



Basics of fuzzy systems

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(Engelbrecht: Computational Intelligence: An Introduction (2nd ed.), John Willey & Sons, 2007)

Motivation

- Limitations of binary logic
- Necessity to process incomplete, imprecise, vague or uncertain information in problem solving
- Common use of linguistic terms with good understanding: e.g. “it is partly cloudy”, or “Jane is very tall”
- Tools for human reasoning: fuzzy logic, fuzzy set theory
 - from two-valued to infinite-valued logic (Zadeh)
- Many application areas of fuzzy logic (e.g. in control)

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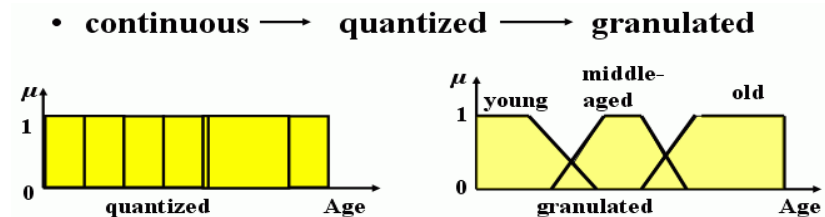
Fuzzy sets

- Concept of **partial truth** introduced (not in binary/crisp logic)
- Can help express the vagueness of natural language:
 - e.g. “when it is very cloudy, it will most probably rain”
- Formalization: domain X , element $x \in X$, fuzzy set A
- A is characterized by **membership function**: $\mu_A: X \rightarrow [0,1]$
- $\mu_A(x)$ – means certainty to which x belongs to A .
- Fuzzy sets are defined for discrete and continuous domains.

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Fuzzy logic

- Core concepts: **graduation** and **granulation**
- In FL, everything is (or is allowed to be) graduated, i.e. fuzzy.
- Furthermore, everything is (or is allowed to be) granulated.

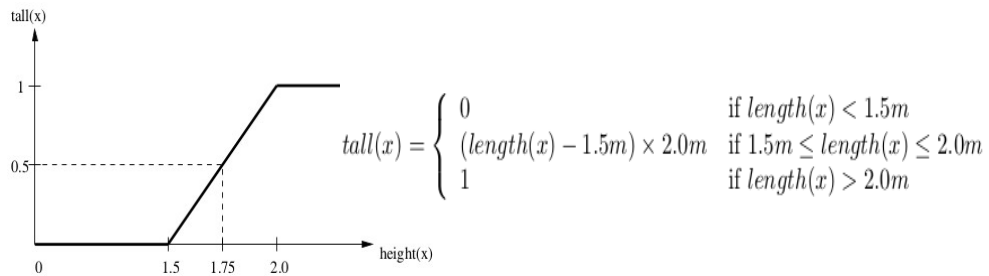


(Zadeh: Fuzzy logic, Scholarpedia)

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Membership functions

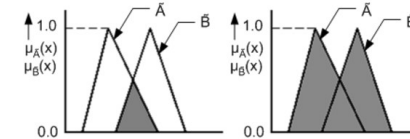
- MF ~ characteristic function of the fuzzy set
- associates a degree of membership of each of the elements of the domain to the corresponding fuzzy set
- MF must be bounded from below by 0 and from above by 1.
- for each $x \in X$, $\mu_A(x)$ must be unique (it is a function).



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Fuzzy operators

- **Equality:** $A = B$ (are equal) $\equiv \mu_A(x) = \mu_B(x)$ for all $x \in X$.
- **Containment:** $A \subset B$ (A is subset of B) $\equiv \mu_A(x) \leq \mu_B(x), \forall x \in X$
- **Complement (NOT):** A' is complement of $A \equiv \mu_{A'}(x) = 1 - \mu_A(x)$
- **Intersection (AND):** set of elements occurring in both sets
 - Min operator: $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$
 - Product operator: $\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x), \forall x \in X$
- **Union (OR):** set of elements occurring in either set
 - Max operator: $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$



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Fuzziness and probability

- Both theories use similar (or same) tools but the key difference is **meaning**.
- Fuzziness is about **facts** expressed by their degrees of truth (partial truth), trying to capture the essence of vagueness.
- Probability is about **events** that either occur or not. Two views:
 - Frequentists talk about inherent probability of event (in %)
 - Bayesians say that probability is assessment of our state of knowledge (e.g. there is x% certainty that event occurs)
- Fuzziness is about **partial truth**, whereas probability is about **partial knowledge**.
- Possibility theory (Zadeh) = marriage of the two.

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Fuzzy logic and reasoning

- Simple example: Peter, Carl, and 3 fuzzy sets
 - $\mu_{tall}(Peter) = 0.9$ and $\mu_{good-athlete}(Peter) = 0.8$
 - $\mu_{tall}(Carl) = 0.9$ and $\mu_{good-athlete}(Carl) = 0.5$
- Which of the two is the better basketball player?
 - $\mu_{good-basketball-player}(Peter) = \min\{0.9, 0.8\} = 0.8$
 - $\mu_{good-basketball-player}(Carl) = \min\{0.9, 0.5\} = 0.5$
- Fuzzy logic + inferencing system needed

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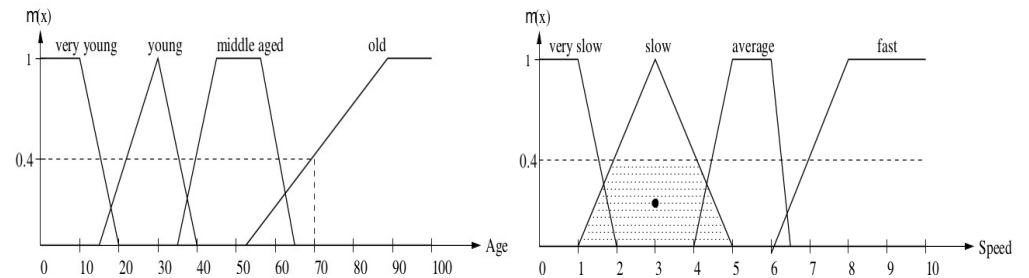
Linguistic variables and hedges

- **Categories of linguistic variables:**
 - *Quantification variables:* all, most, many, none, etc.
 - *Usuality variables:* sometimes, frequently, seldom, etc.
 - *Likelihood variables:* possible, likely, certain, etc.
- **Hedge** is a modifier of fuzzy values: e.g. *very tall*
 - changes the membership, e.g. $\mu_{\text{very-tall}}(x) = \mu_{\text{tall}}(x)^2$
 - Types: concentration (e.g. *very*), dilation (*somewhat*), contrast intensification (*extremely*), vague (*seldom*), probabilistic (*likely*).

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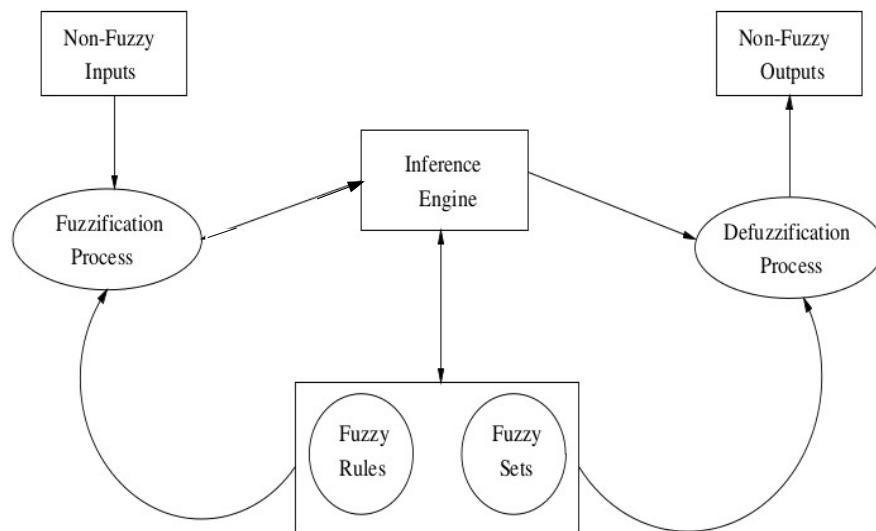
Fuzzy rules

- Linguistic fuzzy rules, based on the knowledge and experience of a human expert within that domain.
- General form: **if antecedent(s) then consequent(s)**
- if *A* is *a* and *B* is *b*, then *C* is *c*
- Example: if “*Age is Old*”, then “*Speed is Slow*.” *Age=70*. How slow is the person?



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Fuzzy rule-based reasoning system



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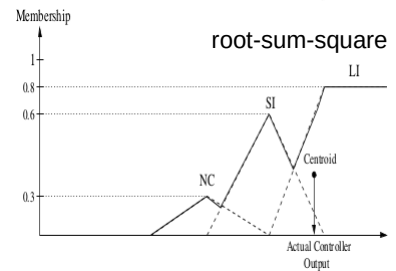
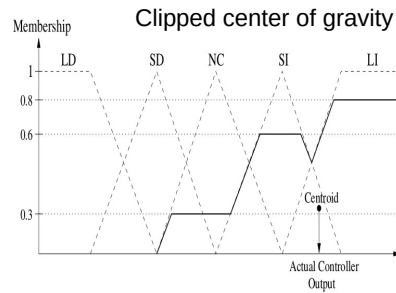
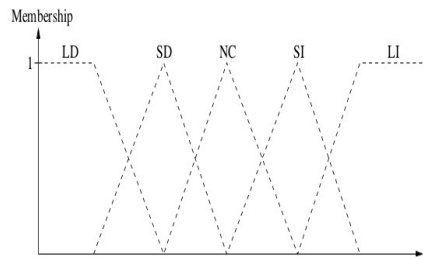
Fuzzy inferencing

- Antecedents of the fuzzy rules form the fuzzy “input space,” while the consequents form the fuzzy “output space”.
- **Fuzzification** process is concerned with finding a fuzzy representation of non-fuzzy input values.
- **Inferencing** process maps fuzzified inputs to the rule base, and produces a fuzzified output for each rule.
- **Defuzzification** process converts the output of the fuzzy rules into a scalar, or non-fuzzy value.
 - Various methods can be used for centroid calculation

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Examples of defuzzification methods

Assume linguistic variable C , given by these MFs: large decrease (LD), slight decrease (SD), no change (NC), slight increase (SI), and large increase (LI). Assume 3 rules with the following C membership values: $\mu_{LI} = 0.8$, $\mu_{SI} = 0.6$ and $\mu_{NC} = 0.3$.

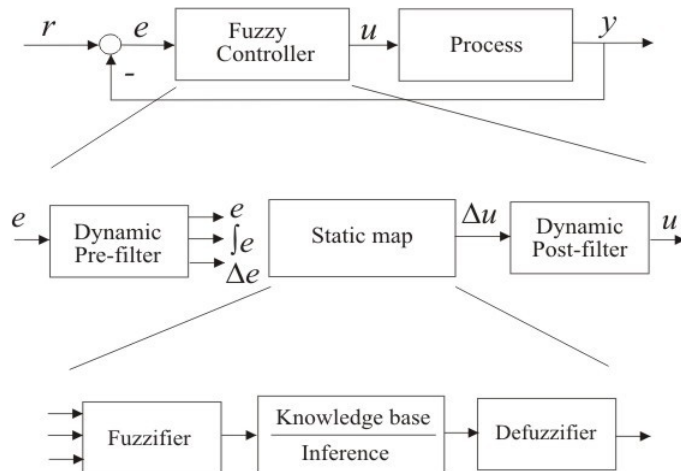


Four principal facets of fuzzy logic

- **Fuzzy-set-theoretic:** fuzzy sets as basis (Zadeh, 1965)
- **Logical:** In a narrow sense, fuzzy logic is a logical system.
- **Epistemic:** concerned with knowledge representation and semantics of natural languages.
- **Relational:** focused on fuzzy relations
- **Fuzzy logic is not fuzzy, it is precise** (in a broad sense, it is much more than a logical system).

(Zadeh: Fuzzy logic, Scholarpedia)

Fuzzy control



PID controller:

$$u(t) = Pe(t) + I \int_0^t e(\tau) d\tau + D \frac{de(t)}{dt}$$

(Scholarpedia: Fuzzy control)

Summary of key points

- Why are fuzzy systems interesting
- Fuzzy sets
- Facets of fuzzy logic
- Membership function
- Fuzziness and probability
- fuzzy operators, fuzzy rules
- Linguistic variables, hedges
- Fuzzy inferencing, defuzzification
- Fuzzy control