Introduction to computational intelligence

Basics of fuzzy systems

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(Engelbrecht: Computational Intelligence: An Introduction (2nd ed.), John Willey & Sons, 2007)

Motivation

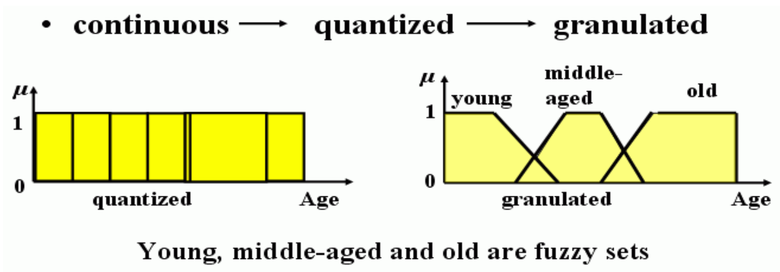
- Limitations of binary logic
- Necessity to process incomplete, imprecise, vague or uncertain information in problem solving
- Common use of linguistic terms with good understanding: e.g. *"it is partly cloudy"*, or *"Jane is very tall"*
- Tools for human reasoning: fuzzy logic, fuzzy set theory
  - from two-valued to infinite-valued logic (Zadeh)
- Many application areas of fuzzy logic (e.g. in control)

Fuzzy sets

- Concept of partial truth introduced (not in binary/crisp logic)
- Can help express the vagueness of natural language:
  - e.g. *"when it is very cloudy, it will most probably rain"*
- Formalization: domain X, element $x \in X$, fuzzy set A
- A is characterized by membership function: $\mu_A: X \rightarrow [0,1]$
- $\mu_A(x)$ – means certainty to which x belongs to A, *e.g. tall*(M)
- Fuzzy sets are defined for discrete and continuous domains.

Fuzzy logic

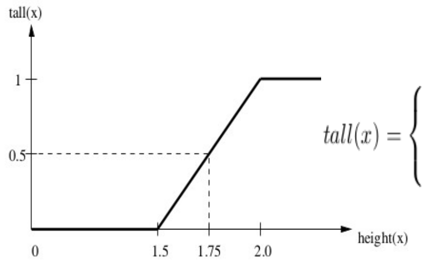
- Core concepts: graduation and granulation
- In FL, everything is (or is allowed to be) graduated, i.e. fuzzy.
- Furthermore, everything is (or is allowed to be) granulated.



(Zadeh: Fuzzy logic, Scholarpedia)

Membership functions

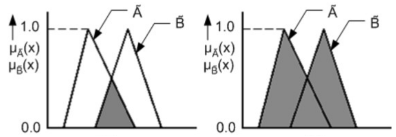
- MF ~ characteristic function of the fuzzy set
- associates a degree of membership of each of the elements of the domain to the corresponding fuzzy set
- MF must be bounded from below by 0 and from above by 1.
- for each $x \in X$, $\mu_A(x)$ must be unique (it is a function).



$$tall(x) = \begin{cases} 0 & \text{if } length(x) < 1.5m \\ (length(x) - 1.5m) \times 2.0m & \text{if } 1.5m \leq length(x) \leq 2.0m \\ 1 & \text{if } length(x) > 2.0m \end{cases}$$

Fuzzy operators

- Equality: $A = B$ (are equal) $\equiv \mu_A(x) = \mu_B(x)$ for all $x \in X$.
- Containment: $A \subset B$ (A is subset of B) $\equiv \mu_A(x) \leq \mu_B(x), \forall x \in X$
- Complement (NOT): A' is complement of $A \equiv \mu_{A'}(x) = 1 - \mu_A(x)$
- Intersection (AND): set of elements occurring in both sets
  - Min operator: $\mu_{A \cap B}(x) = \min\{ \mu_A(x), \mu_B(x) \}, \forall x \in X$
  - Product operator: $\mu_{A \cap B}(x) = \mu_A(x) \,.\, \mu_B(x), \forall x \in X$
- Union (OR): set of elements occurring in either set
  - Max operator: $\mu_{A \cup B}(x) = \max\{ \mu_A(x), \mu_B(x) \}, \forall x \in X$



Fuzziness and probability

- Both theories use similar (or same) tools but the key difference is meaning.
- Fuzziness is about facts expressed by their degrees of truth (partial truth), trying to capture the essence of vagueness.
- Probability is about events that either occur or not. Two views:
  - Frequentists talk about inherent probability of event (in %)
  - Bayesians say that probability is assessment of our state of knowledge (e.g. there is x% certainty that event occurs)
- Fuzziness is about partial truth, whereas probability is about partial knowledge.
- Possibility theory (Zadeh) = marriage of the two.

Fuzzy logic and reasoning

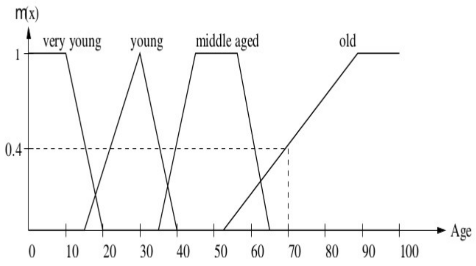
- Simple example: Peter, Carl, and 3 fuzzy sets
  - $\mu_{tall}(Peter) = 0.9$ and $\mu_{good\text{-}athlete}(Peter) = 0.8$
  - $\mu_{tall}(Carl) = 0.9$ and $\mu_{good\text{-}athlete}(Carl) = 0.5$
- Which of the two is the better basketball player?
  - $\mu_{good\text{-}basketball\text{-}player}(Peter) = \min\{0.9, 0.8\} = 0.8$
  - $\mu_{good\text{-}basketball\text{-}player}(Carl) = \min\{0.9, 0.5\} = 0.5$
- Fuzzy logic + inferencing system needed

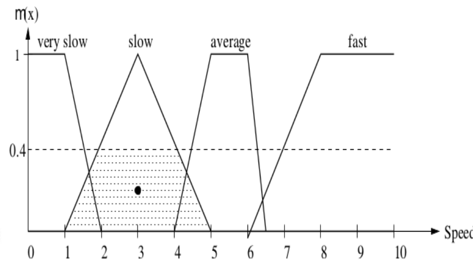
Linguistic variables and hedges

- Categories of linguistic variables:
  - *Quantification variables*: all, most, many, none, etc.
  - *Usuality variables*: sometimes, frequently, seldom, etc.
  - *Likelihood variables*: possible, likely, certain, etc.
- Hedge is a modifier of fuzzy values: e.g. *very* tall
  - changes the membership, e.g. $\mu_{\text{very-tall}}(x) = \mu_{\text{tall}}(x)^2$
  - Types: concentration (e.g. *very*), dilation (*somewhat*), contrast intensification (*extremely*), vague (*seldom*), probabilistic (*likely*).

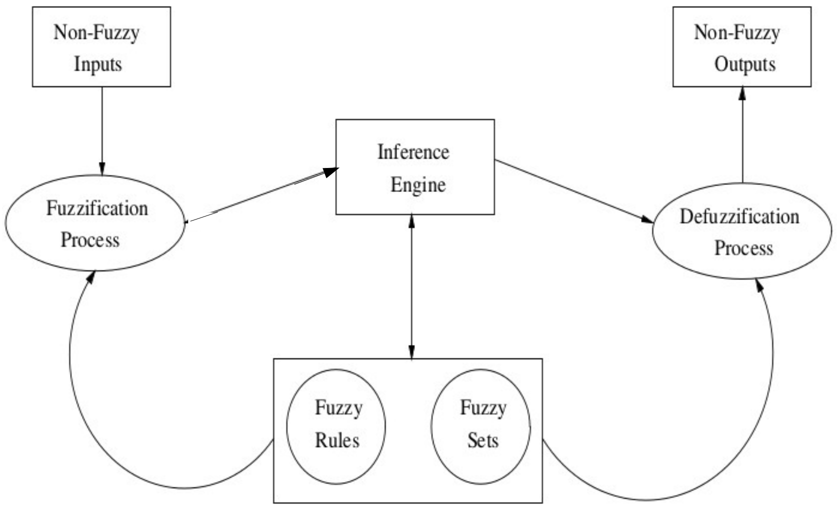
Fuzzy rules

- Linguistic fuzzy rules, based on the knowledge and experience of a human expert within that domain.
- General form: if antecedent(s) then consequent(s)
- if *A* is *a* and *B* is *b,* then *C* is *c*
- Example: if "*Age* is *Old",* then "*Speed* is *Slow.*" *Age*=70. How slow is the person?





Fuzzy rule-based reasoning system

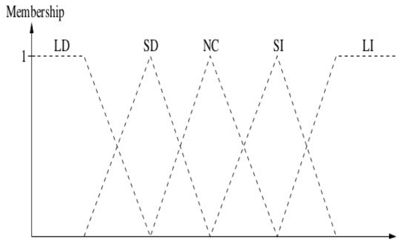


Fuzzy inferencing

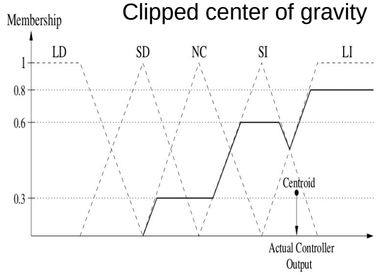
- Antecedents of the fuzzy rules form the fuzzy "input space," while the consequents form the fuzzy "output space".
- Fuzzification process is concerned with finding a fuzzy representation of non-fuzzy input values.
- Inferencing process maps fuzzified inputs to the rule base, and produces a fuzzified output for each rule.
- Defuzzification process converts the output of the fuzzy rules into a scalar, or non-fuzzy value.
  - Various methods can be used for centroid calculation

Examples of defuzzification methods

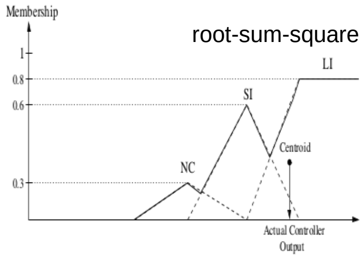
Assume linguistic variable C, given by these MFs: large decrease (LD), slight decrease (SD), no change (NC), slight increase (SI), and large increase (LI). Assume 3 rules with the following C membership values: $\mu_{LI} = 0.8$, $\mu_{SI} = 0.6$ and $\mu_{NC} = 0.3$.



Clipped center of gravity



root-sum-square



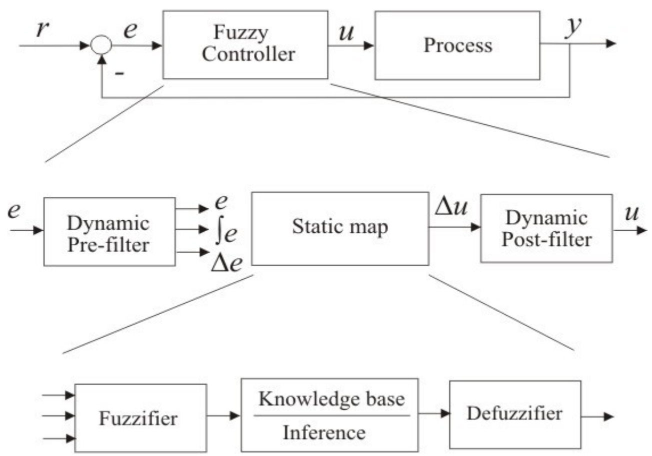
Four principal facets of fuzzy logic

- Fuzzy-set-theoretic: fuzzy sets as basis (Zadeh, 1965)
- Logical: In a narrow sense, fuzzy logic is a logical system.
- Epistemic: concerned with knowledge representation and semantics of natural languages.
- Relational: focused on fuzzy relations

- Fuzzy logic is not fuzzy, it is precise (in a broad sense, it is much more than a logical system).

(Zadeh: Fuzzy logic, Scholarpedia)

Fuzzy control



PID controller:

$$u(t) = Pe(t) + I\int_0^t e(\tau)d\tau + D\frac{de(t)}{dt}$$

(Scholarpedia: Fuzzy control)

Summary of key points

- fuzzy systems = 3rd pillar of computational intelligence
- Built on fuzzy sets and fuzzy logic
- Key concept: membership function
- Additional components: fuzzy operators, fuzzy rules
- Linguistic variables
- Fuzzy inferencing, defuzzification
- Difference between fuzziness and probability
- Major application area: Fuzzy control
- Typically, no learning involved :-(