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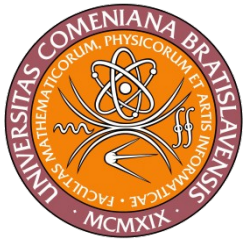


Neural Networks

Lecture 7

Sequential modeling

Igor Farkaš

2024

Temporal processing with neural networks

Required for tasks with temporal structure – where the same input can be associated with different outputs

e.g. $A \rightarrow \alpha$, $B \rightarrow \beta$, $B \rightarrow \alpha$, $B \rightarrow \gamma$, ...

Types of tasks:

- Sequence recognition (classification)
- Sequence prediction
- Sequence generation

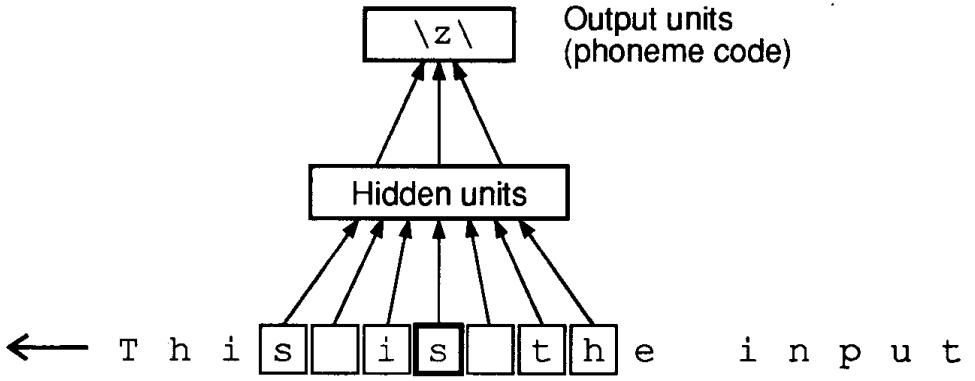
Incorporating time into a NN:

- tapped-delay input ~ time as spatial dimension
- using recurrent NN architecture
  - associative memory
  - temporal input-output mapping
- new: attention

Time as spatial dimension: NETtalk

NN learns to read English text

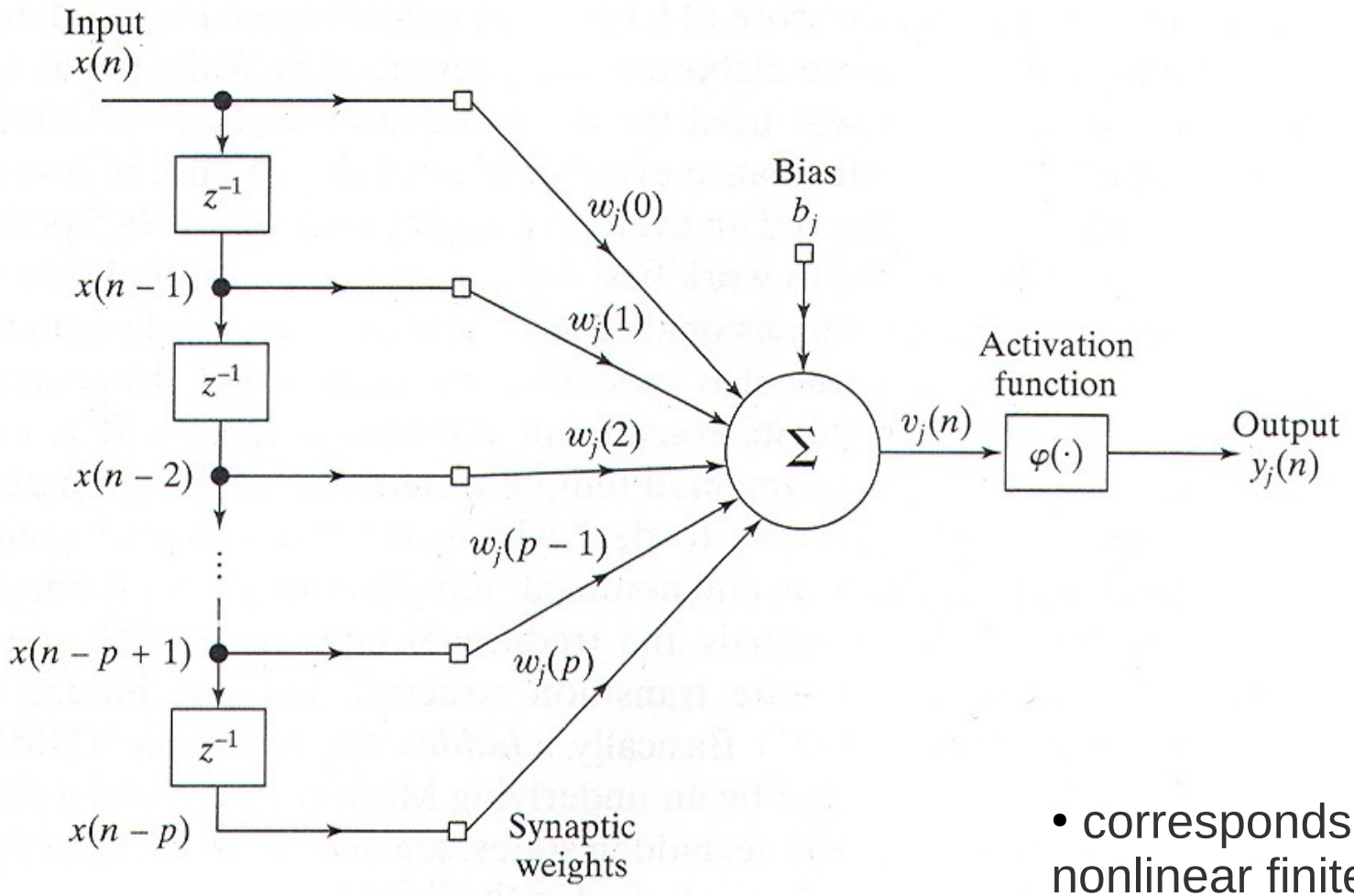
Input 7×29 units encoding 7 chars,
80 hidden and 26 output units
encoding phonemes.



- training on 1024 words, intelligible speech after 10 epochs, 95% accuracy after 50 epochs
- 78% generalization accuracy, quite intelligible speech
- NN first learned gross features (e.g. word boundaries) and then gradually refined its discrimination (psycholinguistic plausibility).
- graceful degradation
- meaningful internal representations (e.g. vowel-consonant distinction)
- General question: How to estimate delay from training data?

(Sejnowski & Rosenberg, 1987)

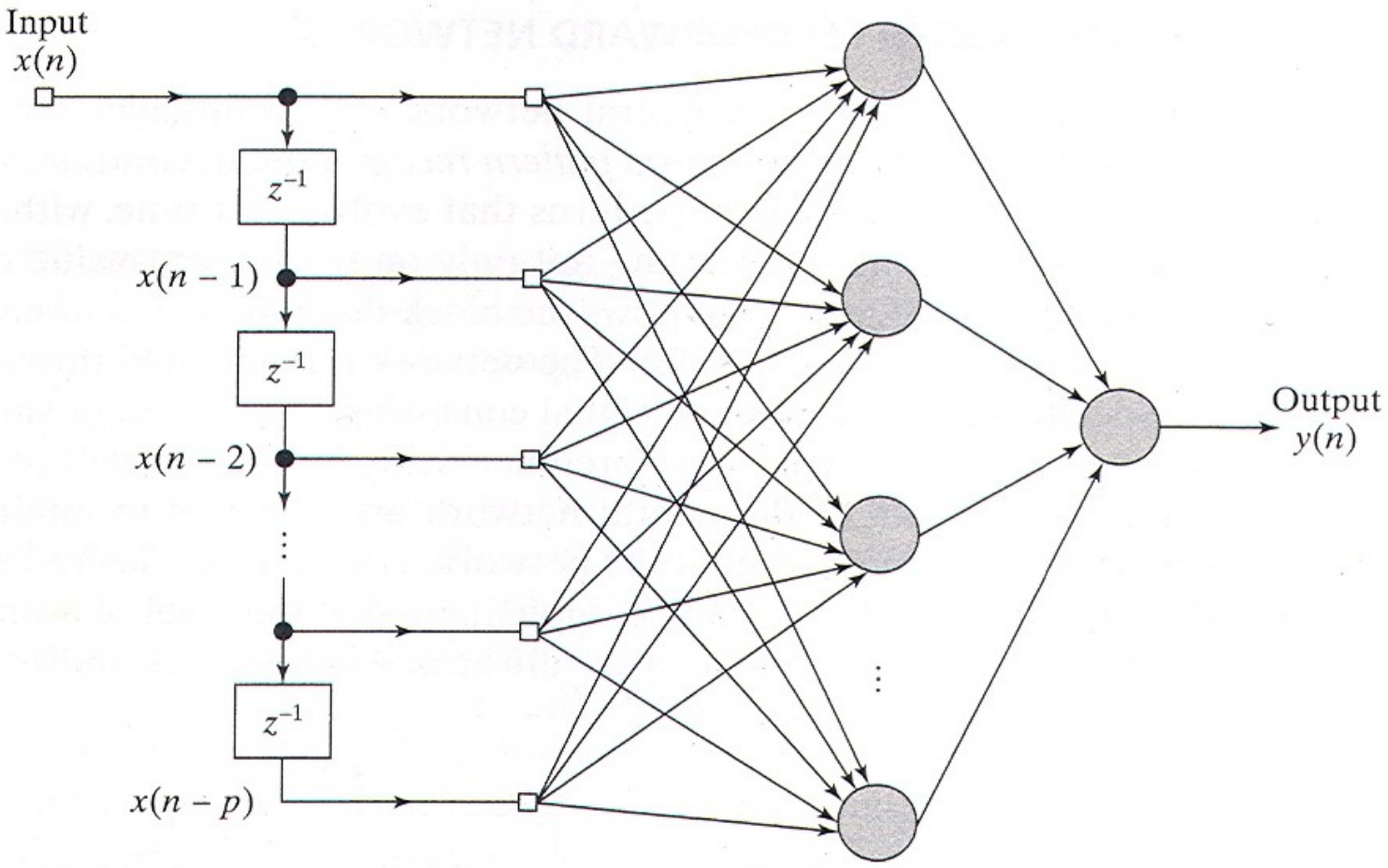
Focused neuronal filter



- focused because all memory comes from the input
- can be trained as ordinary feedforward NN

- corresponds to a nonlinear finite impulse response (FIR) filter (in digital signal processing)

Focused time feedforward NN

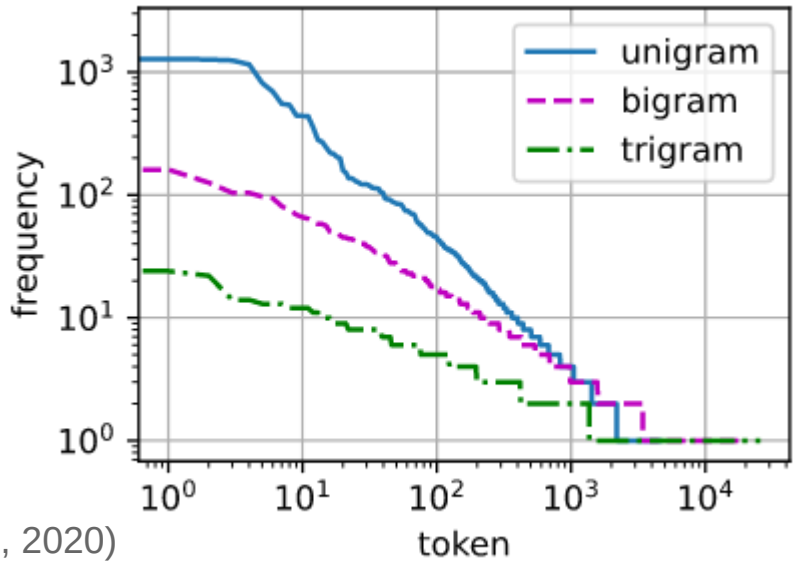


- usable for stationary input-output mapping tasks
- can be trained as ordinary feedforward NN
- time a as spatial dimension

Link to statistical n-gram models

- Assume 1D time series $X = x_1, x_2, \ldots, x_{t-1}, \ldots$
- Autoregressive predictive model: $p(x_t \mid x_{t-1}, \ldots, x_1)$
- Simplified under Markov assumption: $p(x_t \mid x_{t-1}, \ldots, x_{t-n})$ for $n > 0$
- Useful in language models: estimate $p(w_1, w_2, \ldots, w_T)$
- Probability of the next word (in text): $w_t \sim p(w_t \mid w_{t-1}, \ldots, w_{t-n})$

- Zipf's law governs the word distribution for not only unigrams but also the other *n*-grams:
- high accuracy of NLP models thank to large corpora
- time-lagged NNs ~ *n*-grams



(Zhang et al, 2020)

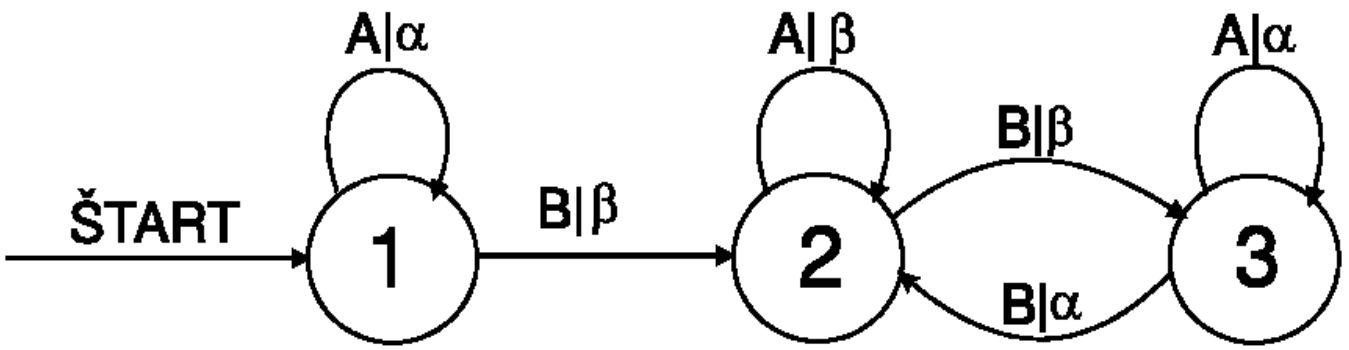
Tasks for time-lagged NNs

- time series prediction (single-step, multi-step)
- noise cancellation
- adaptive control
- speech recognition
- system identification

BUT... what if the required memory may be unlimited?

- time-lagged NNs have no feedback
- global feedback = facilitator of computational intelligence
  (Haykin, 2009)

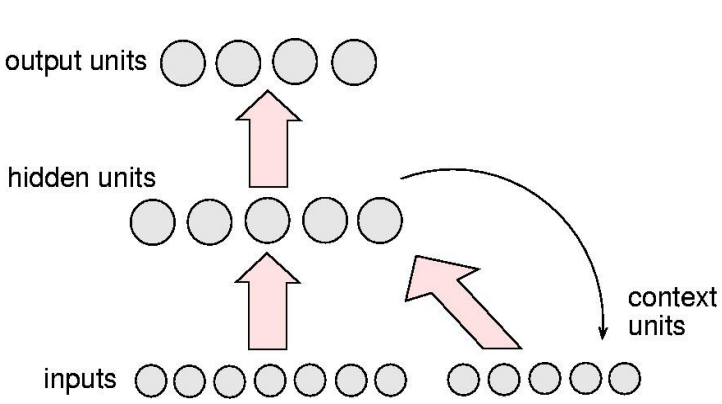
Example: Mealy automaton



(Tiňo, 1997)

- Inputs: $\{A,B\}$, outputs: $\{\alpha,\beta\}$
- Training set: $A\rightarrow\alpha,\ A\rightarrow\alpha,\ B\rightarrow\beta,\ B\rightarrow\beta,\ B\rightarrow\alpha,\ A\rightarrow\beta,\ A\rightarrow\beta,\ A\rightarrow\beta$
- no sufficient tapped-line can reliably be set, so as to learn the behavior
- State representation of temporal context is more appropriate than “past window” → recurrent models

Simple recurrent network (SRN)



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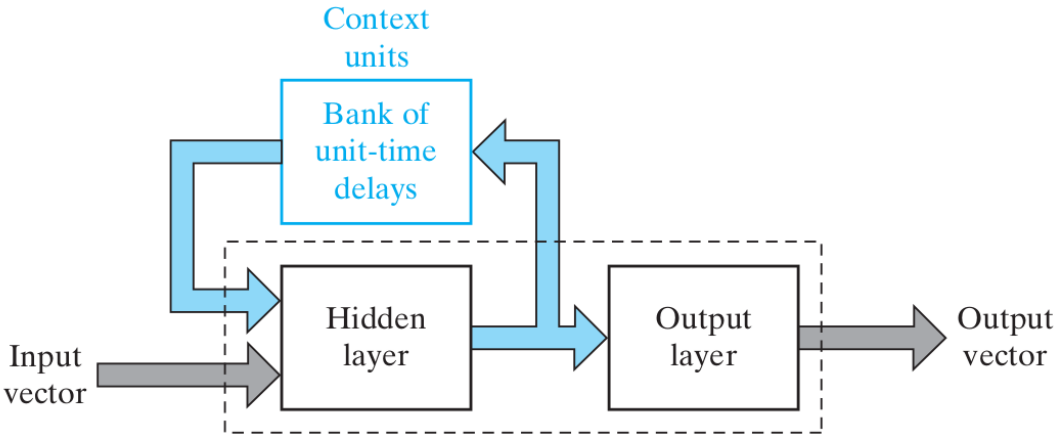


Hidden state activation:

$$h_k(t) = f\left(\sum_j w_{kj} x_j(t) + \sum_l c_{kl} h_l(t-1)\right)$$

Output activation: $$y_i(t) = f\left(\sum_k v_{ik} h_k(t)\right)$$

Unit's activation function:

$$f(u) = \frac{1}{1+\exp(-u)}$$

Latent variable model: $p(x_t \mid x_{t-1}, \ldots, x_1) \approx p(x_t \mid h_{t-1})$

Can be trained by various gradient algorithms (BP, BPTT, RTRL,…)

- SRN is a partially recurrent, state-space model
- time is implicitly represented

(Elman, 1990)

Example: Next letter prediction task

Task: letter-in-word prediction (sequence of symbols), 5-bit inputs

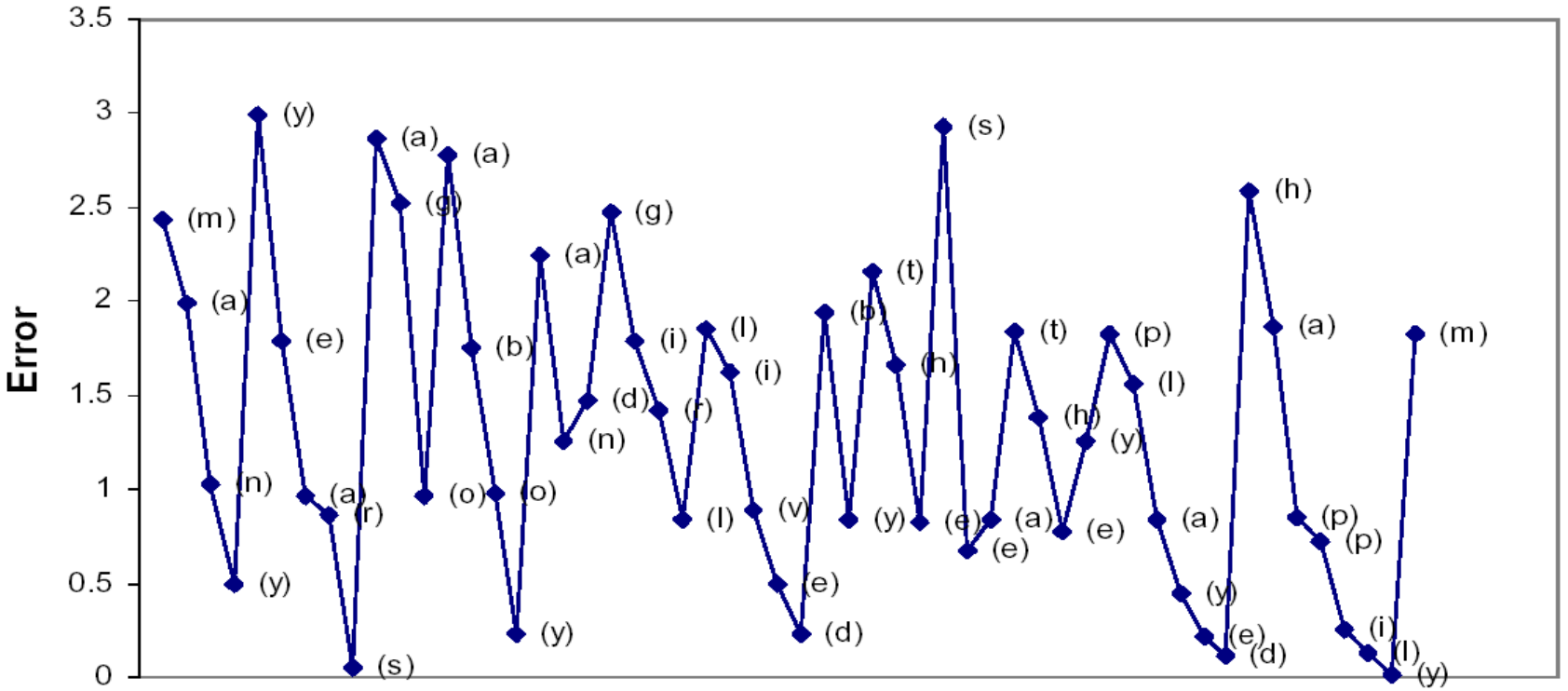
Data: 200 sentences, 4 to 9 words in a sentence

SRN: 5-20-5 units, trained by back-propagation (Rumelhart, Hinton & Williams, 1986)

- SRN discovers the notion of “word”

Many years ago boy and girl lived by the sea ...

(Elman, 1990)



Example: Next word prediction task

Categories of lexical items used

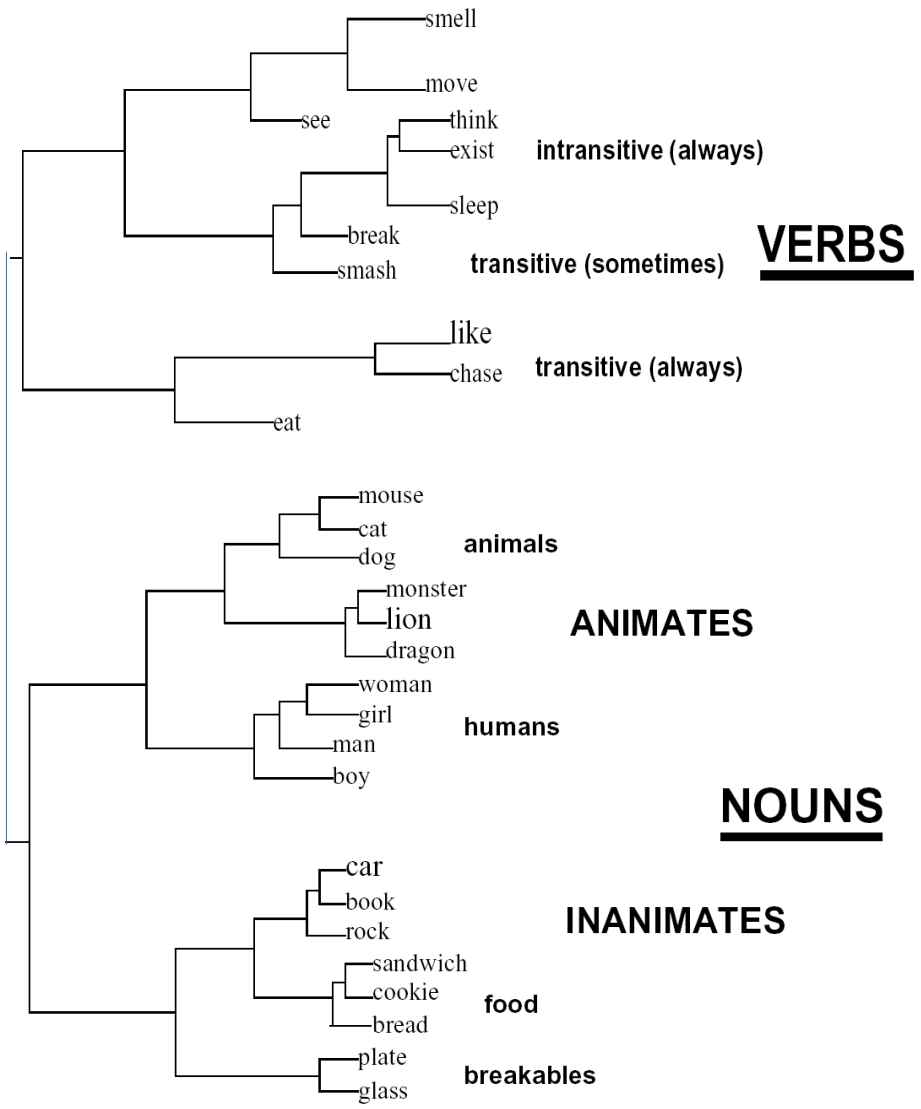
Category	Examples
NOUN-HUM	man, woman
NOUN-ANIM	cat, mouse
NOUN-INANIM	book, rock
NOUN-AGRESS	dragon, monster
NOUN-FRAG	glass, plate
NOUN-FOOD	cookie, sandwich
VERB-INTRAN	think, sleep
VERB-TRAN	see, chase
VERB-AGPA	move, break
VERB-PERCEPT	smell, see
VERB-DESTROY	break, smash
VERB-EA	eat

Templates for sentence generator

WORD 1	WORDS	WORD 3
NOUN-HUM	VERB-EAT	NOUN-FOOD
NOUN-HUM	VERB-PERCEPT	NOUN-INANIM
NOUN-HUM	VERB-DESTROY	NOUN-FRAG
NOUN-HUM	VERB-INTRAN	
NOUN-HUM	VERB-TRAN	NOUN-HUM
NOUN-HUM	VERB-AGPAT	NOUN-INANIM
NOUN-HUM	VERB-AGPAT	
NOUN-ANIM	VERB-EAT	NOUN-FOOD
NOUN-ANIM	VERB-TRAN	NOUN-ANIM

SRN: 31-150-31, localist encoding of words

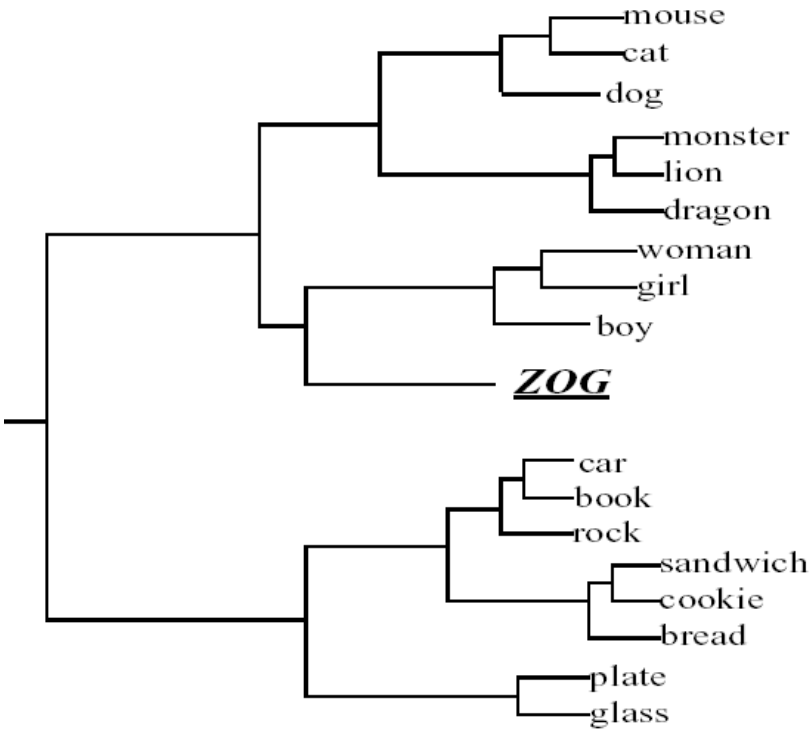
Averaged hidden-unit activation vectors

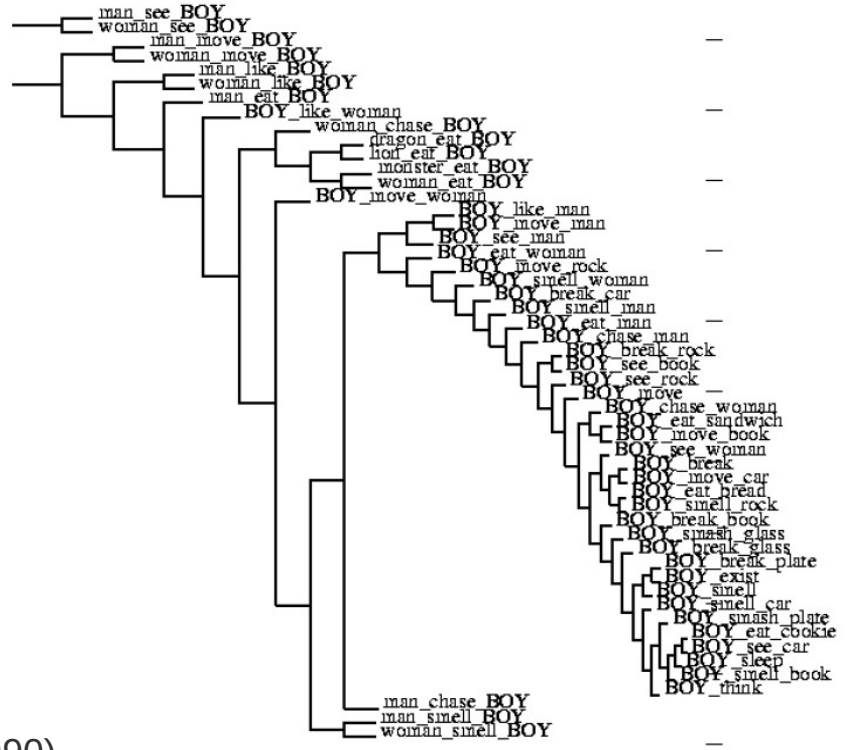


(Elman, 1990)

Properties of hidden-unit activations after training

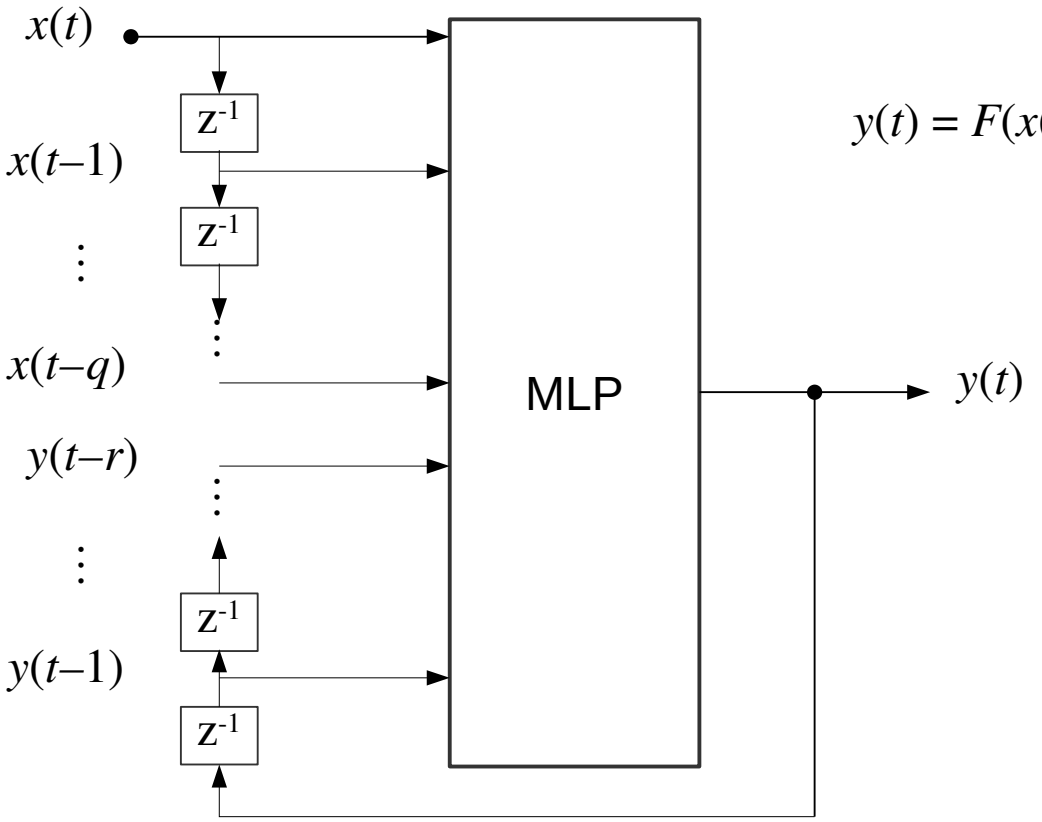
- activations show structure (clusters) in 150-dim. space
- types/tokens distinction: types = centroids of tokens
- representations are hierarchically structured
- representation space would not grow with a growing lexicon
- type vector for a novel word (*zog*) consistent with previous knowledge





(Elman, 1990)

Input–output recurrent model (NARX)



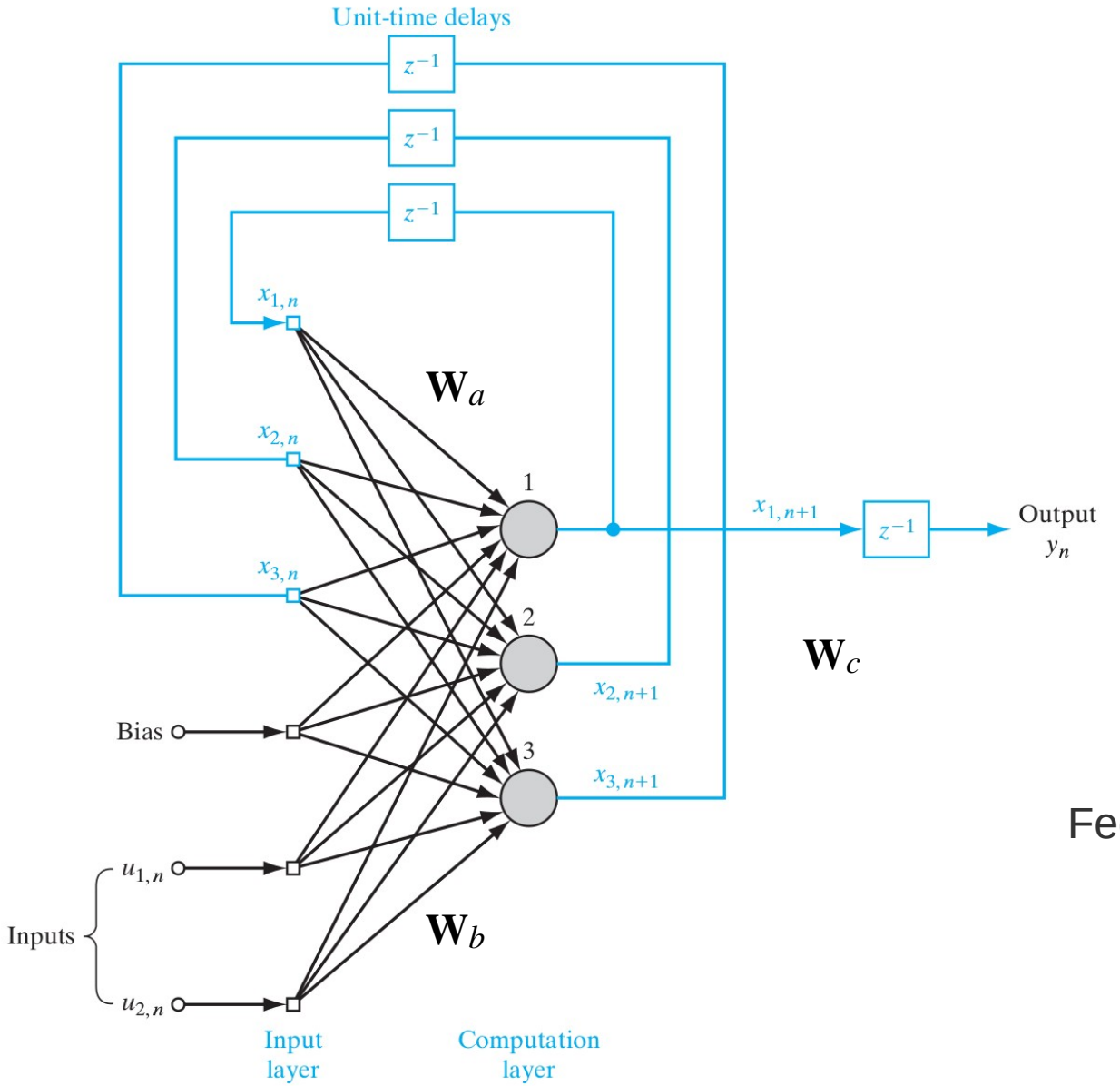
$$y(t) = F(x(t), x(t-1), \ldots, x(t-q), y(t-1), \ldots, y(t-r))$$

- Generic single-input single-output recurrent architecture
- NARX (nonlinear autoregressive model with exogenous inputs
  (Chen et al, 1990)

Learning algorithms for fully recurrent NNs

- dynamically driven RNNs, with a global feedback
  - acquire (internal) state representations
- (similarly to spatial tasks) two modes: (Williams & Zipser, 1995)
  - epochwise training: epoch = sequence
  - continuous training
- We mention two gradient based algorithms:
  - BPTT (epochwise or continuous), and RTRL (continuous)
- Heuristics: (Giles, 1996)
  - start with shorter sequences, then increase length
  - update weights only if training error is larger than threshold
  - consider regularization (e.g. weight decay)

Example of a fully recurrent NN



$$\mathbf{W}_a = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$$

$$\mathbf{W}_b = \begin{bmatrix} b_1 & w_{14} & w_{15} \\ b_2 & w_{24} & w_{25} \\ b_3 & w_{34} & w_{35} \end{bmatrix}$$

$$\mathbf{W}_c = [1, \quad 0, \quad 0]$$

Feedback connections are global

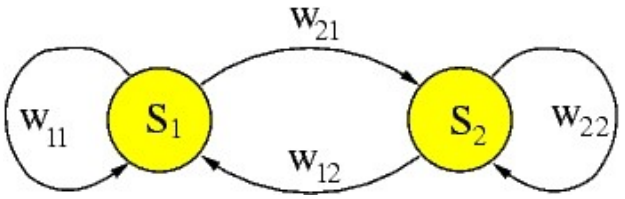
(Haykin, 2009)

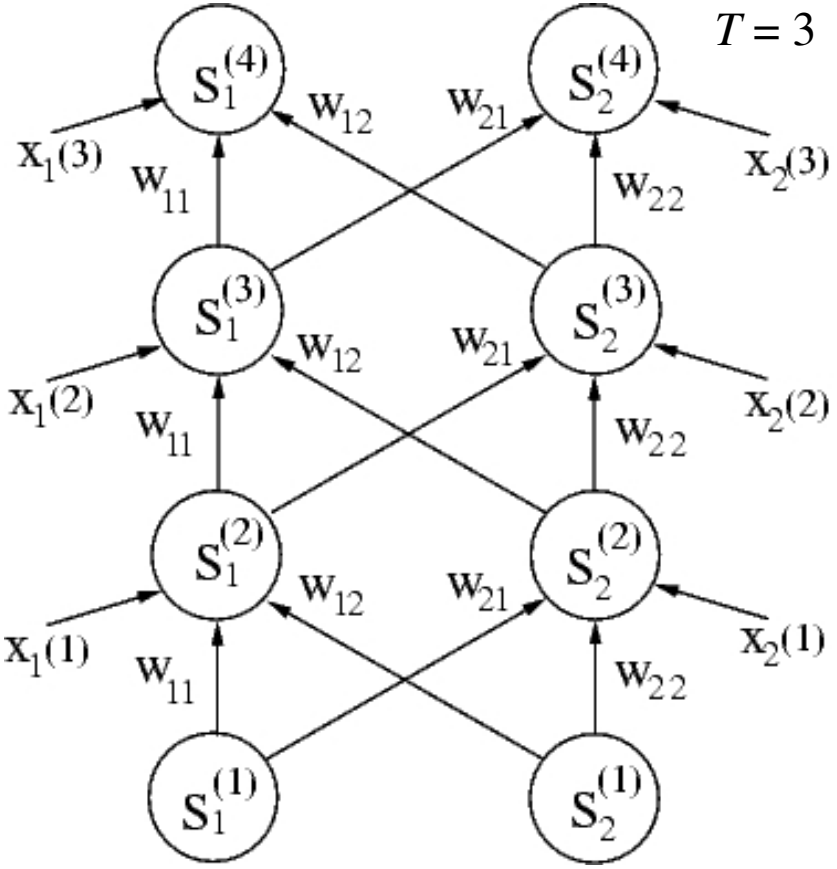
Back-propagation through time

- extension of standard BP algorithm – unfolding in time into a feedforward NN with identical weights (Werbos, 1990)
- sequence with inputs $\boldsymbol{x}(1), \boldsymbol{x}(2), \ldots, \boldsymbol{x}(T)$

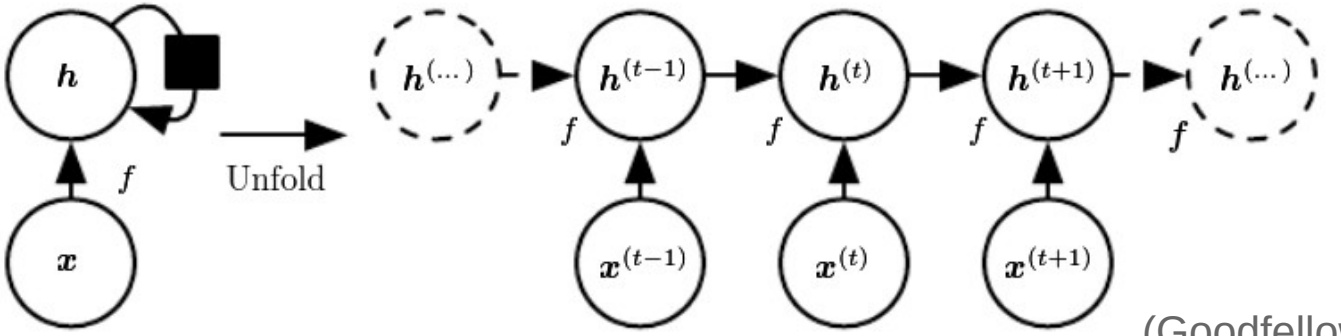
State equation:

$$s_i(t+1) = f\left(\sum_j w_{ij} s_j(t) + x_i(t)\right),$$
$$[in\ example\ \ i, j \in \{1,2\}]$$





(Tiňo, 1997)



(Goodfellow et al, 2016)

BPTT algorithm

- applied after processing each sequence (of length T)
- during single forward pass through a sequence: $\boldsymbol{x}(1), \boldsymbol{x}(2), ..., \boldsymbol{x}(T)$
  - record inputs, local gradients δ
- Overall error: $E_{\text{total}}(T) = \frac{1}{2} \sum_{t=1}^{T} \sum_{i \in Output} e_i^2(t)$
- for $t = T$: $\delta_i(t) = f'(o_i)\, e_i(t)$

  for $1 < t < T$: $\delta_i(t) = f'(o_i)\, [e_i(t) + \sum_{l \in Output} w_{il}\, \delta_l(t+1)]$
- Update weights: $\Delta w_{ij} = -\alpha\, \partial E_{\text{total}}(T) / \partial w_{ij} = \alpha \sum_{t=2}^{T} \delta_i(t)\, s_j(t-1)$

- impractical for longer sequences (of unknown length)
- truncated BPTT possible (Williams and Peng, 1990)

Real-time recurrent learning (RTRL)

- Units in discrete time: $s_i(t+1) = f\left(\sum_j w_{ij} s_j(t) + x_i(t)\right)$
- Instantaneous output error: $e_i(t) = d_i(t) - s_i(t);\ \ i \in O$ (where targets exist)

  $E(t) = \frac{1}{2} \sum_{k \in O} e_k^2(t)$
- Update weights:

$$\Delta w_{ij}(t) = -\alpha \frac{\partial E(t)}{\partial w_{ij}} = \alpha \sum_{k \in O} e_k(t) \frac{\partial s_k(t)}{\partial w_{ij}}$$

$$\frac{\partial s_k(t)}{\partial w_{ij}} = f'(o_k(t)).\left[\delta_{ik}^{kr} s_j(t-1) + \sum_l w_{kl} \frac{\partial s_l(t-1)}{\partial w_{ij}}\right]$$

$l \in$ units feeding to unit k, and $\delta^{kr}_{ik} = 1$, if $k = i$, else 0.

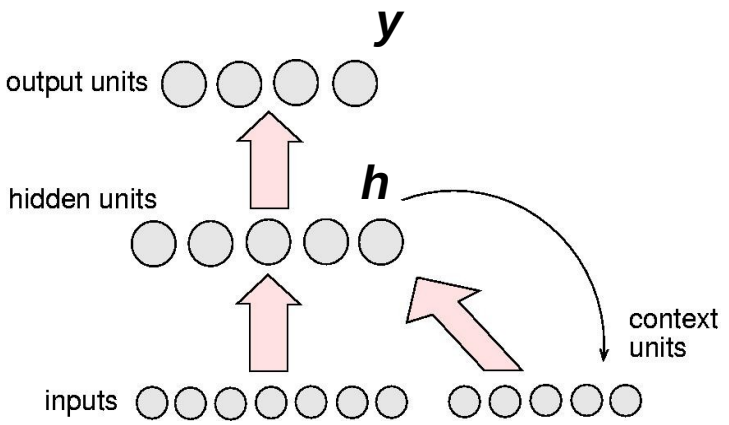
- if j pertains to an external input, $x_j(t-1)$ is used instead of $s_j(t-1)$

- Teacher forcing – replace actual output with desired whenever available (may lead to faster training and enhance learning capability)
- Very large time and memory requirements (with N neurons, each iteration): N^3 derivatives, $O(N^4)$ updates to maintain

(Williams & Zipser, 1989)

RNN state space organization

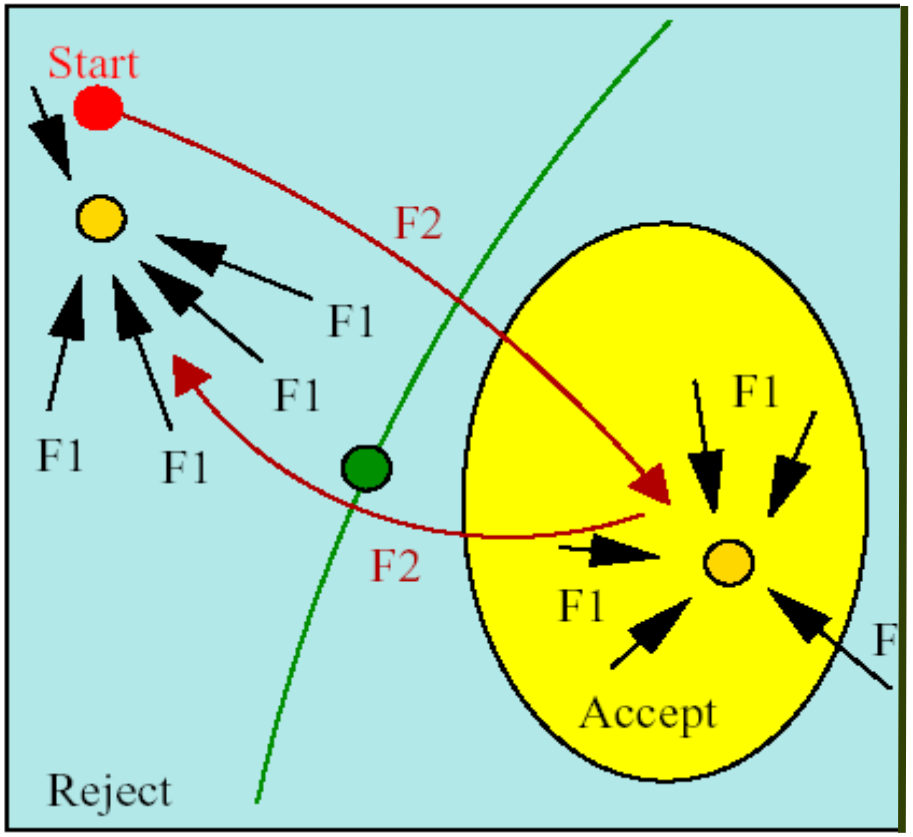
- ***y*** = **F**(***h***) is a squashed version of a linear transformation => it is smooth and monotonic
- Activations ***h*** leading to the same/similar output ***y*** are forced to lie close to each other in the RNN state space



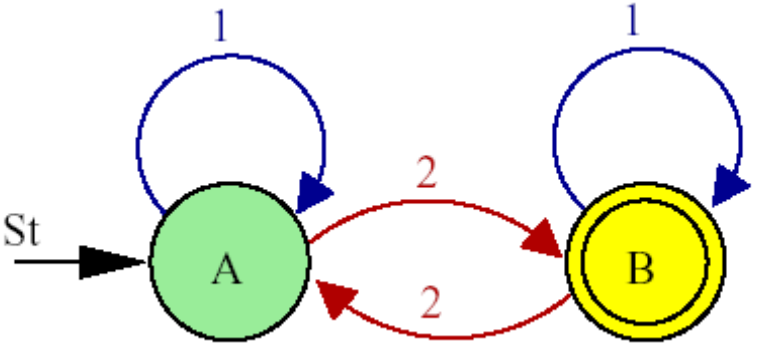
- Heuristics for enhancing RNN generalization: cluster RNN state space into a finite number of clusters
- Each cluster will represent an abstract information-processing state => knowledge extraction from RNN (e.g. learning finite state automata with RNNs)
- Symbolic dynamics helps understand RNNs

Example: Learning a finite state automaton

- State-space activations in RNN – neural memory – code the entire history of symbols we have seen so far.
- To latch a piece of information for a potentially unbounded number of time steps, we need attractive sets.



RNN state space

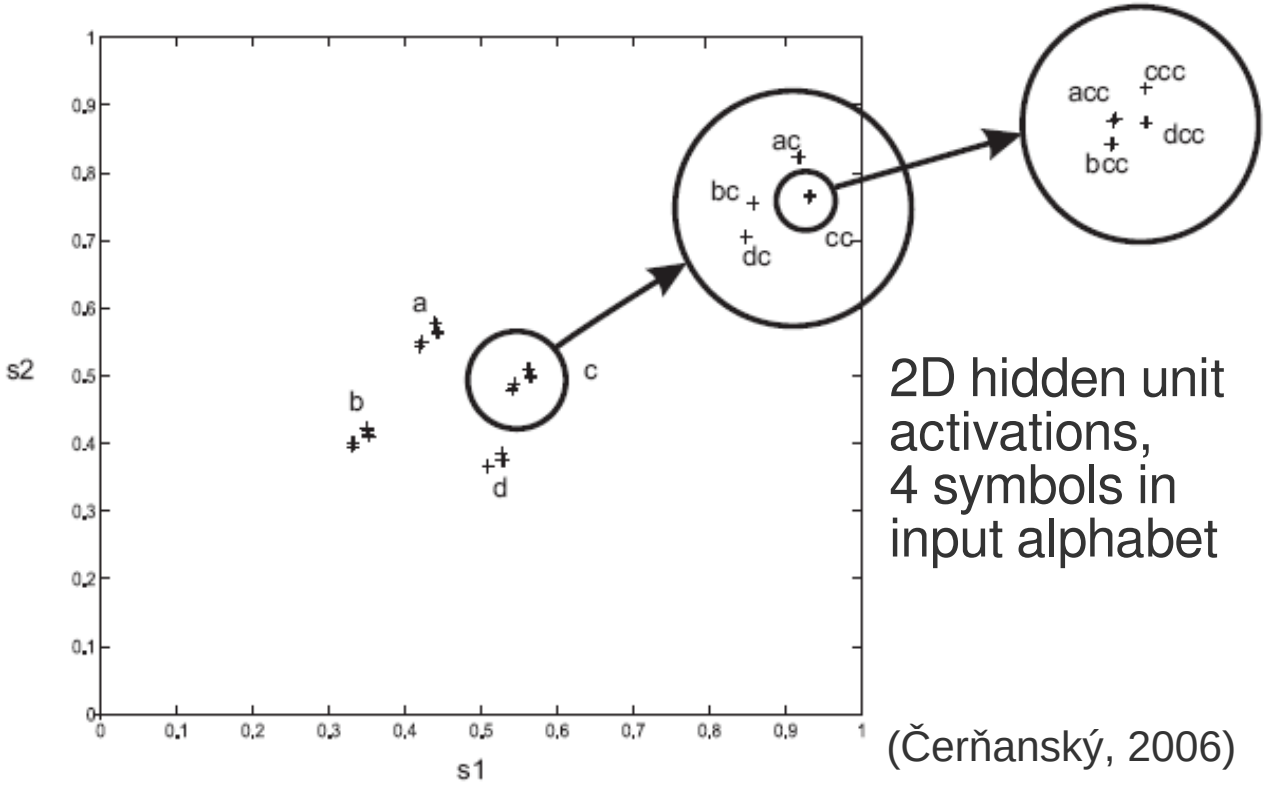


Tiňo (2003)

Grammatical:
all strings containing odd number of 2's

Architectural bias

- Structured hidden-unit activations in RNN exist prior to training, i.e. model predictions overcome random guess:
- clusters of recurrent activations that emerge prior to training correspond to Markov prediction contexts – histories of symbols are grouped according to number of symbols they share in their suffix (Tiňo, Čerňanský, Beňušková, 2004)



2D hidden unit activations, 4 symbols in input alphabet

(Čerňanský, 2006)

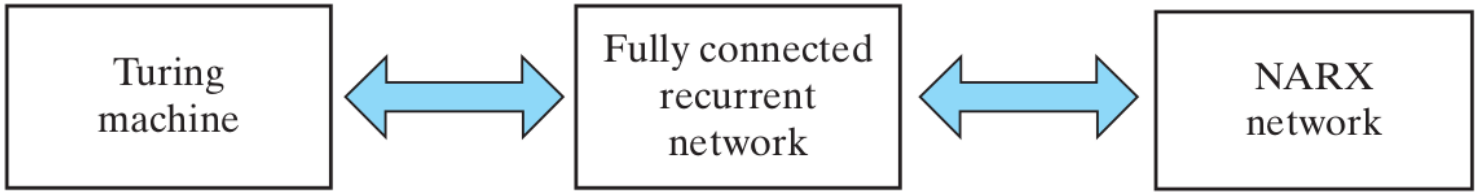
RNNs can outperform finite memory models, but to assess the contribution of training, RNN accuracy should be compared with that of variable-length Markov models extracted before training as the “null” base models.

Computational power of recurrent networks I

- recurrent NNs (e.g. NARX, SRN...) are able to simulate finite-state automata, e.g. by learning
  - regular grammars
  - context-free grammars (e.g. $a^n b^n$ ~ saddle-point attractive set)
  - simple context-sensitive grammars (LSTM)
- "Every finite-state machine is equivalent to, and can be 'simulated' by, some neural net (NN). That is, given any finite-state machine M, we can build a certain NN which, regarded as a black-box machine, will behave precisely like M!" (Minsky, 1967)
- Pivotal work of learning Reber grammar with a SRN (Cleeremans, Servan-Schreiber & McClelland, 1989)
- *Theorem I:* "All Turing machines may be simulated by fully connected recurrent networks built on neurons with sigmoidal activation functions." (Siegelmann & Sontag, 1991)
  - TM is a more abstract mathematical model than FSM

Computational power of recurrent networks II

- *Theorem II:* "NARX networks with one layer of hidden neurons with bounded, one-sided saturated activation functions and a linear output neuron can simulate fully connected RNN with bounded, one-sided saturated (BOSS) activation functions, except for a linear slowdown." (Siegelmann, 1997)
  - A "linear slowdown" means that if the fully connected RNN with N neurons computes a task of interest in time T, then the total time taken by the equivalent NARX network is $(N+1)T$.
- *Corollary:* NARX networks with one hidden layer of neurons with BOSS activation functions and a linear output neuron are Turing equivalent.



Summary

- two classes of architectures (time-lagged, partially or fully recurrent)
- time-lagged models are good for tasks with limited memory
- link to statistical n-gram models
- recurrent models with global feedback learn internal state representations
- links to nonlinear dynamical systems, signal processing and control theory
- more complex learning algorithms: BPTT, RTRL (gradient-based)
- despite theoretical potential, difficulties to learn more complex tasks
- architectural bias allows Markovian predictions
- remaining problem: long-term dependencies