Cognitive semantics and cognitive theories of representation:

Session 6: Conceptual spaces

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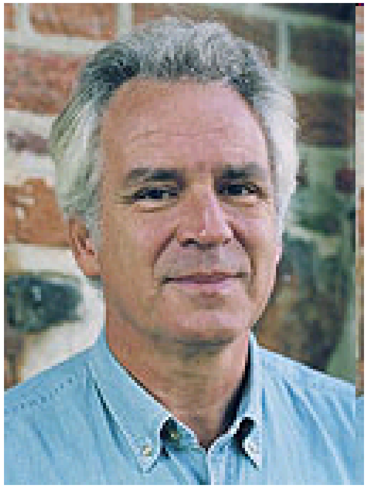
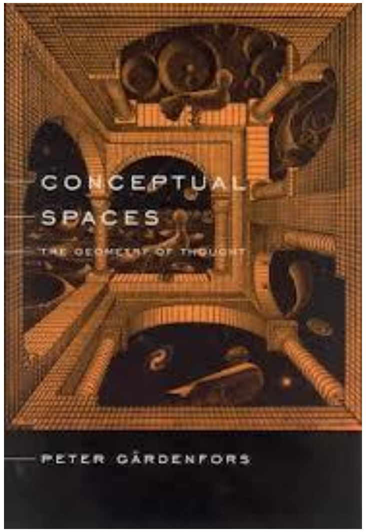
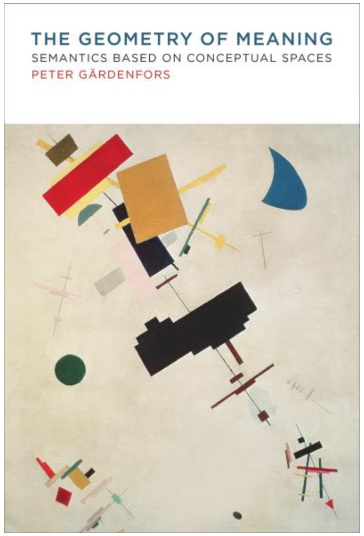
Conceptual spaces

Peter Gärdenfors



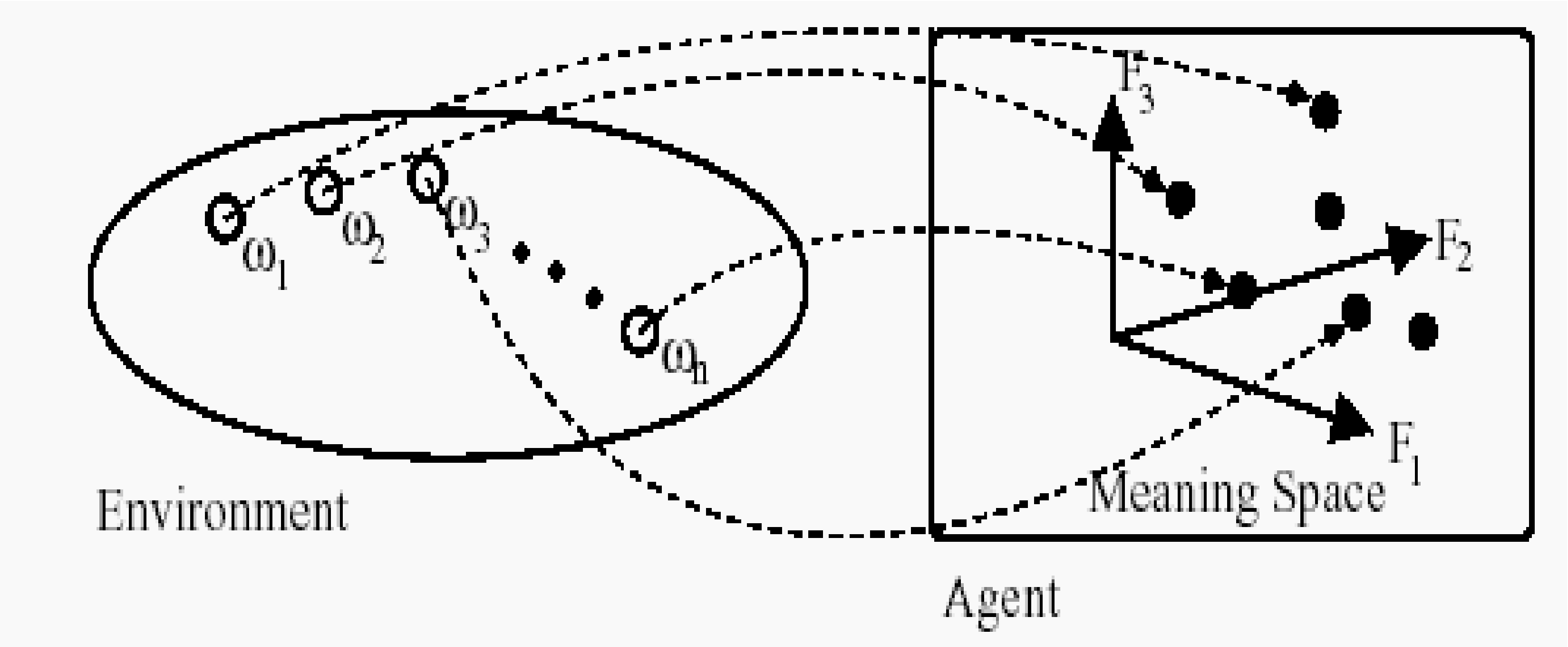




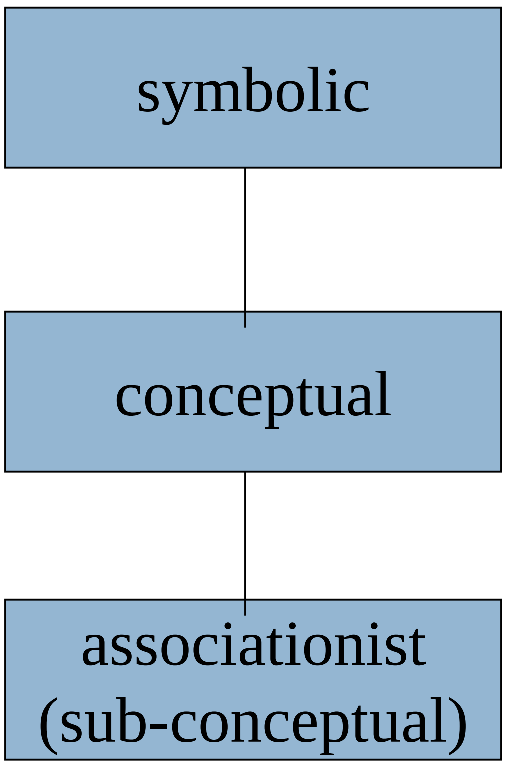
(MIT Press, 2000, 2014)

Semantic elements are geometrical structures

- Particular objects map to points. Representations of **similar** objects are **close** to each other in a meaning space formed by quality dimensions.



Gärdenfors' cognitive model



Propositional representation

Geometric representation

Connectionist representation

Conceptual spaces

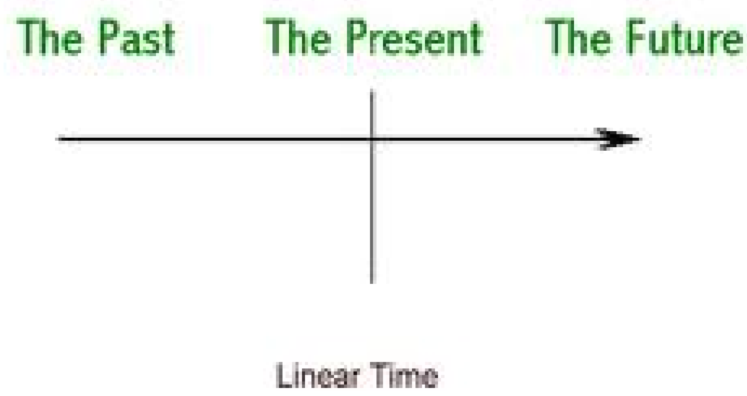
- consist of a number of **quality dimensions** that represent various qualities of objects.
- Quality dimensions correspond to the different ways stimuli can be judged similar or different.
  - Weight, temperature, brightness, pitch, height, width, depth
  - Abstract non-sensory dimensions too.

Dimensions

- Innate – hardwired in nervous system
- Learned
  - Learning involves expanding conceptual space with new quality dimensions
  - E.g. size vs volume
- Culturally dependent
  - Time, Kinship
- Scientific
  - Weight vs. mass

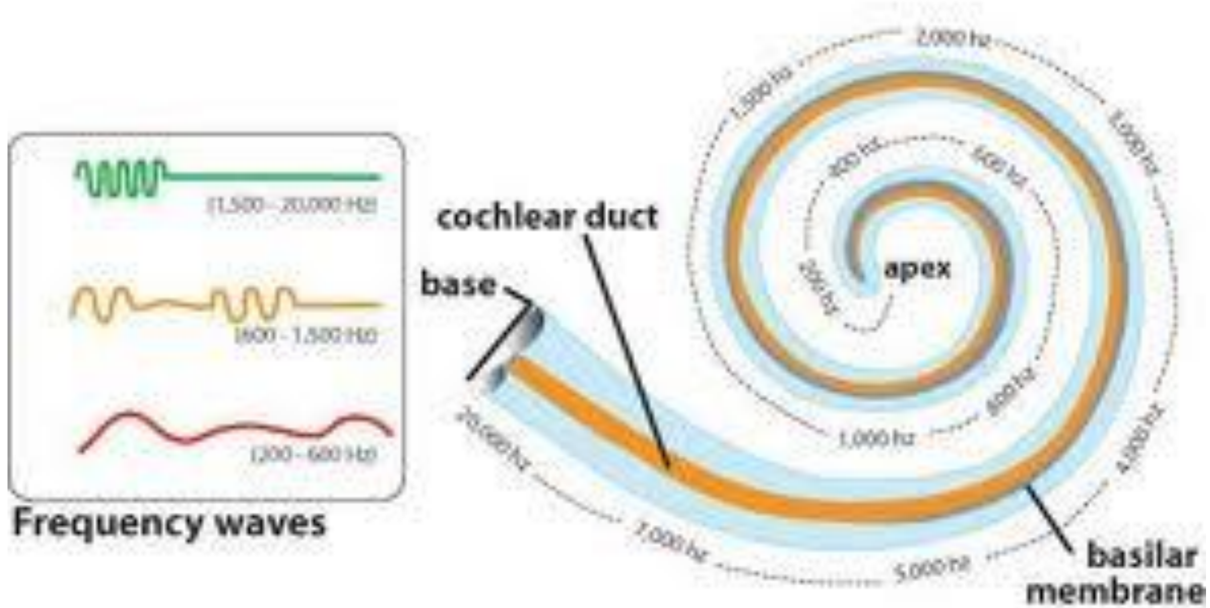
Dimension shape: ‘time’

- In our culture and in science
  - One-dimensional structure isomorphic to the line of real numbers
- In other cultures
  - Circular structure



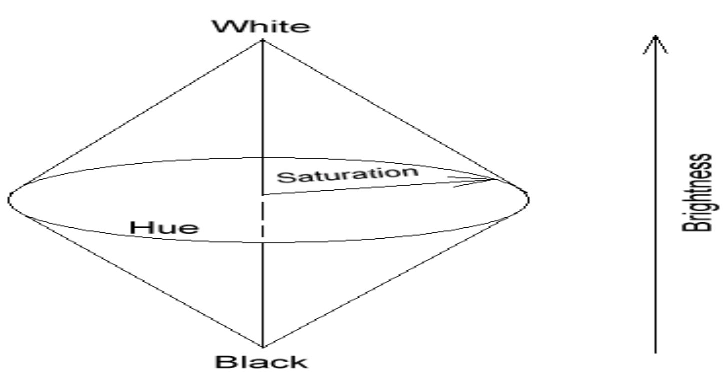
Dimension shape: ‘pitch’

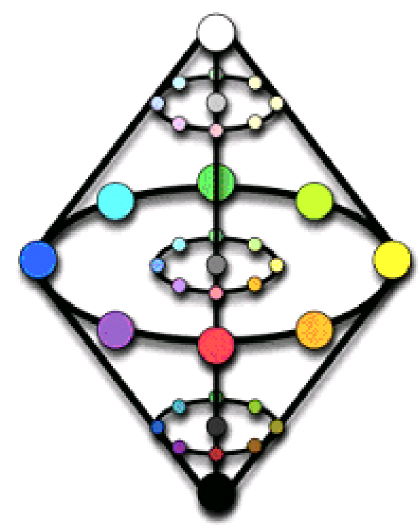
- 1-D structure from low tones to high
- Logarithmic scale
- Acoustic frequency is spatially coded in cochlea



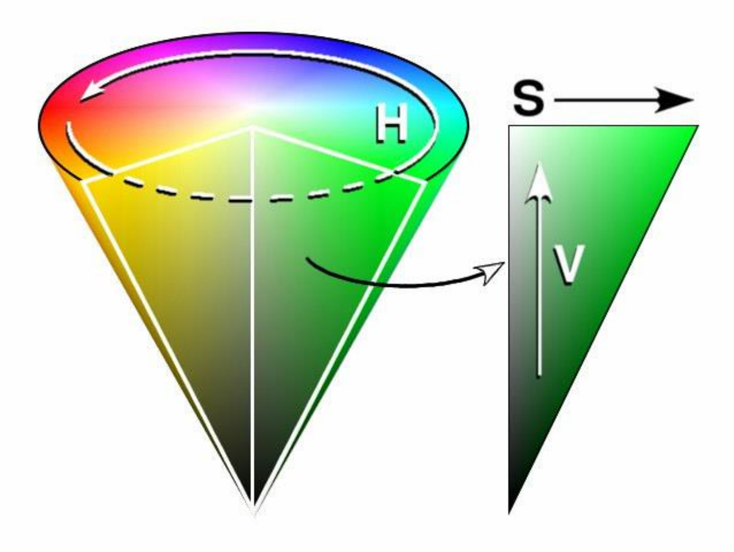
Example: Phenomenology of colour

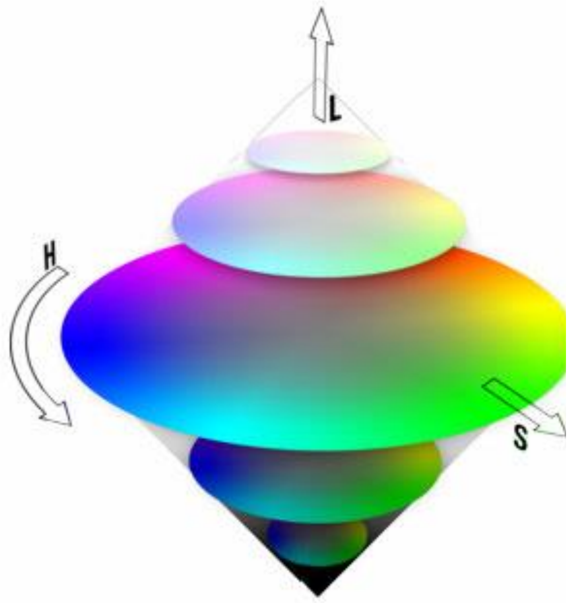
- Hue- the particular shade of colour
  - Geometric structure: circle
  - Value: polar coordinate
- Chromaticity- the saturation of the colour; from grey to higher intensities
  - Geometric structure: segment of reals
  - Value: real number
- Brightness: black to white
  - Geometric structure: reals in [0,1]
  - Value: real number





Phenomenological colour space





Integral and separable dimensions

- Dimensions are integral if an object cannot be assigned a value in one dimension without giving it a value in another:
  - E.g. cannot distinguish hue without brightness, or pitch without loudness
- Dimensions that are not integral, are said to be separable
- Psychologically, integral and separable dimensions are assumed to differ in cross dimensional similarity –
  - integral dimensions are higher in cross-dimensional similarity than separable dimensions.
  - (This point will motivate how similarities in the conceptual space are calculated depending on whether dimensions are integral or separable.)

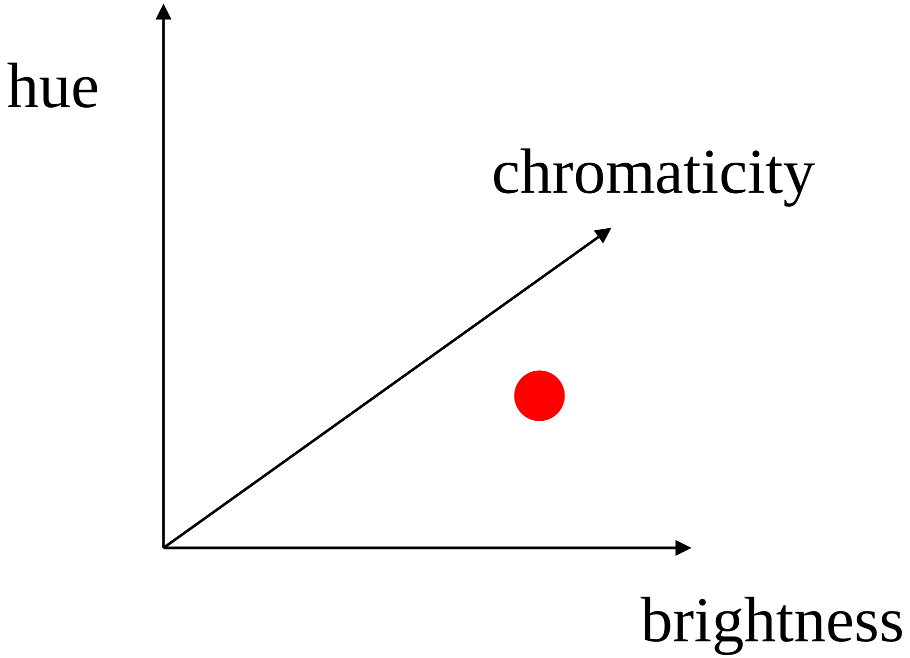
Domains and conceptual space

- A domain is set of integral dimensions- a *separable subspace* (e.g., hue, chromaticity, brightness)
- A conceptual space is a collection of quality dimensions divided into domains
  - Cognitive structure is defined in terms of domains as it is assumed that an object can be ascribed certain properties **independently** of other properties
- Not all domains are assumed to be metric – a domain may be an ordering with no distance defined
- Domains are not independent, but may be correlated, e.g., the ripeness and colour domains co-vary in the space of fruits

Properties and concepts: general idea

- A property is a region in a domain
- A concept is based on several domains

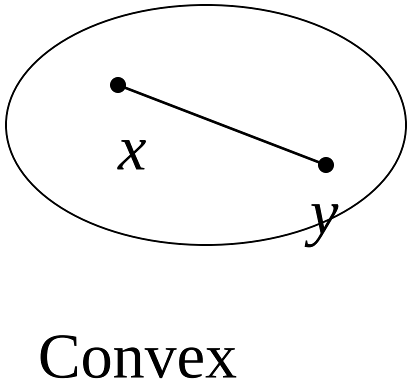
Example property: "red"

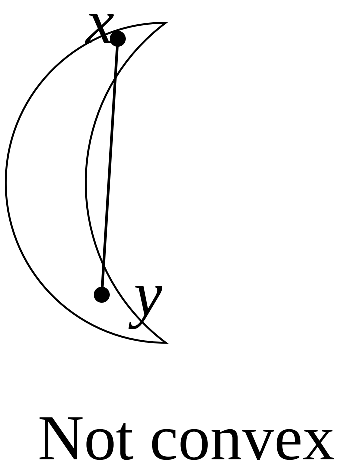


Criterion P: A *natural* property is a convex region of a domain (subspace)

"natural" – those properties that are natural for the purposes of problem solving, planning, communicating, etc

Motivation for convex regions





x and *y* are points (objects) in the conceptual space

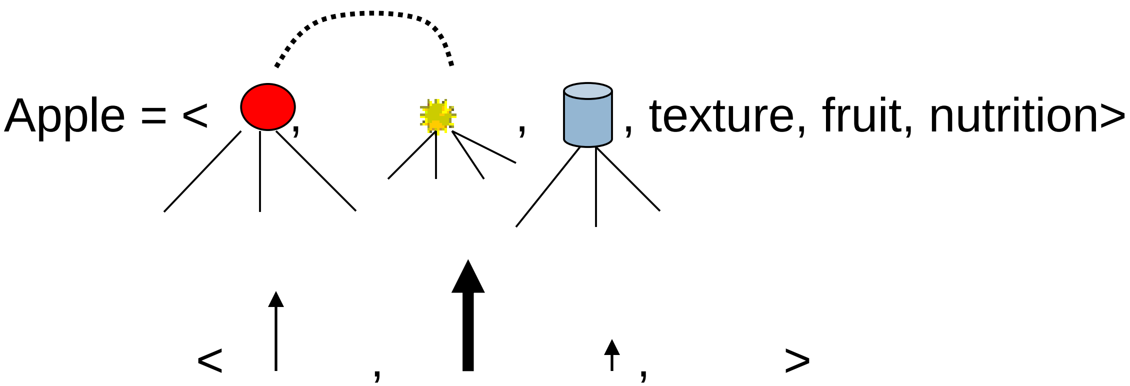
If *x* and *y* both have property P, then any object between *x* and *y* is assumed to have property P

Remarks about Criterion P

Criterion P: A *natural* property is a convex region of a domain (subspace)

- Assumption: “Most properties expressed by simple words in natural languages can be analyzed as natural properties”
- “The semantics of the linguistic constituents (e.g. “red”) is constrained by the underlying conceptual space”
- Strong connection between convex regions and *prototype theory* (categorization)

Example concept: “apple”



Criterion C: A natural concept is represented as a set of regions in a number of domains together with an assignment of salience weights to the domains and information about how the regions in the different domains are correlated

Concepts and inference

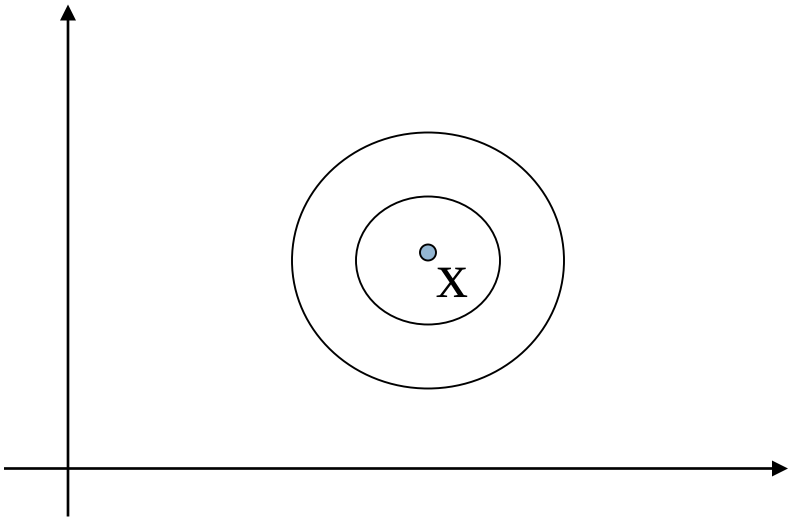
- The salience of different domains determines which associations can be made, and which inferences can be triggered
  - Context: moving a piano – leads to association “heavy”

Similarity: introductory remarks

- Similarity is central to many aspects of cognition: concept formation (learning), memory and perceptual organization
- Similarity is not an absolute notion but relative to a particular domain (or dimension)
  - "an apple and orange are similar as they have the same shape"
  - Similarity defined in terms of the "number of shared properties" leads to arbitrary similarity – "a writing desk is like a raven"
- Similarity is an exponentially decreasing function of distance

Equi-distance under the Euclidean metric

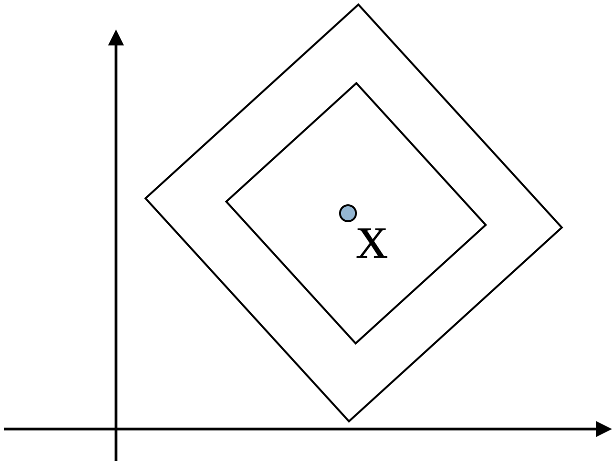
$$d_E(x,y) = \sqrt{\sum_i (x_i - y_i)^2}$$



Set of points at distance *d* from a point *x* form a circle
Points between *x* and *y* are on a straight line

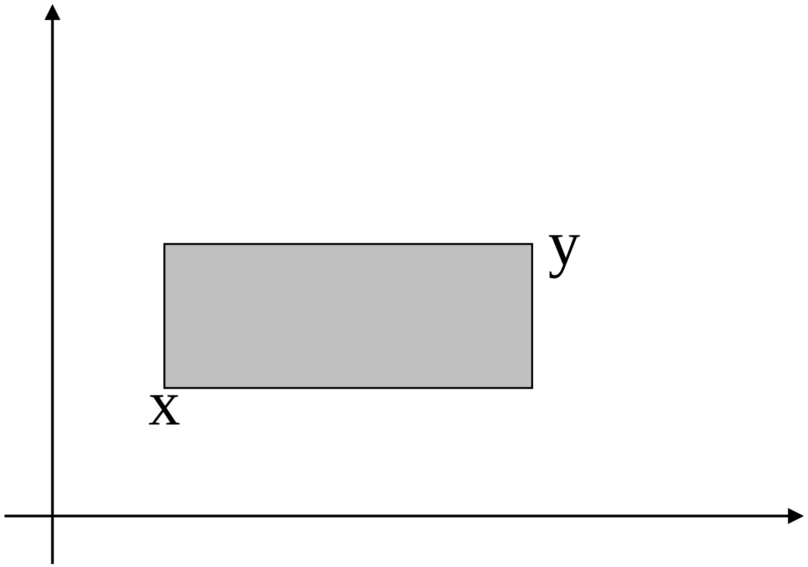
Equi-distance under the city-block metric

$$d_C(x,y) = \sum_i |x_i - y_i|$$



The set of points at distance d from a point *x* form a diamond
The set of points between *x* and *y* is a rectangle generated by *x* and *y* and the directions of the axes

Between-ness in the city-block metric



All points in the rectangle are considered to be between *x* and *y*

Metrics: integral and separable dimensions

- For separable dimensions, calculate the distance using the city-block metric:
  - “If two dimensions are separable, the dissimilarity of two stimuli is obtained by adding the dissimilarity along each of the two dimensions”
- For integral dimensions, calculate distance using the Euclidean metric:
  - “When two dimensions are integral, the dissimilarity is determined both dimensions taken together

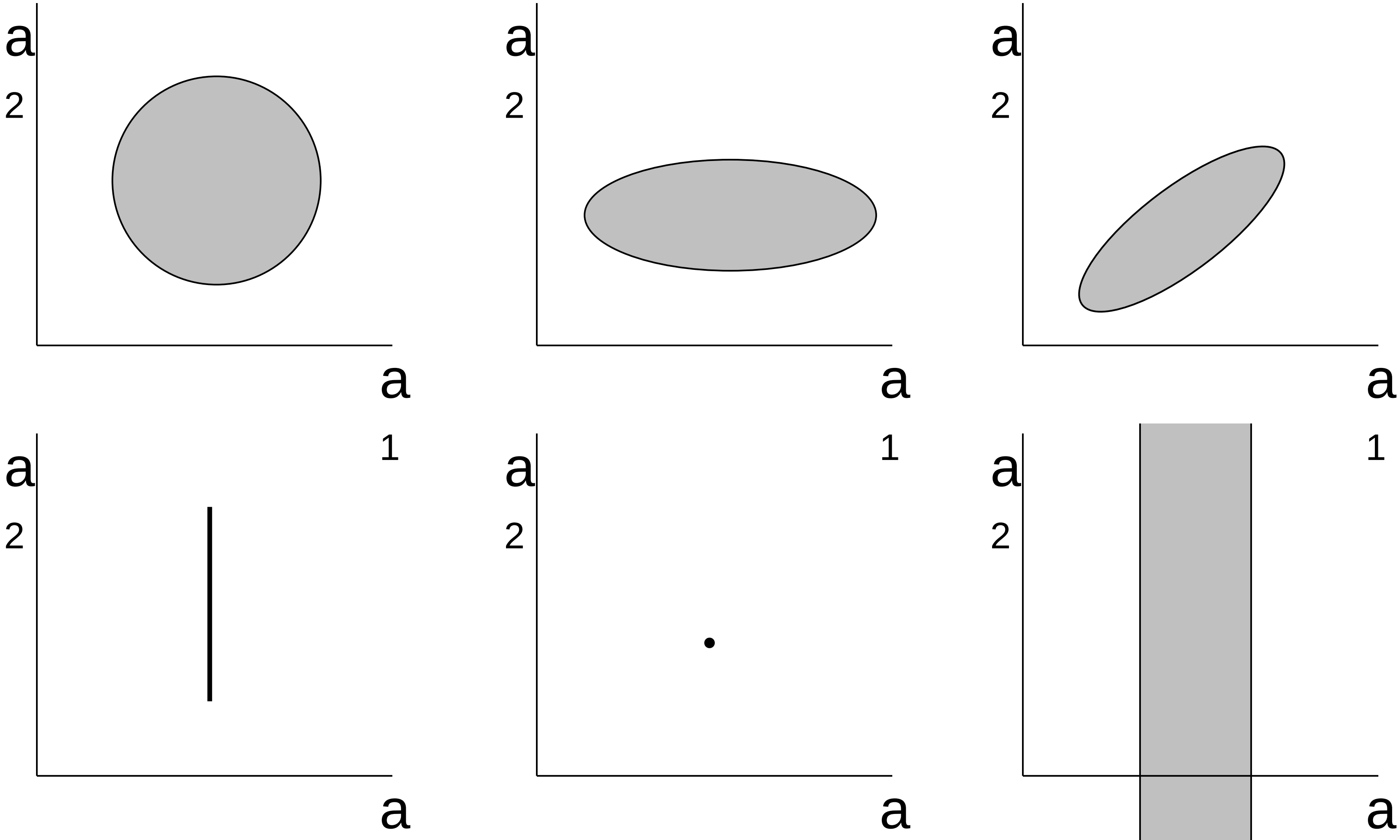
Scaling dimensions

Due to context, the scales of the different dimensions cannot be assumed identical.

$$d_k(x, y) = \sqrt[r]{\sum_i w_i |x_i - y_i|^r}$$

Dimensional scaling factor

Example of scaling (and other distortions)



Similarity as a function of distance

A common assumption in psychological literature is that similarity is an exponentially decaying function of distance:

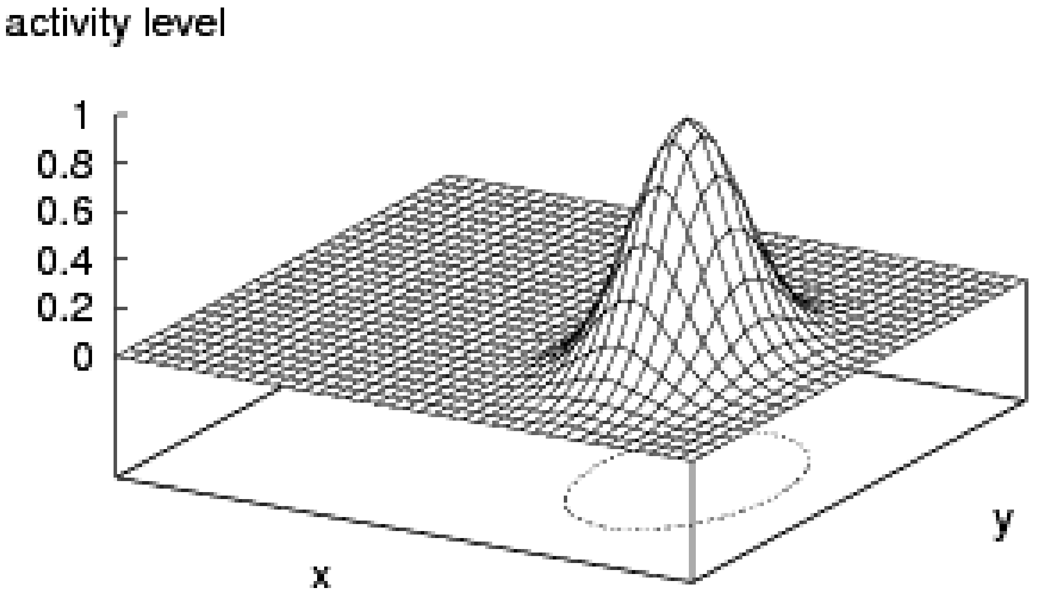
$$s(x, y) = e^{-c.d(x,y)}$$

The constant *c* is a sensitivity parameter.

The similarity between *x* and *y* drops quickly when the distance between the objects is relatively small, while it drops more slowly when the distance is relatively large.

The formula captures the similarity-based generalization performances of human subjects in a variety of settings.

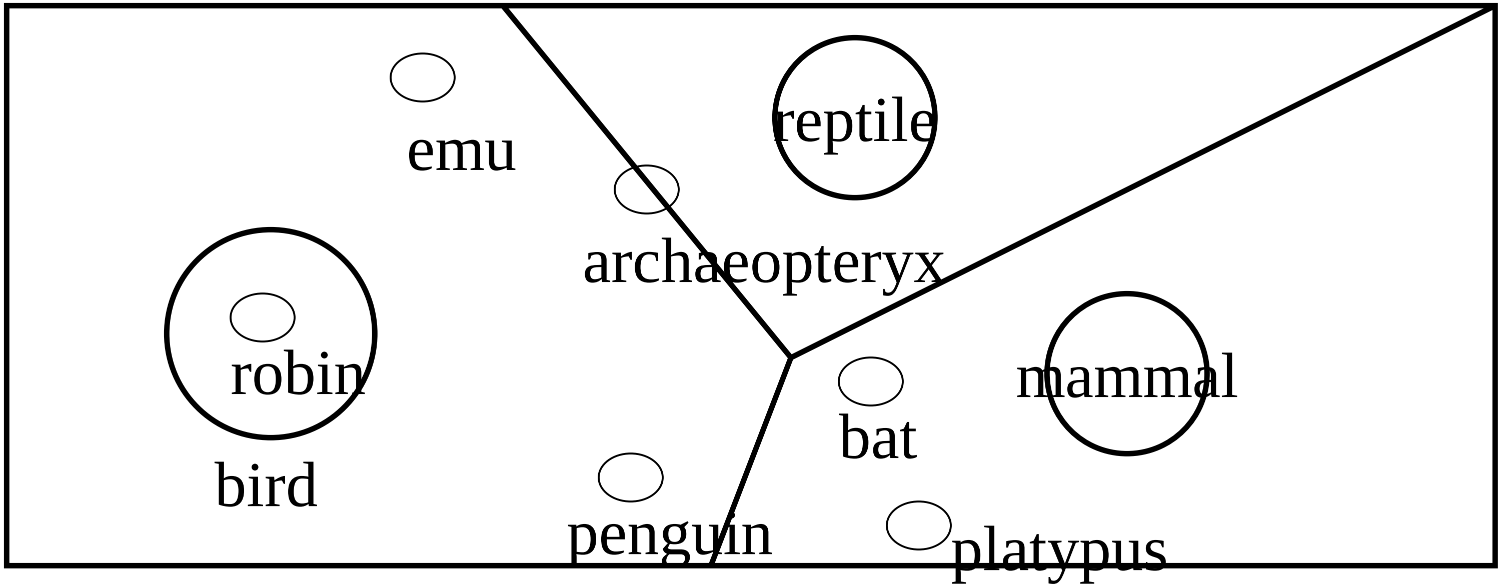
Gaussian similarity



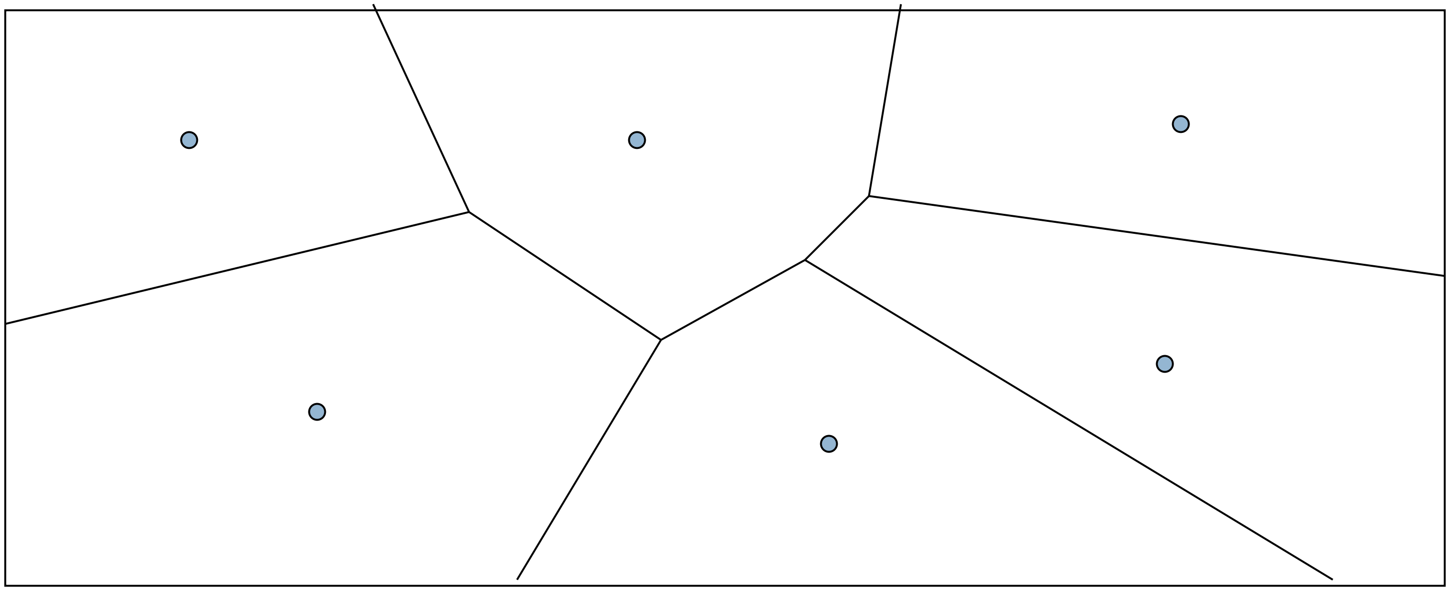
Prototypes and categorical perception: introductory remarks

- Human subjects judge “a robin as a more prototypical bird than a penguin”
- Classifying an object is accomplished by determining its similarity to the prototype:
  - Similarity is judged w.r.t a reference object/region
  - Similarity is context-sensitive: a robin is a prototypical bird, but a canary is a prototypical *pet* bird
- Continuous perception: membership to a category is graded

Prototype regions in animal space

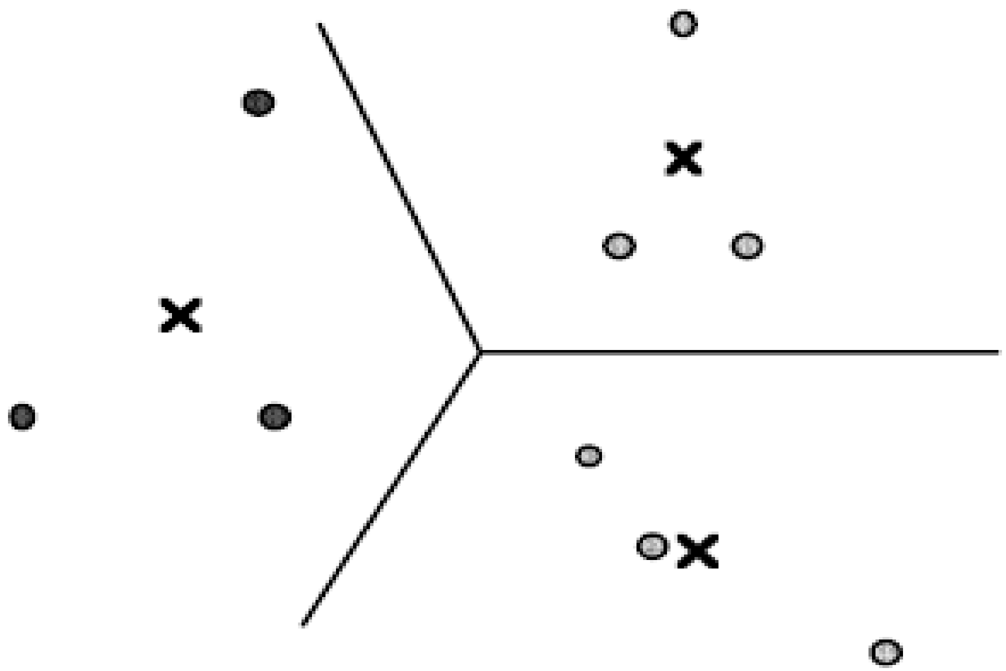


Computing categories in conceptual space: Voronoi tessellations

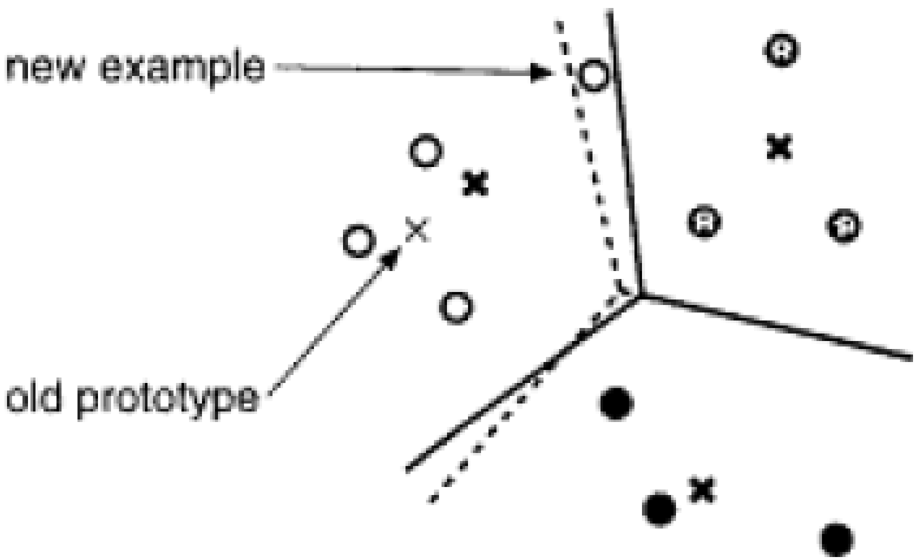


Given prototypes $p_1, \ldots, p_n$ require that q be in the same category as its most similar prototype.
Consequence: partitioning of the space into convex regions

Voronoi Tessellations – learnability

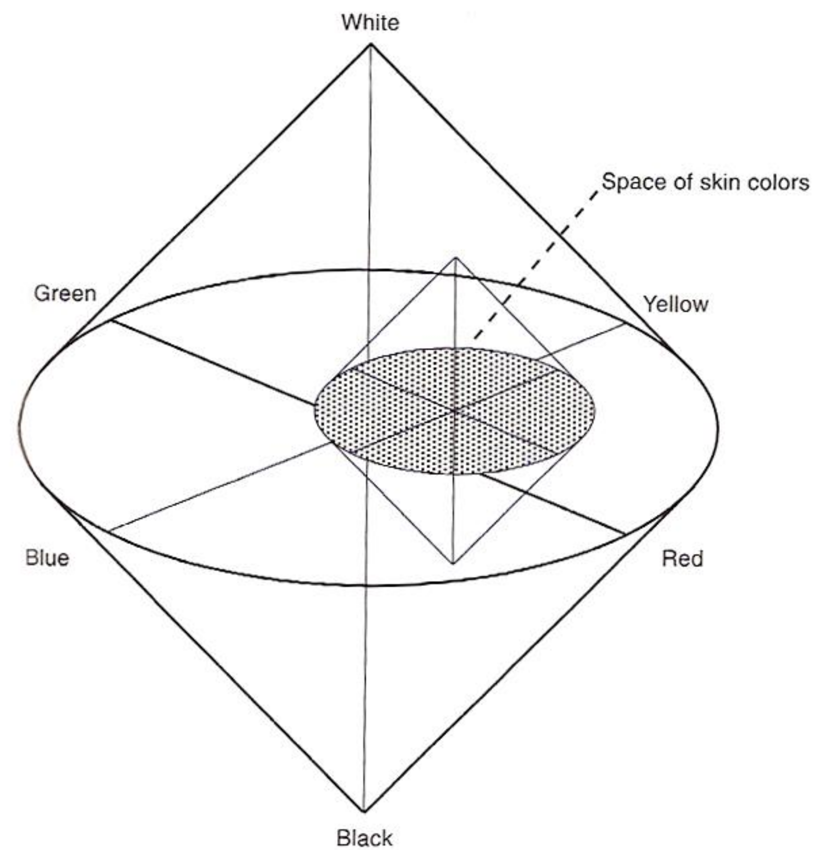


Concept change



Contrast classes

- Skin color
  - Possible colors are the subset of the full color space
  - Can be irregular
  - Subset "stretched" to form a space with the same **topology**
    - Color terms can be used even if they do not correspond to the original hues
    - "Metaphor"



Metaphors in conceptual space

- A metaphor expresses a similarity in topological or metrical structure between different quality dimensions
  - A word that represent a particular structure in one quality dimension can be used as a metaphor to express a similar structure about another dimension
- Metaphors transfer knowledge about one conceptual dimension to another
  - E.g. space mapped to time

Shapes (Marr 1982)

- Cylinders
  - Length
  - Width
  - Angle between the dominating and the other one
  - Position of the added cylinder
- Prototypical vector for an object – **image schema**
- Subordinate cat. – subregions of the convex region

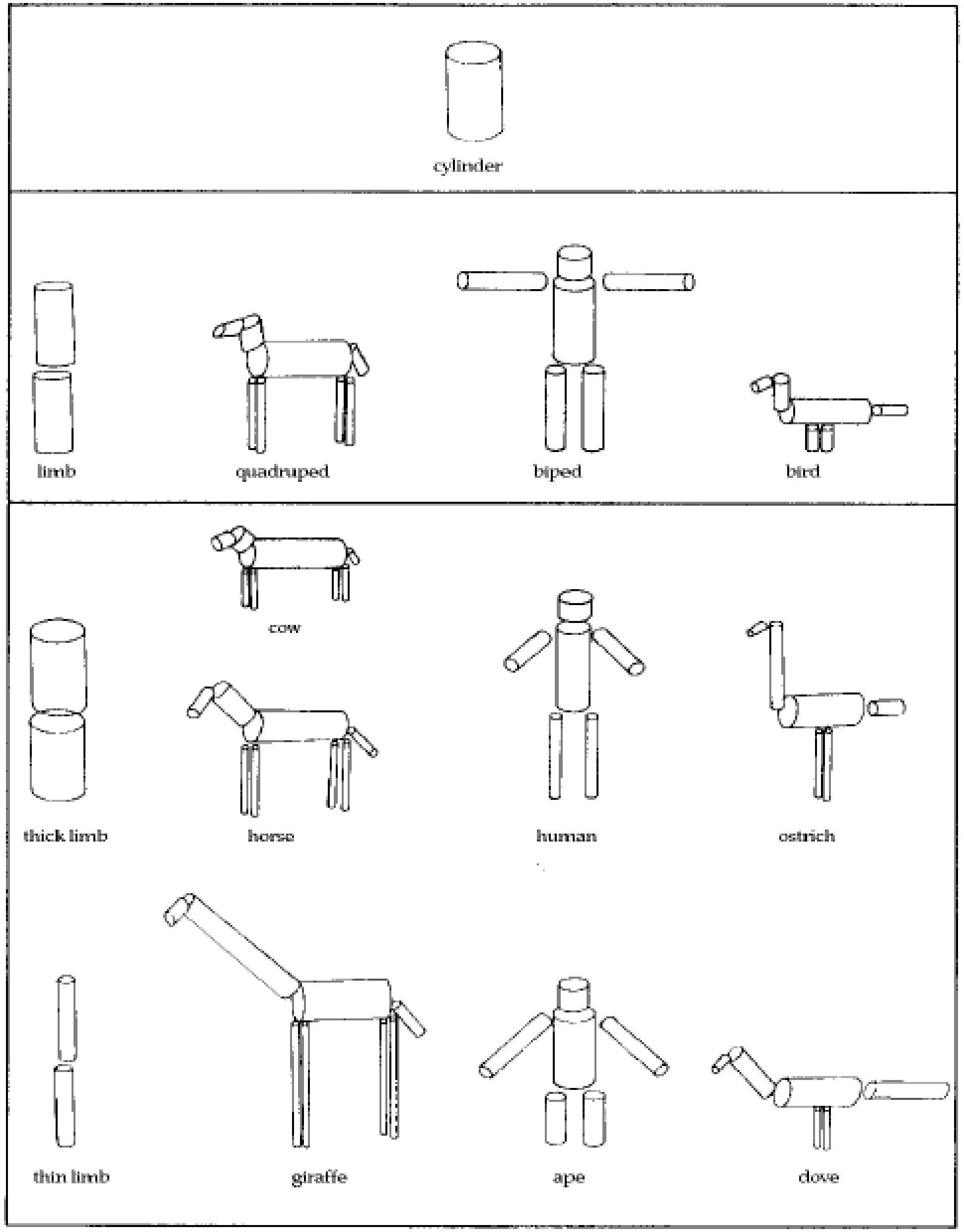
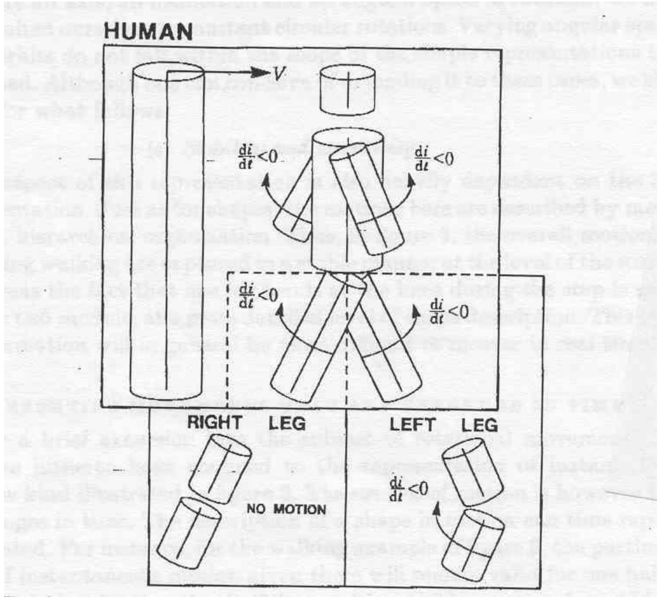


Figure 6: Representation of animal shapes using cylinders as modelling primitives (from Marr (1982)).

Action space

- Spatio-temporal **patterns of forces** that generate the movement
- Not only physical, but also emotional and social forces



Functional concepts

- Function of an object can be analysed
  - Actions it affords

- Functional concept = convex region in action space

Events

- Involve agent and patient
  - Represented as points in CS
- An agent is described by an *agent space* that at minimum contains a force domain in which the action performed by the agent can be represented (as a force vector *f* – a pattern of forces).
- The patient is described by a *patient space* that contains the domains needed to account for its properties relevant to the modelled event (often location and/or emotional state). A force vector c associated with the patient represents the (counter-)force exerted by the patient in relation to the agent's action.

Events

- The resultant force vector $r = f + c$
- An event is a mapping between an action in an agent space and a resulting change in a patient space that is the result of applying *r*.

Event categories

- Represented by *vector fields*.
- An event category then represents how the agent space potentially affects the patient vector field.
  - For example, the event category of pushing a table should represent the effect of different, albeit similar, patterns of force on the different points in the table patient space.

Semantics of verbs

- Simple sentences typically express events.
- A single verb can never completely describe an event, but only bring out one aspect of it.
- A verb represents *one of the vectors in the model of an event*.
- If the verb focuses on the force vector of the agent (e.g. “push”, “hit”), it is a *manner* verb.
- If it focuses on the change vector of the patient (e.g. “move”, “stretch”), it is a *result* verb.

Questions?