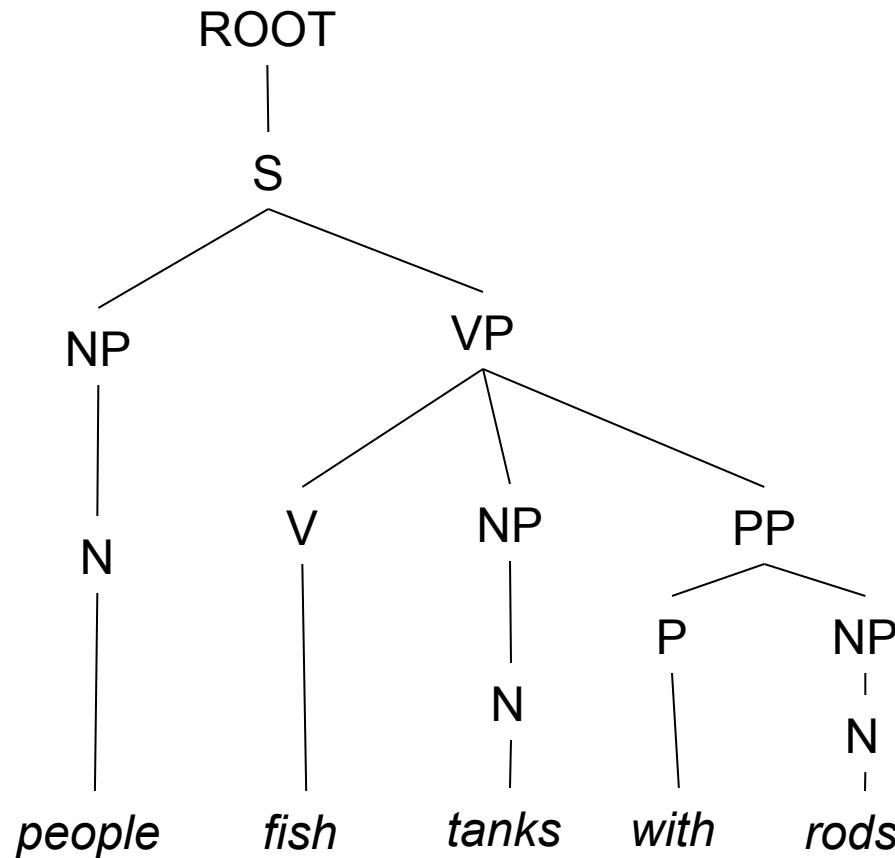


# Chomsky Normal Form

- You should think of this as a transformation for efficient parsing
- With some extra book-keeping in symbol names, you can even reconstruct the same trees with a detransform
- In practice full Chomsky Normal Form is a pain
  - Reconstructing n-aries is easy
  - Reconstructing unaries/empties is trickier
- **Binarization** is crucial for cubic time CFG parsing
- The rest isn't necessary; it just makes the algorithms cleaner and a bit quicker

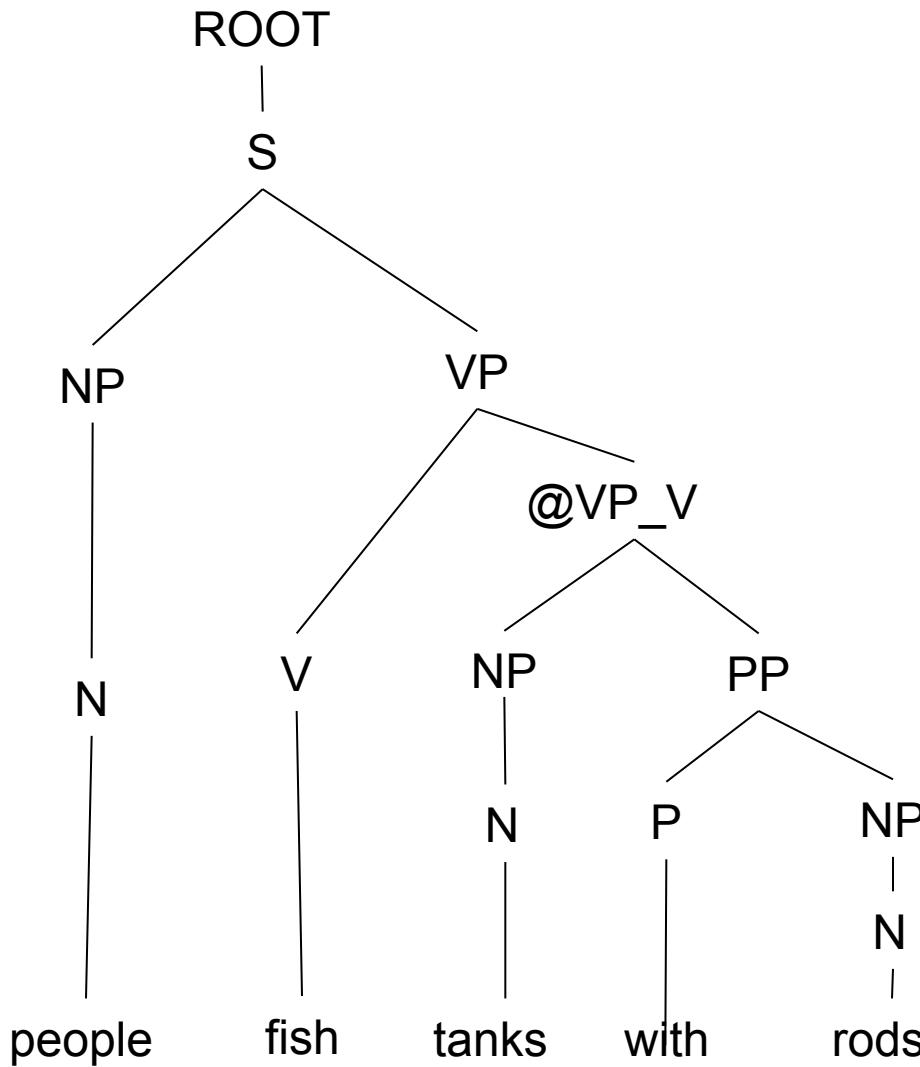


# An example: before binarization...



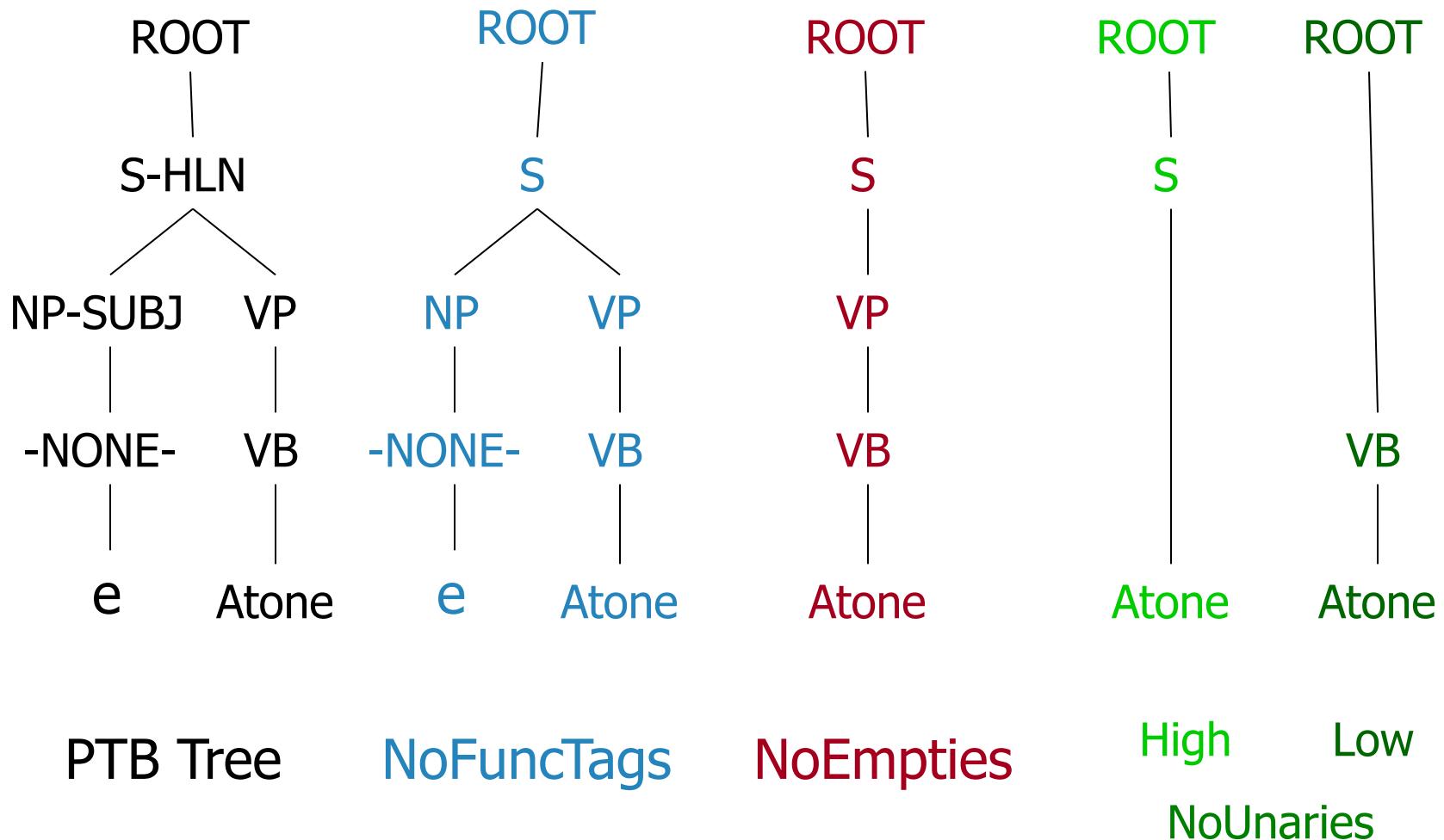


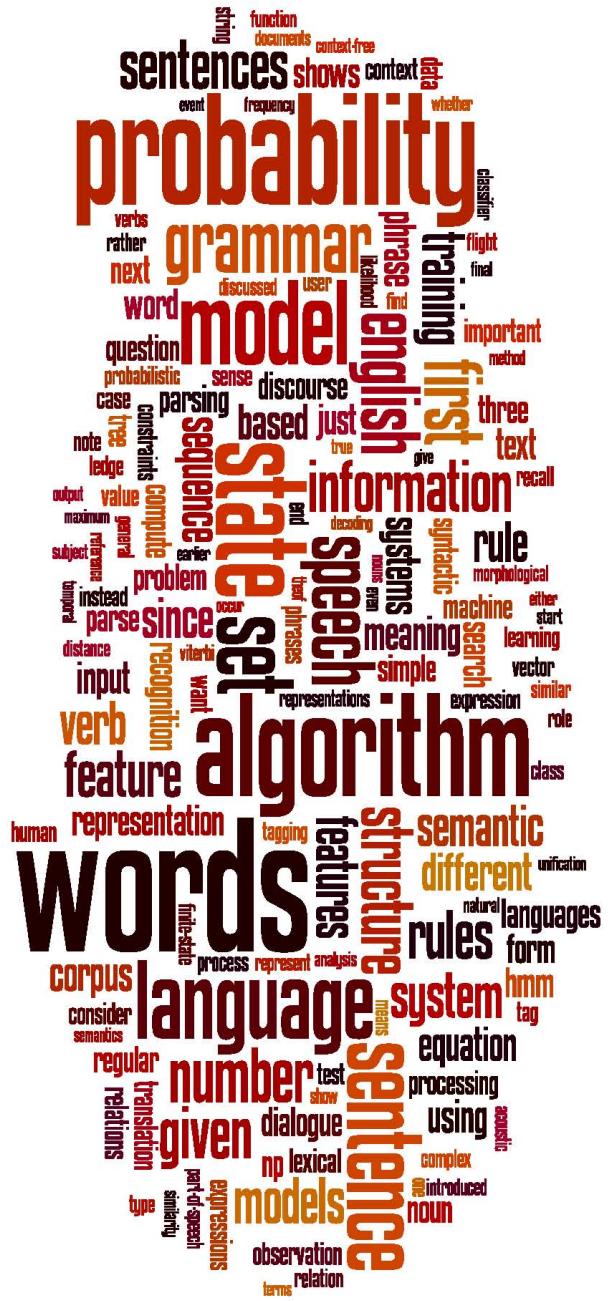
# After binarization...





# Treebank: empties and unaries



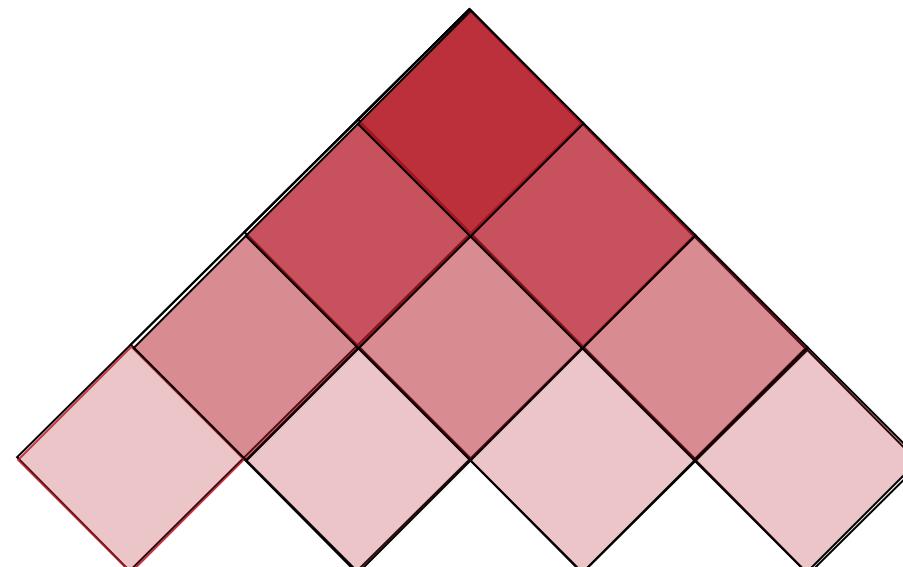


## CKY Parsing

Exact polynomial  
time parsing of  
(P)CFGs



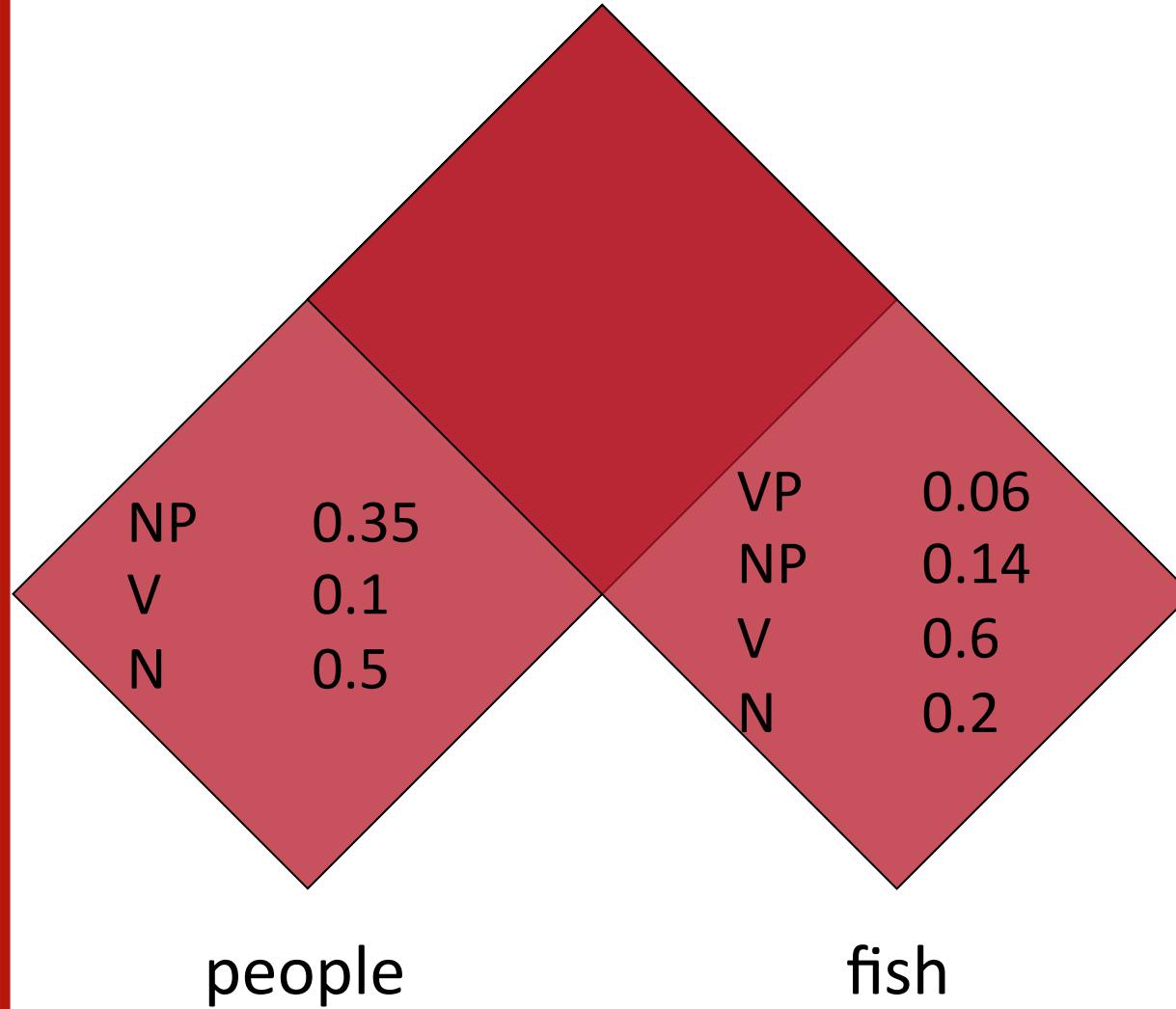
# Cocke-Kasami-Younger (CKY) Constituency Parsing



fish people fish tanks



# Viterbi (Max) Scores



$S \rightarrow NP\ VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V\ NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V\ PP$	0.1
$@VP\_V \rightarrow NP\ PP$	1.0
$NP \rightarrow NP\ NP$	0.1
$NP \rightarrow NP\ PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P\ NP$	1.0



# Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts
  - Doesn't increase complexity; essentially like unaries
- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar
    - Binarization may be an explicit transformation or implicit in how the parser works (Early-style dotted rules), but it's always there.



# The CKY algorithm (1960/1965)

## ... extended to unaries

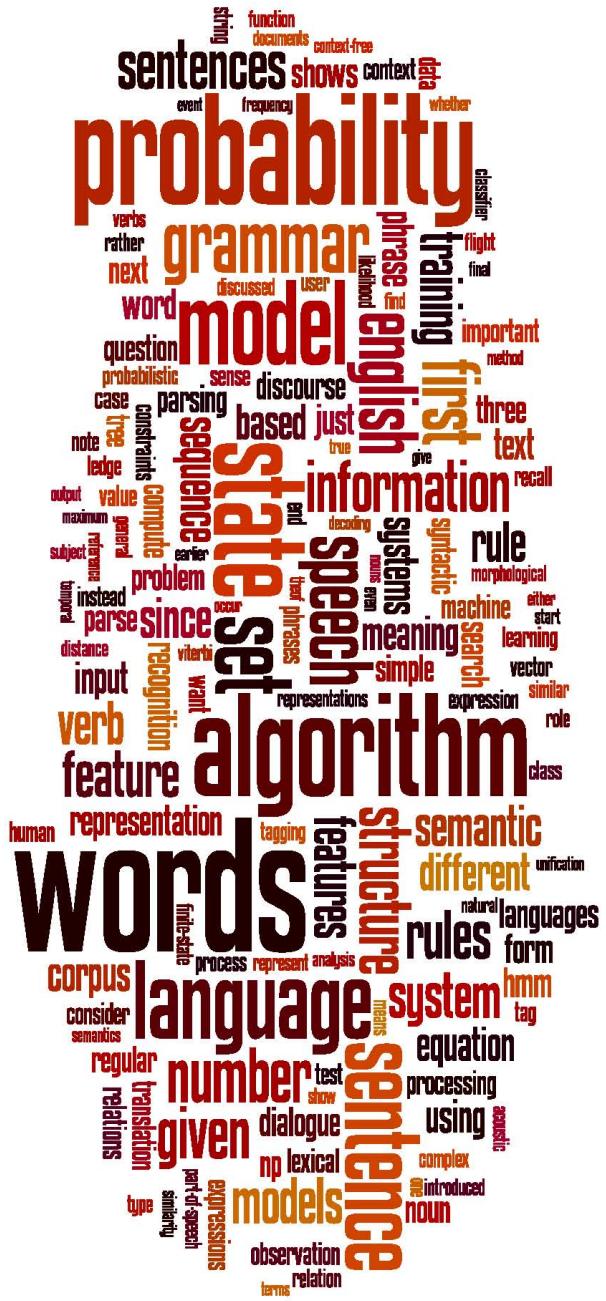
```
function CKY(words, grammar) returns [most_probable_parse, prob]
    score = new double[#{words}+1][#{words}+1][#{nonterms}]
    back = new Pair[#{words}+1][#{words}+1][#{nonterms}]
    for i=0; i<#{words}; i++
        for A in nonterms
            if A -> words[i] in grammar
                score[i][i+1][A] = P(A -> words[i])
        //handle unaries
        boolean added = true
        while added
            added = false
            for A, B in nonterms
                if score[i][i+1][B] > 0 && A->B in grammar
                    prob = P(A->B)*score[i][i+1][B]
                    if prob > score[i][i+1][A]
                        score[i][i+1][A] = prob
                        back[i][i+1][A] = B
                        added = true
```



# The CKY algorithm (1960/1965)

## ... extended to unaries

```
for span = 2 to #(words)
    for begin = 0 to #(words)- span
        end = begin + span
        for split = begin+1 to end-1
            for A,B,C in nonterms
                prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
                if prob > score[begin][end][A]
                    score[begin]end][A] = prob
                    back[begin][end][A] = new Triple(split,B,C)
//handle unaries
boolean added = true
while added
    added = false
    for A, B in nonterms
        prob = P(A->B)*score[begin][end][B];
        if prob > score[begin][end][A]
            score[begin][end][A] = prob
            back[begin][end][A] = B
            added = true
return buildTree(score, back)
```



# CKY Parsing

# A worked example



# The grammar: Binary, no epsilons,

$S \rightarrow NP\ VP$	0.9	$N \rightarrow people$	0.5
$S \rightarrow VP$	0.1	$N \rightarrow fish$	0.2
$VP \rightarrow V\ NP$	0.5	$N \rightarrow tanks$	0.2
$VP \rightarrow V$	0.1	$N \rightarrow rods$	0.1
$VP \rightarrow V @VP\_V$	0.3	$V \rightarrow people$	0.1
$VP \rightarrow V\ PP$	0.1	$V \rightarrow fish$	0.6
$@VP\_V \rightarrow NP\ PP$	1.0	$V \rightarrow tanks$	0.3
$NP \rightarrow NP\ NP$	0.1	$P \rightarrow with$	1.0
$NP \rightarrow NP\ PP$	0.2		
$NP \rightarrow N$	0.7		
$PP \rightarrow P\ NP$	1.0		

	fish	1	people	2	fish	3	tanks	4
0	score[0][1]		score[0][2]		score[0][3]		score[0][4]	
1								
2			score[1][2]		score[1][3]		score[1][4]	
3					score[2][3]		score[2][4]	
4							score[3][4]	

$S \rightarrow NP VP$

0.9

$S \rightarrow VP$

0.1

$VP \rightarrow V NP$

0.5

$VP \rightarrow V$

0.1

$VP \rightarrow V @VP_V$

0.3

$VP \rightarrow V PP$

0.1

$@VP_V \rightarrow NP PP$

1.0

$NP \rightarrow NP NP$

0.1

$NP \rightarrow NP PP$

0.2

$NP \rightarrow N$

0.7

$PP \rightarrow P NP$

1.0

$N \rightarrow people$

0.5

$N \rightarrow fish$

0.2

$N \rightarrow tanks$

0.2

$N \rightarrow rods$

0.1

$V \rightarrow people$

0.1

$V \rightarrow fish$

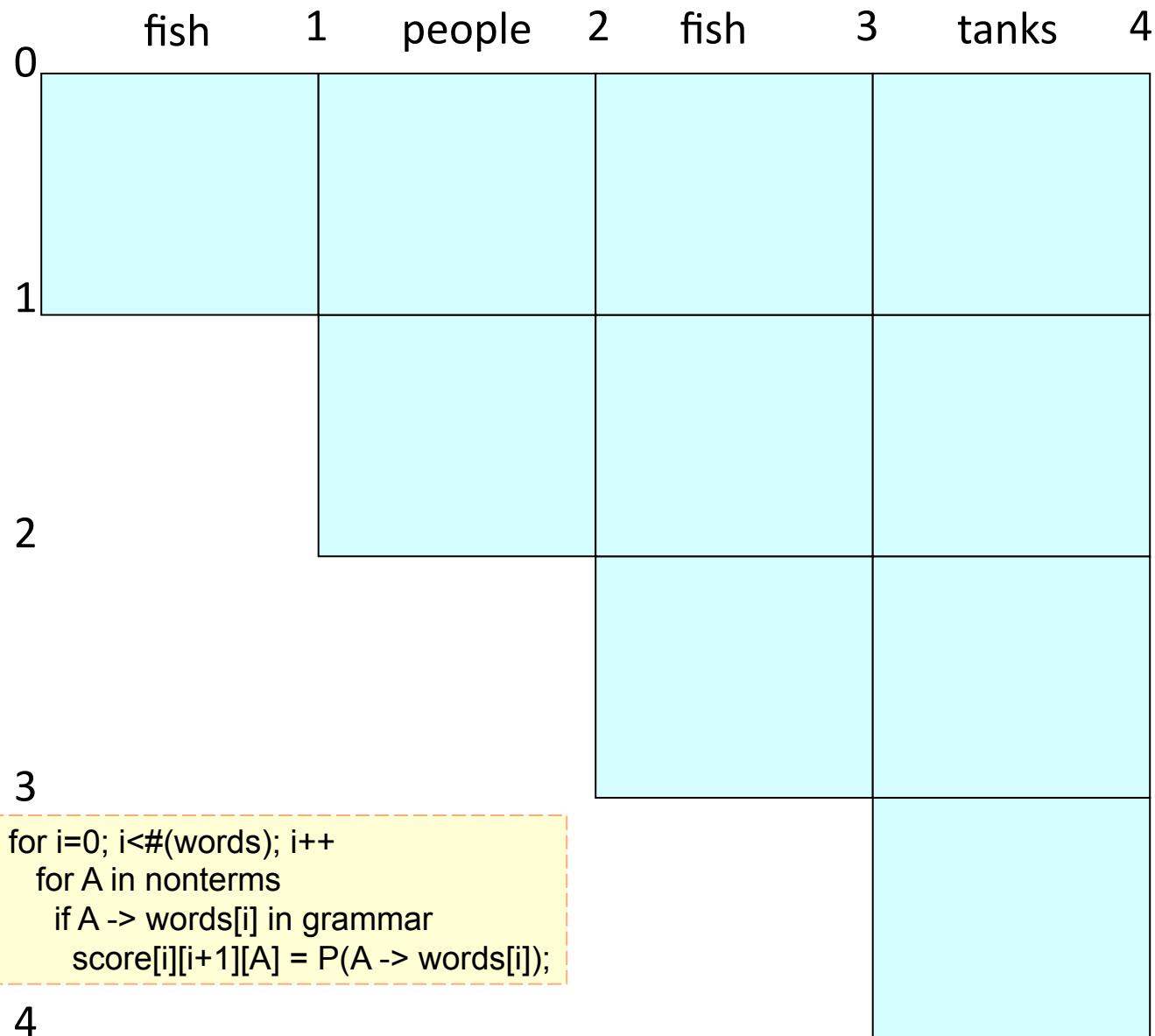
0.6

$V \rightarrow tanks$

0.3

$P \rightarrow with$

1.0



		0	fish	1	people	2	fish	3	tanks	4
$S \rightarrow NP VP$	0.9									
$S \rightarrow VP$	0.1									
$VP \rightarrow V NP$	0.5									
$VP \rightarrow V$	0.1									
$VP \rightarrow V @VP_V$	0.3									
$VP \rightarrow V PP$	0.1									
$@VP_V \rightarrow NP PP$	1.0									
$NP \rightarrow NP NP$	0.1									
$NP \rightarrow NP PP$	0.2									
$NP \rightarrow N$	0.7	2								
$PP \rightarrow P NP$	1.0									
$N \rightarrow people$	0.5									
$N \rightarrow fish$	0.2									
$N \rightarrow tanks$	0.2									
$N \rightarrow rods$	0.1									
$V \rightarrow people$	0.1									
$V \rightarrow fish$	0.6									
$V \rightarrow tanks$	0.3									
$P \rightarrow with$	1.0									
		0	N $\rightarrow$ fish 0.2 V $\rightarrow$ fish 0.6							
		1		N $\rightarrow$ people 0.5 V $\rightarrow$ people 0.1						
		2				N $\rightarrow$ fish 0.2 V $\rightarrow$ fish 0.6				
		3					N $\rightarrow$ tanks 0.2 V $\rightarrow$ tanks 0.1			
		4								

```
// handle unaries
boolean added = true
while added
    added = false
    for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
            prob = P(A->B)*score[i][i+1][B]
            if(prob > score[i][i+1][A])
                score[i][i+1][A] = prob
                back[i][i+1][A] = B
                added = true
```

		fish	people	fish	tanks	
	0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006				4
S → NP VP	0.9					
S → VP	0.1					
VP → V NP	0.5					
VP → V	0.1					
VP → V @VP_V	0.3					
VP → V PP	0.1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001			
@VP_V → NP PP	1.0					
NP → NP NP	0.1					
NP → NP PP	0.2					
NP → N	0.7					
PP → P NP	1.0		N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006			
N → people	0.5					
N → fish	0.2					
N → tanks	0.2					
N → rods	0.1	prob=score[begin][split][B]*score[split][end][C]*P(A->BC) if (prob > score[begin][end][A]) score[begin][end][A] = prob back[begin][end][A] = new Triple(split,B,C)				
V → people	0.1					
V → fish	0.6					
V → tanks	0.3					
P → with	1.0					

		fish	people	fish	tanks	
	0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → NP VP 0.00126			4
S → NP VP	0.9					
S → VP	0.1					
VP → V NP	0.5					
VP → V	0.1					
VP → V @VP_V	0.3					
VP → V PP	0.1					
@VP_V → NP PP	1.0					
NP → NP NP	0.1					
NP → NP PP	0.2					
NP → N	0.7					
PP → P NP	1.0					
N → people	0.5	<pre>//handle unaries boolean added = true while added     added = false     for A, B in nonterms         prob = P(A-&gt;B)*score[begin][end][B];         if prob &gt; score[begin][end][A]             score[begin][end][A] = prob             back[begin][end][A] = B             added = true</pre>				
N → fish	0.2					
N → tanks	0.2					
N → rods	0.1					
V → people	0.1					
V → fish	0.6					
V → tanks	0.3					
P → with	1.0					

		fish	1	people	2	fish	3	tanks	4
$S \rightarrow NP VP$	0.9	0	$N \rightarrow fish 0.2$ $V \rightarrow fish 0.6$ $NP \rightarrow N 0.14$ $VP \rightarrow V 0.06$ $S \rightarrow VP 0.006$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105					
$S \rightarrow VP$	0.1								
$VP \rightarrow V NP$	0.5								
$VP \rightarrow V$	0.1								
$VP \rightarrow V @VP\_V$	0.3	1							
$VP \rightarrow V PP$	0.1			$N \rightarrow people 0.5$ $V \rightarrow people 0.1$ $NP \rightarrow N 0.35$ $VP \rightarrow V 0.01$ $S \rightarrow VP 0.001$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189				
$@VP\_V \rightarrow NP PP$	1.0								
$NP \rightarrow NP NP$	0.1								
$NP \rightarrow NP PP$	0.2								
$NP \rightarrow N$	0.7	2							
$PP \rightarrow P NP$	1.0								
$N \rightarrow people$	0.5								
$N \rightarrow fish$	0.2								
$N \rightarrow tanks$	0.2	3							
$N \rightarrow rods$	0.1								
$V \rightarrow people$	0.1								
$V \rightarrow fish$	0.6								
$V \rightarrow tanks$	0.3								
$P \rightarrow with$	1.0	4							

```

for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```

		fish	1	people	2	fish	3	tanks	4
$S \rightarrow NP VP$	0.9	0	$N \rightarrow fish 0.2$ $V \rightarrow fish 0.6$ $NP \rightarrow N 0.14$ $VP \rightarrow V 0.06$ $S \rightarrow VP 0.006$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.00147 $S \rightarrow NP VP$ 0.000882				
$S \rightarrow VP$	0.1			$N \rightarrow people 0.5$ $V \rightarrow people 0.1$ $NP \rightarrow N 0.35$ $VP \rightarrow V 0.01$ $S \rightarrow VP 0.001$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189				
$VP \rightarrow V NP$	0.5				$N \rightarrow fish 0.2$ $V \rightarrow fish 0.6$ $NP \rightarrow N 0.14$ $VP \rightarrow V 0.06$ $S \rightarrow VP 0.006$	$NP \rightarrow NP NP$ 0.00196 $VP \rightarrow V NP$ 0.042 $S \rightarrow VP$ 0.0042			
$VP \rightarrow V$	0.1								
$VP \rightarrow V @VP\_V$	0.3								
$VP \rightarrow V PP$	0.1								
$@VP\_V \rightarrow NP PP$	1.0								
$NP \rightarrow NP NP$	0.1								
$NP \rightarrow NP PP$	0.2								
$NP \rightarrow N$	0.7	2							
$PP \rightarrow P NP$	1.0								
$N \rightarrow people$	0.5								
$N \rightarrow fish$	0.2								
$N \rightarrow tanks$	0.2	3							
$N \rightarrow rods$	0.1								
$V \rightarrow people$	0.1								
$V \rightarrow fish$	0.6								
$V \rightarrow tanks$	0.3								
$P \rightarrow with$	1.0	4							

```

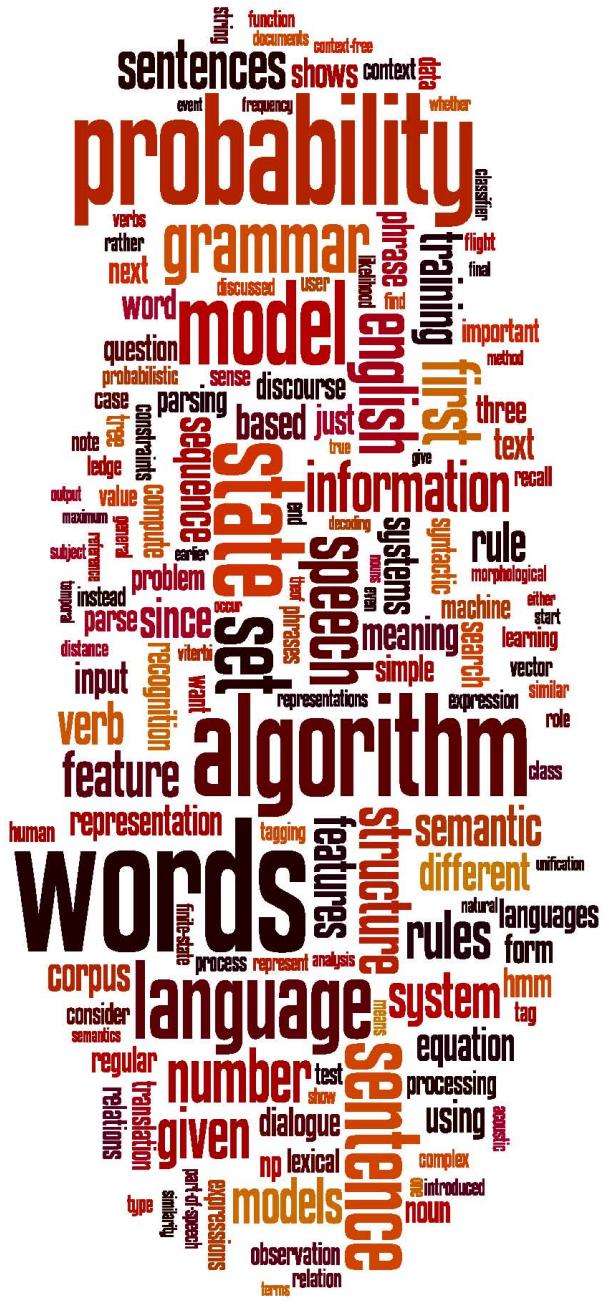
for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```

		fish	1	people	2	fish	3	tanks	4
$S \rightarrow NP VP$	0.9	0	$N \rightarrow fish 0.2$ $V \rightarrow fish 0.6$ $NP \rightarrow N 0.14$ $VP \rightarrow V 0.06$ $S \rightarrow VP 0.006$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.00147 $S \rightarrow NP VP$ 0.000882				
$S \rightarrow VP$	0.1			$N \rightarrow people 0.5$ $V \rightarrow people 0.1$ $NP \rightarrow N 0.35$ $VP \rightarrow V 0.01$ $S \rightarrow VP 0.001$	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.000098 $S \rightarrow NP VP$ 0.01323			
$VP \rightarrow V NP$	0.5				$N \rightarrow fish 0.2$ $V \rightarrow fish 0.6$ $NP \rightarrow N 0.14$ $VP \rightarrow V 0.06$ $S \rightarrow VP 0.006$	$NP \rightarrow NP NP$ 0.00196 $VP \rightarrow V NP$ 0.042 $S \rightarrow VP$ 0.0042			
$VP \rightarrow V$	0.1								
$VP \rightarrow V @VP\_V$	0.3								
$VP \rightarrow V PP$	0.1								
$@VP\_V \rightarrow NP PP$	1.0								
$NP \rightarrow NP NP$	0.1								
$NP \rightarrow NP PP$	0.2								
$NP \rightarrow N$	0.7	2							
$PP \rightarrow P NP$	1.0								
$N \rightarrow people$	0.5								
$N \rightarrow fish$	0.2								
$N \rightarrow tanks$	0.2	3							
$N \rightarrow rods$	0.1								
$V \rightarrow people$	0.1								
$V \rightarrow fish$	0.6								
$V \rightarrow tanks$	0.3								
$P \rightarrow with$	1.0	4							

```

for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```

		fish	people	fish	tanks	
	0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105	NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882	NP → NP NP 0.0000009604 VP → V NP 0.00002058 S → NP VP 0.00018522	4
	1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323	2
	2			N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	3
	3				N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	4
P → with	4		Call buildTree(score, back) to get the best parse			
S → NP VP	0.9					
S → VP	0.1					
VP → V NP	0.5					
VP → V	0.1					
VP → V @VP_V	0.3					
VP → V PP	0.1					
@VP_V → NP PP	1.0					
NP → NP NP	0.1					
NP → NP PP	0.2					
NP → N	0.7					
PP → P NP	1.0					
N → people	0.5					
N → fish	0.2					
N → tanks	0.2					
N → rods	0.1					
V → people	0.1					
V → fish	0.6					
V → tanks	0.3					
P → with	1.0					

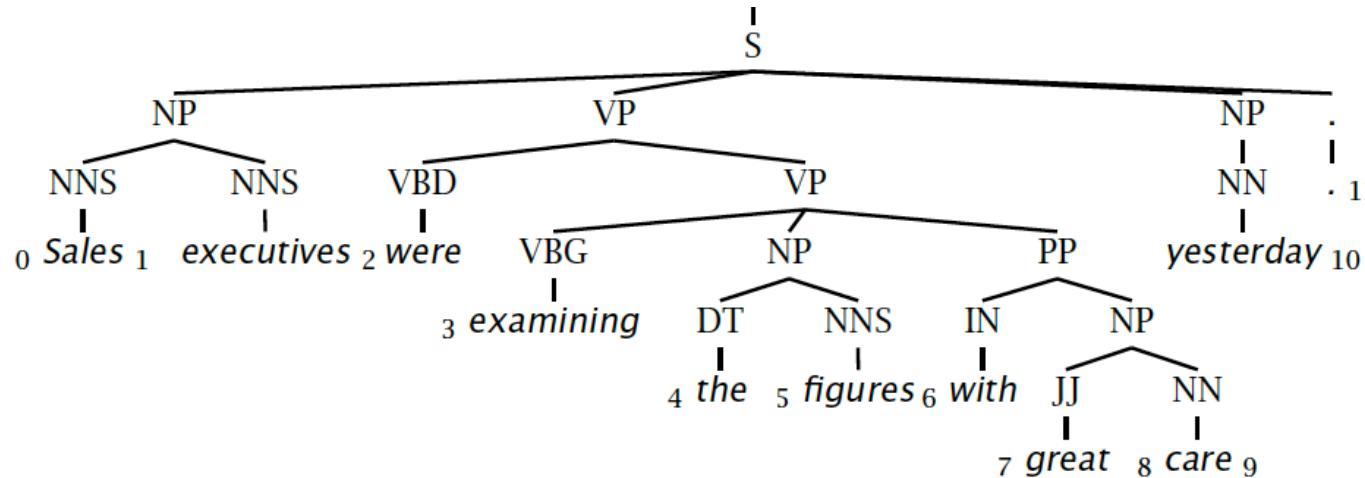


# Constituency Parser Evaluation

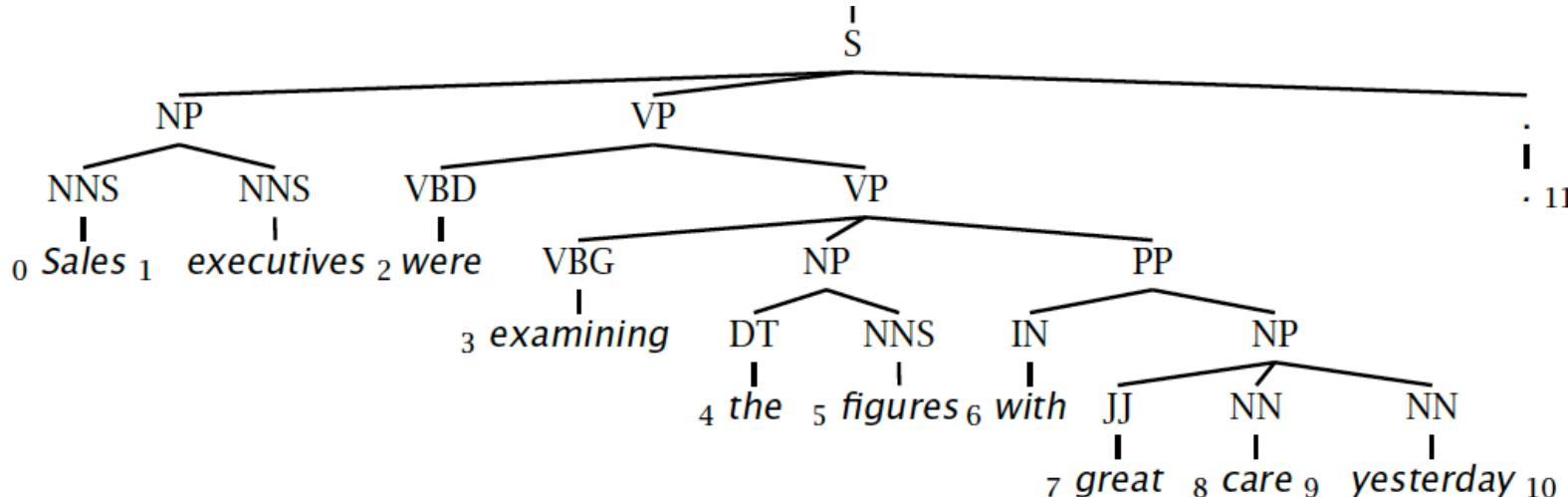


# Evaluating constituency parsing

Gold standard brackets: S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), NP-(4:6), PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:6), PP-(6-10), NP-(7,10)





# Evaluating constituency parsing

**Gold standard brackets:**

**S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), NP-(4:6), PP-(6-9), NP-(7,9), NP-(9:10)**

**Candidate brackets:**

**S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:6), PP-(6-10), NP-(7,10)**

Labeled Precision             $3/7 = 42.9\%$

Labeled Recall               $3/8 = 37.5\%$

LP/LR F1                    40.0%

Tagging Accuracy             $11/11 = 100.0\%$



# How good are PCFGs?

- Penn WSJ parsing accuracy: about 73% LP/LR F1
- Robust
  - Usually admit everything, but with low probability
- Partial solution for grammar ambiguity
  - A PCFG gives some idea of the plausibility of a parse
  - But not so good because the independence assumptions are too strong
- Give a probabilistic language model
  - But in the simple case it performs worse than a trigram model
- The problem seems to be that PCFGs lack the lexicalization of a trigram model